Beautiful hadronic contributions to FCNCs via Lattice QCD

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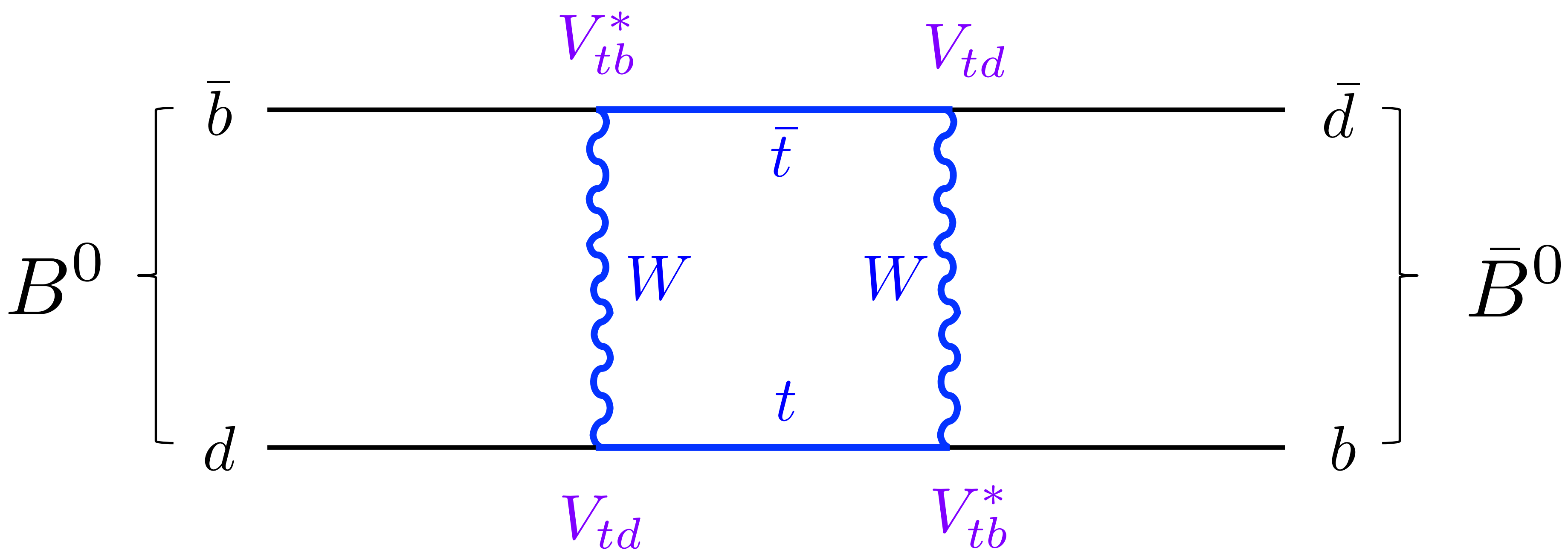
Beauty 2018, Isola d'Elba

08 May 2018

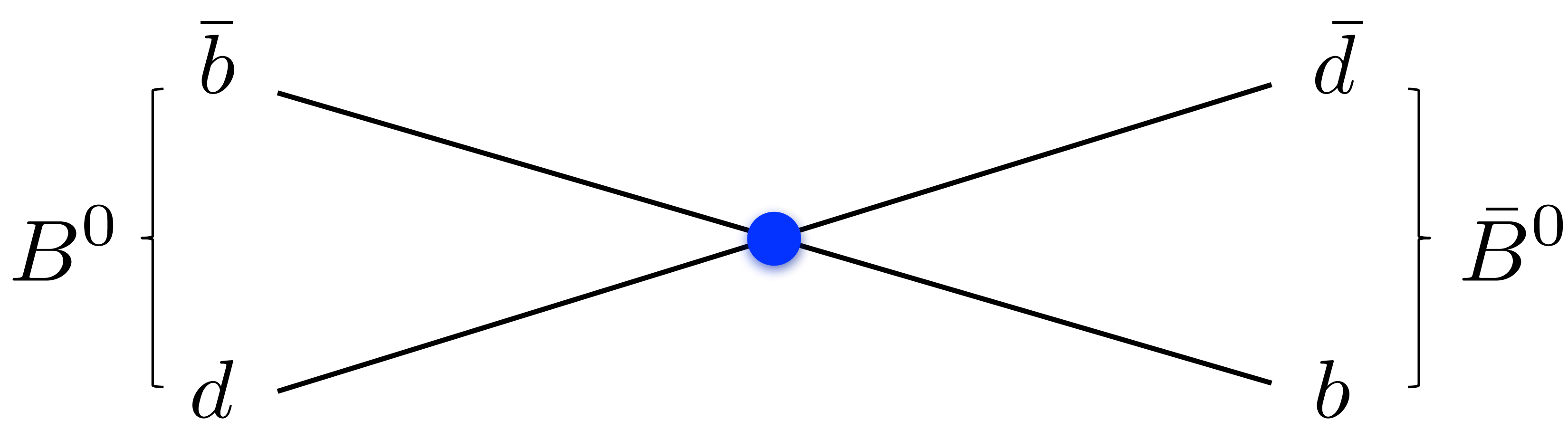
- Introduction
  - role of LQCD
- $B_{(s)} - \bar{B}_{(s)}$
  - state of the art $\Delta M_{d,s}$
  - work in progress on $\Delta\Gamma_s$
- $B \to K$
  - state of the art
  - restricted to large q^2
- $B \to \pi$
  - state of the art
  - work in progress, getting around the limitation
- Outlook

