Exotic Tetraquark Mesons in Large-$N_c$ Limit: an Unexpected Great Surprise

W. Lucha,1 D. Melikhov1,2,3 & H. Sazdjian4

1 Institute for High Energy Physics, Austrian Academy of Sciences, Vienna, Austria
2 D. V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Russia
3 Faculty of Physics, University of Vienna, Austria
4 Institut de Physique Nucléaire, CNRS/IN2P3, Université Paris-Sud, Université Paris-Saclay, Orsay, France

Multiquark Spectra from $1/N_c$ Expansions

Tetraquarks are hypothetical meson bound states of two antiquarks and two quarks predicted by quantum chromodynamics (QCD). We infer qualitative information on their overall features by analyzing the possible appearance of associated poles in amplitudes for the scattering of two ordinary mesons into two ordinary mesons by exploiting a variant of QCD dubbed large-$N_c$ QCD.

QCD is a particular case, $N_c = 3$, in a set of quantum field theories invariant under SU($N_c$) gauge transformations, with fermions transforming according to the fundamental SU($N_c$) representation of dimension $N_c$. Large-$N_c$ QCD is a limiting case of the latter quantum field theory, defined by its number $N_c$ of colour degrees of freedom rising beyond bounds, $N_c \to \infty$, and the strong fine-structure coupling $\alpha_s \equiv g_s^2/4\pi$ simultaneously scaling as $\alpha_s \propto 1/N_c$.[1]

All expansions in powers of $1/N_c$ are underpinned by plausible assumptions:

★ Considering the large-$N_c$ limit makes sense; our study of tetraquarks by means of the $1/N_c$ expansion is justified and entails reliable conclusions.
★ For $N_c \to \infty$, tetraquark-associated poles in the complex-$s$ plane exist.
★ For $N_c \to \infty$, tetraquark masses don’t rise without limit but stay finite.
In order to isolate, in a perturbative expansion of some scattering amplitude (with incoming external momenta \( p_1 \) and \( p_2 \)) in powers of both \( 1/N_c \) and \( \alpha_s \), those “tetraquark-phile” Feynman diagrams that may support a tetraquark pole (with constituents of mass \( m_1, m_2, m_3, m_4 \)), we propose to impose a set of two selection criteria on the analytic specifics of a potential contributor to a tetraquark pole as a function of the Mandelstam variable \( s \equiv (p_1 + p_2)^2 \): 
- The graph has a nontrivial, more exactly, nonpolynomial \( s \) dependence.
- The graph allows for four-quark intermediate states with cut starting at 
  \[ s = (m_1 + m_2 + m_3 + m_4)^2 \].

Interpolating an ordinary meson \( M_{ij} \) composed of antiquark \( \bar{q}_i \) and quark \( q_j \) with flavour quantum numbers \( i, j = 1, 2, 3, 4 \) by appropriate bilinear quark currents \( j_{ij} \), we extract from four-current correlators the large-\( N_c \) behaviour of the relevant tetraquark-phile correlators (indicated by a subscript \( T \)), the amplitudes \( A \) for transitions between a tetraquark and two ordinary mesons as well as the implied decay rate \( \Gamma \) for two interesting types of tetraquark [2].

**Tetraquark of Exotic Flavour Composition**

Tetraquarks \( (\bar{q}_1 q_2 \bar{q}_3 q_4) \) involving four different quark flavours are genuinely exotic. Two classes of graphs emerge, differing in whether the quarks present in such a tetraquark are or are not redistributed among the external mesons.

\( N_c \)-leading \((a,b)\) and \( N_c \)-subleading \((c)\) graphs for correlator \( \langle j^\dagger_{12} j^\dagger_{34} j_{12} j_{34} \rangle \), where quarks are indicated by solid lines and gluon exchanges by curly lines:
At large $N_c$, the two types of tetraquark-phile correlators behave differently:

$$\langle j_{12}^\dagger j_{34}^\dagger j_{12} j_{34} \rangle_T = O(N_c^0) , \quad \langle j_{14}^\dagger j_{32}^\dagger j_{14} j_{32} \rangle_T = O(N_c^0) ,$$

$$\langle j_{14}^\dagger j_{32}^\dagger j_{12} j_{34} \rangle_T = O(N_c^{-1}) ;$$

their pole terms’ $N_c$ consistency enforces the existence of (not less than) two tetraquark states, $T_A$ and $T_B$, favouring different two-meson decay channels, but, from their preferred decays, with parametrically identical decay widths:

