Global Dyson-Schwinger-Bethe-Salpeter Approach to Mesons with Open Flavour

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Poincaré-Covariant Analysis: Physics Case

Within quantum field theories, the Bethe–Salpeter framework, underpinned by the Dyson–Schwinger equation controlling the dressed quark propagator, enables the Poincaré-covariant description of quark–antiquark bound states. This quark Dyson–Schwinger equation is part of the infinite tower of coupled Dyson–Schwinger equations, which requires to truncate this tower to a finite set of coupled relations. The merits of such a covariant approach are evident:

- ★ Quark models constitute a convenient framework for the comprehensive investigation of hadron states by comparatively simple technical means.
- ★ Technical/computational constraints limit nonperturbative approaches.
- ★ The covariant analyses use QCD input and modelling to bridge this gap.

Understandably, the first target of the covariant approach usually is the case of quarkonia, bound states of a quark and its antiquark, and thus flavourless. In order to gain a comprehensive picture, we complete this kind of studies by applying a single common framework to all conceivable flavour combinations and fathom its implications for the predicted meson masses, decay constants and in-meson condensates by comparison with experiment or other findings:

- ★ Covariant studies of open-flavour mesons have been and remain limited.
- ★ Nonetheless, a covariant study yields utmost extensive sets of results [1].

Dyson–Schwinger–Bethe–Salpeter Liaison

The Bethe–Salpeter framework represents a bound state of total momentum P, composed of quark and antiquark of relative momentum p, by such state's Bethe–Salpeter amplitude $\Gamma(p;P)$ or Bethe–Salpeter wave function $\chi(p;P)$, related by the dressed propagators of the two bound-state constituents, $S_{1,2}$:

$$\chi(p;P) \equiv S_1(p+\eta\,P)\,\Gamma(p;P)\,S_2(p-(1-\eta)\,P)\ , \qquad \eta \in \mathbb{R}\ .$$

These propagators may be obtained as the solutions of the Dyson–Schwinger equation for the quark two-point function, in rainbow truncation of the form

$$S^{-1}(p) = Z_2\,(\mathrm{i}\,\gamma\cdot p + m_\mathrm{b}) + \frac{4}{3}\,Z_2^2 \int_q^\Lambda \mathcal{G}\big((p-q)^2\big)\,T_{\mu\nu}(p-q)\,\gamma_\mu\,S(q)\,\gamma_\nu\ ,$$

adopting current-quark wave-function renormalization constant Z_2 and bare mass m_b, the free Landau-gauge gluon-propagator transverse-projector part

$$T_{\mu\nu}(k) \equiv \delta_{\mu\nu} - \frac{k_\mu\,k_\nu}{k^2}\ ,$$

translationally invariant integration measure $\int_q^\Lambda$ Pauli–Villars regularized at scale Λ, and an effective coupling $k^2\,\mathcal{G}(k^2)$ mimicking the effects of full gluon propagator and full quark–gluon vertex; the mass renormalization factor Z_m relates bare quark mass m_b and renormalized quark mass $m_q(\mu)$ at a scale μ:

$$m_\mathrm{b} = Z_m\,m_q(\mu)\ .$$

A bound state's Bethe–Salpeter amplitude or wave function is governed by a homogeneous Bethe–Salpeter equation, which in ladder truncation (in order to satisfy the QCD axial-vector Ward–Takahashi identity) reads, for mesons,