Neutral B and Bs meson mixing



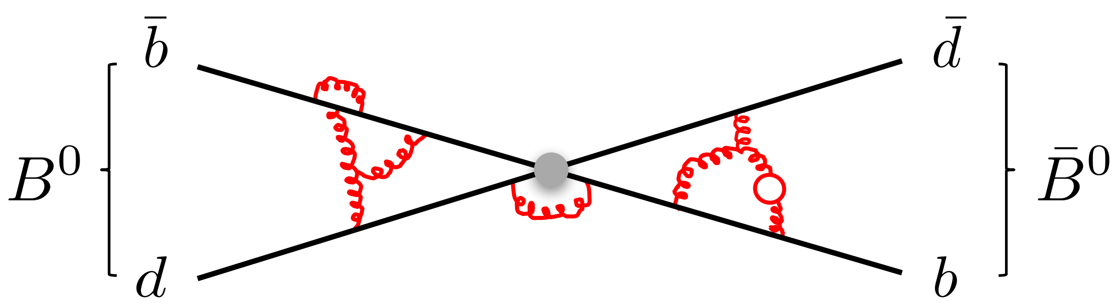
- depends on $|V_{td}V_{tb}^*|^2$ ($|V_{ts}V_{tb}^*|^2$ for B_s mixing)
- only in SM through quantum fluctuations

Flavor-changing interactions are "short distance".