$$A(T_A \leftrightarrow M_{12} M_{34}) = O(N_c^{-1}) , \quad A(T_A \leftrightarrow M_{14} M_{32}) = O(N_c^{-2}) ,$$

$$\Rightarrow \quad \Gamma(T_A) = O(N_c^{-2})$$

$$A(T_B \leftrightarrow M_{12} M_{34}) = O(N_c^{-2}) , \quad A(T_B \leftrightarrow M_{14} M_{32}) = O(N_c^{-1}) ,$$

$$\Rightarrow \quad \Gamma(T_B) = O(N_c^{-2}) .$$

Of course, being composed of the same four quarks, $T_A$ and $T_B$ may undergo mixing, by large-$N_c$ analysis with strength falling off at least as fast as $1/N_c$.

**Tetraquarks of Nonexotic Flavour Content**

Tetraquarks ($\bar{q}_1 q_2 \bar{q}_2 q_3$), including the flavours of a quark and its antiquark, have the net flavour content of the ordinary mesons $M_{13} \equiv (\bar{q}_1 q_3)$; they may be called *cryptoexotic*. Therefore, among the tetraquark-phile contributions graphs of new topologies appear at either the same or even lower $1/N_c$ order.
$N_c$-leading tetraquark-phile contributions to correlator $\langle j_{12}^\dagger j_{23}^\dagger j_{12} j_{23}\rangle$, with the quarks forming the tetraquark identified by crosses on their propagators:

$\sim N_c^2 \alpha_s^2$

$N_c$-subleading and $N_c$-leading tetraquark-phile contributions to correlator $\langle j_{13}^\dagger j_{22}^\dagger j_{12} j_{23}\rangle$, where crosses indicate tetraquark constituents’ propagators:

$\sim N_c \alpha_s^2$

Both tetraquark-phile correlator types exhibit the same large-$N_c$ behaviour,

\[
\langle j_{12}^\dagger j_{23}^\dagger j_{12} j_{23}\rangle_T = O(N_c^0), \quad \langle j_{13}^\dagger j_{22}^\dagger j_{12} j_{23}\rangle_T = O(N_c^0),
\]

\[
\langle j_{13}^\dagger j_{22}^\dagger j_{12} j_{23}\rangle_T = O(N_c^0),
\]

whence a single cryptoexotic tetraquark, $T_C$, fulfils all constraints at its pole:

\[
A(T_C \leftrightarrow M_{12} M_{23}) = O(N_c^{-1}), \quad A(T_C \leftrightarrow M_{13} M_{22}) = O(N_c^{-1}) \quad \Rightarrow \quad \Gamma(T_C) = O(N_c^{-2})
\]
Count: always two there are, . . . , no less [3]

Large-$N_c$ QCD proves to be a powerful tool in the analysis of tetraquarks[2]:

★ Exotic tetraquarks come in pairs, differing in the dominant decay mode.
★ Exotic and cryptoexotic tetraquarks $T$ have narrow decay widths $\Gamma(T)$:

$$\Gamma(T) \propto \frac{1}{N_c^2} \xrightarrow{N_c \to \infty} 0 \quad \text{for} \quad T = T_A, T_B, T_C .$$

Comparison: upper bounds on large-$N_c$ behaviour of tetraquark decay rates.

<table>
<thead>
<tr>
<th>Author Collective</th>
<th>Exotic Tetraquarks</th>
<th>Cryptoexotic Tetraquarks</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our findings</td>
<td>$O(1/N_c^2)$</td>
<td>$O(1/N_c^2)$</td>
<td>[2]</td>
</tr>
<tr>
<td>Knecht &amp; Peris</td>
<td>$O(1/N_c^2)$</td>
<td>$O(1/N_c)$</td>
<td>[4]</td>
</tr>
<tr>
<td>Cohen &amp; Lebed</td>
<td>$O(1/N_c^2)$</td>
<td>—</td>
<td>[5]</td>
</tr>
<tr>
<td>Maiani et al.</td>
<td>$O(1/N_c^3)$</td>
<td>$O(1/N_c^3)$</td>
<td>[6]</td>
</tr>
</tbody>
</table>

With respect to the minimal number of exotic tetraquarks of a given flavour, for tetraquark-phile graphs it is not compulsory to arise at $N_c$-leading order: diagrams that do not require flavour redistribution are of even powers of $N_c$, whereas diagrams demanding flavour reshuffle are of odd powers of $N_c$. That mismatch calls for two exotic states, even if they appear at subleading order.

Acknowledgment. D.M. was supported by the Austrian Science Fund (FWF) under project P29028.