Forward Particle Production in Proton-nucleus Collisions at NLO

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Iancu, Mueller, DT JHEP12 (2016) 41

Ducloué, Iancu, Lappi, Mueller, Soyez, DT, Zhu PRD97 (2018) 054020

Outline

- Resummation of perturbative series in field theory
- Gluon saturation (Color Glass Condensate)
- Process and problem: dA or pA and negative σ
- Process at LO and phenomenology
- Process at NLO : positive defined cross section
- Negative cross section as an artefact of $k_\perp$-factorization

Perturbation Theory in Quantum Mechanics

Energy levels in quantum mechanics, e.g. add perturbation $-2gx^3 + gx^4$ to simple harmonic oscillator

$$E_n = E_n^{(0)} + g^2 E_n^{(1)} + g^4 E_n^{(2)} + \cdots$$

Coefficients $E_n^{(i)}$ are numbers (for given m and ω)
Good approximation to keep few terms $n \leq n_0$

[NB: This example needs $n_0 \lesssim 1/g^2$: asymptotic series due to non-perturbative effects and instantons formation]

Perturbation Theory in Field Theory

Quantity in field theory with interaction (particle creation)

$$\sigma = g^{2k}\sigma_0 + g^{2k+2}\sigma_1 + g^{2k+4}\sigma_2 + \cdots$$

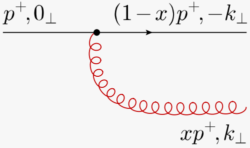
E.g. diff. cross section: σ_i functions of particle 4-momenta
At short distance $\alpha_s = g^2/4\pi \ll 1$, series looks meaningful

For some momenta could be $\alpha_s \sigma_{k+1} \sim \sigma_k$: fixed order series not enough, resum in such kinematic domains

[QCD at large distance, series is bad, use NP methods]

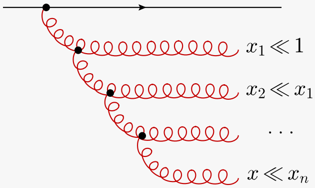
Parton Emission in QCD

Emission of gluon from parent parton (quark or gluon)



$$\mathrm{d}P = C_\mathrm{R} \frac{\bar{\alpha}_s}{2\pi} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$$

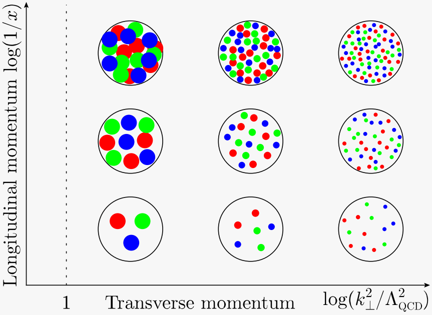
Integrate intermediate particles in cascade, two types of large logarithms: transverse for DGLAP, longitudinal for BFKL

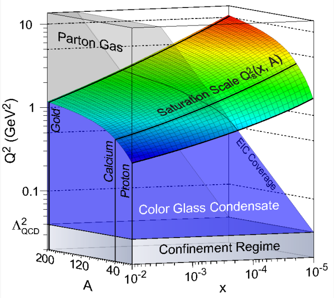


$$xG(x, Q^2) \sim \frac{1}{x^\lambda}$$

$$\lambda \sim 0.2 \div 0.3$$

Parton Saturation/Color Glass Condensate





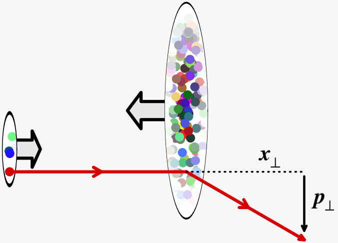
High density, weak coupling, non-linear dynamics

Saturation when $xG(x, Q_s^2)/Q_s^2 R^2 \sim 1/\alpha_s$

Dynamical perturbative (semi-hard) scale

$$Q_s^2(x) \sim Q_0^2 A^{1/3} x^{-\lambda} \gg \Lambda_{\rm QCD}^2$$

Collinear quark with proton picks up transverse momentum by multiple scattering with gluons in target nucleus



$$\eta = -\ln\tan\theta/2$$

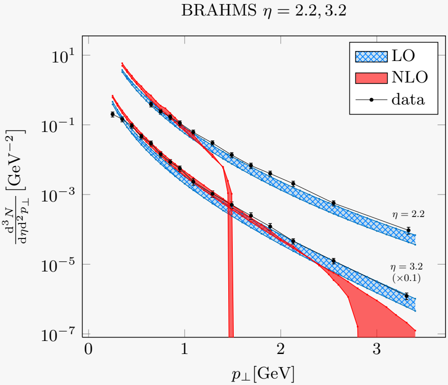
$$x_p = \frac{p^+}{Q^+} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

$$X_g = \frac{p^-}{P^-} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

- η: quark rapidity in COM frame
- x_p: longitudinal fraction of quark in proton
- X_g: longitudinal fraction of gluon in nucleus

