OPE in Wilson lines with sub-eikonal spin corrections for TMDs and g_1 structure function

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Outline

- Motivation
- Brief review of Operator Product Expansion at high-energy
- Operator Product Expansion at high-energy with sub-eikonal corrections
  - Quark propagator with sub-eikonal corrections
- Leading Order Impact Factor for sub-eikonal spin correction
- Conclusions

Motivation

- Unpolarized DIS at low-x_B: dynamics is driven by gluon structure functions
  - gluon structure function grows as $(1/x_B)^\lambda$ with $\lambda > 1$.
- Polarized DIS at low-x_B: polarized gluon structure function grows as $(1/x_B)^\lambda$ with λ close to 0.
  - This implies that polarized quark and gluon structure functions are equally relevant.
- At Electron Ion Collider low-x_B spin TMDs and g_1 structure function are relevant
  - Highly polarized ($\sim 70\%$) electron and nucleon beams
- Understand how the proton's spin arises from the intrinsic and orbital angular momenta of the constituent quarks and gluons.

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- Unpolarized DIS at low-x_B: dynamics is driven by gluon structure functions
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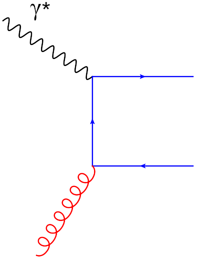
- At Electron Ion Collider low-x_B spin TMDs and g_1 structure function are relevant
  - Highly polarized ($\sim 70\%$) electron and nucleon beams
- Understand how the proton's spin arises from the intrinsic and orbital angular momenta of the constituent quarks and gluons.
- Compare with results obtained in the Leading Log approximation by Bartels-Ermolaev-Ryskin-(1995-1996) and recent work in Saturation formalism obtained by Kovchegov-Pytoniak-Sievert (20016-2017)

Unpolarized vs. Polarized DIS at low-x

- DGLAP: resums $\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$ BFKL: resums $\left(\alpha_s \ln \frac{1}{x_B}\right)^n$
- overlap region resums $\left(\alpha_s \ln \frac{1}{x_B} \ln \frac{Q^2}{\mu^2}\right)^n$
- Scattering amplitude with fermion in t-channel in Regge limit we have $\left(\alpha_s \ln^2 \frac{1}{x_B}\right)^n$ contributions
  - such contribution not included in DGLAP asymptotic $x_B \to 0$
- Double Log of energy of quark distribution
  - unpolarized case: are not relevant since are suppressed by gluon distribution
  - polarized case: are relevant Bartels-Ermolaev-Ryskin-(1995-1996)

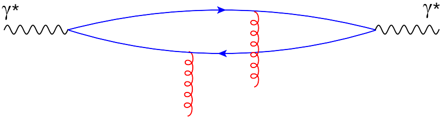
DGLAP vs BFKL dynamics

- DGLAP dynamics



- Incoherent interactions
- x_B fixed; resum $\ln \frac{Q^2}{\mu^2}$

- BFKL dynamics



- life-time $q\bar{q}$-pair $\tau \sim (m_N x_B)^{-1}$
- Coherent (multiple) interactions
- $x_B \to 0$; resum $\ln \frac{1}{x_B}$

Propagation in the shock wave: Wilson line (Spectator frame)

Boost of the fields $x_\bullet = \sqrt{\frac{s}{2}}x^-$ $x_* = \sqrt{\frac{s}{2}}x^+$ $x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

$$
\begin{aligned}
A_\bullet(x_\bullet, x_*, x_\perp) &\rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\
A_*(x_\bullet, x_*, x_\perp) &\rightarrow \lambda^{-1} A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\
A_\perp(x_\bullet, x_*, x_\perp) &\rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)
\end{aligned}
$$

λ is the boost parameter.

$$
\langle x|\frac{i}{\hat{\not{P}} + i\epsilon}|y\rangle \rightarrow \langle x|\frac{i}{\hat{\not{p}} + \alpha\frac{2}{s}\not{p}_2\hat{A}_\bullet + i\epsilon}|y\rangle
$$

$[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}p^+$ and $\not{p}_2 \propto \gamma^+$

