Exotic Tetraquark Mesons in Large-N_c Limit: an Unexpected Great Surprise

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Multiquark Spectra from $1/N_c$ Expansions

Tetraquarks are hypothetical meson bound states of two antiquarks and two quarks predicted by quantum chromodynamics (QCD). We infer qualitative information on their overall features by analyzing the possible appearance of associated poles in amplitudes for the scattering of two ordinary mesons into two ordinary mesons by exploiting a variant of QCD dubbed large-N_c QCD.

QCD is a particular case, $N_c = 3$, in a set of quantum field theories invariant under $SU(N_c)$ gauge transformations, with fermions transforming according to the fundamental $SU(N_c)$ representation of dimension N_c. Large-N_c QCD is a limiting case of the latter quantum field theory, defined by its number N_c of colour degrees of freedom rising beyond bounds, $N_c \to \infty$, and the strong fine-structure coupling $\alpha_s \equiv g_s^2/4\pi$ simultaneously scaling as $\alpha_s \propto 1/N_c$ [1].

All expansions in powers of $1/N_c$ are underpinned by plausible assumptions:

- ★ Considering the large-N_c limit makes sense; our study of tetraquarks by means of the $1/N_c$ expansion is justified and entails reliable conclusions.
- ★ For $N_c \to \infty$, tetraquark-associated poles in the complex-s plane exist.
- ★ For $N_c \to \infty$, tetraquark masses don't rise without limit but stay finite.

In order to isolate, in a perturbative expansion of some scattering amplitude (with incoming external momenta p_1 and p_2) in powers of both $1/N_c$ and α_s, those "tetraquark-phile" Feynman diagrams that may support a tetraquark pole (with constituents of mass m_1, m_2, m_3, m_4), we propose to impose a set of two selection criteria on the analytic specifics of a potential contributor to a tetraquark pole as a function of the Mandelstam variable $s \equiv (p_1+p_2)^2$ [2]:

★ The graph has a nontrivial, more exactly, nonpolynomial s dependence.

★ The graph allows for four-quark intermediate states with cut starting at

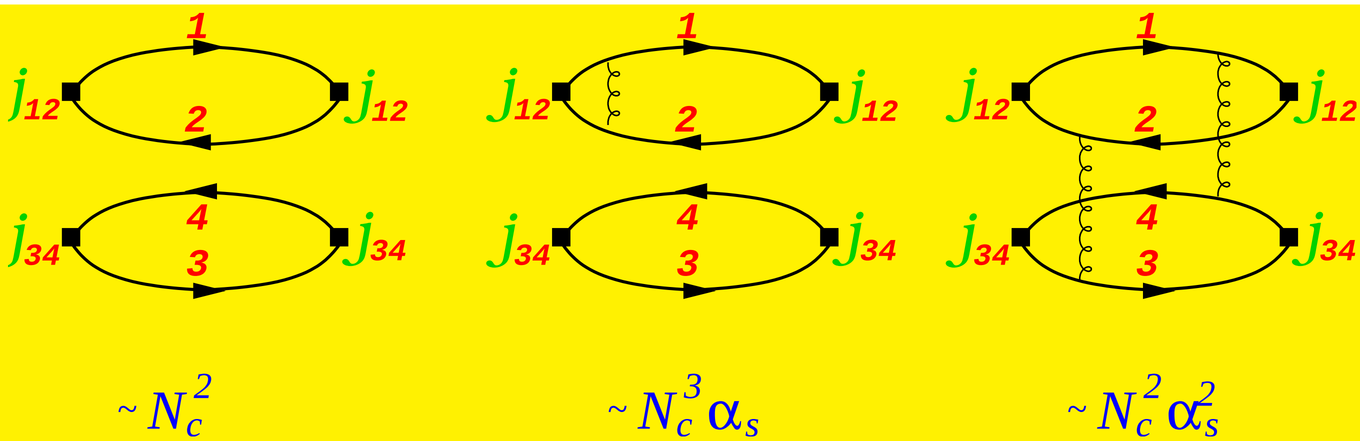
$$s = (m_1 + m_2 + m_3 + m_4)^2 \ .$$

Interpolating an ordinary meson M_{ij} composed of antiquark $\bar{q}_i$ and quark q_j with flavour quantum numbers $i, j = 1, 2, 3, 4$ by appropriate bilinear quark currents j_{ij}, we extract from four-current correlators the large-N_c behaviour of the relevant tetraquark-phile correlators (indicated by a subscript T), the amplitudes A for transitions between a tetraquark and two ordinary mesons as well as the implied decay rate Γ for two interesting types of tetraquark [2].

