On the "unreasonable" effectiveness of Homogeneous Transport of Intensity imaging

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CSIRO Data61

5 September 2017

What is the main question considered in this talk?

- Common knowledge tells us that any low-pass filtering can increase signal-to-noise ratio (SNR) or contrast-to-noise ratio (CNR) in an image, but it (almost) inevitably comes together with image blurring (unless some *a priori* information is used in the process, either explicitly or implicitly).
- The trade-off between SNR/CNR and spatial resolution has been recently expressed mathematically in the form of the noise-resolution uncertainty (NRU) principle, which is inherent to optics, scattering and even QED.
- In spite of this, the reconstructive imaging with Homogeneous Transport of Intensity equation is capable of "magically" increasing SNR (or CNR) while preserving the spatial resolution. This capability has been conclusively demonstrated both theoretically and experimentally.

Question: what is the solution of this apparent paradox?

Answer: will be given in this talk!

Denoising: better SNR => less sharpness

Image with 20% Poisson noise
SNR=5
n_{phot} =25 pp

5-pixel wide Gaussian filter
SNR=40
n_{phot} =25 pp

9-pixel wide median filter
SNR=32
n_{phot} =25 pp

9-pixel wide average filter
SNR=43
n_{phot} =25 pp

Increase in SNR comes at the expense of image sharpness

Sharpening: better sharpness => lower SNR

10-pixel blur
1% noise
SNR=60
ΔX=10 pix

Wiener deconv.
reg.=0.52 (optim.)
SNR=47
ΔX=7 pix
(n_{phot} unchanged)

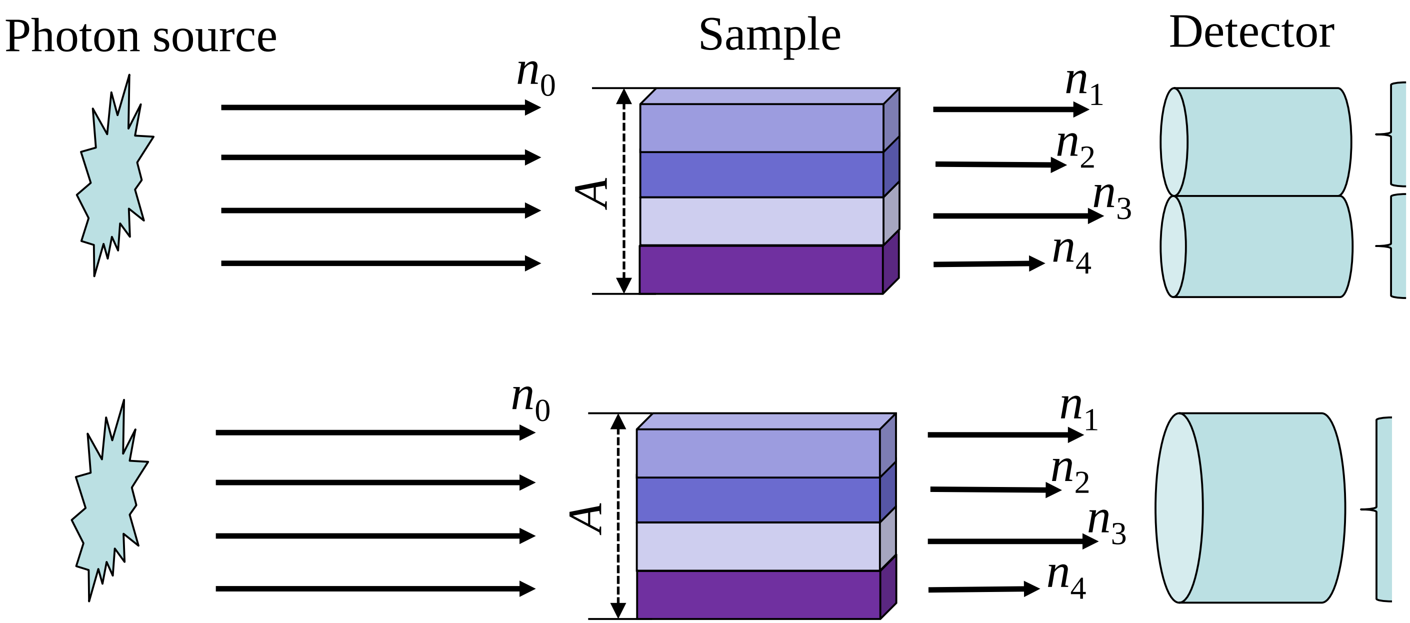
Richardson-Lucy
iter.=214
SNR=54
ΔX=8 pix
(n_{phot} unchanged)

Iterative Wiener
reg.=1, iter.=2
SNR=47
ΔX=7 pix
(n_{phot} unchanged)

Increase in image sharpness always comes at the expense of SNR (increased noise)

Duality between signal-to-noise and spatial resolution under the fixed dose condition

Near field imaging (geometrical optics)



$$SNR^2 = n_1 + n_2$$
$$\Delta x = A/2$$
$$SNR^2 = n_3 + n_4$$
$$\Delta x = A/2$$

$$SNR'^2 = n_1 + n_2 + n_3 + n_4$$
$$\Delta x' = A$$

$$\boxed{\frac{<SNR^2>}{\Delta x} = \frac{[(n_1+n_2)+(n_3+n_4)]/2}{(A/2)} = \frac{<SNR'^2>}{\Delta x'} = \frac{N_{phot}}{A} = \overline{S}}$$

$$\boxed{\frac{<SNR^2>}{\Delta x\,\overline{S}} = 1}$$

The ratio of SNR^2 to spatial resolution depends only on the photon fluence (if the photon statistics is fixed).