Infinite boost: particle does not have time to deviate from straight line

Eikonal interactions give a Wilson lines

$$U_z = [\infty p_1 + z_\perp, -\infty p_1 + z_\perp]$$

$$[x, y] = Pe^{ig\int_0^1 du(x-y)^\mu A_\mu(ux+(1-u)y)} \qquad p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

Propagation in the shock wave: Wilson line (Spectator frame)

Quark propagator with eikonal interactions

$$\langle x|\frac{i}{\hat{\not P}+i\epsilon}|y\rangle = \left[\int_0^{+\infty}\frac{đ\alpha}{2\alpha}\theta(x_*-y_*) - \int_{-\infty}^{0}\frac{đ\alpha}{2\alpha}\theta(y_*-x_*)\right] e^{-i\alpha(x_\bullet - y_\bullet)}$$

$$\times\frac{1}{\alpha s}\,\langle x_\perp|\,e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\hat{\not p}\,\not p_2\,[x_*,y_*]\,\hat{\not p}e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}\,|y_\perp\rangle$$

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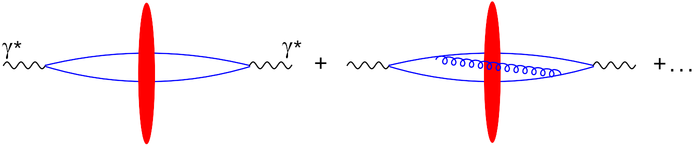
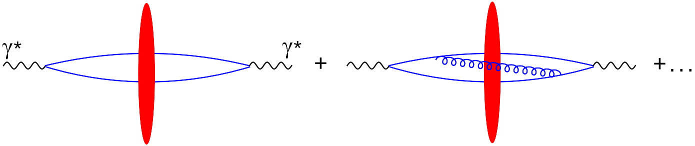
Semi-classical approach: Background field method

Diagrammatic representation Operator Product Expansion at High-energy



- The target is highly boosted

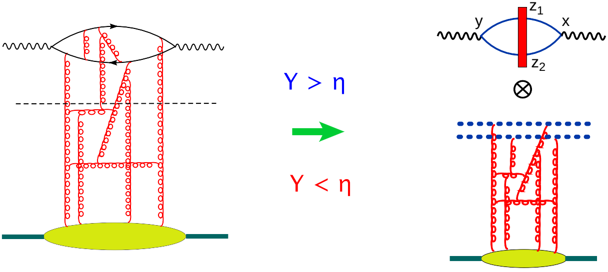
Operator Product Expansion at high energy



$$\langle B|\mathrm{T}\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle \simeq \int d^2z_1 d^2z_2\; I^{LO}_{\mu\nu}(z_1,z_2;x,y)\langle B|\mathrm{tr}\{U^\eta_{z_1}U^{\dagger\,\eta}_{z_2}\}|B\rangle$$
$$+\frac{\alpha_s}{\pi}\int d^2z_1 d^2z_2 d^2z_3\; I^{NLO}_{\mu\nu}(z_1,z_2,z_3;x,y)$$
$$\times\langle B|\Big[\mathrm{tr}\{U^\eta_{z_1}U^{\dagger\eta}_{z_3}\}\mathrm{tr}\{U^\eta_{z_3}U^{\dagger\eta}_{z_2}\} - N_c\mathrm{tr}\{U^\eta_{z_1}U^{\dagger\eta}_{z_2}\}\Big]|B\rangle$$

High-Energy Operator Product Expansion

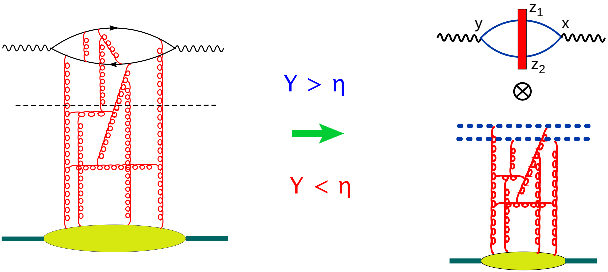
DIS amplitude is factorized in rapidity: η



$|B\rangle$ is the target state.

