From SYK to tensor models

The Sachdev-Ye-Kitaev (SYK) model and the tensor models have recently attracted a considerable amount of attention. Their popularity is mainly due to the fact that they are examples of quantum mechanical models which are exactly solvable in the large N limit but nevertheless they have interesting and non-trivial physical properties. They are also believed to be toy models to study in a quantitative way important aspects of quantum black holes.

In the first part of the leture, after a brief overview of the key properties of quantum black holes which these models are able to reproduce, we will provide an overview to the SYK model and its flavored generalization, introduced by Gross and Rosenhaus. To be more precise, we will review the computations of the two-point function and of the four-point function. As we will see, the computation is greatly simplified by the fact that, in the large N limit, only a very particular set of diagrams survives (the so-called "melonic" diagrams). This simplification is the key ingredient that makes the SYK model solvable. From the four-point function we will be able to compute the Lyapunov exponent and we will see that SYK is maximally chaotic. We will also make some comments about the main drawback of the SYK model (and of its flavored generalization): they are defined in terms of couplings which are *random* gaussian variables. This property makes problematic the interpretation of the SYK a standard, quantum mechanical system.

For this reason, in the second part of the talk, we will discuss tensor models. They are a natural generalization of matrix models, and it has been known since a few years that they have a large-N limit dominated by melonic diagrams. Therefore, quantum mechanical models of fermions with tensor indices are natural candidates for models with the same interesting features of the SYK model, without the drawbacks of quenched disorder. In the second half of the lecture we will review the basics of tensor models and their large-N limit, we will discuss the construction of tensorial analogues of the SYK model and some key differences between the two, and we will present some of their most recent developments.

Recommended readings

- For the first part of the talk, the most complete reference is [1]. One could also take a look at the analogous paper by Polchinski and Rosenhaus [2].
- The flavored generalization of SYK has been introduced in [3].
- The fact that the main results of SYK can be obtained by a model of tensors, without disorder, was pointed out by Witten in [4]. Simpler tensor models with the same properties were introduced and studied by Klebanov and Tarnopolsky in [5].
- The models introduced by Witten and by Klebanov and Tarnopolsky are one-dimensional generalizations of the the zero-dimensional tensor models studied in [6] and [7, 8], respectively. These papers provide more details on the combinatorial aspects of such models.
- Some recent developments on tensor models that will be discussed during the lectures were presented in [9] and [10, 11].

References

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