Forward Particle Production in Proton-nucleus Collisions at NLO

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Iancu, Mueller, DT JHEP12 (2016) 41
Resummation of perturbative series in field theory

Gluon saturation (Color Glass Condensate)

Process and problem: $dA$ or $pA$ and negative $\sigma$

Process at LO and phenomenology

Process at NLO: positive defined cross section

Negative cross section as an artefact of $k_\perp$-factorization
Energy levels in quantum mechanics, e.g. add perturbation $-2gx^3 + gx^4$ to simple harmonic oscillator

$$E_n = E_n^{(0)} + g^2 E_n^{(1)} + g^4 E_n^{(2)} + \cdots$$

Coefficients $E_n^{(i)}$ are numbers (for given $m$ and $\omega$)

Good approximation to keep few terms $n \leq n_0$

[NB: This example needs $n_0 \lesssim 1/g^2$ : asymptotic series due to non-perturbative effects and instantons formation]
Quantity in field theory with interaction (particle creation)

\[ \sigma = g^{2k} \sigma_0 + g^{2k+2} \sigma_1 + g^{2k+4} \sigma_2 + \cdots \]

E.g. diff. cross section: \( \sigma_i \) functions of particle 4-momenta

At short distance \( \alpha_s = \frac{g^2}{4\pi} \ll 1 \), series looks meaningful

For some momenta could be \( \alpha_s \sigma_{k+1} \sim \sigma_k \): fixed order series not enough, resum in such kinematic domains

[QCD at large distance, series is bad, use NP methods]
Emission of gluon from parent parton (quark or gluon)

\[ p^+, 0_\perp \rightarrow (1 - x)p^+, -k_\perp \]

\[ xp^+, k_\perp \]

Integrate intermediate particles in cascade, two types of large logarithms: transverse for DGLAP, longitudinal for BFKL

\[ xG(x, Q^2) \sim \frac{1}{x^\lambda} \]

\[ \lambda \sim 0.2 \div 0.3 \]
High density, weak coupling, non-linear dynamics

Saturation when $xG(x, Q_s^2)/Q_s^2 R^2 \sim 1/\alpha_s$

Dynamical perturbative (semi-hard) scale

$$Q_s^2(x) \sim Q_0^2 A^{1/3} x^{-\lambda} \gg \Lambda_{QCD}^2$$
Collinear quark with proton picks up transverse momentum by multiple scattering with gluons in target nucleus.

\[ \eta = -\ln \tan \frac{\theta}{2} \]

\[ x_p = \frac{p^+}{Q^+} = \frac{p_\perp}{\sqrt{s}} \, e^\eta \]

\[ X_g = \frac{p^-}{P^-} = \frac{p_\perp}{\sqrt{s}} \, e^{-\eta} \]

- \( \eta \): quark rapidity in COM frame
- \( x_p \): longitudinal fraction of quark in proton
- \( X_g \): longitudinal fraction of gluon in nucleus

Forward (\( \eta \gg 1 \)) probes soft saturated modes.
Negative $\sigma$ at NLO

BRAHMS $\eta = 2.2, 3.2$

NLO theory [Chirilli, Xiao, Yuan '12] Numerics [Stasto, Xiao, Zaslavsky '13]

Issue appears for $p_\perp \sim Q_s$, where CGC should apply.
Multiple Scattering

Multiple scattering off target color field $A$ to all orders
Eikonal: fixed transverse coordinate, color changing

Amplitude: $\mathcal{M}_{ij} = \int d^2k e^{-ik \cdot x} V_{ij}(x), \quad V(x) = P \exp \left[ ig \int dx^+ A_a(x^+, x) t^a \right]$

Cross section: $\frac{dN}{d\eta d^2k} = x_p q(x_p) \frac{1}{N_c} \left\langle \sum_{ij} |\mathcal{M}_{ij}(k)|^2 \right\rangle X_g$
Leads to elastic $S$-matrix for $q\bar{q}$ dipole (NLO) \cite{Mueller, Munier '12}

\[
\frac{dN}{d\eta d^2k} = x_p q(x_p) \int d^2x d^2y e^{-i(x-y) \cdot k} S(x, y; X_g)
\]

\[
S(x, y; X_g) = \frac{1}{N_c} \langle \text{tr}[V(x)V^{\dagger}(y)] \rangle_{X_g}
\]

