$|V_{us}|$ from kaon decays with the KLOE detector

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1 Introduction

While much emphasis is placed on the search for new physics, we still lack precise information on the validity of certain aspects of the Standard Model itself. In the Standard Model, the coupling of the W boson to the weak charged current is

$$\frac{g}{\sqrt{2}}W^+_\alpha(\overline{\mathbf{U}}_L\,\mathbf{V}_{\rm CKM}\gamma^\alpha\,\mathbf{D}_L+\overline{e}_L\gamma^\alpha\nu_{e\,L}+\overline{\mu}_L\gamma^\alpha\nu_{\mu\,L}+\overline{\tau}_L\gamma^\alpha\nu_{\tau\,L})\ +\ \text{h.c.},\tag{1}$$

where $\mathbf{U}^{\rm T}=(u,c,t)$, $\mathbf{D}^{\rm T}=(d,s,b)$ and L is for lefthanded. In the coupling above there is only one coupling constant for leptons and quarks. Quarks are mixed by the Cabibbo-Kobayashi-Maskawa matrix, $\mathbf{V}_{\rm CKM}$, which must be unitary. In low energy processes the Fermi coupling constant G_F is related to the gauge coupling g by $G_F=g^2/(4\sqrt{2}\,M_W^2)$. In the early sixties only two elements of $\mathbf{V}_{\rm CKM}$ were known. From nuclear β decay it was known that $|V_{ud}|{\sim}0.98$ and from strangeness changing decays, $|V_{us}|{\sim}0.26$, [1].

Precise measurements of leptonic and semileptonic kaon decay rates provide information about lepton universality. Combined with results from nuclear β decay and pion decays, such measurements also provide information about the unitarity of the mixing matrix. Ultimately they tell us whether quarks and leptons do indeed carry the same weak charge. The partial rates $\Gamma(K\to\pi e\nu)$ and $\Gamma(K\to\pi\mu\nu)$ provide measurements of $g^4|V_{us}|^2$, which, combined with $g^4|V_{ud}|^2$ from nuclear β decay and the muon decay rate, test the unitarity condition $|V_{ud}|^2+|V_{us}|^2+V_{ub}^2=1\simeq|V_{ud}|^2+|V_{us}|^2$. The ratio $\Gamma(K\to\mu\nu)/\Gamma(\pi\to\mu\nu)$ provides an independent measurement of $|V_{us}|^2/|V_{ud}|^2$.

1.1 Leptonic and Semileptonic kaon decays

The semileptonic decay rates, fully inclusive of radiation, are given by

$$\Gamma(K_{\ell3(\gamma)})=\frac{C_K^2G_F^2M_K^5}{192\pi^3}\,S_{\rm EW}\,|V_{us}|^2\,|f_+(0)|^2\,I_{K\ell}\,\left(1+\delta_K^{\rm SU(2)}+\delta_{K\ell}^{\rm EM}\right)^2.\tag{2}$$

In the above expression, the index K denotes $K^0\to\pi^\pm$ and $K^\pm\to\pi^0$ transitions, for which $C_K^2=1$ and $1/2$, respectively. M_K is the appropriate kaon mass, $S_{\rm EW}$

is the universal short-distance electroweak correction [2] and $\ell = e,\ \mu$. Following a common convention, $f_+(0) \equiv f_+^{K^0\pi^-}(0)$. The mode dependence is contained in the δ terms: the long-distance electromagnetic (EM) corrections, which depend on the meson charges and lepton masses and the SU(2)-breaking corrections, which depend on the kaon species [3]. $I_{K\ell}$ is the integral of the dimensionless Dalitz-plot density.

If the form factors are expanded in powers of t up to t^2 as

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0}\ \frac{t}{m^2_{\pi^+}} + \frac{1}{2}\ \lambda''_{+,0}\ \left(\frac{t}{m^2_{\pi^+}}\right)^2, \tag{3}$$

four parameters (λ'_+, λ''_+, λ'_0 and λ''_0) need to be determined from the decay pion spectrum in order to be able to compute the phase-space integral.