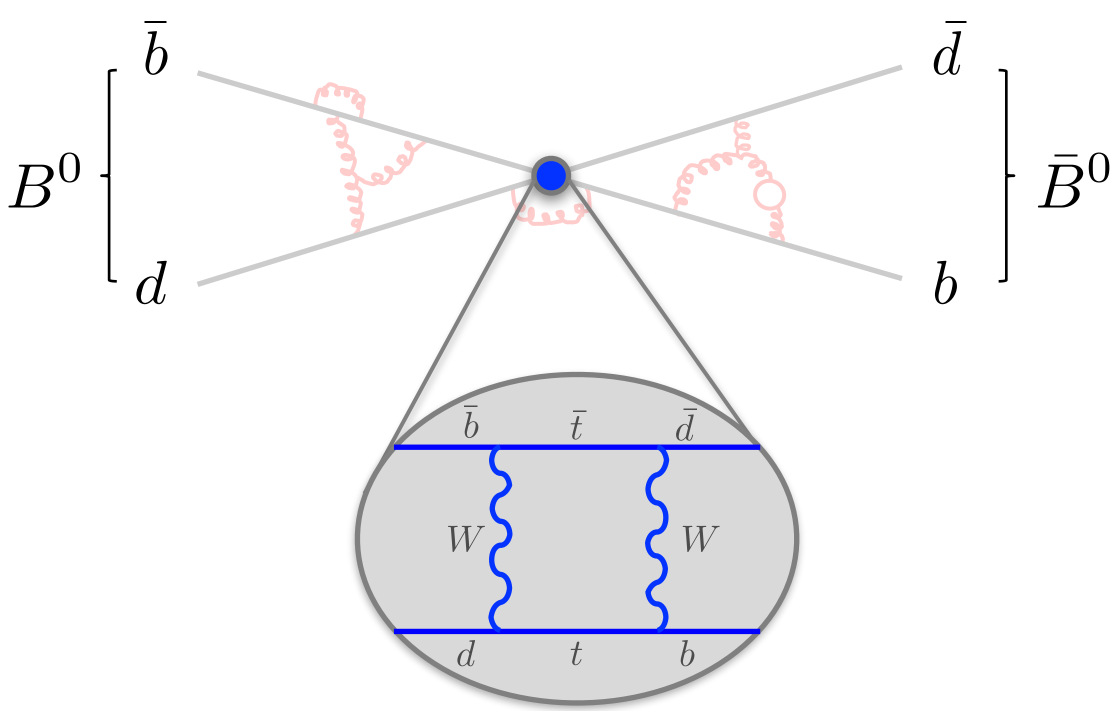


$B_{(s)}$ mixing occurs via local, effective four quark interactions.

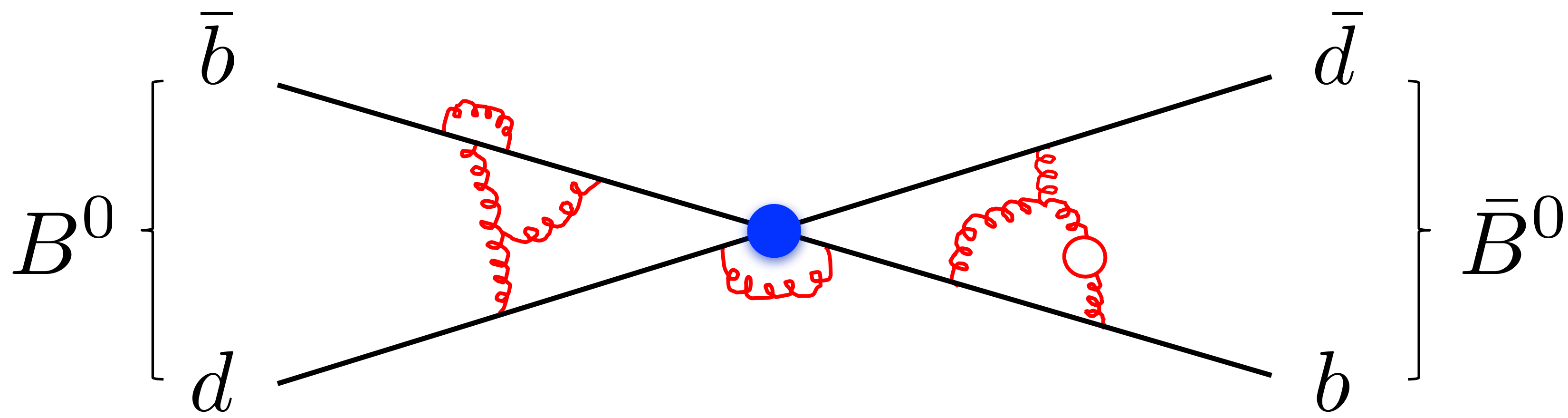
Role of lattice QCD: a tale of two scales



$\Lambda_{\rm QCD} \sim$ few $\times$ 100 MeV



$\Lambda_{\rm EW} \sim 100$ GeV



Physics at disparate scales factorizes

$$\Delta M = C(V)\,\langle B^0|\mathcal{O}|\bar{B}^0\rangle + \ldots$$

- Wilson coefficients: short distance, *perturbative*
- hadronic matrix elements: long distance, *nonperturbative*

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Weak eigenstates are convenient for the calculation...

$$i\frac{d}{dt}\begin{pmatrix}|B^0\rangle \\ |\bar{B}^0\rangle\end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix}|B^0\rangle \\ |\bar{B}^0\rangle\end{pmatrix}$$

hermitian $\mathbf{M}, \mathbf{\Gamma}$

$$\begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Mixing characterized by off-diagonal term, $M_{12} - \frac{i}{2}\Gamma_{12}$.

M_{12}: "absorptive", short distance

Γ_{12} : "dispersive", long distance

Observables: $|M_{12}|$, $|\Gamma_{12}|$, and $\phi_{12} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$

Experimentally, mass eigenstates are more convenient.

Diagonalize $\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}$ ...