Evaluated at softest target scale $X_g$, resum $(\bar{\alpha}_s \ln 1/X_g)^n$

Lowest order proportional to unintegrated gluon distribution
BK Equation

Closed evolution equation for dipole $S$-matrix

$$\frac{\partial S_{xy}}{\partial \ln(1/x)} = \frac{\bar{\alpha}_s}{2\pi} \int d^2z \frac{(x-y)^2}{(x-z)^2(z-y)^2} [S_{xz}S_{zy} - S_{xy}]$$

dipole kernel

Saturation momentum from unitarity

$$S(r_\perp = 1/Q_s(x), x) = 1/2$$
Hybrid Formalism and LO Phenomenology

\[ \frac{dN}{d\eta d^2k} \bigg|_{LO} = K^h \int_{x_p}^1 \frac{dz}{z^2} \frac{x_p}{z} q \left( \frac{x_p}{z} \right) S \left( \frac{k}{z}; X_g \right) D_{h/q}(z) \]

Fit parameters: IC for rcBK and \( K \)-factors

[Dumitru, Hayashigaki, Jalilian-Marian '05] [Albacete, Dumitru, Fujii, Nara '12]
LO in Integral Form

One explicit emission close to dipole, most evolution in nucleus

\[ S_{xy}(X_g) = S_{xy}(X_0) + \frac{\tilde{\alpha}_s}{2\pi} \int_{X_g}^{1} \frac{dx}{x} \int d^2z \frac{(x-y)^2}{(x-z)^2(z-y)^2} [S_{xz}S_{zy} - S_{xy}] \bigg|_{X(x)} \]

Energy fraction in target \( X(x) = \frac{X_g}{x} \)
NLO Corrections to Particle Production

- $\mathcal{O}(\bar{\alpha}_s)$ correction to evolution: soft partons with $x_1 \sim x_2$
- $\mathcal{O}(\bar{\alpha}_s)$ correction to impact factor: first gluon $x \sim \mathcal{O}(1)$

Non-eikonal emission, exact kinematics
Impact Factor Correction

\[
\frac{dN}{d\eta d^2k} = S_0(k) + \bar{\alpha}_s \int_{X_g}^1 \frac{dx}{x} \mathcal{K}(x) S_{q\bar{q}g}(k, X(x)) \quad X(x) = \frac{X_g}{x}
\]

Distribution and fragmentation functions implicit
Floating scale \(X(x)\), generalized factorization
Tree-level + positive defined. Where was the problem?
Recovering LO

Let $\mathcal{K}(x) \rightarrow \mathcal{K}(0)$ to get LO

\[
\frac{dN}{d\eta d^2k} \bigg|_{\text{LO}} = S_0(k) + \bar{\alpha}_s \int_{X_g}^{1} \frac{dx}{x} \mathcal{K}(0) S_{qqg}(k, X(x)) = S(k, X_g)
\]

Just Fourier transform of BK equation (integral form)
Add and subtract LO result, free to do so

\[
\frac{dN}{d\eta d^2k} = S(k, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{dx}{x} [K(x) - K(0)] S_{q\bar{q}g}(k, X(x))
\]

Correct but dangerous: add and subtract large contribution
**Recovering $k_\perp$-factorization (bad)**

At NLO consistent to let $S(X(x)) \to S(X_g)$ [integrand not dominated by small-$x$] and $X_g \to 0$ in limit [plus prescription]

$$\frac{dN}{d\eta d^2k} = S(k, X_g) + \bar{\alpha}_s \int_0^1 \frac{dx}{x} [K(x) - K(0)] S_{q\bar{q}g}(k, X_g)$$

Local in $X_g$, $k_\perp$-factorized. Mistreats kinematics at low-$x \sim \sigma < 0$. 
Numerics

Ducloué, Lappi, Zhu '17

- Generalized factorization, same for sub and unsub, $\sigma > 0$
- Large but controlled NLO correction, $50\%$ at $k_\perp \sim 5\text{GeV}$
- $k_\perp$-factorized result: $\sigma < 0$
**Conclusions**

- $pA$ collision at large $\eta$ at NLO in CGC:
  Process suited to study gluon saturation
- “Strict” $k_\perp$-factorized expression leads to negative $\sigma$
- Generalized factorization based on skeleton expansion:
  - No explicit separation between LO and NLO
  - Non-local in longitudinal space
- Well-defined result: positive $\sigma$ at NLO, smaller than LO
Energy conservation:
\[ \frac{k_\perp^2}{2(1-x)q_0^+} + \frac{p_\perp^2}{2xq_0^+} = XP^- \]

Simplifies for \( k_\perp \sim p_\perp \gtrsim Q_s \):
\[ X \simeq \frac{k_\perp^2}{xs} = \frac{Xg}{x} \]