$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \dots$$

High-Energy Operator Product Expansion

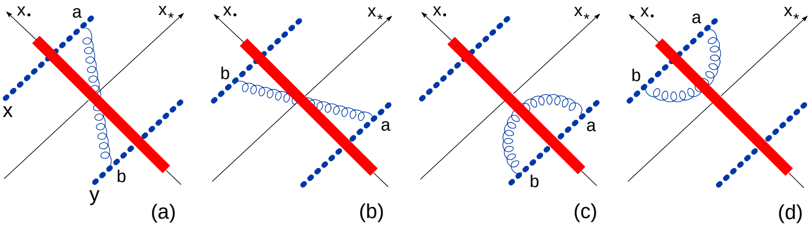


$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \dots$$

- If we use a model to evaluate $\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ we can calculate the DIS cross-section.
- If we want to include energy dependence to the DIS cross section, we need to find the evolution of $\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ with respect to the rapidity parameter η.

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \ldots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $x_* = \sqrt{\frac{s}{2}}x^+$

Non linear evolution equation: Balitsky-Kovchegov equation

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

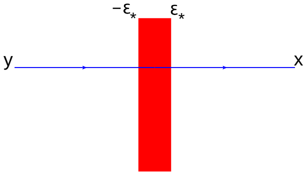
$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) \;=\; \frac{\alpha_s N_c}{2\pi^2}\int\frac{d^2z\;(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): describes proliferation of gluons.

- LLA for DIS in semi-classical-QCD $\Rightarrow$ BK eqn
  - background field method: describes recombination process.

- Note: if $x_\perp \;\to\; z_\perp$ or $y_\perp \;\to\; z_\perp$ divergences cancel out.

Shock-wave with finite width



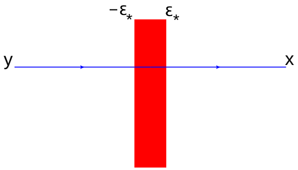
$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^-$

$$A_\bullet(x_\bullet, x_*, x_\perp) \rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$
$$A_*(x_\bullet, x_*, x_\perp) \rightarrow \lambda^{-1} A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$
$$A_\perp(x_\bullet, x_*, x_\perp) \rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$

λ is the boost parameter

- $p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$
- small α gluons are classical fields   large α gluons are quantum fields.
- Longitudinal sized classical fields: $\epsilon_* = \frac{\alpha s}{l_\perp^2}$ with $l_\perp$ trans. mom. of classical fileds
- Distance traveled by quantum fields: $z_* = \frac{\alpha s}{k_\perp^2}$ with $k_\perp$ trans. mom. of classical fileds
- We are in the case $l_\perp \sim k_\perp$

Shock-wave with finite width



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λ is the boost parameter

sub-eikonal terms go like $\frac{1}{\lambda}$

$$\langle x|\frac{i}{\not{P} + i\epsilon}|y\rangle \rightarrow \langle x|\not{P}\frac{i}{p^2 + 2\alpha A_\bullet + ig\frac{2}{s}\not{p}_2\gamma^i F_{\bullet i} + \frac{1}{2}F_{ij}\sigma^{ij} + ... + i\epsilon}|y\rangle$$

- Note: $[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}p^+$ and $\not{p}_2 \propto \gamma^+$

$$e^{i\frac{\hat{p}^2_\perp}{\alpha s}z_*}\hat{A}_\bullet(z_*)\,e^{-i\frac{\hat{p}^2_\perp}{\alpha s}z_*} \simeq A_\bullet(z_*) - \frac{z_*}{\alpha s}\{p^i, F_{\bullet i}(z_*)\} - \frac{z_*^2}{2\alpha^2 s^2}\{p^j, \{p^i, D_j F_{\bullet i}(z_*)\}\} + ...$$

Quark propagator with sub-eikonal corrections

$x_* = \sqrt{\frac{s}{2}}\, x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}\, x^- \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

$$\langle x|\frac{i}{\hat{\not P} + i\epsilon}|y\rangle = \left[\int_0^{+\infty}\frac{{\not d}\alpha}{2\alpha}\theta(x_* - y_*) - \int_{-\infty}^0 \frac{{\not d}\alpha}{2\alpha}\theta(y_* - x_*)\right] e^{-i\alpha(x_\bullet - y_\bullet)}$$
$$\times \frac{1}{\alpha s}\langle x_\perp|\, e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\Big\{\hat{\not p}\,\not p_2\,[x_*, y_*]\,\hat{\not p} + \hat{\not p}\,\not p_2\,\hat{\mathcal{O}}_1(x_*, y_*; p_\perp)\,\hat{\not p} + \frac{1}{2}\hat{\not p}\,\not p_2\,\hat{\mathcal{O}}_2(x_*, y_*)$$
$$-\frac{1}{2}\hat{\mathcal{O}}_2(x_*, y_*)\,\not p_2\,\hat{\not p} + \hat{\not p}\not p_2\hat{\mathcal{O}}_3(x_*, y_*) + \hat{\mathcal{O}}_3(x_*, y_*)\not p_2\hat{\not p}\Big\} e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle$$
$$+O(\lambda^{-2})$$