- eigenstates: $|B_{H,L}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle, \text{ where } |p|^2 + |q|^2 = 1$
- eigenvalues: $M_{H,L} - \frac{i}{2}\Gamma_{H,L}$

Observables

$\Delta M = M_H - M_L$: oscillation frequency

$\Delta\Gamma = \Gamma_L - \Gamma_H$: decay width difference [and $\Gamma = (\Gamma_L + \Gamma_H)/2$]

$a_{\rm sl} = \dfrac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2}$: flavor-specific asymmetry

Observables in two bases are related:

lattice QCD focus to date

$$\Delta M = 2|M_{12}| \left(1 - \frac{1}{8}\left|\frac{\Gamma_{12}}{M_{12}}\right|^2 \sin^2\phi_{12} + \ldots\right)$$

$\sim 10^{-8}$ in SM B^0 mixing
$\sim 10^{-10}$ in SM B_s^0 mixing

$$\Delta\Gamma = 2|\Gamma_{12}|\cos\phi_{12} \left(1 + \frac{1}{8}\left|\frac{\Gamma_{12}}{M_{12}}\right|^2 \sin^2\phi_{12} + \ldots\right)$$

new work underway

$$a_{\rm sl} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{12} + \ldots$$

Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi, 1603.07770

MEs of 5 operators span space of local 4-quark interactions in and beyond the SM:

Standard Model

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma_\mu L q^\alpha \ \bar{b}^\beta \gamma_\mu L q^\beta$$

$$\mathcal{O}_2^q = \bar{b}^\alpha L q^\alpha \ \bar{b}^\beta L q^\beta$$

$$\mathcal{O}_3^q = \bar{b}^\alpha L q^\beta \ \bar{b}^\beta L q^\alpha$$

NP models

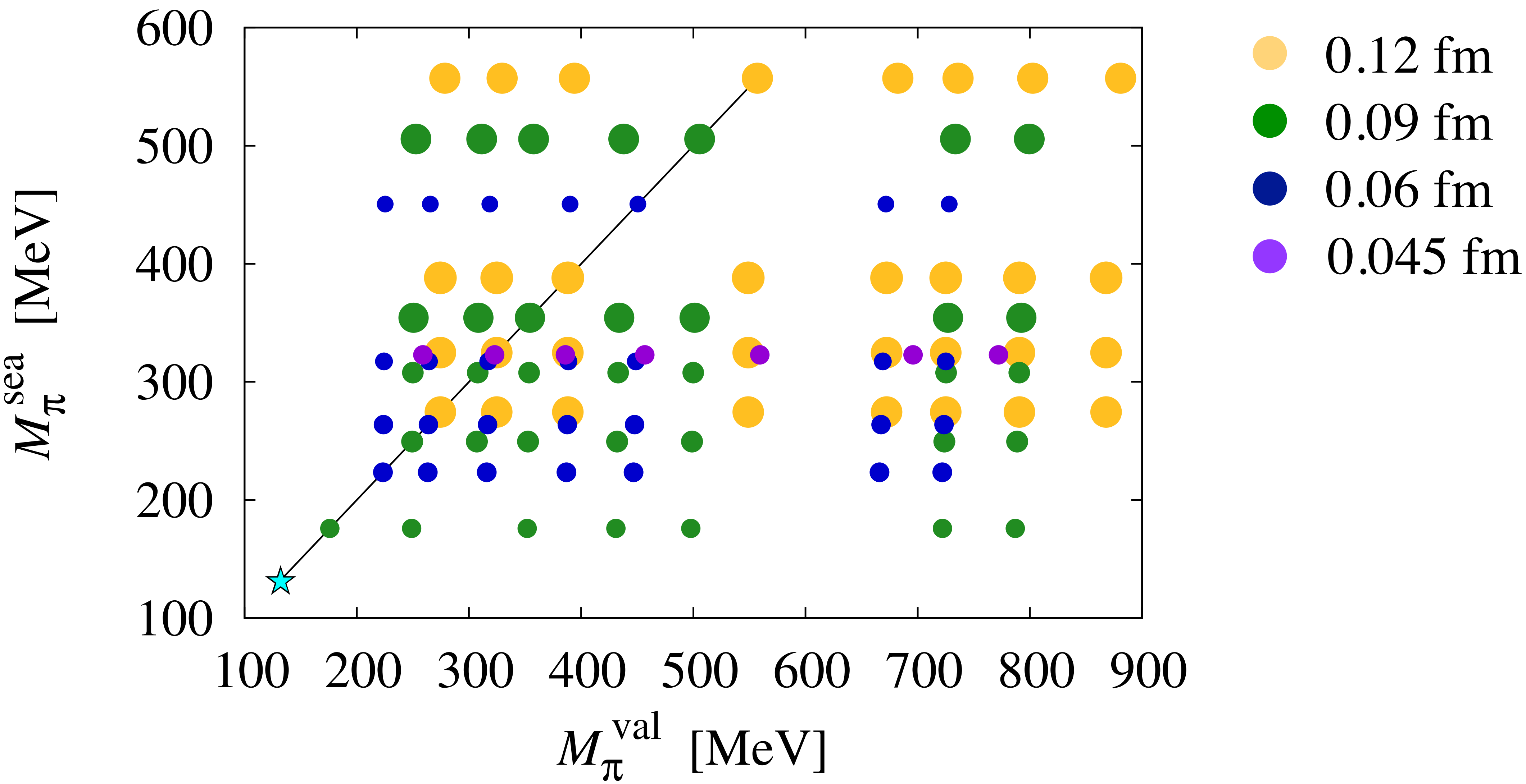
$$\mathcal{O}_4^q = \bar{b}^\alpha L q^\alpha \ \bar{b}^\beta R q^\beta$$