Forward ($\eta \gg 1$) probes soft saturated modes

Negative σ at NLO



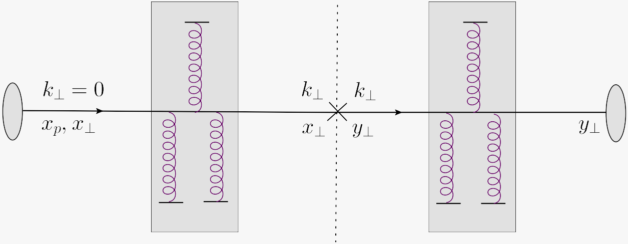
NLO theory [Chirilli, Xiao, Yuan '12] Numerics [Stasto, Xiao, Zaslavsky '13]

Issue appears for $p_\perp \sim Q_s$, where CGC should apply.

Multiple Scattering

Multiple scattering off target color field A to all orders

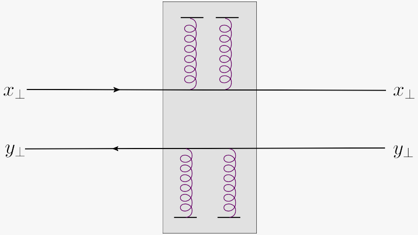
Eikonal: fixed transverse coordinate, color changing



Amplitude: $\mathcal{M}_{ij} = \int \mathrm{d}^2\boldsymbol{k}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\, V_{ij}(\boldsymbol{x}),\;\; V(\boldsymbol{x}) = P\exp\left[\mathrm{i}g\int \mathrm{d}x^+\, A_a^-(x^+,\boldsymbol{x})t^a\right]$

Cross section: $\dfrac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = x_p q(x_p)\dfrac{1}{N_c}\left\langle \sum_{ij} |\mathcal{M}_{ij}(\boldsymbol{k})|^2 \right\rangle_{X_g}$

Leads to elastic S-matrix for $q\bar{q}$ dipole (NLO) [Mueller, Munier '12]



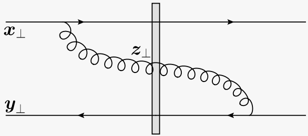
$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = x_p q(x_p) \int \mathrm{d}^2\boldsymbol{x}\,\mathrm{d}^2\boldsymbol{y}\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\, S(\boldsymbol{x},\boldsymbol{y};X_g)$$

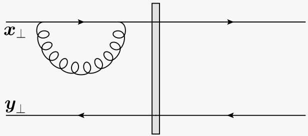
$$S(\boldsymbol{x},\boldsymbol{y};X_g) = \frac{1}{N_c}\left\langle \mathrm{tr}[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})]\right\rangle_{X_g}$$

Evaluated at softest target scale X_g, resum $(\bar{\alpha}_s \ln 1/X_g)^n$

Lowest order proportional to unintegrated gluon distribution

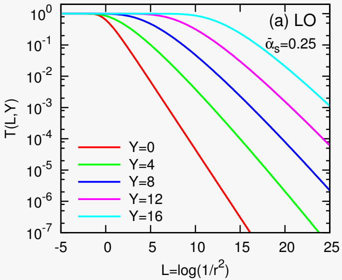
BK Equation





Closed evolution equation for dipole S-matrix

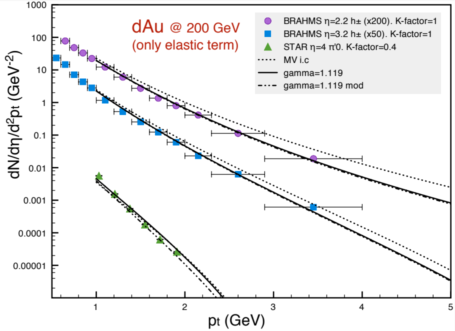
$$\frac{\partial S_{\boldsymbol{xy}}}{\partial \ln(1/x)} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2\boldsymbol{z} \, \underbrace{\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}}_{\text{dipole kernel}} [S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}]$$