High-precision lattice quantum chromodynamics (QCD) results have recently become available and are rapidly improving [4]. The availability of precise values for the pion- and kaon-decay constants f_π and f_K allows use of a relation between $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$ and $|V_{us}|^2/|V_{ud}|^2$, with the advantage that lattice-scale uncertainties and radiative corrections largely cancel out in the ratio [5]:

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2}\ \frac{f_K^2}{f_\pi^2}\ \frac{m_K\left(1 - m_\mu^2/m_K^2\right)^2}{m_\pi\left(1 - m_\mu^2/m_\pi^2\right)^2} \times (0.9930 \pm 0.0035), \tag{4}$$

where the uncertainty in the numerical factor is dominantly from structure-dependent radiative corrections and may be improved. This ratio can be combined with direct measurements of $|V_{ud}|$ to obtain $|V_{us}|$.

2 KLOE's role

The KLOE detector is operated at DAΦNE, the Frascati ϕ factory. DAΦNE is an e^+e^- collider running at a center of mass energy $W = m_\phi \sim 1019.45$ MeV. ϕ mesons are produced with a cross section of $\sim$3 μb and decay mostly to charged kaon pairs (49%) and neutral kaon pairs (34%).

The neutral kaon pair from $\phi \to K^0\bar{K}^0$ is in a pure $J^{PC} = 1^{--}$ state. Detection of a K_S thus signals the presence of, "tags", a K_L and vice versa. Thus at DAΦNE we have pure K_S and K_L beams of precisely known momenta (event by event) and flux, which can be used to measure absolute K_S and K_L branching ratios.

2.1 K_L decays

We search for K_L decays using a beam tagged by detection of $K_S \to \pi^+\pi^-$ decays. The $\pi^+\pi^-$ decays observed near the origin count the number of K_L mesons, providing the direction and momentum of each. We have used this technique to measure the BRs for the four main K_L decay modes, as well as the K_L lifetime [6].

The errors on the KLOE BR values are dominated by the uncertainty on the K_L lifetime τ_L; since the dependence of the geometrical efficiency on τ_L is known, KLOE can solve for τ_L by imposing $\sum_x \mathrm{BR} K_L \to x = 1$ (using previous averages for the minor BRs), thereby greatly reducing the uncertainties on the BR values obtained. Our fit makes use of the KLOE BR values before application of this constraint: BR(K_{e3}) = 0.4049(21), BR($K_{\mu 3}$) = 0.2726(16), BR(K_{e3}) = 0.2018(24), and BR(K_{e3}) = 0.1276(15). The dependence of these values on τ_L and the correlations between the errors are taken into account. KLOE has also measured τ_L directly, by fitting the proper decay time distribution for $K_L \to 3\pi^0$ events, for which the reconstruction efficiency is high and uniform over a fiducial volume of $\sim 0.4\lambda_L$. We obtain τ_L = 50.92(30) ns [6].

2.2 K_S decays

We have measured the ratios BR($K_S \to \pi e \nu$)/BR($K_S \to \pi^+\pi^-$) separately for each lepton charge, using $\phi \to K_L K_S$ decays in which the K_L is recognized by its interaction in the calorimeter barrel. Semileptonic K_S decays are identified by time of flight (TOF) of both pion and electron. Our most recent analysis [6] gives about 13,600 signal events.

In a separate analysis, we have used K_S decays tagged by the K_L interaction in the EMC barrel to measure $\mathrm{BR} K_S \to \pi^+\pi^- / \mathrm{BR} K_S \to \pi^0\pi^0 = 2.2549 \pm 0.0054$ [6]. Together, these measurements completely determine the main K_S BRs and give: BR($K_S \to \pi e \nu$)=$(7.046 \pm 0.091) \times 10^{-4}$ [6].

In our evaluation of $|V_{us}|$, we use the KLOE value for BR($K_S \to \pi e \nu$) together with the lifetime value $\tau_S = 0.08958 \pm 0.00005$ ns from the PDG [7].

2.3 $K^{\pm}$ decays

At KLOE, $\phi \to K^+K^-$ events are identified by detecting the abundant two-body decay (BR($K^\pm \to \pi^\pm\pi^0 + K^\pm \to \mu^\pm\nu$)$\sim$84%) of one of the kaons. As in the analysis of neutral kaon decays, this provides tagging of the kaon of opposite charge. As noted above, the decay $K \to \mu\nu$ is of interest in its own right for the determination of $|V_{us}|$.

We measure BR($K^+ \to \mu^+\nu$) using $K^- \to \mu^-\overline{\nu}$ decays as tags [6]. In $\sim$34% of some four million tagged events, we find $\sim$865,000 signal events giving BR($K^+ \to \mu^+\nu(\gamma)$) $= 0.6366 \pm 0.0009 \pm 0.0015$.

