Exploring the structure of LFU violations at colliders

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Anomalies in the decay of the B meson were reported through the measurements of the $b \to s ll$ transitions in the form of foll. ratio:

$$R_K = \frac{\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \to K^+ e^+ e^-)} \bigg|_{q^2=1-6 \text{ GeV}^2} = 0.745^{+0.090}_{-0.074} \text{ (stat) } \pm 0.036 \text{ (syst)}$$

This was further corroborated by the measurement of the following ratio:

$$R_{K^*} = \frac{\mathcal{B}(B^0 \to K^{*0} \mu^+ \mu^-)}{\mathcal{B}(B^0 \to K^{*0} e^+ e^-)} = \begin{cases} 0.660^{+0.110}_{-0.070} \text{ (stat) } \pm 0.024 \text{ (syst)}, & 0.045 \leq q^2 \leq 1.1 \text{ GeV}^2 \\ 0.685^{+0.113}_{-0.069} \text{ (stat) } \pm 0.047 \text{ (syst)}, & 1.1 \leq q^2 \leq 6.0 \text{ GeV}^2 \end{cases}$$

This corresponds to a 2.6 - 2.7σ event.

**Flavour physics is one of the best probes of BSM physics**

**There are several indications pointing towards the possible existence of NP**

**Anomalies in the decay of the B meson were reported through the measurements**

**Hiller, Kruger 0310219**

**2.6 - 2.7σ**

**LHCB, BELLE**
Motivated by the $P_5'$ anomaly, it is not uncommon to consider NP purely in the muon sector.

However, this will not necessarily constitute the holy grail for our analysis, leaving the door open for electrons as well.

As a model building exercise, we focus on custodial models of RS and present example where electron and muons contribute.

Electrons or muons or both? We try to address this question for the structure of solutions to the anomalies at the LHC.
Description of custodial RS models

Muons and electrons both play a role

From indirect to direct searches

Possible hint on the structure of the solutions

$R(k), R(K^*)$ to

$B \to K \tau \tau$
Randall Sundrum Model

$S_1/Z_2$ compactified

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \]

effective 4D scale depends on the position in the bulk

One Fundamental gravity scale!!

Hierarchy problem Solved!!

\[ M_{ew} = e^{-kL} M_{Pl} \]

- Provides insight on strongly coupled theories #win
- Solution to the Yukawa hierarchy problem #win
Elements of the framework 1:

**Gauge bosons in RS**

The bulk gauge symmetry is: $SU(2)_L \times SU(2)_R \times U(1)_X$

KK excitations of the corresponding bulk gauge fields lead to a tower of states:

We consider the lowest scale with mass $M_{KK} = 3$ TeV

In the mass basis there are three neutral states with similar mass contributing to the $Z', Z_X, A^{(1)}$ FCNC

They have a similar wave function profile which is peaked near the IR brane: Origin of non-universal couplings
Elements of the framework 2.:

Fermions in RS

We consider fermion field with a bulk mass parametrised as: \( m_\Psi = c k \)

These bulk masses control the localisation of the fermion zero mode (SM fermions) in the bulk

\[
Y^{(4)} = Y^{(5)} \int_0^{\pi R} dy \, f_{0}^{(0)}(b, y) f_{1/2}^{(0)}(c_L, y) f_{1/2}^{(0)}(c_R, y)
\]

The choices are governed by the proximity to the Higgs field and hence a relatively larger effective Yukawa coupling

Except for the third generation doublet and top singlet, other fields are away from the IR brane
Elements of the framework 3.:

Non-universal couplings

Since the fermions are at different points in the bulk: Non universality is in built

The third generation quarks are likely to be closer to the Higgs and hence the gauge KK states $\rightarrow$ Larger coupling

The coupling of a pair of SM fermions to KK states can be expressed as

$$ I(c) = \frac{1}{\pi R} \int_0^{\pi R} d\sigma(y) (f_i^0(y, c))^2 \xi(y)_{Z, Z'} $$

We assume universal coupling between the first two generations: U(2)

The flavour violating couplings are:

$$ a^{12} = \tilde{g} (D_{21}^* D_{22} (I(2) - I(1)) + D_{31}^* D_{32} (I(3) - I(1))) $$

$$ a^{23} = \tilde{g} (D_{12}^* D_{13} (I(1) - I(2)) + D_{32}^* D_{33} (I(3) - I(2))) $$

$$ a^{13} = \tilde{g} (D_{21}^* D_{23} (I(2) - I(1)) + D_{31}^* D_{33} (I(3) - I(1))) $$
We are now in a position to understand the contributions to b-sll transitions.