$$\mathcal{O}_5^q = \bar{b}^\alpha L q^\alpha \ \bar{b}^\beta R q^\alpha$$

We calculate hadronic matrix elements of these 5 operators,

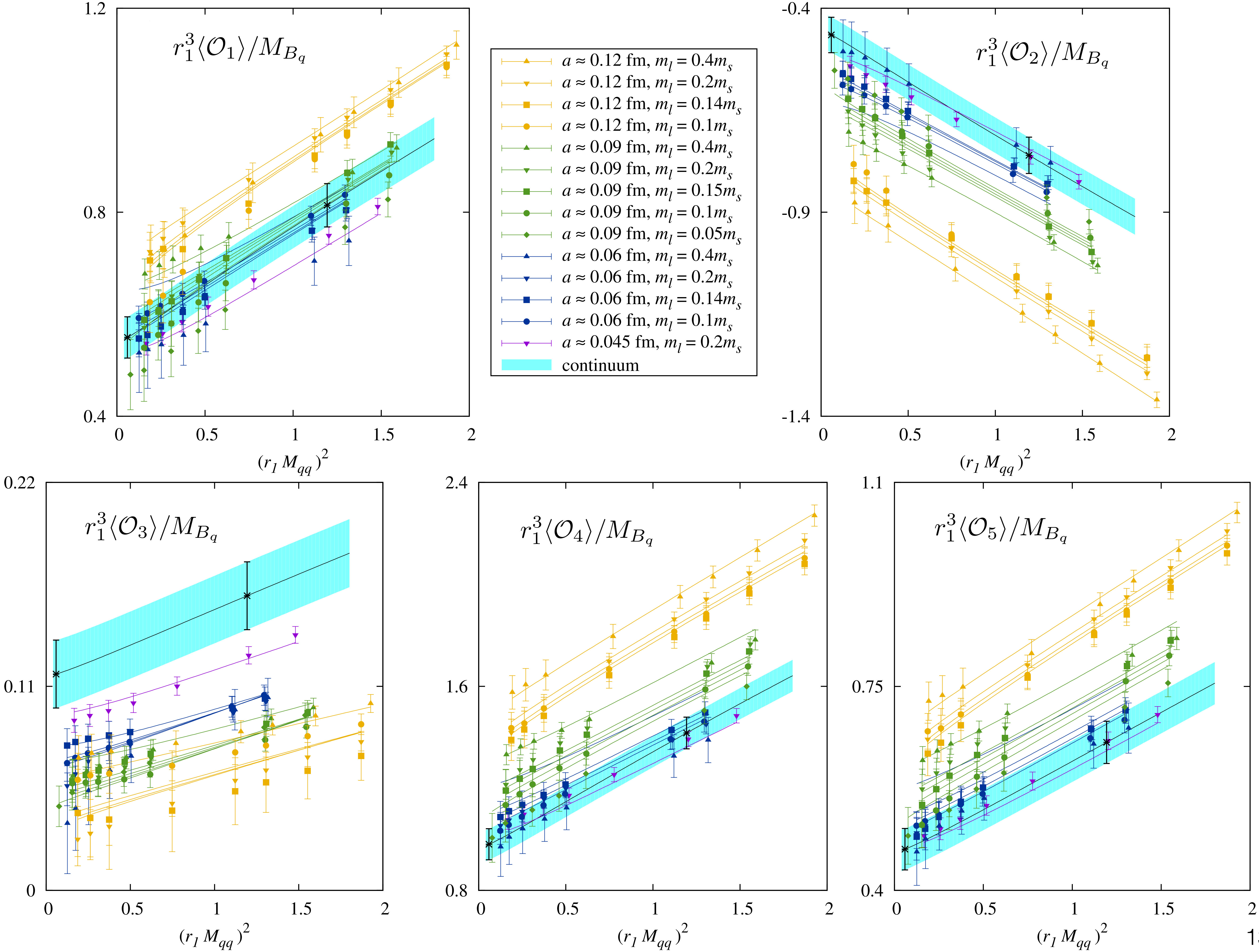
$$\langle B_q | \mathcal{O}_i^q | \bar{B}_q \rangle_{\mathrm{QCD}}; \quad q = s, d.$$