- Leading-eikonal term
- Sub-eikonal terms

Operators $\hat{\mathcal{O}}_1$, $\hat{\mathcal{O}}_2$ and $\hat{\mathcal{O}}_3$ *measure* the deviation from the straight line.

Quark propagator with sub-eikonal corrections

$x_* = \sqrt{\frac{s}{2}}\,x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}\,x^- \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

$$\hat{\mathcal{O}}_1(x_*,y_*;p_\perp) = \frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big([x_*,\omega_*]\frac{1}{2}\sigma^{ij}F_{ij}[\omega_*,y_*] + \{\hat{p}^i,[x_*,\omega_*]\,\frac{2}{s}\,\omega_*\,F_{i\bullet}(\omega_*)\,[\omega_*,y_*]\}$$

$$+g\int_{\omega_*}^{x_*} d\frac{2}{s}\,\omega'_*\,\frac{2}{s}(\omega_* - \omega'_*)\,[x_*,\omega'_*]F^i{}_\bullet[\omega'_*,\omega_*]\,F_{i\bullet}\,[\omega_*,y_*] + [x_*,\omega_*]\,i\,\frac{2}{s}\,F_{\bullet *}\,[\omega_*,y_*]\Big)$$

$$\hat{\mathcal{O}}_2(x_*,y_*) = \frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big[[x_*,\omega_*](\frac{1}{2}\,i\not{D}_\perp\sigma^{ij}F_{ij})[\omega_*,y_*] + \alpha s\,[x_*,\omega_*]\,i\,\frac{4}{s^2}\not{p}_1\,F_{\bullet *}\,[\omega_*,y_*]$$

$$+g\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\Big([x_*,\omega'_*](\frac{2}{s}\omega'_* i\not{D}_\perp F^i{}_\bullet + i\gamma^j F^i{}_j)[\omega'_*,\omega_*]F_{i\bullet}[\omega_*,y_*]$$

$$-[x_*,\omega'_*]F_{i\bullet}[\omega'_*,\omega_*](\frac{2}{s}\omega_* i\not{D}_\perp F^i{}_\bullet + i\gamma^j F^i{}_j)[\omega_*,y_*]\Big)\Big]$$

$$\hat{\mathcal{O}}_3(x_*,y_*) = -\frac{ig}{4\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big[[x_*,\omega_*]\,i\,\frac{2}{s}\,(i\not{D}_\perp F_{\bullet *})\,[\omega_*,y_*]$$

$$-g\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\,\gamma^j\Big([x_*,\omega'_*]F_{j\bullet}[\omega'_*,\omega_*]\,i\,\frac{2}{s}F_{\bullet *}\,[\omega_*,y_*] + [x_*,\omega'_*]\,i\,\frac{2}{s}F_{\bullet *}\,[\omega'_*,\omega_*]F_{j\bullet}[\omega_*,y_*]\Big)\Big]$$

Quark propagator with sub-eikonal corrections

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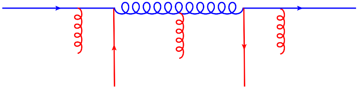
$$\hat{\mathcal{O}}_1(x_*, y_*; p_\perp) = \frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_* \Big([x_*, \omega_*]\frac{1}{2}\sigma^{ij}F_{ij}[\omega_*, y_*] + \{\hat{p}^i, [x_*, \omega_*]\,\frac{2}{s}\,\omega_*\, F_{i\bullet}(\omega_*)\,[\omega_*, y_*]\}$$
$$+ g\int_{\omega_*}^{x_*} d\frac{2}{s}\,\omega'_*\,\frac{2}{s}(\omega_* - \omega'_*)\,[x_*, \omega'_*]F^i{}_\bullet[\omega'_*, \omega_*]\,F_{i\bullet}\,[\omega_*, y_*] + [x_*, \omega_*]\,i\,\frac{2}{s}\,F_{\bullet *}\,[\omega_*, y_*]\Big)$$