The effective operator contributing to this process is given as

$$\mathcal{L} \supset \frac{V_{tb}^* V_{ts} G_F \alpha}{\sqrt{2\pi}} \sum_i C_i O_i$$

$$O_9 = (\bar{s}_L \gamma^\mu b_L)(\bar{l} \gamma_\mu l) \quad O_9' = (\bar{s}_R \gamma^\mu b_R)(\bar{l} \gamma_\mu l)$$

$$O_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{l} \gamma_\mu \gamma^5 l) \quad O_{10}' = (\bar{s}_R \gamma^\mu b_R)(\bar{l} \gamma_\mu \gamma^5 l)$$

The tree level contributions to b-sll is simply

$$\mathcal{L}_{NP} \subset \sum_{X=Z_{SM},Z_H,Z_X,\gamma^{(1)}} X_\mu \left[ \alpha^b_{L}(X)(\bar{s}_L \gamma^\mu b_L) + \alpha^b_{R}(X)(\bar{s}_R \gamma^\mu b_R) + \bar{l}(\alpha^l_V(X)\gamma^\mu - \alpha^l_A(X)\gamma^\mu \gamma^5) l \right]$$

Using this the WC are simply

$$\Delta C_9 = -\frac{\sqrt{2\pi}}{M_X^2 G_F \alpha} \alpha^b_{L}(X)\alpha^l_V(X), \quad \Delta C'_9 = -\frac{\sqrt{2\pi}}{M_X^2 G_F \alpha} \alpha^b_{R}(X)\alpha^l_V(X)$$

$$\Delta C_{10} = \frac{\sqrt{2\pi}}{M_X^2 G_F \alpha} \alpha^b_{L}(X)\alpha^l_A(X), \quad \Delta C'_{10} = \frac{\sqrt{2\pi}}{M_X^2 G_F \alpha} \alpha^b_{R}(X)\alpha^l_A(X)$$
Two scenarios are possible

Scenario A: The muon singlets are closer to the gauge KK states (couple more). The lepton doublets are universal.

Unorthodox scenario as there are contributions to the WC from the lepton doublets as well

These are largely due to ensure fits to the muon mass with $O(1)$ Parameters.

The fit in this case is 4D scenario with $C_9$, $C_{10}$ for both electron and muon contributing

Scenario B: The lepton singlets now have near universal coupling and smaller coupling to the gauge KK states. The muon doublets are now closer to the KK states and hence larger coupling

The fits to muon mass is better with $O(1)$ Parameters.

Mainly $C_9$ and $C_{10}$ for the muon contribute with a possibility of $C_9=-C_{10}$
The following ranges were used in the scan:

\[ c_{\mu_R} \in [0.45, 0.55] \quad c_{Q_3} \in [0.4, 0.5] \quad c_L \in [0.45, 0.55] \]

The Z- mu mu coupling is not a problem as the singlets are also embedded in custodial representations!

The \( c \) values for the lepton doublets are chosen such that to ensure an extension into 5D leptonic MFV.
This is a 4D fit to b-s ll data.

A model independent fit along these lines was performed in Hurth, Mahmoudi, Neshatpour 1603.00865

It was shown to relax the allowed ranges on the WC required to fit the data.
From indirect searches to colliders

Electrons or muons or both?

We found that solutions are possible in a consistent model with even electrons playing a role.

These were associated with fits to data on 2D or 4D plane involving both electrons & muons.

That brings us to the question: Is it possible to get a hint on the structure of WC from colliders.

We present an explicit example with an effective Z' model.
Consider a $Z'$ model with the following effective lagrangian

$$
\mathcal{L}_{eff} = \frac{\lambda_{bs}\lambda_e}{M^2} [(\bar{s}\gamma_\mu b)(\bar{e}\gamma^\mu e)] + \frac{\lambda_{bs}\lambda_\mu}{M^2} [(\bar{s}\gamma_\mu b)(\bar{\mu}\gamma^\mu \mu)] + \frac{\lambda_{bs}\lambda_\tau}{M^2} [(\bar{s}\gamma_\mu b)(\bar{\tau}\gamma^\mu \tau)] \\
+ \frac{\lambda_b\lambda_\mu}{M^2} [2V_{cb}(\bar{c}\gamma_\mu b)(\bar{\tau}\gamma^\mu \nu_\tau) + (\bar{b}\gamma_\mu b)(\bar{\tau}\gamma^\mu \tau)] \\
+ \left[ \frac{\lambda_b\lambda_\mu}{M^2} (\bar{b}\gamma_\mu b)(\bar{\mu}\gamma^\mu \mu) + \frac{\lambda_c\lambda_\mu}{M^2} (\bar{c}\gamma_\mu c)(\bar{\mu}\gamma^\mu \mu) \right]
$$