For all 5 O_i , we obtain values for $\langle B_q|O_i^q|\bar{B}_q\rangle$ ...



... and extrapolate to the real world ☆ .

FNAL/MILC, PRD93 (2016) 113016



$r_1^3\langle\mathcal{O}_1\rangle/M_{B_q}$
$r_1^3\langle\mathcal{O}_2\rangle/M_{B_q}$
$r_1^3\langle\mathcal{O}_3\rangle/M_{B_q}$
$r_1^3\langle\mathcal{O}_4\rangle/M_{B_q}$
$r_1^3\langle\mathcal{O}_5\rangle/M_{B_q}$
$(r_1 M_{qq})^2$
$a \approx 0.12$ fm, $m_l = 0.4m_s$
$a \approx 0.12$ fm, $m_l = 0.2m_s$
$a \approx 0.12$ fm, $m_l = 0.14m_s$
$a \approx 0.12$ fm, $m_l = 0.1m_s$
$a \approx 0.09$ fm, $m_l = 0.4m_s$
$a \approx 0.09$ fm, $m_l = 0.2m_s$
$a \approx 0.09$ fm, $m_l = 0.15m_s$
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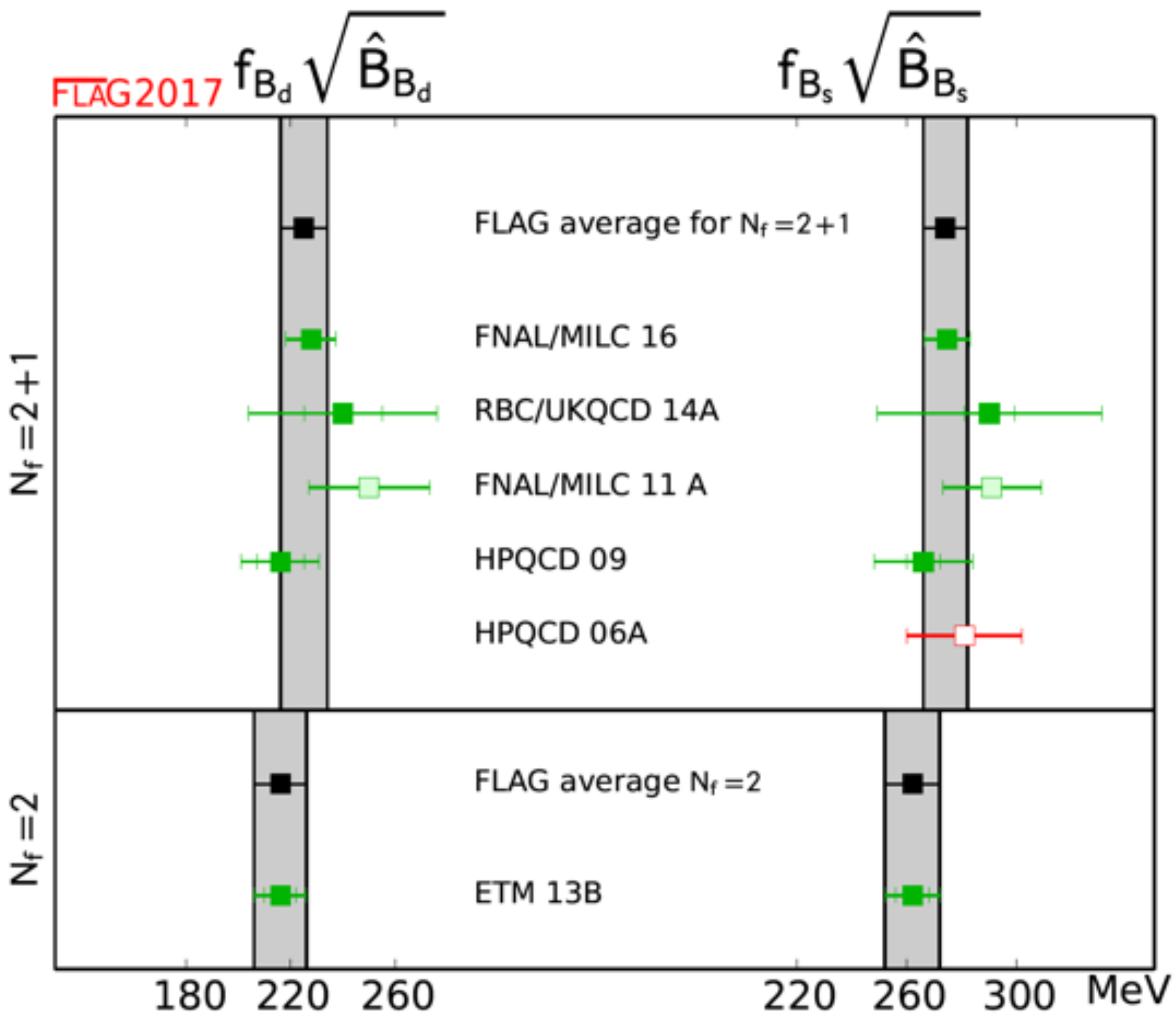
$a \approx 0.09$ fm, $m_l = 0.05m_s$
$a \approx 0.06$ fm, $m_l = 0.4m_s$
$a \approx 0.06$ fm, $m_l = 0.2m_s$
$a \approx 0.06$ fm, $m_l = 0.14m_s$
$a \approx 0.06$ fm, $m_l = 0.1m_s$
$a \approx 0.045$ fm, $m_l = 0.2m_s$
continuum