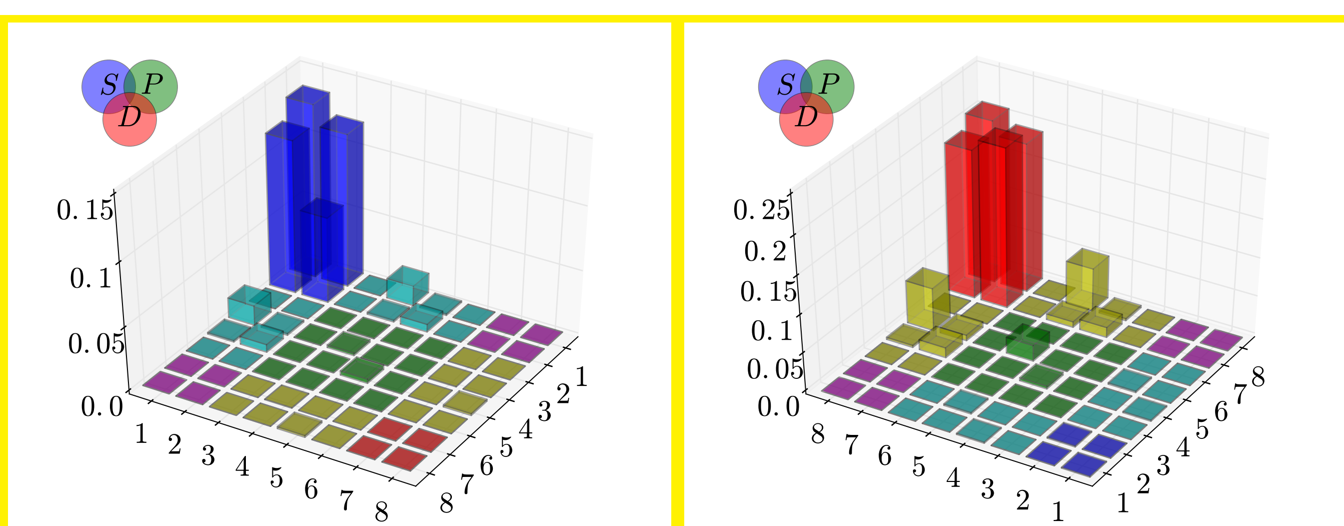
$$\Gamma(p;P) = -\frac{4}{3}\,Z_2^2 \int_q^\Lambda \mathcal{G}\big((p-q)^2\big)\,T_{\mu\nu}(p-q)\,\gamma_\mu\,\chi(q;P)\,\gamma_\nu\ .$$

Expansion of $\Gamma(p;P)$ in Lorentz covariants recasts this bound-state equation into a system of four (for bound states of spin zero) or eight (for bound states of non-zero spin) coupled equations. An estimate of (some of) the systematic uncertainties inherent to such treatment can be acquired by adopting for the effective couplings $k^2\,\mathcal{G}(k^2)$ at least two different (rather popular) models [2]. From the solution for a bound state's Bethe–Salpeter amplitude $\Gamma(p;P)$ and mass M, we find its decay constant f and in-hadron condensate $|\langle\bar q\,q\rangle|^{1/3}$ [3], the hadron-to-vacuum matrix element of the relevant quark-bilinear density.

It's a Long Way to ~~Tipperary~~ Bound States

Within a covariant approach, the classification of predicted states in terms of quantum numbers is not as straightforward as in nonrelativistic frameworks.

