

Strings and Number Theory

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Abstract

In this session we review three topics that lie at the intersection between string theory and number theory.

(1) **Moonshine**

The name moonshine refers to a series of mysterious coincidences relating modular forms and finite groups. The classical example, Monstrous moonshine, led to the discovery of new unexpected connections between string theory, conformal field theory, algebra and geometry. After a brief description of Monstrous moonshine, we discuss the more recent and still unexplained examples of Mathieu and umbral moonshine, and their possible relations with string theory on K3 surfaces.

(2) **Mock modular forms and Black holes**

We review the connection between mock Jacobi forms and generating functions for immortal 1/4-BPS dyons in $\mathcal{N} = 4$ string models, as described by Dabholkar-Murthy-Zagier.

(3) **3-Manifold invariants and Quantum modular forms**

We introduce the concept of quantum modular form in relation to Chern-Simons theory, as first analyzed by Lawrence and Zagier. This is the starting point to investigate the appearance of quantum, false and mock modular forms in the context of 3-manifolds.

References:

- (1) Eguchi, Ooguri, Tachikawa, 1004.0956.
Cheng, Duncan, Harvey, 1204.2779, 1307.5793.
Kachru 1605.00697 (review).
- (2) Dabholkar, Murthy, Zagier, 1208.4074.
- (3) Zagier, Quantum Modular Forms, Clay Mathematics Proceedings (2010).
Lawrence and Zagier, Modular forms and quantum invariants of 3-manifolds, Asian J. Math. (1999).