The Wilson co-efficients for the $R(K)$ and $R(K^*)$ anomalies are given as

$$
C_9^e = -\frac{\sqrt{2}\pi}{G_F \alpha} \frac{\lambda_{bs}\lambda_e}{M^2} \quad C_9^\mu = -\frac{\sqrt{2}\pi}{G_F \alpha} \frac{\lambda_{bs}\lambda_\mu}{M^2}
$$

The ratio of WC is simply

$$
\frac{\lambda_e}{\lambda_\mu}
$$

A key part of this ratio is that the quark dependance cancels out as it is common for both
Example of a fit to the anomalies with both electron and muon

The fit admits a wide parameter space of WC. Is there a way to explore the structure of these WC at colliders?
Consider the on-shell production of $Z'$ at colliders and consider the following ratio

$$\delta = \frac{\sigma_{Z'} \lambda_{\mu}^2 \mathcal{L} \epsilon_{\mu}}{\sigma_{Z'} \lambda_{e}^2 \mathcal{L} \epsilon_{e}} = \frac{N_{\mu}}{N_{e}}$$

Now the electron and muon are in general associated with different acceptance efficiencies

Is there a way for the above ratio to roughly reflect the ratio of WC

Its clear that if $\epsilon_{\mu} \simeq \epsilon_{e}$ then

$$\delta \simeq \frac{\lambda_{\mu}^2 \lambda_{e}^2 \mathcal{L} \epsilon_{e}}{\mathcal{L} \epsilon_{e}} = \left( \frac{C_{e}}{C_{e}} \right)^2$$
Typically muons have a larger acceptance than electrons

\[ m_{Z'} = 1500 \text{ GeV} \]

<table>
<thead>
<tr>
<th></th>
<th>( Z \rightarrow \mu\mu )</th>
<th>( Z \rightarrow ee )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Isolation (&gt; 1 leptons)</td>
<td>71.73</td>
<td>51.4</td>
</tr>
<tr>
<td>Mass cuts (&gt; 800GeV)</td>
<td>67.85</td>
<td>48.50</td>
</tr>
</tbody>
</table>

\[ m_{Z'} = 3000 \text{ GeV} \]

<table>
<thead>
<tr>
<th></th>
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<th>( Z \rightarrow ee )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Isolation (&gt; 1 leptons)</td>
<td>59.33</td>
<td>39.79</td>
</tr>
<tr>
<td>Mass cuts (&gt; 1000GeV)</td>
<td>58.79</td>
<td>39.61</td>
</tr>
</tbody>
</table>

Is there a way to get them as close to each other as possible!!
So the analysis is democratic?

Move from conventional electrons to electron jets!
One way to pull up `electron' efficiency is to use electron-jets

\textbf{`jets'}

- E-cal
  - Calorimetric four-vectors
  - Track four-vectors
  - Cluster them using anti-kt 0.4,
    \( pT=100 \text{ GeV} \)

- H-cal

- Track four vectors are scaled by an arbitrary small number to avoid over counting

Different samples can be distinguished by studying the properties of jets:- \textbf{JET SUBSTRUCTURE}
\[ \theta_J = \frac{1}{E_J} \sum_{i \in H_{\text{cat}} \in J} E_i \]

Select jets with very small hadron content

\[ \log[\theta_J] < -0.5 \]
To extract maximum information from the electron jet system we make the following selection

Use substructure variables to distinguish electron jets from QCD jets

Leading jet has exactly one track: Takes care of photon fakes
Sub-Leading jet may have either one or zero track. This is to capture events lost by tracker.

We put a min invariant mass cut of 1000 GeV on leading jets to ensure a democratic analysis

QCD fake rate was found to be < 1 event in 300000

<table>
<thead>
<tr>
<th>$m_{Z'}$ (GeV)</th>
<th>$Z \rightarrow \mu\mu$</th>
<th>$Z \rightarrow ee$ (Electron jets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>71.45</td>
<td>64.75</td>
</tr>
<tr>
<td>2500</td>
<td>66.35</td>
<td>63.06</td>
</tr>
<tr>
<td>3000</td>
<td>58.79</td>
<td>60.37</td>
</tr>
<tr>
<td>3500</td>
<td>51.68</td>
<td>59.50</td>
</tr>
</tbody>
</table>

Table 2: Comparison of efficiencies of electron jets and muons for $m_{Z'} = 3000$ GeV
A Brief comment on \( Z' \rightarrow \tau\tau \)

This could give a rough estimate of \( B \rightarrow K^*\tau\tau \ (C_{9,10}^\tau) \)

We look at hadronic decay of tau.