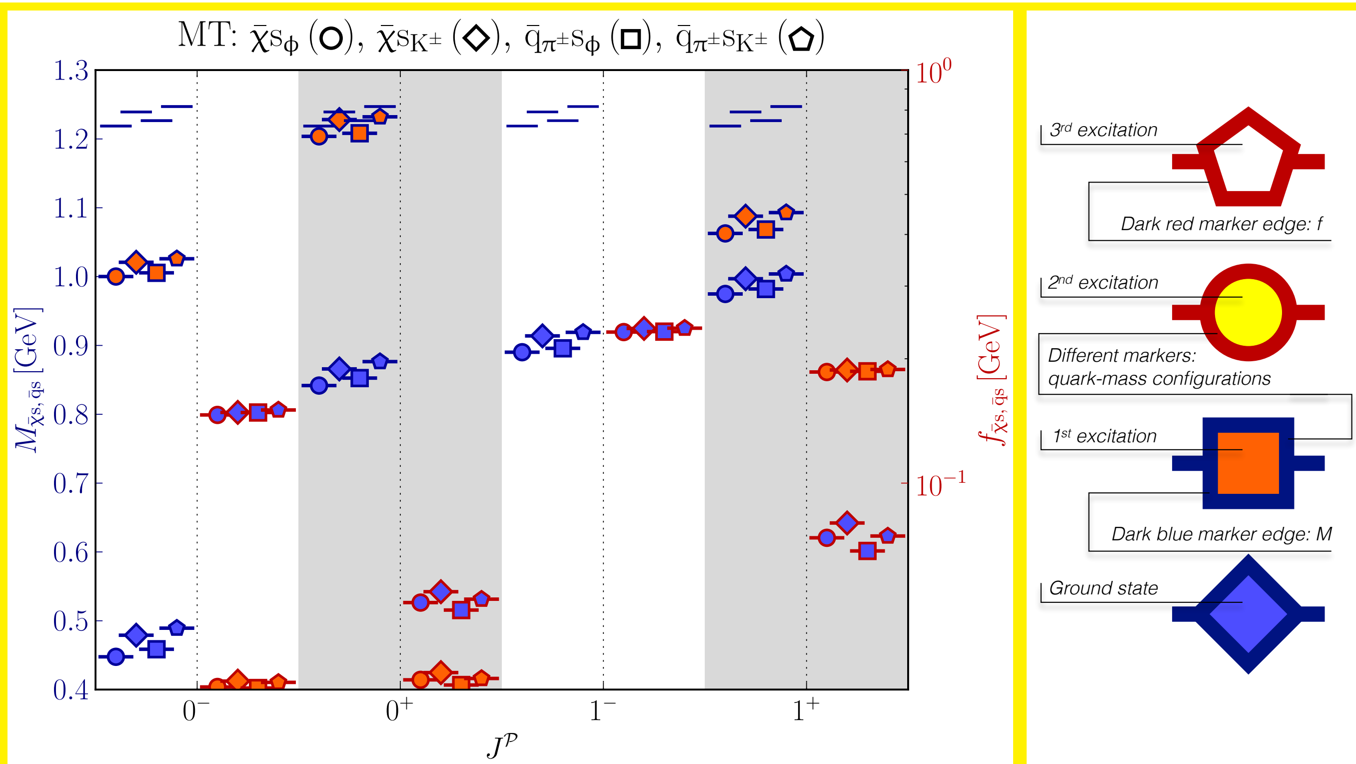
- ★ In quark models: construction of a quark-bilinear bound state with total spin $s = 0, 1$ and relative orbital angular momentum ℓ of its ingredients.
- ★ Permitted bound-state spectrum identified by total angular momentum J, parity $P = (-1)^{\ell+1}$, and charge-conjugation parity $C = (-1)^{\ell+s}$ (for states with well-defined C), constrained by $|\ell - s| \leq J \leq |\ell + s|$ to some assignment $J^{PC} \in \{0^{++}, 0^{-+}, 1^{++}, 1^{+-}, 1^{--}, 2^{++}, 2^{-+}, 2^{--}, 3^{++}, \ldots\}$.
- ★ States with $J^{PC} \in \{0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots\}$ are viewed as exotic.
- ★ So far, just hints of isovector 1^{-+} states have been found experimentally.
- ★ This situation carries over to open-flavour mesons where one encounters quasi-exotic mesons, mirroring exotic mesons in the equal-mass case [4].
- ★ Bethe–Salpeter amplitude: more complex than solution of quark model.
- ★ So, a covariant approach predicts more meson states than quark models.
- ★ Orbital angular momentum ℓ can be identified also in the covariant case, e.g., for the ρ meson and its excitations [5]. This can be visualized via the contributions to the norm of $\chi(p; P)$ of the various Lorentz covariants of different values of ℓ attributable to them in the given meson's rest frame.
- ★ For ground and first excited state of the ρ, one finds [5, Figs. 7 and 12], for the 8×8 combinations of Lorentz covariants in the vector-meson case