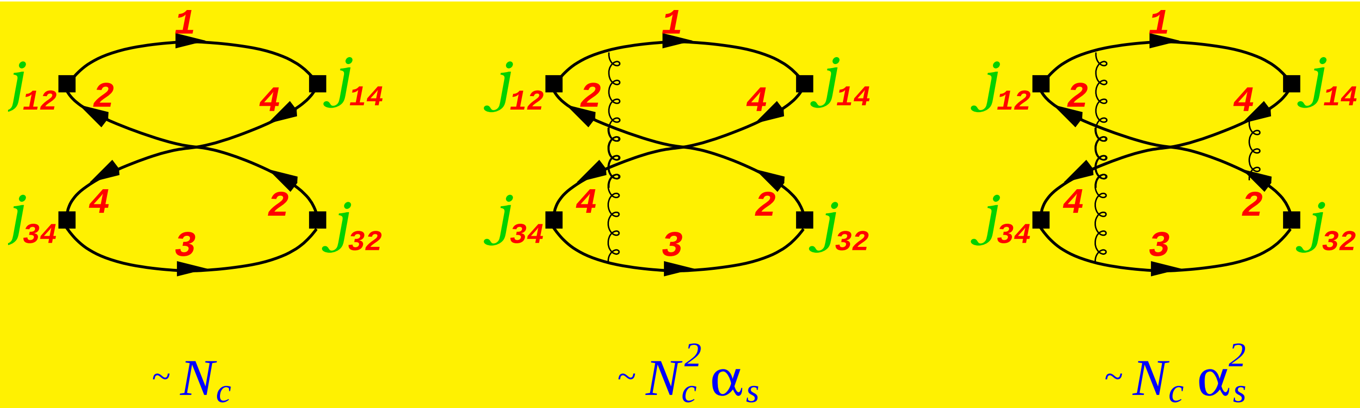
Tetraquark of Exotic Flavour Composition

Tetraquarks $(\bar{q}_1\, q_2\, \bar{q}_3\, q_4)$ involving four different quark flavours are genuinely exotic. Two classes of graphs emerge, differing in whether the quarks present in such a tetraquark are or are not redistributed among the external mesons.

N_c-leading (a,b) and N_c-subleading (c) graphs for correlator $\langle j_{12}^\dagger\, j_{34}^\dagger\, j_{12}\, j_{34}\rangle$, where quarks are indicated by solid lines and gluon exchanges by curly lines:



N_c-leading (a,b) and N_c-subleading (c) graphs for correlator $\langle j^\dagger_{14}\, j^\dagger_{32}\, j_{12}\, j_{34}\rangle$:



At large N_c, the two types of tetraquark-phile correlators behave differently:

$$\langle j^\dagger_{12}\, j^\dagger_{34}\, j_{12}\, j_{34}\rangle_{\rm T} = O(N_c^0)\ , \qquad \langle j^\dagger_{14}\, j^\dagger_{32}\, j_{14}\, j_{32}\rangle_{\rm T} = O(N_c^0)\ ,$$
$$\langle j^\dagger_{14}\, j^\dagger_{32}\, j_{12}\, j_{34}\rangle_{\rm T} = O(N_c^{-1})\ ;$$

their pole terms' N_c consistency enforces the existence of (not less than) two tetraquark states, T_A and T_B, favouring different two-meson decay channels, but, from their preferred decays, with parametrically identical decay widths:

$$\underbrace{A(T_A \longleftrightarrow M_{12}\, M_{34}) = O(N_c^{-1})}_{\Longrightarrow \qquad \Gamma(T_A) = O(N_c^{-2})}\ , \qquad A(T_A \longleftrightarrow M_{14}\, M_{32}) = O(N_c^{-2})\ ,$$