Saturation momentum from unitarity

$$S(r_\perp = 1/Q_s(x), x) = 1/2$$

Hybrid Formalism and LO Phenomenology

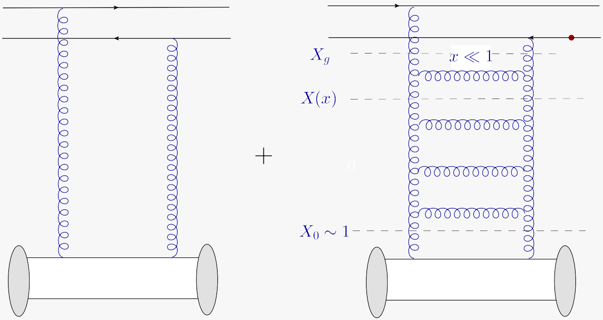


$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}}\right|_{\mathrm{LO}} = K^h \int_{x_p}^{1} \frac{\mathrm{d}z}{z^2}\, \frac{x_p}{z}\, q\left(\frac{x_p}{z}\right) \mathcal{S}\left(\frac{\boldsymbol{k}}{z}; X_g\right) \mathcal{D}_{h/q}(z)$$

Fit parameters: IC for rcBK and K-factors

[Dumitru, Hayashigaki, Jalilian-Marian '05] [Albacete, Dumitru, Fujii, Nara '12]

LO in Integral Form



One explicit emission close to dipole, most evolution in nucleus

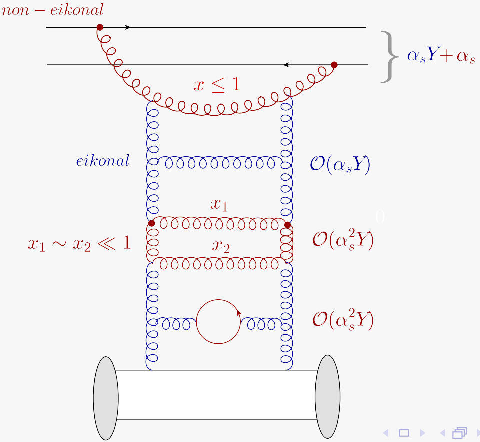
$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \frac{\bar{\alpha}_s}{2\pi} \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \int \mathrm{d}^2\boldsymbol{z} \, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \left[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\right]\Big|_{X(x)}$$

Energy fraction in target $X(x) = X_g/x$

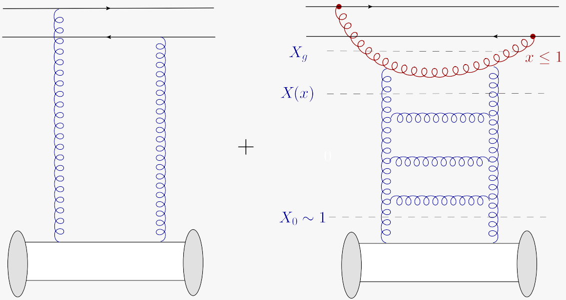
NLO Corrections to Particle Production

- $\mathcal{O}(\bar{\alpha}_s)$ correction to evolution: soft partons with $x_1 \sim x_2$
- $\mathcal{O}(\bar{\alpha}_s)$ correction to impact factor: first gluon $x \sim \mathcal{O}(1)$

  Non-eikonal emission, exact kinematics



Impact Factor Correction



$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \mathcal{K}(x) \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big) \qquad X(x) = \frac{X_g}{x}$$

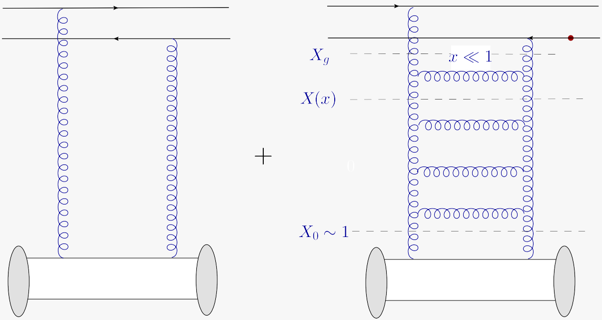
Distribution and fragmentation functions implicit

Floating scale $X(x)$, generalized factorization

Tree-level + positive defined. Where was the problem?