One way to possibly distinguish it from qcd jets is to look at track multiplicity

For leading jets we look at jets with either 1 or 0 tracks but with large hadronic content.
This distinguishes it from the electron jets.

<table>
<thead>
<tr>
<th>( m_{Z'} ) (GeV)</th>
<th>( Z \rightarrow \mu\mu )</th>
<th>( Z \rightarrow ee ) (Electron jets)</th>
<th>( Z \rightarrow \tau\tau ) (tau jets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>71.45</td>
<td>64.75</td>
<td>31.25</td>
</tr>
<tr>
<td>2500</td>
<td>66.35</td>
<td>63.06</td>
<td>37.28</td>
</tr>
<tr>
<td>3000</td>
<td>58.79</td>
<td>60.37</td>
<td>40.88</td>
</tr>
<tr>
<td>3500</td>
<td>51.68</td>
<td>59.50</td>
<td>43.98</td>
</tr>
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This technique is useful on many accounts:

For broad with resonances, observations of width is not easy. Measurement of the ratio enables us to get a signal for NP-Discovery.

For narrow width resonances, this can help us extract the extent of NP contributions-CONFIRMATION.

For narrow width resonances, but beyond the reach of LHC capturing events at the left end of the tail-TOWARDS DISCOVERY.
What about leptoquarks!!

Consider the following effective lagrangian

\[ \mathcal{L} = \lambda^1_{\alpha k} \bar{Q}^c_{\alpha} i \tau_2 L_k (\Phi_1)^\dagger + \lambda^3_{\alpha k} \bar{Q}^c_{\alpha} i \tau_2 (\tau. \Phi_3)^\dagger L_k + h.c. \]

With the following hierarchy of couplings

\[ \lambda^3_{sl} = \frac{m_s}{m_b} \lambda^3_{bl}, \quad \lambda^3_{cl} = \frac{m_c}{m_t} \lambda^3_{tl} \]

Using this the WC are simply

\[ C^e_9 \propto \frac{\lambda_b \lambda_e}{M^2_{LQ}}, \quad C^\mu_9 \propto \frac{\lambda_b \lambda_{\mu}}{M^2_{LQ}} \]

With the ratio

\[ \frac{C^e_9}{C^\mu_9} = \frac{\lambda_{be}^2}{\lambda_{b\mu}^2} \]

With this hierarchy, b quark fusion dominates over the charm contribution

We are interested in the T channel production: The cross-section goes as \( \lambda^4 \)

Implying a pattern

\[ \delta \sim \left( \frac{C^\mu_9}{C^e_9} \right)^2 \]

Similar to Z'
To Conclude...

We considered a scenario in a warped framework where both Muon and electron couple to NP

Extent of muon and Electron contribution can be extracted at LHC

The techniques can also be extended to di-tau final states with some hints on other flavour experiments.
Elements of the framework 2.: 
Fermions in RS

Bulk fermionic lagrangian in a warped background is written as

\[ \mathcal{L}_{\text{fermion}} = e^{-3\sigma} \overline{\Psi} \left[ i\gamma^\mu \partial_\mu - \gamma_5 e^{-\sigma} (\partial_5 - 2\sigma') \right] \Psi \]

where \( \sigma = k|y| \). Expanding the bulk field as

\[ \Psi(x, y) = \frac{1}{\sqrt{\pi R}} \sum_n \left[ \psi^{(n)}_L(x) f^{(n)}_L(y) + \psi^{(n)}_R(x) f^{(n)}_R(y) \right] \]

But

5D theory is non-chiral
Scenario B:

The $Z$-mu mu coupling is not a problem as the doublets are also embedded in custodial representations!