To measure BR($K^\pm_{e3}$) and BR($K^\pm_{\mu 3}$), we use both $K \to \mu\nu$ and $K \to \pi\pi^0$ decays as tags.

In all, we find about 300,000 K_{e3} and 160,000 $K_{\mu 3}$ events. We obtain $BR(K_{e3}) = (4.965 \pm 0.038 \pm 0.037)\%$ and $BR(K_{\mu 3}) = (3.233 \pm 0.029 \pm 0.026)\%$, with a correlation of 62.7% [6].

At KLOE, two methods are used to measure charged kaon lifetime. The first is to obtain the decay time from the kaon path length in the DC, accounting for the continuous change in the kaon velocity due to ionization energy losses. A fit to the proper-time distribution in the interval from 15–35 ns ($1.6\lambda_\pm$) gives the result $\tau_\pm = 12.364 \pm 0.031 \pm 0.031$ ns. Alternately, the decay time can be obtained from the precise measurement of the arrival times of the photons from $K^+ \to \pi^+\pi^0$ decays. In this case, a fit to the proper-time distribution in the interval from 13–42 ns ($2.3\lambda_\pm$) gives the result $\tau_\pm = 12.337 \pm 0.030 \pm 0.020$ ns. Taking into account the statistical correlation between these two measurements ($\rho = 0.307$), we obtain the average value $\tau_\pm = 12.347 \pm 0.030$ ns, see [6].

2.4 Form factor parameters

K_{e3} form factor(FF) parameters have also been measured [6]. We obtain the vector form factor parameters from binned log-likelihood fits to the t distribution. Using the quadratic parametrization of eq. 3, we obtain $\lambda'_+ = (25.5 \pm 1.5 \pm 1.0) \times 10^{-3}$ and $\lambda''_+ = (1.4 \pm 0.7 \pm 0.4) \times 10^{-3}$, where the total errors are correlated with $\rho = -0.95$.

The measurement of the vector and scalar FF parameters using $K_L \to \pi\mu\nu$ decays is reported in Ref. [6]. The FF parameters have been obtained from fits to the distribution of the neutrino energy E_ν after integration over the pion energy.

The result of this fit when combined with those from our K_{e3} analysis is:

$$\begin{aligned} \lambda'_+ &= (25.6 \pm 1.7) \times 10^{-3} \\ \lambda''_+ &= (1.5 \pm 0.8) \times 10^{-3} \\ \lambda'_0 &= (15.4 \pm 2.2) \times 10^{-3} \end{aligned} \qquad \begin{pmatrix} 1 & -0.95 & 0.29 \\ & 1 & -0.38 \\ & & 1 \end{pmatrix} \tag{5}$$

The values of the phase-space integrals for $K_{\ell 3}$ decays are listed in table 1.

Parameters	$I(K^0_{e3})$	$I(K^0_{\mu 3})$	$I(K^+_{e3})$	$I(K^+_{\mu 3})$
λ'_+, λ''_+, λ'_0	0.15483(40)	0.10271(52)	0.15919(41)	0.10568(54)

Table 1: Phase-space integrals for $K_{\ell 3}$ decays.

3 Test of CKM unitarity

Using all of the experimental and theoretical inputs discussed above and the SU(2)-breaking correction from Ref. [8], the values of $|f_+(0)\,V_{us}|$ have been evaluated for the K_{Le3}, $K_{L\mu 3}$, K_{Se3}, $K^\pm_{e3}$, and $K^\pm_{\mu 3}$ decay modes,as shown in table 2. Statistical and systematic uncertainties are added in quadrature everywhere.

Channel	$\lvert f_+(0)\,V_{us}\rvert$	Correlation coefficients				
K_{Le3}	0.2155(7)	1				
$K_{L\mu3}$	0.2167(9)	0.28	1			
K_{Se3}	0.2153(14)	0.16	0.08	1		
$K^{\pm}_{e3}$	0.2152(13)	0.07	0.01	0.04	1	
$K^{\pm}_{\mu3}$	0.2132(15)	0.01	0.18	0.01	0.67	1

Table 2: KLOE results for $\lvert f_+(0)\,V_{us}\rvert$.

with $\chi^2/\mathrm{ndf} = 7.0/4$ (CL=13%). It is worth noting that the only external experimental input to this analysis is the K_S lifetime. All other experimental inputs are KLOE results.

