Rare radiative leptonic decays $B_{d,s} \rightarrow \gamma \ell^+\ell^-$:
- Induced by weak flavour changing neutral currents (FCNCs)
- Forbidden at tree level in the Standard Model and occur only via loop diagrams
- Have small branching ratios of order $10^{-8} - 10^{-10}$
- New particles can contribute to the loops

Similar B-decays are actively studied experimentally ($B \rightarrow K(\ast)\ell^+\ell^-$ and others), and 2-3 $\sigma$ deviations from the Standard Model have been observed.

1. Effective Hamiltonian

Penguin diagram as an example:

In the loop $u,c,t$ quarks are possible

Exchange of heavy virtual particles ($t, W, Z$) can be reduced to effective quasilocal interactions

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_i C_i(\mu) O_i(\mu),$$

$C_i(\mu)$ - Wilson coefficients,
$O_i(\mu)$ - set of basis operators

Light particles in the loops (u and c quarks) are dynamical
3. The ratio $R_{\mu/e} = \Gamma(B_s \rightarrow \mu\mu\gamma)/\Gamma(B_s \rightarrow ee\gamma)$

Test of lepton universality: possibility to study New Physics effects?
- For small $q^2$, visible effect of $\phi(\rho,\omega)$-resonance
- For large $q^2$, the ratio strongly depends on hadron form factors

4. Forward-backward asymmetry

Forward-backward asymmetry has the standard form and contains the cc-resonances in the kinematical region

5. Main goals
- We calculated the form factors which parametrize the amplitudes of the processes via relativistic quark model including the constraints obtained from gauge invariance.
2. Charm contribution

- The W boson line can be reduced to a point in both cases \((M_W >> M_B)\)
- In (a) we have the contribution of the c-quark loop for \(0 < q^2 < M_B^2\) and the cc-resonances appear in the kinematical region of the process.

2.1. Factorizable contribution

Takes into account only factorizable gluon exchanges

\[
A_{\mu\nu} \sim \frac{3C_1 + C_2}{3} \Pi_{\mu\nu}^{cc}(k) \ast (\text{Rest})
\]

Perturbative QCD + results of the measurements

\[
\Pi_{\mu\nu}^{cc}(k') = i \int dx e^{ikx} \langle 0 | T\{\bar{c}\gamma_\mu c(x), \bar{c}\gamma_\nu c(0)\} | 0 \rangle = (-g_{\mu\nu}k^2 + k_\mu k_\nu) \Pi^{cc}(k^2).
\]

Factorizable contribution can be represented as an addition to Wilson coefficient \(C_9\):

\[
C_{9V} \rightarrow C_{9V}^{\text{eff}}(q^2) = C_{9V} + \frac{16\pi^2}{9} (3C_1 + C_2) \Pi_{cc}(q^2)
\]

2.2. Nonfactorizable contribution

- Nonfactorizable and factorizable contributions are of the same order (experimental data)
- It is impossible to calculate the charm contribution in the region of cc-resonances
- Nonfactorizable contributions are not universal and depend on concrete processes

We used the results for \(B \rightarrow V\ell\ell\), combined with the concept of vector meson dominance to obtain predictions for \(B \rightarrow \gamma\ell\ell\)
These form factors satisfy restrictions from QCD in the heavy quark limit. Comparing our results with the results from lattice QCD and sum rules we obtained that the accuracy of our results is about 10%.

We analyzed the charm contribution and obtained restrictions for this contribution imposed by gauge invariance.

We used sum rules results for $B \rightarrow V\ell\ell$ combined with the concept of vector meson dominance to take into account the charm contribution to $B \rightarrow \gamma\ell\ell$.

We demonstrated that the results of the nonfactorizable contribution in the region of small $q^2$ do not provide any conclusions about the phases of the cc-resonances. The experimental measurement of the forward-backward asymmetry in the region between the $\psi$ and $\psi'$ resonances will probably clarify the situation with the relative contribution of the narrow cc-resonances.

We obtained numerical predictions for different differential distributions of the decays $B \rightarrow \gamma\ell^+\ell^-$. In particular, we demonstrated that the measurement of the ratio $R_{\mu/e}(q^2)$ in the region of large $q^2$ will clarify the $q^2$ dependence of the $B \rightarrow \gamma$ transition form factors if the lepton universality is established from the data for small $q^2$.

We calculated the branching ratios for the region $q^2 \in [1, 6] \text{GeV}^2$, where the form factors are reliably known and the contribution of charm is at the level of few percent:

\[
\mathcal{B}(\bar{B}_s \rightarrow \gamma\ell^+\ell^-)|_{q^2 \in [1, 6] \text{GeV}^2} = (6.01 \pm 0.08 \pm 0.70) \times 10^{-9},
\]

\[
\mathcal{B}(\bar{B}_d \rightarrow \gamma\ell^+\ell^-)|_{q^2 \in [1, 6] \text{GeV}^2} = (1.02 \pm 0.15 \pm 0.05) \times 10^{-11}
\]

The first error in these predictions reflects the uncertainty of the $B \rightarrow \gamma$ transition form factors; the second error reflects the uncertainty in the contributions of the light vector resonances ($\rho$; $\omega$; $\phi$).

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