$c_{\mu R} \in [0.5, 0.6]$  \quad $c_{Q_3} \in [0.4, 0.5]$  \quad $c_{L_2} \in [0, 0.5]$
From B anomalies to rare Kaon decays

Rare Kaon decays are likely to constitute the next probe for NP

The SM expectation is

\[ B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 8.3 \pm 0.3 \pm 0.3 \times 10^{-11} \quad B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2.9 \pm 0.2 \pm 0.0 \times 10^{-11} \]

The current experimental bound is

\[ B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 17.3^{+11.5}_{-10.5} \times 10^{-11} \quad B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 2.6 \times 10^{-8} \quad (90\% \ C.L.) \]

The NA62 aims to achieve 15% precision wrt SM in 2018

The KOTO experiment is focussed at measuring the KL decays


**Scenario A:**

The effective lagrangian for \( s \to d\nu\nu \) transitions is given as

\[
\mathcal{L} = \frac{4G_F\alpha}{2\sqrt{2}\pi} V^*_{ts}V_{td} C_{d,s,l} \left( \bar{s}_L \gamma_{\mu} d_L \right) \left( \bar{\nu}_l \gamma^{\mu} \nu_l \right)
\]

\(c_L = 0.51\)

\(c_{Q_3}\)

\(c_{Q_3}\)

**Figure 6:** Scenario A: Plots depicting the excess over the SM expectation for the \( K \) decays modes. The \( c \) parameters for the doublets is universal and chosen to be \( c_L = 0.51 \).

Due to universality of lepton doublets, the contributions cannot be enhanced beyond a point!
Scenario B:

The larger contributions in this case are primarily due $c_{L3}$ is free compared to Scenario A.
How do we reproduce chiral SM?

\[ Z_2 \]

\[ \Psi = \begin{bmatrix} \psi_L(+) \\ \psi_R(-) \end{bmatrix} \]

- Even - massless zero mode
- Odd - no zero mode

Zero mode for the $Z_2$ even field say $f^{(0)}_L$ satisfies

\[ e^{-\sigma} (\partial_y - 2\sigma') f^{(0)}_L = 0 \]

Using orthonormality

\[ f^{(0)}_L = Ne^{k0.5(y-\pi R)} \]

Localized profiles!!

Introducing a bulk mass term $m_{1/2} = c\sigma' = ck$ modifies the solution to

\[ f^{(0)}_L = Ne^{(0.5-c)\sigma(y)} \]
SM Couplings are given by the `overlap' of these profiles:

\[ Y^{(4)} = Y^{(5)} \int_0^{\pi R} dy \ f_0^{(0)}(b, y) f_{1/2}^{(0)}(c_L, y) f_{1/2}^{(0)}(c_R, y) \]
How to get rid of QCD

Select jets with zero tracks

In the previous section, we gave a brief description of the variables which can potentially constitute the axes for the multi-dimensional space. The choice of these variables would primarily depend on the behaviour of the SM samples under them.

Tracks: In Fig. 1, we show the track distribution for each of the SM background as well as signal samples discussed above. From the figure, it is evident that photon peaks at zero, while electron and $\tau$ dominantly peak around unity. The $\tau$ samples also have a fair amount of three-track events due to three charged pions. QCD jets have a large number of tracks with a peak around 6. On account of its discrete nature, it offers the best discrimination between different samples. The right panel of Fig. 1 shows the track distribution for the different SM samples.

Log$(\mathcal{J})$: For QCD and $\tau$-initiated jets, the corresponding Log$(\mathcal{J})$ is closer to 1 as they deposit most of their energy in the $H_{\text{cal}}$. On the other hand, electron and photon-initiated jets deposit most of their energy in the electromagnetic calorimeter leading to much smaller values of Log$(\mathcal{J})$. The right panel of Fig. 1 shows the distribution of Log$(\mathcal{J})$ for different signal samples.

Both the variables put the electron and photon samples in the same region.

N-subjettiness: In the left panel of Fig. 2, we display the log$(\mathcal{J})$ estimated for the SM samples (left). Predominantly single-lobed samples like $e$ and $\mu$ jets peak at a much smaller value of log$(\mathcal{J})$ since $\tau \sim 0$. However, multilobed samples for which $\tau \sim O(1)$, it peaks near zero. From the left panel, one can see that there is a small but significant overlap between the $\tau$ and $e/\mu$ jets, the reason being the leptonic decay of the $\tau$-leptons. In addition to $\tau$, it is also useful to study various N-subjettiness ratios given by $\tau_{N,N-1}=\tau_N/\tau_{N-1}$, which are also small for a N-lobe configuration. One can define $\tau_{31}=\tau_2/\tau_1$ the $\tau$ and other di-samples from the QCD jets as shown in right panel Fig. 2.

For the samples with single photon, single electron, and (large fraction) single tau, both $\tau_{21}$ and $\tau_{32}$ peak around smaller values. On the other hand, di-electron, di-photon, and di-tau samples also peak at smaller values of $\tau_{31}$. For QCD multijet events, we do not expect any specific pattern in the energy distribution among the subjets, and thus $\tau_1, \tau_2$, and $\tau_3$ all are expected to be of the same value.
Some Tracks getting lost?