General principle: noise - resolution duality

If the total number of quanta (e.g. photons) forming an image is fixed, then:

1) increase in SNR comes at the expense of image sharpness (i.e. blurring increases = number of resolvable units decreases = spatial resolution deteriorates)

2) increase in image sharpness (increase in the number of resolvable units = improvement in the spatial resolution) comes at the expense of SNR

Can this "noise-resolution" trade-off in imaging be formulated quantitatively in a general form?

Intrinsic quality characteristic of imaging systems

We have introduced a dimensionless "intrinsic imaging quality characteristic" Q_S which incorporates both the average signal-to-noise ratio and the spatial resolution of an imaging system:

$$Q_S^2 = \frac{SNR^2(\mathbf{x})}{\bar{S}_{in}(\mathbf{x})(\Delta x)^d} = <\ldots \times \frac{A^d}{A^d}> = \frac{M_{pix} < SNR^2 >}{N_{phot}}$$

where

- $N_{phot} = <\bar{S}_{in}> A^d$ is the total number of incident photons or other imaging quanta
- $M_{pix} = (A / \Delta x)^d$ is the effective number of pixels (spatial resolution units)
- d is the dimensionality of the data (d=2 corresponds to planar images)
- A is the linear size of the image (illuminated aperture)

Compare this with the information capacity per imaging particle (a quantity introduced following the approach used by Shannon, Gabor and others):

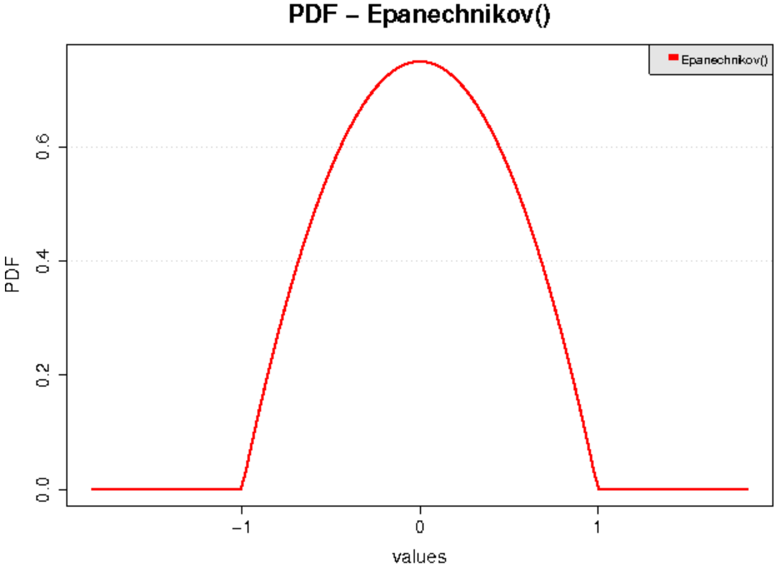
$$C_S \cong 0.5(M_{pix} / N_{phot}) \log_2 < SNR^2 >$$

Noise – resolution uncertainty principle

Mathematical result proved:

$$\boxed{Q_S[T] \le C_d^{-1/2}}$$

Unlike the case of the Heisenberg uncertainty principle, where the minimal phase-space volume is achieved for Gaussian distributions, the minimum of the new uncertainty functional is achieved for the Epanechnikov kernel (which is well known in statistics): $T_{Ep}(\mathbf{x}) = (1 - |\mathbf{x}|^2)_+$



The absolute constants C_d are smaller than one, which means, that $Q_S^{(d)}$ can be slightly larger than 1, for example for *d*=1,2,3 we have:

$$C_1^{-1/2} \cong 1.025 \qquad C_2^{-1/2} \cong 1.061 \qquad C_3^{-1/2} \cong 1.104$$

$$Q_S[T_{Gaussian}] = 1 < Q_S[T_{Ep}] = C_d^{-1/2}$$

Intrinsic spatial resolution

$$\Delta x = \left(\frac{4\pi}{d} \frac{\int |\mathbf{x}|^2 \, P(\mathbf{x}) d\mathbf{x}}{\int P(\mathbf{x}) d\mathbf{x}} \right)^{1/2}$$

- conventional measure of the width of a function; normalization ($4\pi / d$) is consistent with the width of a rectangular function

d =1,2,3… is the dimensionality of the image

$$\boxed{\Delta_2 x = \left(\frac{(\int P(\mathbf{x}) d\mathbf{x})^2}{\int P^2(\mathbf{x}) d\mathbf{x}} \right)^{1/d}}$$

- has the meaning of spatial resolution (dimensionality of length) as a measure of the width of the PSF $P(\mathbf{x})$ (which can depend on the source, the detector, the geometrical parameters, …)

Compare this with the definition of the spectral width (Mandel-Wolf):