$$\hat{\mathcal{O}}_2(x_*, y_*) = \frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_* \Big[[x_*, \omega_*](\frac{1}{2}\,i\not{D}_\perp\sigma^{ij}F_{ij})[\omega_*, y_*] + \alpha s\,[x_*, \omega_*]\,i\,\frac{4}{s^2}\,\not{p}_1\,F_{\bullet *}\,[\omega_*, y_*]$$
$$+ g\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\Big([x_*, \omega'_*](\frac{2}{s}\omega'_* i\not{D}_\perp F^i{}_\bullet + i\gamma^j F^i{}_j)[\omega'_*, \omega_*]F_{i\bullet}[\omega_*, y_*]$$
$$- [x_*, \omega'_*]F_{i\bullet}[\omega'_*, \omega_*](\frac{2}{s}\omega_* i\not{D}_\perp F^i{}_\bullet + i\gamma^j F^i{}_j)[\omega_*, y_*]\Big)\Big]$$

$$\hat{\mathcal{O}}_3(x_*, y_*) = -\frac{ig}{4\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_* \Big[[x_*, \omega_*]\,i\,\frac{2}{s}\big(i\not{D}_\perp F_{\bullet *}\big)[\omega_*, y_*]$$
$$- g\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\Big([x_*, \omega'_*]\gamma^j\, F_{j\bullet}[\omega'_*, \omega_*]\,i\,\frac{2}{s}F_{\bullet *}\,[\omega_*, y_*] + [x_*, \omega'_*]\,i\,\frac{2}{s}F_{\bullet *}\,[\omega'_*, \omega_*]\gamma^j\, F_{j\bullet}[\omega_*, y_*]\Big)\Big]$$

Quark propagator in the background of quark fields



$$\langle \mathrm{T}\psi(x)\bar\psi(y)\rangle_{\psi,\bar\psi} = \frac{ig^2}{\alpha^4 s^4}\Big[\int_0^{+\infty}\frac{đ\alpha}{2\alpha}\int_{y_*}^{x_*}dz_*\int_{y_*}^{z_*}dz'_* - \int_{-\infty}^{0}\frac{đ\alpha}{2\alpha}\int_{y_*}^{x_*}dz'_*\int_{z'_*}^{x_*}dz_*\Big]e^{-i\alpha(x_\bullet - y_\bullet)}$$

$$\times\langle x_\perp| e^{-i\frac{\hat p_\perp^2}{\alpha s}x_*}\not{p}\not{p}_2\Big[\not{p}[x_*,z_*]\gamma^\mu t^a\psi(z_*)\mathcal{G}^{ab}_{\mu\nu}(z_*,z'_*)\bar\psi(z'_*)t^b\gamma^\nu[z'_*,y_*]\not{p}$$

$$+g^2\frac{2}{s}\gamma^i\int_{z_*}^{x_*}d\omega_*[x_*,\omega_*]F_{\bullet i}[\omega_*,z_*]\gamma^\mu t^a\psi(z_*)\mathcal{G}^{ab}_{\mu\nu}(z_*,z'_*)\bar\psi(z'_*)t^b\gamma^\nu\frac{2}{s}\gamma^j\int_{y_*}^{z'_*}d\omega'_*[z'_*,\omega'_*]F_{\bullet j}[\omega'_*,y_*]$$

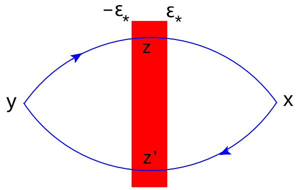
$$-g\frac{2}{s}\gamma^i\int_{z_*}^{x_*}d\omega_*[x_*,\omega_*]F_{\bullet i}[\omega_*,z_*]\gamma^\mu t^a\psi(z_*)\mathcal{G}^{ab}_{\mu\nu}(z_*,z'_*)\bar\psi(z'_*)t^b\gamma^\nu[z'_*,y_*]\not{p}$$

$$-\not{p}[x_*,z_*]\gamma^\mu t^a\psi(z_*)\mathcal{G}^{ab}_{\mu\nu}(z_*,z'_*)\bar\psi(z'_*)t^b\gamma^\nu g\frac{2}{s}\gamma^i\int_{y_*}^{z'_*}d\omega_*[z'_*,\omega'_*]F_{\bullet i}[\omega'_*,y_*]\Big]\not{p}_2\not{p}\, e^{i\frac{\hat p_\perp^2}{\alpha s}y_*}|y_\perp\rangle$$

$$+O(\lambda^{-3})$$

$\mathcal{G}^{ab}_{\mu\nu}$ gluon propagator in the external field.