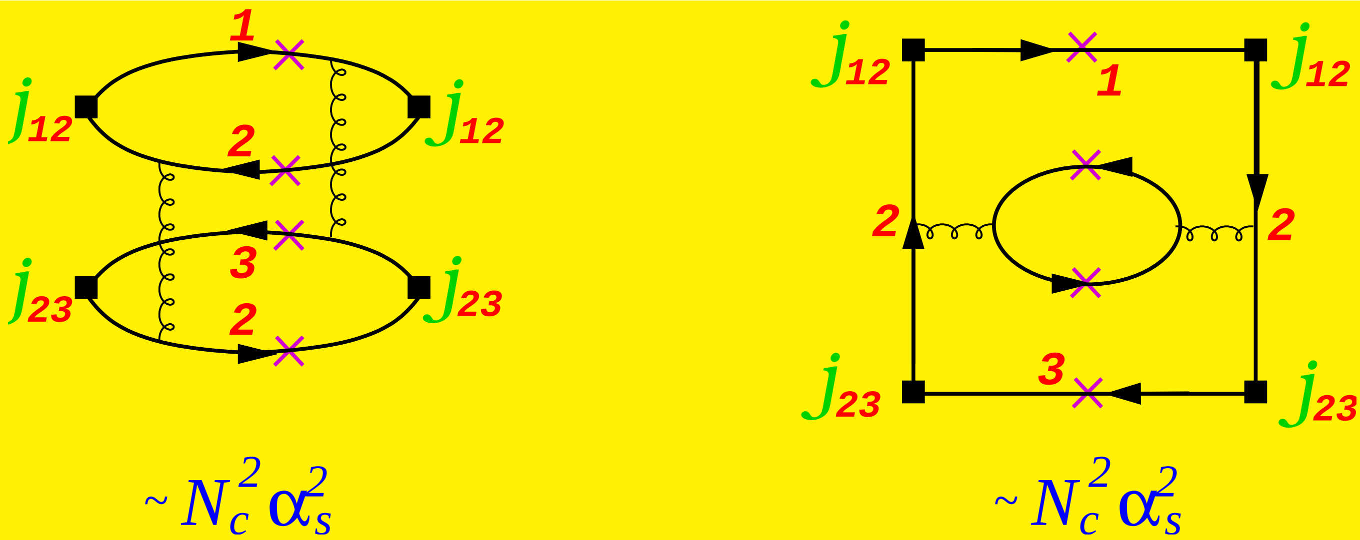
$$A(T_B \longleftrightarrow M_{12}\, M_{34}) = O(N_c^{-2})\ , \qquad \underbrace{A(T_B \longleftrightarrow M_{14}\, M_{32}) = O(N_c^{-1})}_{\Longrightarrow \qquad \Gamma(T_B) = O(N_c^{-2})}\ .$$

Of course, being composed of the same four quarks, T_A and T_B may undergo mixing, by large-N_c analysis with strength falling off at least as fast as $1/N_c$.

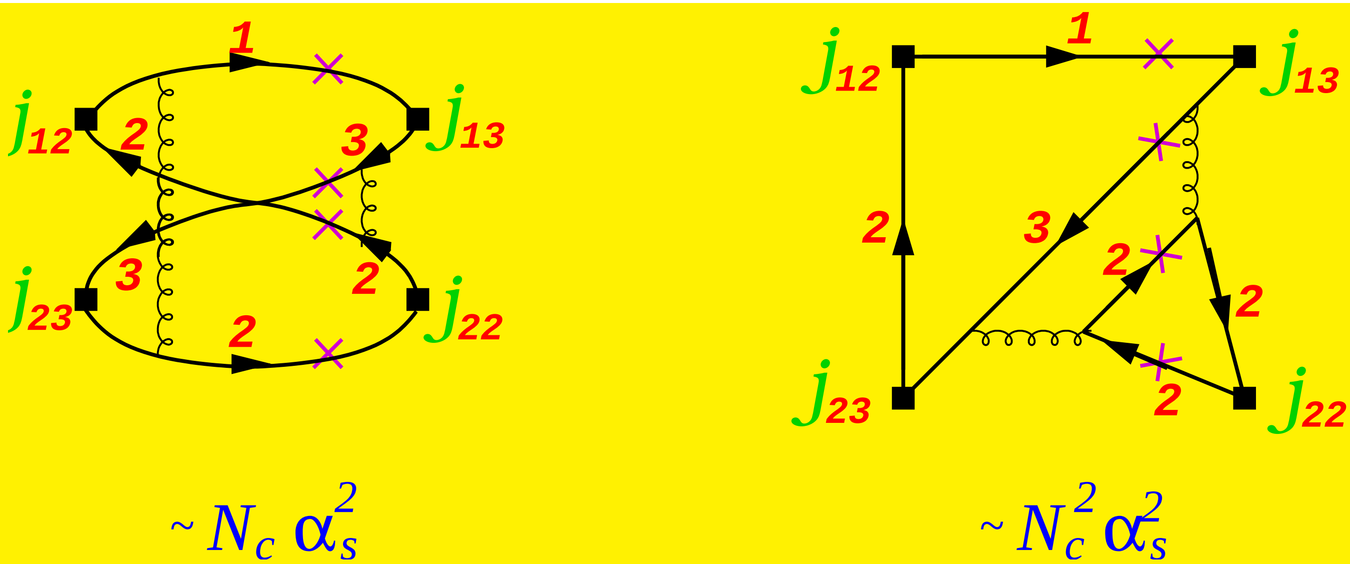
Tetraquarks of Nonexotic Flavour Content