$$\Delta_2 \nu = \frac{(\int s(\nu) d\nu)^2}{\int s^2(\nu) d\nu}$$

and the corresponding definition of coherence time: $T_c = 1 / \Delta \nu$

Two definitions of spatial resolution

PSF	$P(\mathbf{x};\sigma)$	$\hat{P}(\mathbf{u};\sigma)$	$(\Delta x)^2$	$(\Delta_2 x)^2$	$Q_S^{4/d}$
d-dim Gauss	$\frac{1}{(2\pi)^{d/2}\sigma^d} e^{-\frac{\lvert\mathbf{x}\rvert^2}{2\sigma^2}}$	$e^{-2\pi^2\sigma^2\lvert\mathbf{u}\rvert^2}$	$4\pi\sigma^2$	$4\pi\sigma^2$	1
d-dim Rect	$\frac{1}{(2\sigma)^d}\chi_{[-\sigma,\sigma]^d}(\mathbf{x})$	$\prod_{k=1}^{d}\frac{\sin(2\pi\sigma u_k)}{2\pi\sigma u_k}$	$\frac{4\pi}{3}\sigma^2$	$4\sigma^2$	$\frac{3}{\pi}$
Exp	$(\pi/\sigma)e^{-2\pi\lvert x\rvert/\sigma}$	$\frac{1}{1+\sigma^2 u^2}$	$\frac{2}{\pi}\sigma^2$	$\frac{4}{\pi^2}\sigma^2$	$\frac{2}{\pi}$
Lorentz	$\frac{\pi^{-1}\sigma}{\sigma^2+x^2}$	$e^{-2\pi\sigma\lvert u\rvert}$	∞	$4\pi^2\sigma^2$	0
Sech	$\frac{\sigma^{-1}}{e^{\pi x/2\sigma}+e^{-\pi x/2\sigma}}$	$\frac{2}{e^{2\pi\sigma u}+e^{-2\pi\sigma u}}$	$4\pi\sigma^2$	$\pi^2\sigma^2$	$\frac{\pi}{4}$
Epanechnikov [a)]	$\frac{B_d}{\pi^{d/2}\sigma^d}\left(1-\frac{\lvert\mathbf{x}\rvert^2}{\sigma^2}\right)_+$	$B_d\frac{J_{d/2+1}(2\pi\sigma\lvert\mathbf{u}\rvert)}{(\pi\sigma\lvert\mathbf{u}\rvert)^{d/2+1}}$	$\frac{4\pi}{d+4}\sigma^2$	$\frac{4\pi\sigma^2}{(d+4)C_d^{2/d}}$	$\frac{1}{C_d^{2/d}}$

[a)] $B_d=(d/2+1)\Gamma(d/2+1)$ $\quad C_d=2^d\,\Gamma(d/2)d(d+2)/(d+4)^{d/2+1}<1$

Noise / resolution invariance

$$S(\mathbf{x}) = \int P(\mathbf{x}-\mathbf{y}) S_{in}(\mathbf{y}) d\mathbf{y}$$

- action of an LSI system on the input fluence

$$SNR^2(\mathbf{x}) = \frac{\bar{S}^2(\mathbf{x})}{\sigma^2(\mathbf{x})} = SNR_{in}^2(\mathbf{x}) \frac{(\Delta_2 x)^d}{(\Delta_2 x)_{in}^d}$$

- effect of LSI system on the input SNR^2

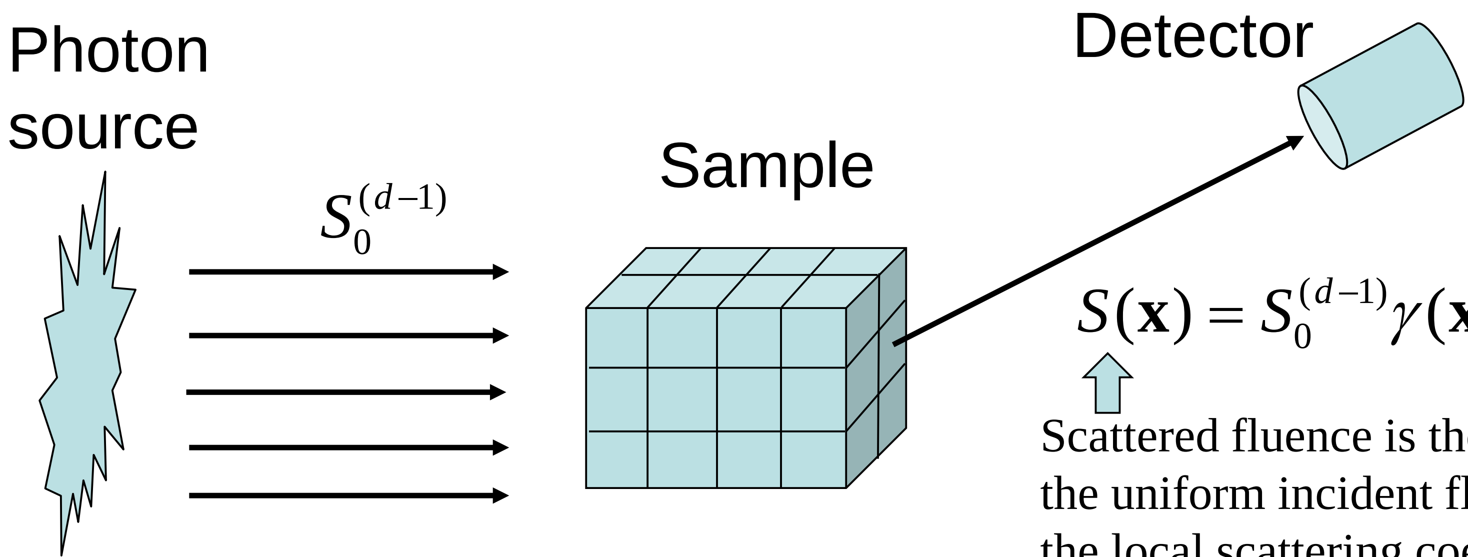
$$\boxed{\frac{SNR^2(\mathbf{x})}{(\Delta_2 x)^d} = \frac{SNR_{in}^2(\mathbf{x})}{(\Delta_2 x)_{in}^d}}$$

- the ratio of SNR^2 and the spatial resolution volume is invariant with respect to the action of LSI transformations, if $P(\mathbf{x})$ is much narrower than $SNR^2(\mathbf{x})$, but much wider than the noise correlation, i.e. if $P(\mathbf{x})$ effectively filters the noise, but does not smear the input signal (best case scenario).

$$\boxed{M_{pix} SNR^2(\mathbf{x}) = M_{pix,in} SNR_{in}^2(\mathbf{x})} \quad M_{pix} = A^d / (\Delta_2 x)^d$$

- the product of SNR^2 and the number of resolution units M_{pix} is a (dimensionless!) invariant.