Quark propagator with sub-eikonal corrections



Let $|B\rangle$ be proton or nuclear target

$$\langle B|J^\mu(x)J^\nu(y)|B\rangle \quad \to \quad \langle J^\mu(x)J^\nu(y)\rangle_A = \mathrm{Tr}\{\gamma^\mu\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle\,\gamma^\nu\langle y|\frac{i}{\not{P}+i\epsilon}|y\rangle\}$$

Quark propagator with sub-eikonal corrections: $\Rightarrow$ New evolution equations

OPE in Wilson lines with sub-eikonal corrections

$$\langle P,S|\bar\psi(x)\gamma^\mu\psi(x)\bar\psi(y)\gamma^\nu\psi(y)|P,S\rangle$$

$$\stackrel{x_*>0>y_*}{=} \int d^2z_1 d^2z_2 \frac{\mathrm{tr}\{\not X_1 \not p_2 \not Y_1 \gamma^\nu \not Y_2 \not p_2 \not X_2 \gamma^\mu\}}{4\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^3} \mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,z_*]_{z_2}\}$$

$$-g \int d^2z_1 d^2z_2 \frac{\mathrm{tr}\{\not X_1 \not p_2 \not Y_1 \gamma^\nu \not Y_2 \not p_2 \sigma^{ij} \not X_2 \gamma^\mu\}}{64\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^2}$$

$$\times \int_{y_*}^{x_*} d\omega_* \Big[\mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,\omega_*]_{z_2} F_{ij}[\omega_*,x_*]_{z_2}\} + \mathrm{tr}\{[x_*,\omega_*]_{z_2} F_{ij}[\omega_*,y_*]_{z_2}[y_*,x_*]_{z_1}\}\Big]$$

$$\mathcal{Z}_i \equiv \frac{(x-z_i)_\perp^2}{x_*} - \frac{(y-z_i)_\perp^2}{y_*} - \frac{4}{s}(x_\bullet - y_\bullet)$$

$$X_i \equiv x - z_i \quad \text{and} \quad Y_i \equiv y - z_i$$

$x_* = \sqrt{\frac{s}{2}}x^+$ and $x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

OPE in Wilson lines with sub-eikonal corrections

$$\langle P,S|\bar\psi(x)\gamma^\mu\psi(x)\bar\psi(y)\gamma^\nu\psi(y)|P,S\rangle$$

$$\stackrel{x_*>0>y_*}{=}\int d^2z_1 d^2z_2\,\frac{\mathrm{tr}\{\not{X}_1\not{p}_2\not{Y}_1\gamma^\nu\not{Y}_2\not{p}_2\not{X}_2\gamma^\mu\}}{4\pi^6\,x_*^4y_*^4\,\mathcal{Z}_1^3\mathcal{Z}_2^3}\,\mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,z_*]_{z_2}\}$$

$$-g\int d^2z_1 d^2z_2\,\frac{\mathrm{tr}\{\not{X}_1\not{p}_2\not{Y}_1\gamma^\nu\not{Y}_2\not{p}_2\sigma^{\rho\sigma}\not{X}_2\gamma^\mu\}}{64\pi^6\,x_*^4y_*^4\,\mathcal{Z}_1^3\mathcal{Z}_2^2}$$

$$\times\int_{y_*}^{x_*}d\omega_*\left[\mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,\omega_*]_{z_2}F^\perp_{\rho\sigma}[\omega_*,x_*]_{z_2}\}+\mathrm{tr}\{[x_*,\omega_*]_{z_2}F^\perp_{\rho\sigma}[\omega_*,y_*]_{z_2}[y_*,x_*]_{z_1}\}\right]$$

- Leading Order Impact Factor for unpolarized case known for 40 years
- Leading Order Impact Factor for polarized case next slide