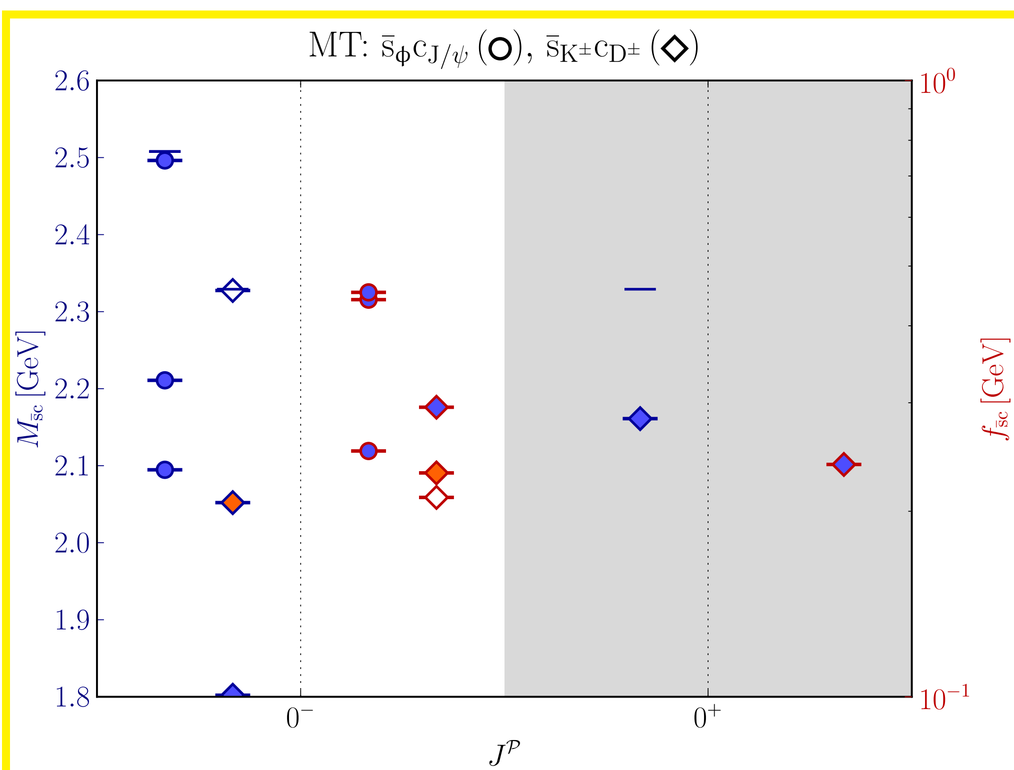
Meson Mass, Decay Constant, Condensate

For a meson composed of antiquark $\bar{q}$ and quark q', we depict our predictions for the masses $M_{\bar{q}\,q'}$ and leptonic decay constants $f_{\bar{q}\,q'}$ of its ground state and lowest radial excitations in single shared plots, as exemplified by two mesons:

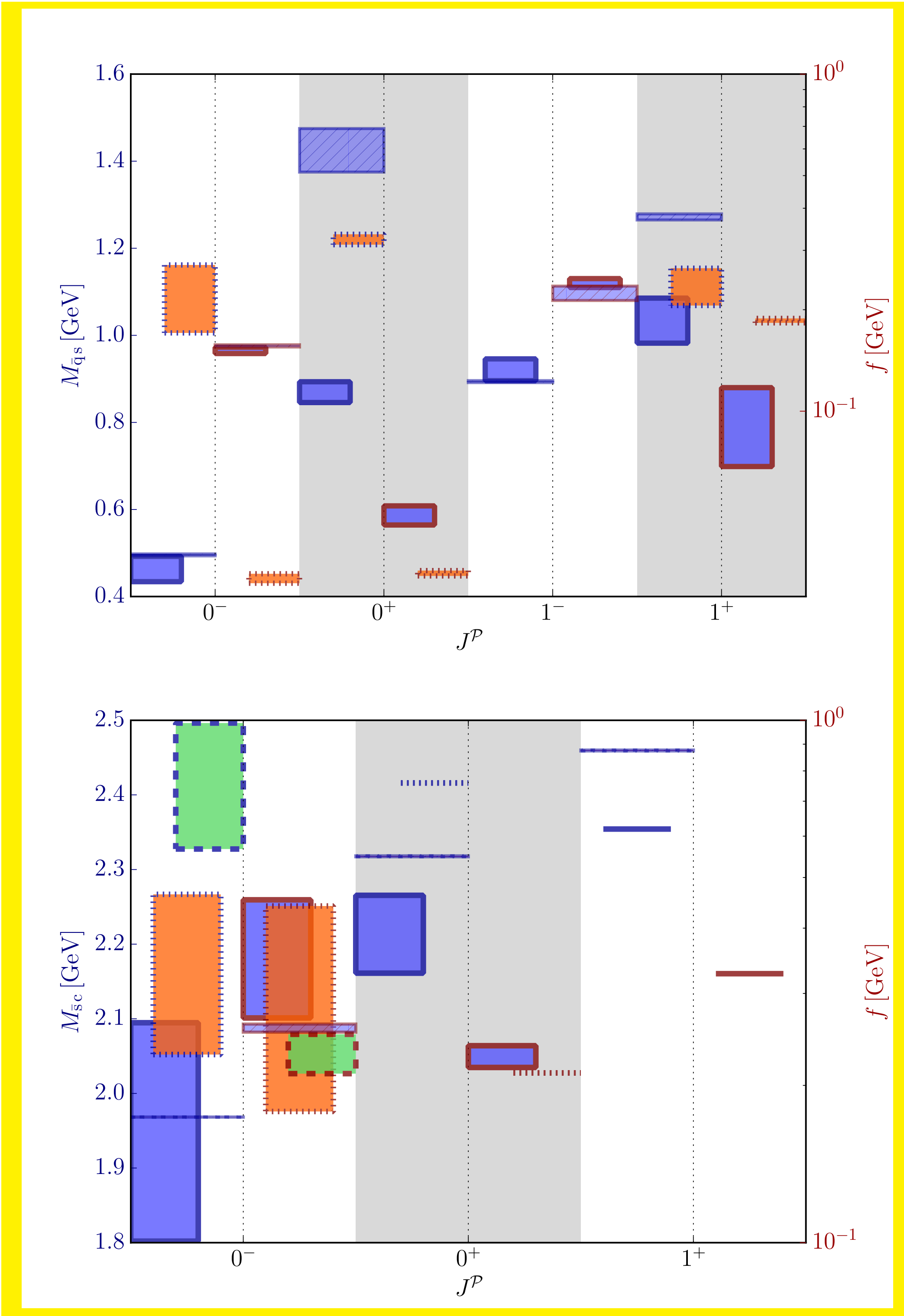
★ strange meson involving massless (χ) or light (q) and s quark [1, Fig. 14]:



★ charmed, strange heavy meson, formed by an s and a c quark [1, Fig. 17]:

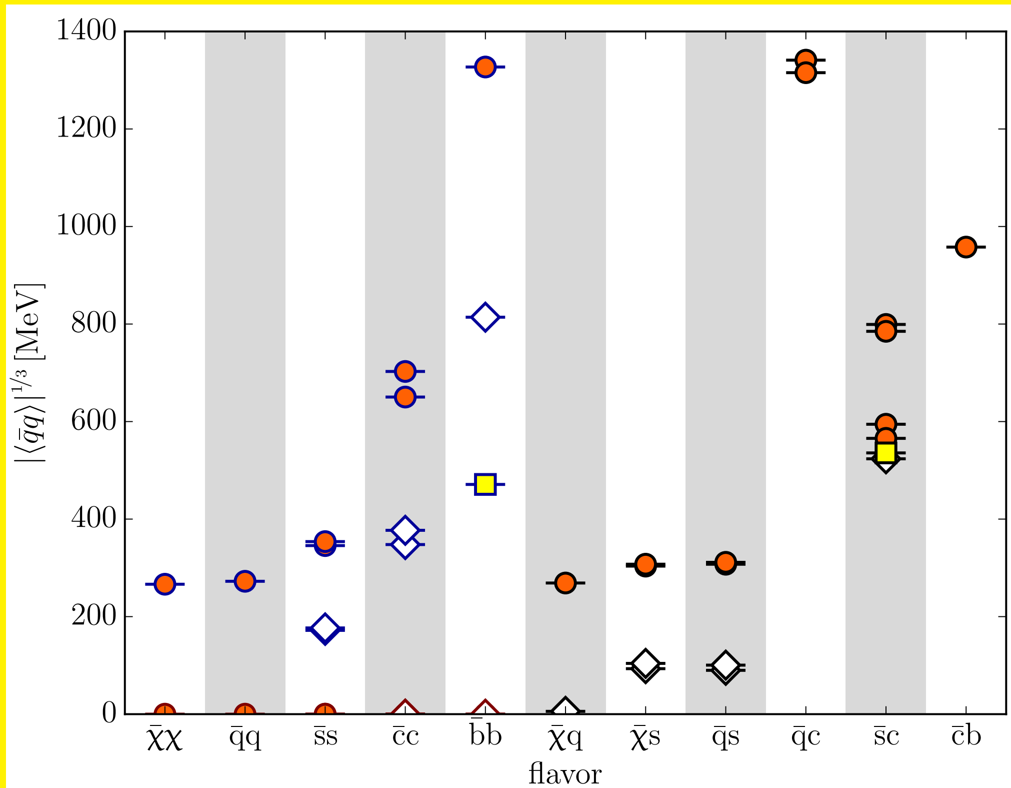


★ For comparison with experiment, we combine our results found from the effective-interaction models under study [2] for two fits [1] of the involved quark masses to single predictions (given by boxes), as illustrated for the strange [1, Fig. 24] (top) or charmed, strange [1, Fig. 25] (bottom) states:



★ It goes without saying that the example findings presented above should merely serve as a teaser: the complete sets of our results may be found in Ref. [1]. A first idea of the size of the systematic uncertainties inherent to the employed approach can be inferred by variation of the model details.

★ Finally, we find the in-hadron condensates $\langle \bar{q}\, q \rangle$ of all mesons [1, Fig. 21]:



Summary: Findings, Conclusions, Outlook

★ The Dyson–Schwinger–Bethe–Salpeter-rooted covariant framework has both qualitative and quantitative features inherited directly from QCD.

★ Therein, model studies are comparatively cheap, as far as the computing power required is concerned, and may be implemented comprehensively.

★ With technical issues overcome, a new class of QCD models will emerge.

★ The models will have scope and comprehension akin to the quark model.

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