Application to idealized generic "black-box" scattering (imaging) experiment



$$S(\mathbf{x}) = S_0^{(d-1)}\gamma(\mathbf{x})$$

Scattered fluence is the product of the uniform incident fluence and the local scattering coefficient

Assumed:

- Uniform coherent illumination (incident photons are uncorrelated)
- Weak elastic single-scattering interaction, $\gamma(\mathbf{x}) = \sigma\rho(\mathbf{x})$.
- Ideal position-sensitive dark-field detection (only scattered photons enter the detector)

The image noise and the spatial resolution are determined primarily by the number and correlation length of detected photons scattered by each elementary "voxel" of the sample!

Information capacity of a scattering experiment (source + sample + detector)

$$\left\langle SNR^2 \right\rangle = \bar{S}(x)(\Delta_2 x)^d = \underbrace{\bar{\gamma} A}_{\Omega} \underbrace{\bar{S}_0^{(d-1)} A^{d-1}}_{N_{phot}} \frac{\overbrace{(\Delta_2 x)^d}^{1/M_{vox}}}{A^d} = \frac{\Omega N_{phot}}{M_{vox}}$$

Registered SNR^2 is proportional to the scattering power $\Omega = \gamma A$ of the sample (dark field) or its square (bright field)

$$\boxed{D_P(M_{vox}, N_{phot}) = 0.5 M_{vox}[\log_2(\Omega N_{phot} / M_{vox}) + o(1)]}$$

information capacity

detector

sample

source

The information capacity of the considered scattering setup is equal to the number of spatial resolution units M_{vox} times the $\log_2$ of the number of scattered photons ΩN_{phot} per unit. In other words, the total number of distinguishable samples is approximately $(\Omega N_{phot} / M_{vox})^{M_{vox}/2}$. This number is maximized, when M_{vox} is large, i.e. for low SNR.

$$< SNR_{CT}^2 > \cong \frac{2\Omega^2 N_{phot}}{M_{vox}^{4/3}}$$

Unlike the simple generic scheme considered here, the registered SNR^2 in CT is proportional to the inverse 4th(!) power of the spatial resolution (noise increases stronger because of the singularity of the inverse Radon transform).

Relationship to Shannon's information capacity

- C. Shannon (1949): Maximum number of bits, N_{bits}, that can be transmitted within a time interval A_t over a communication channel with bandwidth $1/\Delta T$ is limited by

$$N_{bits} \le \frac{A_t}{\Delta T} \log SNR$$

- Our result for 1D case (noise – resolution uncertainty principle) is complementary to the above one:

$$\frac{A_x}{\Delta X} SNR^2 \le C_1^{-1} \; N_{photons}$$

- Combining these results, we can obtain in particular that the information capacity of a communication channel or an imaging system is ultimately limited by the number of signal quanta used:

$$\boxed{N_{bits} \le M_{res.units} \log SNR \le M_{res.units} SNR^2 \le C_d^{-1} N_{photons}}$$

- This relationship provides a link between the information capacity, spatial resolution and SNR (incl. radiation dose) in imaging systems. Note, however, that in principle the right-hand side contains also a quantity $O(M_{res.units})$!

Intermediate conclusions

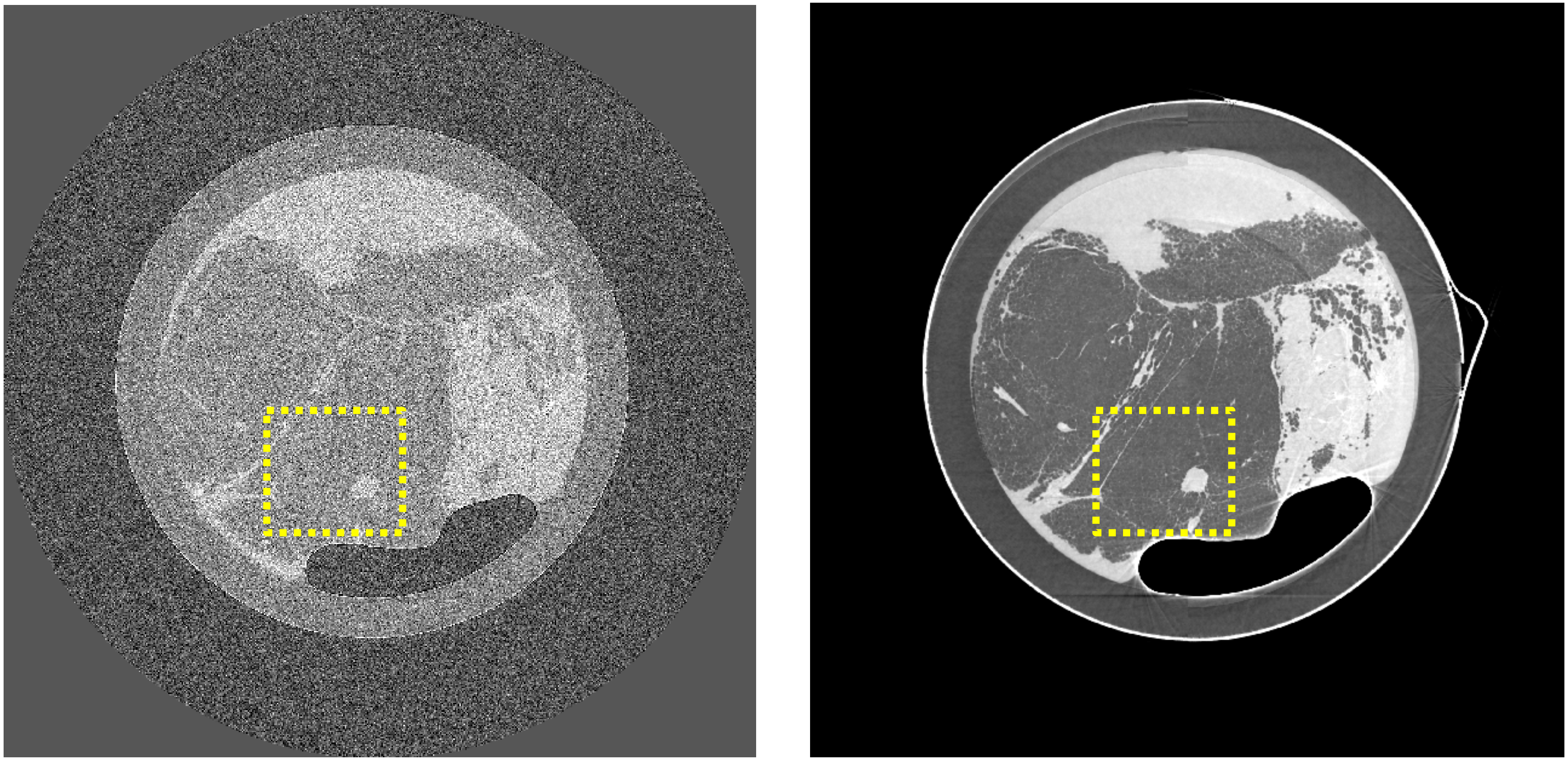
- A duality exists between noise and spatial resolution in LSI imaging systems.