Photon Impact Factor for sub-eikonal spin corrections

$$\frac{\mathrm{tr}\{\not{X}_1 \not{p}_2 \not{Y}_1 \gamma^\nu \not{Y}_2 \not{p}_2 \sigma^{ij} \not{X}_2 \gamma^\mu\}}{64\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^2} = \frac{i}{8\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^2}\Big\{\Phi_1^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_2^{\mu\nu,ij}(x,y;z_1,z_2)$$

$$+\Phi_3^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_4^{\mu\nu,ij}(x,y;z_1,z_2) - (\mu \leftrightarrow \nu, x \leftrightarrow y)$$

$$+\Phi_5^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_6^{\mu\nu,ij}(x,y;z_1,z_2)\Big\} - i \leftrightarrow j$$

$$\Phi_1^{\mu\nu,ij}(x,y;z_1,z_2) = p_2^\nu x_*\Big[\big(Y_{1\perp}^\mu Y_2^j - Y_{2\perp}^\mu Y_1^j\big)(z_1^i - z_2^i) + Y_2^j Y_1^i (X_{2\perp}^\mu + X_{1\perp}^\mu)\Big]$$

$$\Phi_2^{\mu\nu,ij}(x,y;z_1,z_2) = \frac{4}{s} p_1^\mu p_2^\nu x_*^2 Y_1^i Y_2^j$$

$$\Phi_3^{\mu\nu,ij}(x,y;z_1,z_2) = 2x_* y_* g^{j\nu}(z_1^i - z_2^i)\Big(\frac{2}{s} x_* p_1^\mu + X_{2\perp}^\mu\Big)$$

$$\Phi_4^{\mu\nu,ij}(x,y;z_1,z_2) = x_* p_2^\nu g^{\mu i}\Big(Y_2^j (Y_1, z_1 - z_2) - Y_1^j (Y_2, z_1 - z_2) + (Y_1, Y_2)(z_1^j - z_2^j)\Big)$$

$$\Phi_5^{\mu\nu,ij}(x,y;z_1,z_2) = p_2^\nu p_2^\mu \Big[(X_2, Y_1) Y_2^i X_1^j + (X_1, Y_2) X_2^j Y_1^i + (X_2, X_1) Y_2^j Y_1^i + (Y_1, Y_2) X_2^i X_1^j$$

$$+(X_2, Y_2) Y_1^j X_1^i + (Y_1, X_1) X_2^i Y_2^j\Big]$$

$$\Phi_6^{\mu\nu,ij}(x,y;z_1,z_2) = x_* y_* g_\perp^{i\mu} g_\perp^{j\nu} (z_1 - z_2)_\perp^2$$

Photon Impact Factor for sub-eikonal spin corrections

$$\frac{\mathrm{tr}\{\not{X}_1\not{p}_2\not{Y}_1\,\gamma^\nu\not{Y}_2\not{p}_2\,\sigma^{ij}\not{X}_2\gamma^\mu\}}{64\pi^6\,x_*^4 y_*^4\,\mathcal{Z}_1^3\mathcal{Z}_2^2} = \frac{i}{8\pi^6\,x_*^4 y_*^4\,\mathcal{Z}_1^3\mathcal{Z}_2^2}\Big\{\Phi_1^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_2^{\mu\nu,ij}(x,y;z_1,z_2)$$

$$+\Phi_3^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_4^{\mu\nu,ij}(x,y;z_1,z_2) - (\mu\leftrightarrow\nu, x\leftrightarrow y)$$

$$+\Phi_5^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_6^{\mu\nu,ij}(x,y;z_1,z_2)\Big\} - i\leftrightarrow j$$

- Impact factor is
  - Gauge invariant
  - Conformal invariant in SL(2,C)

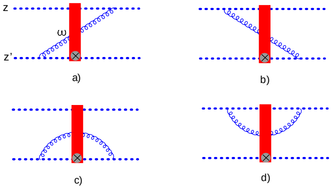
Evolution of sub-eikonal operator

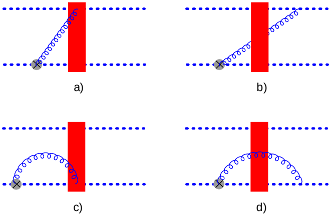
Consider, for example, the following sub-eikonal operator

$$\int_{y_*}^{x_*} d\omega_* \, \mathrm{tr}\{U_z[-\infty, \omega_*]_{z'} \, gF_{ij}(\omega_*, z'_\perp)[\omega_*, +\infty]_{z'}\}$$