Tetraquarks $(\bar q_1\, q_2\, \bar q_2\, q_3)$, including the flavours of a quark and its antiquark, have the net flavour content of the ordinary mesons $M_{13} \equiv (\bar q_1\, q_3)$; they may be called cryptoexotic. Therefore, among the tetraquark-phile contributions graphs of new topologies appear at either the same or even lower $1/N_c$ order.

N_c-leading tetraquark-phile contributions to correlator $\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle$, with the quarks forming the tetraquark identified by crosses on their propagators:



N_c-subleading and N_c-**leading** tetraquark-phile contributions to correlator $\langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle$, where crosses indicate tetraquark constituents' propagators:



Both tetraquark-phile correlator types exhibit the same large-N_c behaviour,

$$\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle_\mathrm{T} = O(N_\mathrm{c}^0)\ , \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{13}\, j_{22}\rangle_\mathrm{T} = O(N_\mathrm{c}^0)\ ,$$

$$\langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle_\mathrm{T} = O(N_\mathrm{c}^0)\ ,$$

whence a single cryptoexotic tetraquark, T_C, fulfils all constraints at its pole:

$$\underbrace{A(T_\mathrm{C} \longleftrightarrow M_{12}\, M_{23}) = O(N_\mathrm{c}^{-1})\ , \qquad A(T_\mathrm{C} \longleftrightarrow M_{13}\, M_{22}) = O(N_\mathrm{c}^{-1})}_{\Longrightarrow \qquad \Gamma(T_\mathrm{C}) = O(N_\mathrm{c}^{-2})}\ .$$

Count: always two there are, . . . , no less [3]

Large-N_c QCD proves to be a powerful tool in the analysis of tetraquarks [2]:

★ Exotic tetraquarks come in pairs, differing in the dominant decay mode.

★ Exotic and cryptoexotic tetraquarks T have narrow decay widths $\Gamma(T)$:

$$\Gamma(T) \propto 1/N_c^2 \xrightarrow[N_c\to\infty]{} 0 \qquad \text{for} \quad T = T_A, T_B, T_C \ .$$

Comparison: upper bounds on large-N_c behaviour of tetraquark decay rates.

Author Collective	Decay Width Γ		Reference
	Exotic Tetraquarks	Cryptoexotic Tetraquarks	
Our findings	$O(1/N_c^2)$	$O(1/N_c^2)$	[2]
Knecht & Peris	$O(1/N_c^2)$	$O(1/N_c)$	[4]
Cohen & Lebed	$O(1/N_c^2)$	—	[5]
Maiani *et al.*	$O(1/N_c^3)$	$O(1/N_c^3)$	[6]

With respect to the minimal number of exotic tetraquarks of a given flavour, for tetraquark-phile graphs it is not compulsory to arise at N_c-leading order: diagrams that do not require flavour redistribution are of even powers of N_c, whereas diagrams demanding flavour reshuffle are of odd powers of N_c. That mismatch calls for two exotic states, even if they appear at subleading order.

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