- This duality ("noise-resolution uncertainty") applies also to (some) deconvolution operations and system transformations described by PDEs with constant coefficients. The ratio of SNR and spatial resolution $(\Delta_2 x)^{d/2}$ is invariant with respect to such transformations.

- Noise-resolution duality is closely related to (Shannon's) information capacity of imaging and communication systems.

- Noise-resolution uncertainty also exists in quantum electrodynamics, where the ultimate lower limit of the product of the spatial width of a mode of an electromagnetic field and the variance of its intensity is determined by vacuum fluctuations.

However …



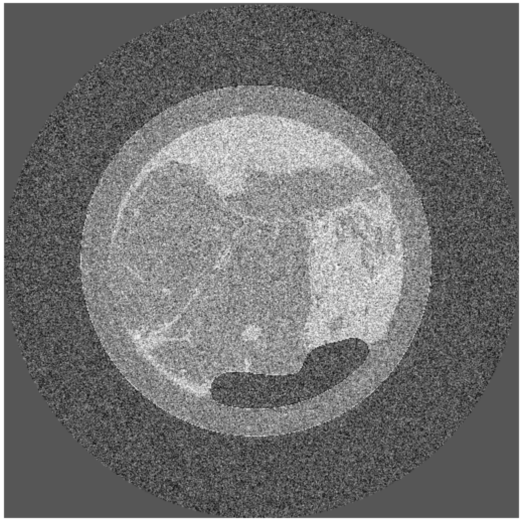
Same area of an axial CT slice of an excised breast tissue sample obtained using conventional CT (left) and in-line phase-contrast CT with TIE-Hom phase retrieval (right) at the same X-ray dose. Factor of x20 improvement in SNR demonstrated, equivalent to x400 reduction in X-ray dose at the same SNR and spatial resolution as in the equivalent absorption image.

How can we explain the "unreasonable" effectiveness of the TIE-Hom imaging

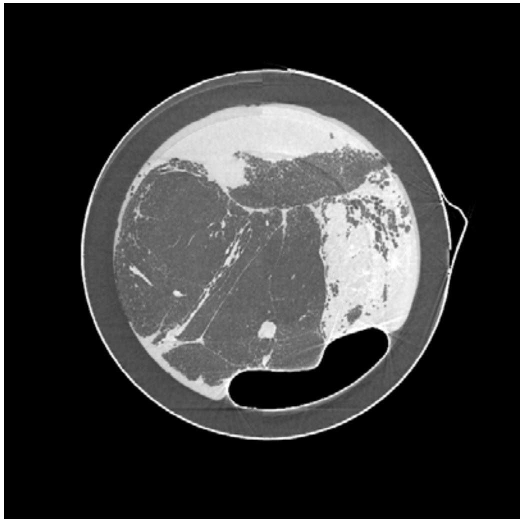
The problem:

1) On one hand, the ratio of SNR to spatial resolution cannot change in any intensity-linear transformation (including TIE-Hom propagation, and TIE-Hom phase retrieval).

2) On the other hand, it has been repeatedly demonstrated in experiments, that a combination of near-field free-space propagation between the object and the detector, with TIE-Hom phase retrieval is capable of increasing SNR by a factor of up to x100s, while not worsening (or only moderately worsening) the spatial resolution.



"Contact" image



Propagated image with TIE-Hom phase retrieval

Variance of exposure-averaged intensity ("self" noise)
1D case expressions considered for simplicity

$$\boxed{V_T(x,R) \equiv < I_T^2(x,R) > - < I_T(x,R) >^2}$$

- definition of (noise) variance

exposure time average   ensemble average

$$< I_T(x,R) > = T^{-1}\int_{-T/2}^{T/2} < |U(x,R;t)|^2 > dt = I(x,R)$$

- the photon flux is assumed to be stationary

$$< I_T^2(x,R) > = < \left(T^{-1}\int_{-T/2}^{T/2} |U(x,R;t)|^2 \, dt \right)^2 > \neq I^2(x,R)$$

$$V_T^{Poisson}(x,R) = < I_T^2(x,R) > - < I_T(x,R) >^2 = I(x,R)$$

- Poisson statistics
(e.g. photo detection process)

$$V_T^{Gauss}(x,R) = < I_T^2(x,R) > - < I_T(x,R) >^2 = I^2(x,R)$$

- Gaussian statistics
(e.g. "intrinsic" or "self" variance of light emitted by thermal light sources)