Recovering LO

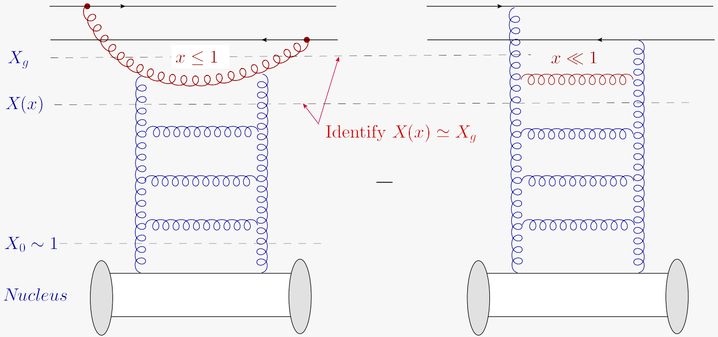
Let $\mathcal{K}(x) \to \mathcal{K}(0)$ to get LO



$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}}\right|_{\mathrm{LO}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^1 \frac{\mathrm{d}x}{x} \mathcal{K}(0) \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big) = \mathcal{S}(\boldsymbol{k}, X_g)$$

Just Fourier transform of BK equation (integral form)

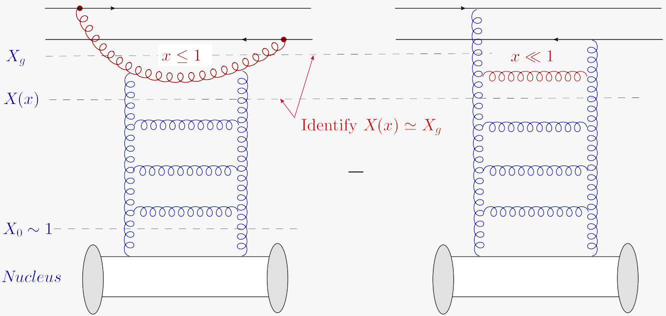
Add and subtract LO result, free to do so



$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\,[\mathcal{K}(x) - \mathcal{K}(0)]\mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)$$

Correct but dangerous: add and subtract large contribution

RECOVERING $k_\perp$-FACTORIZATION (BAD)



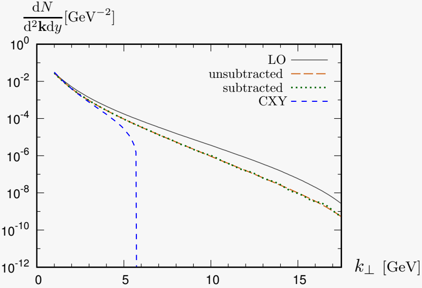
At NLO consistent to let $\mathcal{S}\big(X(x)\big) \to \mathcal{S}\big(X_g\big)$ [integrand not dominated by small-x] and $X_g \to 0$ in limit [plus prescription]

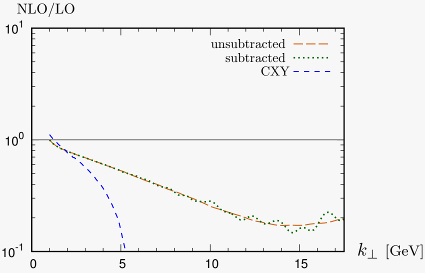
$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_0^1 \frac{\mathrm{d}x}{x}\,[\mathcal{K}(x) - \mathcal{K}(0)]\mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X_g\big)$$

Local in X_g, $k_\perp$-factorized. Mistreats kinematics at low-$x \rightsquigarrow \sigma < 0$.

Numerics

Ducloué, Lappi, Zhu '17



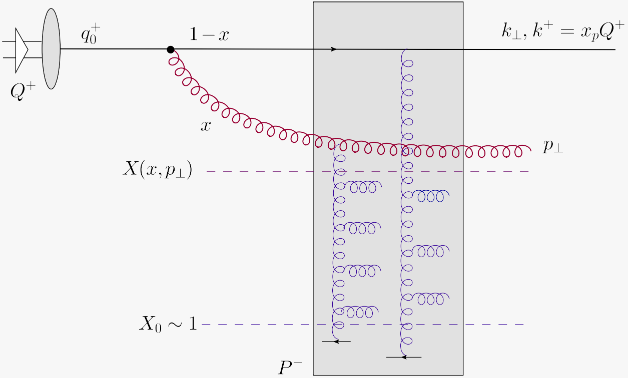


- Generalized factorization, same for sub and unsub, $\sigma > 0$
- Large but controlled NLO correction, 50% at $k_\perp \sim 5$GeV
- $k_\perp$-factorized result: $\sigma < 0$

Conclusions

- pA collision at large η at NLO in CGC:
  Process suited to study gluon saturation
- "Strict" $k_\perp$-factorized expression leads to negative σ
- Generalized factorization based on skeleton expansion:
  - No explicit separation between LO and NLO
  - Non-local in longitudinal space
- Well-defined result: positive σ at NLO, smaller than LO

Kinematics



$$\text{Energy conservation : } \frac{k_\perp^2}{2(1-x)q_0^+} + \frac{p_\perp^2}{2xq_0^+} = XP^-$$

$$\text{Simplifies for } k_\perp \sim p_\perp \gtrsim Q_s : X \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$