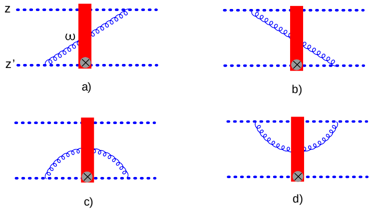
Background field method: split fields in quantum and classical and integrate out the quantum fields

Sample of diagrams:





sample of BK-type diagrams



- $\int_0^{+\infty} \frac{d\alpha}{\alpha}$ rapidity divergence
- if $\omega_\perp \rightarrow z_\perp$ divergence cancel out.
- if $\omega_\perp \rightarrow z'_\perp$ divergence does not cancel out.
  - we have $\left(\alpha_s \ln^2 \frac{1}{x}\right)$ type of contribution

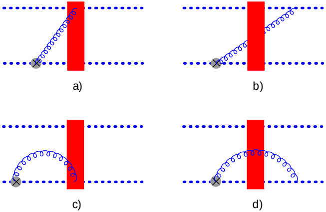
Summing real and virtual diagrams we get

$$\int_{-\infty}^{+\infty} d\omega_* \langle \mathrm{tr}\{[\infty, -\infty]_z [-\infty, \omega_*]_{z'} gF_{ij}(\omega_*)[\omega_*, \infty]_{z'}\}\rangle_{\mathrm{BK-type}}$$

$$= \frac{\alpha_s}{2\pi^2} \int_{-\infty}^{+\infty} d\omega_* \int_0^{+\infty} \frac{d\alpha}{\alpha} \int d^2\omega \, \frac{(z-z')_\perp^2}{(z-\omega)_\perp^2 (z'-\omega)_\perp^2}$$

$$\times \Big[\mathrm{tr}\{U_z U_\omega^\dagger\}\mathrm{tr}\{U_\omega[-\infty, \omega_*]_{z'}\, gF_{ij}(\omega_*, z'_\perp)[\omega_*, \infty]_{z'}\} - N_c \mathrm{tr}\{U_z[-\infty, \omega_*]_{z'}\, gF_{ij}(\omega_*, z'_\perp)[\omega_*, \infty]_{z'}\}\Big]$$

- Diagram with gluon propagator without sub-eikonal corrections has no rapidity divergence



- $\Rightarrow$ Need sub-eikonal corrections also in gluon propagator.

Gluon propagator in the light-cone gauge with sub-eikonal corrections is

$$\langle \mathrm{T}A_\mu(x)A_\nu(y)\rangle = \left[-\int_0^{+\infty}\frac{d\!\!\!{}^-\alpha}{2\alpha}\theta(x_*-y_*) + \int_{-\infty}^{0}\frac{d\!\!\!{}^-}{2\alpha}\theta(y_*-x_*)\right]$$

$$\times\langle x_\perp|\, e^{-i\frac{p_\perp^2}{\alpha s}x_*}\,\mathcal{O}(x_*,y_*)\, e^{i\frac{p_\perp^2}{\alpha s}y_*}\,|y_\perp\rangle + i\langle x|\frac{p_{2\mu}p_{2\nu}}{p_*^2}|y\rangle$$

$$\mathcal{O}(x_*,y_*) = [x_*,y_*] - \frac{2ig}{\alpha s^2}\int_{y_*}^{x_*} dz_*\,\left(z_*\{p^j,[x_*,z_*]F_{\bullet j}[z_*,y_*]\} + \dots\right)$$

see Balitsky – Tarasov 2016

Conclusions

- Quark propagator with sub-eikonal corrections is good for
  - spin-dependent TMDs: SIDIS, Weizsäcker-Williams TMD at low-x
  - spin g_1 structure function at low-x
- New operators appears if we consider sub-eikonal corrections
- OPE at high-energy extended to include sub-eikonal spin corrections
- LO Impact Factor for sub-eikonal spin correction is calculated
- Sub-eikonal corrections to BK-equation are now also possible
  - Although these are suppressed in the unpolarized case
- Future...include NLO corrections with spin