Variance of detected(!) time-averaged intensity

Variance of detected(!) time-averaged intensity in a plane $z = R$:

$$\boxed{V_{P,T}(x,R) = V_{P,T}^{self}(x,R) + V_{P,T}^{det}(x,R) \cong T^{-1}\int S^2(x,R;\nu)d\nu + (\Delta_2 x)^{-1}\int \eta^{-1}(\nu)S(x,R;\nu)d\nu}$$

$\Delta_2 x = 1/\int P^2(x)dx$ - width of the PSF (~1/2 of the correlation length)

$\eta(\nu)$ - detector (quantum) efficiency constant (photons/Joules)

$S(x,R;\nu) = I(x;R)\,s(\nu)$ - cross-spectrally pure case (for simplicity)

$\int s(\nu)d\nu = 1, \quad \int s^2(\nu)d\nu = T_c$ - coherence time (inverse of the spectral width)

$$\boxed{V_{P,T}(x,R) \cong \underbrace{(T_c/T)I^2(x,R)}_{\text{Gaussian self-noise term}} + \underbrace{T^{-1}\eta^{-1}(\Delta_2 x)^{-1}I(x,R)}_{\text{Poissonian (uncorrelated) shot-noise term}}}$$

The source (self) noise term is typically much (much) smaller than the detector (shot) noise term

$$V_{P,T}(x,R) \cong \underbrace{(T_c/T)I^2(x,R)}_{\text{Gaussian self-noise term}} + \underbrace{T^{-1}\eta^{-1}(\Delta_2 x)^{-1}I(x,R)}_{\text{Poissonian shot-noise term}}$$ - $J^2/m^2/s^2$

$$\boxed{T^2\eta^2(\Delta_2 x)^2 \times V_{P,T}(x,R) \cong (T_c/T)[\bar{n}(x)]^2 + \bar{n}(x)}$$ - dimensionless version

$\bar{n}(x) = \eta(\Delta_2 x)T\, I_R(x)$ is the average number of photons registered within the "pixel" with area $\Delta_2 x$ centred at point x, during the exposure time T.

Note that for X-rays with λ=1Å and monochromaticity $\Delta\lambda/\lambda=10^{-4}$, the coherence time $T_c \sim 1/\Delta\nu=\lambda^2/(c\Delta\lambda) \sim 0.3\times 10^{-15}$s! Therefore, if the exposure time is a few ms, then, unless there are 10^{12}(!) or more photons registered per pixel(!) per exposure(!), the self noise term is much smaller than the shot noise term. Therefore,

$$\boxed{V_{P,T}(x,R) \cong T^{-1}\eta^{-1}(\Delta_2 x)^{-1}I(x,R)}$$

Image noise is well approximated by the shot noise alone!

Image noise does not change upon forward propagation

$$V_{P,T}(x,R) \cong T^{-1}\eta^{-1}(\Delta_2 x)^{-1}\int \bar{S}(x,R;\nu)\,d\nu$$ - photodetection shot noise (assuming spatial ergodicity)

$$S(x,R;\nu) \cong [1-\sigma^2\nabla_x^2]S(x,0;\nu)$$ - homogeneous TIE (TIE-Hom)

$$\sigma^2 = \gamma R'\lambda/(4\pi)$$

$$\left|\frac{\sigma^2\nabla^2 S(x,0;\nu)}{S(x,0;\nu)}\right| = \left|1-\frac{S(x,R;\nu)}{S(x,0;\nu)}\right| << 1$$ - TIE validity condition

$$\bar{S}(x,R;\nu) \cong \bar{S}(x,0;\nu) \Longrightarrow \boxed{V_{P,T}(x,R) \cong V_{P,T}(x,0)} \quad SNR(x,R) = \frac{\bar{I}(x,R)}{V_{P,T}^{1/2}(x,R)} \cong \frac{\bar{I}(x,0)}{V_{P,T}^{1/2}(x,0)}$$

Variance of image noise and SNR do not change upon forward free-space propagation in the near-Fresnel region!

Image noise decreases and SNR increases upon TIE-Hom retrieval

$$S_{ret}(x,0;\nu) = \int P_\sigma(x-x')S(x',R;\nu)\,dx'$$ - TIE-Hom "phase" retrieval

$$P_\sigma(x) = \int \frac{\exp(-i2\pi x\xi)}{1+4\pi^2\sigma^2\xi^2}d\xi = \frac{1}{2\sigma}e^{-|x|/\sigma}$$ - 1D case (2D and 3D – qualitatively similar)

$$\boxed{V_{P,T}^{ret}(x,0) = V_{P_\sigma,P,T}(x,R) = \frac{(\Delta_2 x)}{4\sigma}V_{P,T}(x,R)}$$

$$\frac{\Delta_2 x}{4\sigma} = \left(\frac{\pi h^2}{4\gamma\lambda R'}\right)^{1/2} = \left(\frac{\pi}{4}\frac{N_F^{\min}}{\gamma}\right)^{1/2} \equiv G^{-1/2} << 1$$ - provided $N_F << \gamma$

$$SNR_{ret} \equiv \frac{\bar{I}_{ret}(x,0)}{(V_{P,T}^{ret})^{1/2}(x,0)} \cong G^{1/4}\frac{\bar{I}(x,0)}{V_{P,T}^{1/2}(x,0)} \equiv G^{1/4}SNR_0$$

Variance of image noise decreases and SNR increases upon TIE-Hom (phase) retrieval!
The effect is stronger in 2D ($\sim G^{1/2}$), and still stronger in 3D ($\sim G$).