Lattice evaluations of $f_+(0)$ are rapidly improving in precision. The RBC and UKQCD Collaborations have recently obtained $f_+(0) = 0.9644 \pm 0.0049$ from a lattice calculation with $2 + 1$ flavors of dynamical domain-wall fermions [11]. Using their value for $f_+(0)$, our $K_{\ell3}$ results give $\lvert V_{us}\rvert = 0.2237 \pm 0.0013$. A recent evaluation of $\lvert V_{ud}\rvert$ from $0^+ \to 0^+$ nuclear beta decays [10], gives $\lvert V_{ud}\rvert = 0.97418 \pm 0.00026$ which, combined with our result above, gives $\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 - 1 = -0.0009 \pm 0.0008$, a result compatible with unitarity, which is verified to $\sim$0.1%. figure 1 shows a compendium of all the KLOE results.

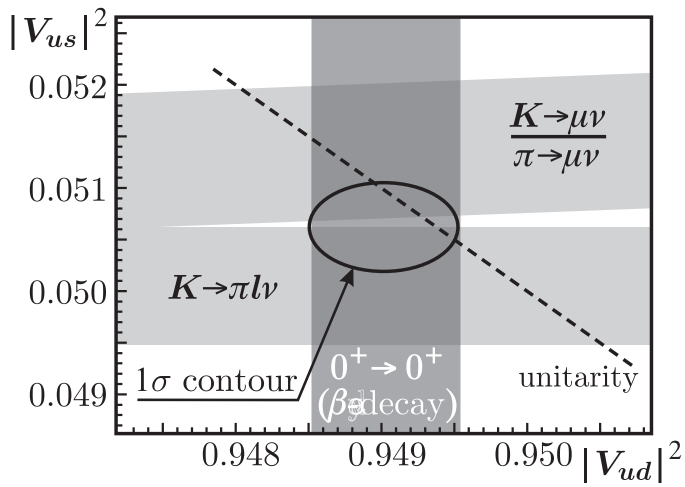

Figure 1: KLOE results for $\lvert V_{us}\rvert^2$, $\lvert V_{us}/V_{ud}\rvert^2$ and $\lvert V_{ud}\rvert^2$ from β-decay measurements, shown as 1σ wide grey bands. The ellipse is the 1 σ contour from the fit. The unitarity constraint is illustrated by the dashed line.

Additional information is provided by the determination of the ratio $\lvert V_{us}/V_{ud}\rvert$, following the approach of eq. 4. From our measurements of BR($K_{\mu 2}$) and $\tau_\pm$ and using $\Gamma(\pi_{\mu 2})$ from ref. [7], we find $|V_{us}/V_{ud} \times f_K/f_\pi|^2 = 0.07650(33)$. Using the recent lattice determination of f_K/f_π from the HPQCD/UKQCD collaboration, f_K/f_π=1.189 ± 0.007 [9], we obtain $|V_{us}/V_{ud}|^2$=0.0541 ± 0.0007. The best estimate of $|V_{us}|^2$ and $|V_{ud}|^2$ can be obtained from a fit to the above ratio and our result $|V_{us}|^2$=0.05002 ± 0.00057 together with the result $|V_{ud}|^2 = 0.9490 \pm 0.0005$ from super-allowed β-decays. The fit gives $|V_{us}|^2 = 0.0506 \pm 0.0004$ and $|V_{ud}|^2 = 0.9490 \pm 0.0005$ with a correlation of 3%. The fit CL is 13% (χ^2/ndf = 2.34/1). The values obtained confirm the unitarity of the CKM quark mixing matrix as applied to the first row. We find

$$1 - |V_{us}|^2 - |V_{ud}|^2 = 0.0004 \pm 0.0007 \quad (\sim 0.6\sigma)$$

i.e. the unitarity condition is verified to $\mathcal{O}(0.1\%)$, see figure 1. In a more conventional form, the results of the fit are:

$$\begin{aligned} |V_{us}| &= 0.2249 \pm 0.0010 \\ |V_{ud}| &= 0.97417 \pm 0.00026 \end{aligned} \tag{6}$$

Imposing unitarity as a constraint, $|V_{us}|^2 + |V_{ud}|^2 = 1$, on the values above and performing a constrained fit we find $|V_{us}| = 0.2253 \pm 0.0007$ with χ^2/dof=0.46/1 corresponding to a CL of 50%.

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