Spatial resolution improves upon forward propagation, and then deteriorates upon TIE-Hom retrieval

$$(\Delta x)[P] = \left(4\pi \int x^2 P(x) dx\right)^{1/2}$$ - spatial resolution (defined as *std* of the PSF)

$$4\pi \int x^2 (1-\sigma^2 \nabla_x^2) P(x) dx = (\Delta x)_0^2 [P] - 4\pi\sigma^2 \int (\nabla_x^2 x^2) P(x) dx$$

$$\boxed{(\Delta x)_R^2 [P] = (\Delta x)_0^2 [(1-\sigma^2 \nabla_x^2) P] = (\Delta x)_0^2 [P] - 8\pi\sigma^2}$$

Spatial resolution improves upon ”forward” free-space propagation

$$\boxed{(\Delta x)_{0,ret}^2 [P] = (\Delta x)_R^2 [(1-\sigma^2 \nabla_x^2)^{-1} P] = (\Delta x)_R^2 [P] + 8\pi\sigma^2 = (\Delta x)_0^2 [P]}$$

Spatial resolution deteriorates upon TIE-Hom retrieval, back to the level it had in the contact plane z=0.

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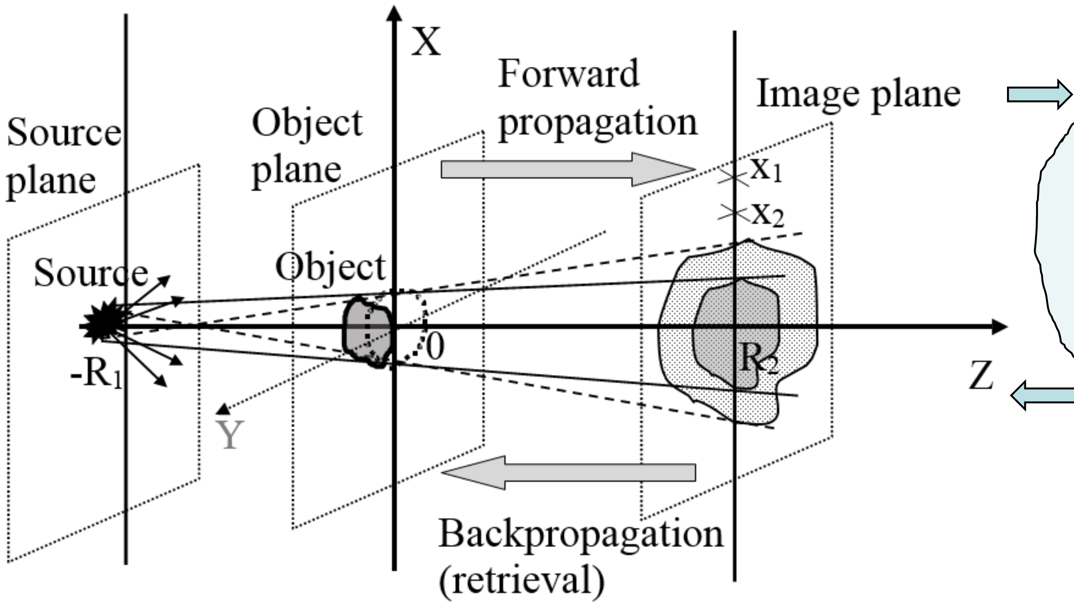
"Unreasonable" effectiveness of TIE-Hom imaging explained at last



Magic!

1. *SNR does not change upon forward free-space propagation in the near-Fresnel region.*
Spatial resolution improves upon forward free-space propagation.

2. SNR inreases upon TIE-Hom (phase) retrieval.
Spatial resolution deteriorates upon TIE-Hom retrieval, back to the level it had in the contact plane $z = 0$.

Total effect: $\dfrac{SNR_{ret}(x,0)}{(\Delta x)_{0,ret}} = G^{1/4}\dfrac{SNR(x,0)}{(\Delta x)_0}$ $\qquad G \cong \dfrac{4}{\pi}\dfrac{\gamma\lambda R'}{h^2}$

<u>Total effect</u>: the ratio of SNR and spatial resolution increases in TIE-Hom imaging. Note that all the "magic" happens during free-space propagation, phase retrieval does not do anything really special! This "magic" is possible only because the image noise is dominated by the photodection noise.

Gain in SNR in PCT (without loss of spatial resolution) at a fixed dose, as a function of *R* and detector pixel size

SNR gains for ideal detector with a single-pixel PSF.

$h \Downarrow \backslash R \Rightarrow$	**1.5m**	**2m**	**6m**	**9m**
30µm	24	31	80	114
50µm	11	**14**	33	47
70µm	6	8	19	26
100µm	4	**6**	11	15

E=32keV

$\gamma(E)$ corresponds to glandular relative to adipose tissue:

$$\gamma = \frac{\delta_{gland} - \delta_{fat}}{\beta_{gland} - \beta_{fat}}$$

SNR gains for detector with a Gaussian PSF, std σ = pixel size.

$h \Downarrow \backslash R \Rightarrow$	**1.5m**	**2m**	**6m**	**9m**
30µm	4	8	15	20
50µm	2	4	7	10
70µm	1.8	2	4	6
100µm	1.6	1.8	3	4

$$Gain = \frac{SNR_{ret}(x,0)}{SNR(x,0)} \sim G$$

$$G = \frac{4}{\pi}\frac{\gamma\lambda R'}{h^2}$$

Dose reduction factor in PCT (at a fixed SNR and spatial resolution), as a function of *R* and detector pixel size

DRFs for an ideal detector with a single-pixel PSF.

$h \Downarrow \backslash R \Rightarrow$	1.5m	2m	6m	9m
30µm	576	961	6400	13000(!)
50µm	121	196	1089	2209
70µm	36	64	361	676
100µm	16	36	121	225

E=32keV

$\gamma(E)$ corresponds to glandular relative to adipose tissue:

$$\gamma = \frac{\delta_{gland} - \delta_{fat}}{\beta_{gland} - \beta_{fat}}$$

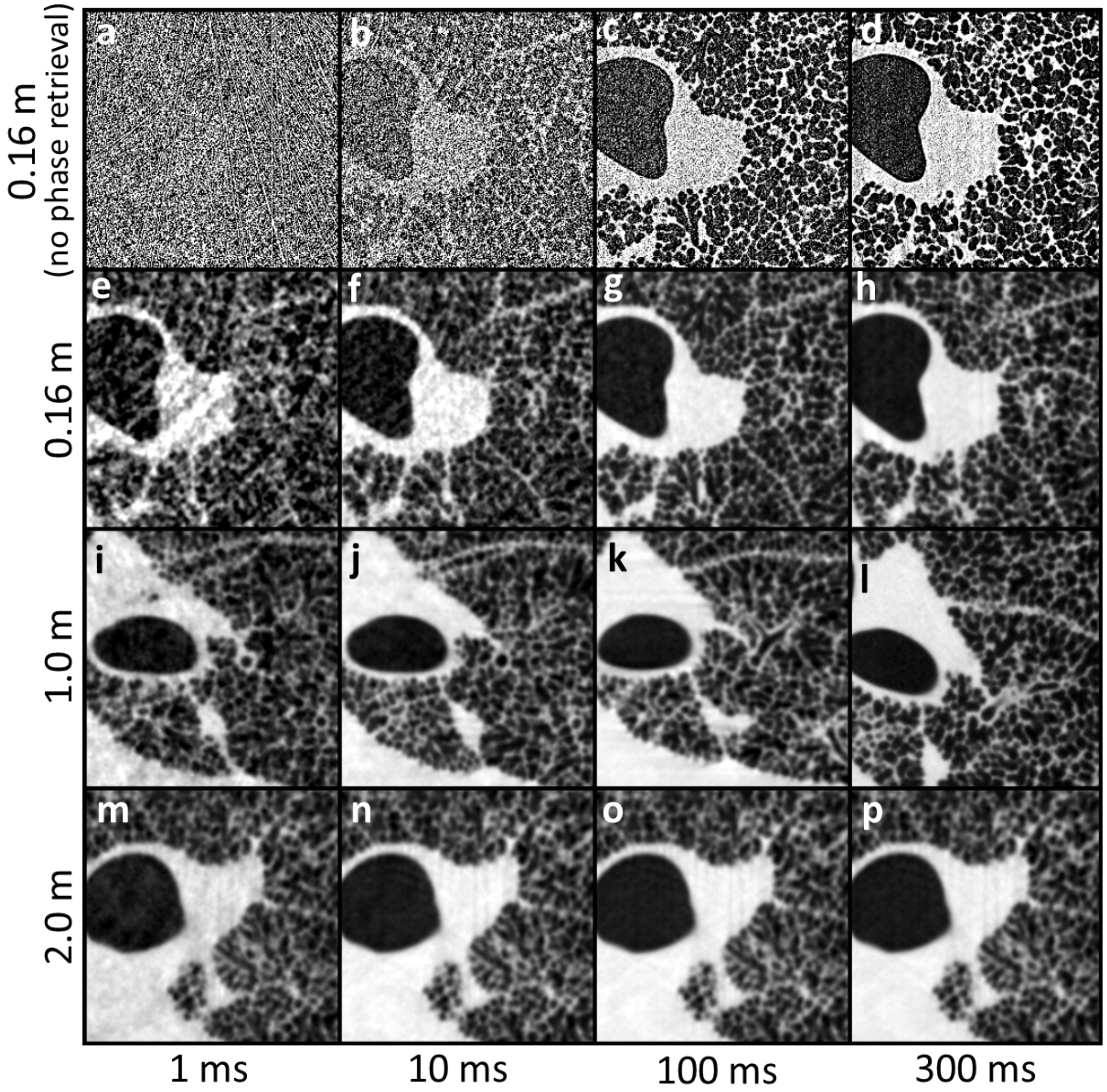
DRFs for a detector with a Gaussian PSF, std σ = pixel size.

$h \Downarrow \backslash R \Rightarrow$	1.5m	2m	6m	9m
30µm	16	64	225	400
50µm	4	16	49	100
70µm	3.2	4	16	36
100µm	2.6	3.2	9	16

$$DRF = \frac{Dose(x,0)}{Dose_{ret}(x,0)} \sim G^2$$

$$G = \frac{4}{\pi} \frac{\gamma \lambda R'}{h^2}$$

Advantage of short exposures and large number of projections (from *Kitchen et al, arXiv:1704.03556, 2017*)



TIE-Hom imaging of mouse lungs as a function of exposure time (dose) and propagation distance (phase retrieval).

X-ray energy E = 24 keV
Detector pixel size h = 15.3 μm
γ=1773

Dose Reduction Factor of 27600 demonstrated at a fixed SNR and spatial resolution.

At these E,h,R and γ, theoretical SNR Gain Factor for an ideal detector is 243, corresponding to DRF = 59000!

Conclusions

- TIE-Hom imaging is a unique method which allows one to increase SNR in images at a fixed radiation dose, without a loss of spatial resolution.

- The "magic" of TIE-Hom imaging happens during the forward free-space propagation, which improves the spatial resolution (reduces the photon correlation length) without increasing the noise variance.

- This "magic" is only possible because the image noise is dominated by photodetection shot noise, which is "added" only at the image detection stage and does not depend on the propagation distance.

- With good photon-counting detectors (high DQE, relatively small pixels, single-pixel PSF), SNR gain factors of the order of 10^2 may be possible, which translates into dose reduction factors of the order of 10^4, in PB-CT of breast tissue.

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The end