- FNAL/MILC: next generation B mixing calculation underway
- HPQCD: calculating Bs mixing, dim-7 operators for Γ_{12}

C. Davies et al., 0712.09934

- Current state of the art via Flavor Lattice Averaging Group (FLAG)

Aoki et al., 1607.00299



Bazavov et al., PRD93 (2016) 113016

Aoki et al., PRD91 (2015) 114505

Gamiz et al., PRD80 (2009) 014503

Carrasco et al., JHEP 03 (2014) 016

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At LO in heavy quark expansion for Γ_{12}, only dim-6 operators contribute.

$$\Gamma_{12}^{\mathrm{cc}} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[\left(G + \frac{\alpha_2}{2} G_s \right) \langle \bar{B}_s | \mathcal{O}_1 | B_s \rangle + \alpha_1 G_s \langle \overline{B}_s | \mathcal{O}_3 | B_s \rangle \right] + \tilde{\Gamma}^{\mathrm{cc}}_{12,\frac{1}{m_b}}$$

At NLO, there are dim-7 operators. Work underway by HPQCD to calculate these matrix elements

$$\tilde{\Gamma}^{\mathrm{cc}}_{12,\frac{1}{m_b}} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left\{ g_0^{\mathrm{cc}} \langle \bar{B}_s | R_0 | B_s \rangle + \sum_{j=1}^{3} \left[g_j^{\mathrm{cc}} \langle \bar{B}_s | R_j | B_s \rangle + \tilde{g}_j^{\mathrm{cc}} \langle \bar{B}_s | \tilde{R}_j | B_s \rangle \right] \right\}$$

$R_{0,1}$ are linear combinations of $\mathcal{O}_{1,2,3,4}$. Only two new matrix elements...

$$R_2 = \frac{1}{m_b^2} \left(\bar{b} \overleftarrow{D}_\rho \gamma_\mu (1-\gamma_5) D^\rho s \right) \left(\bar{b} \gamma_\mu (1-\gamma_5) s \right)$$

$$R_3 = \frac{1}{m_b^2} \left(\bar{b} \overleftarrow{D}_\rho (1-\gamma_5) D^\rho s \right) \left(\bar{b} (1-\gamma_5) s \right)$$

Davies et al (HPQCD), 0712.09934

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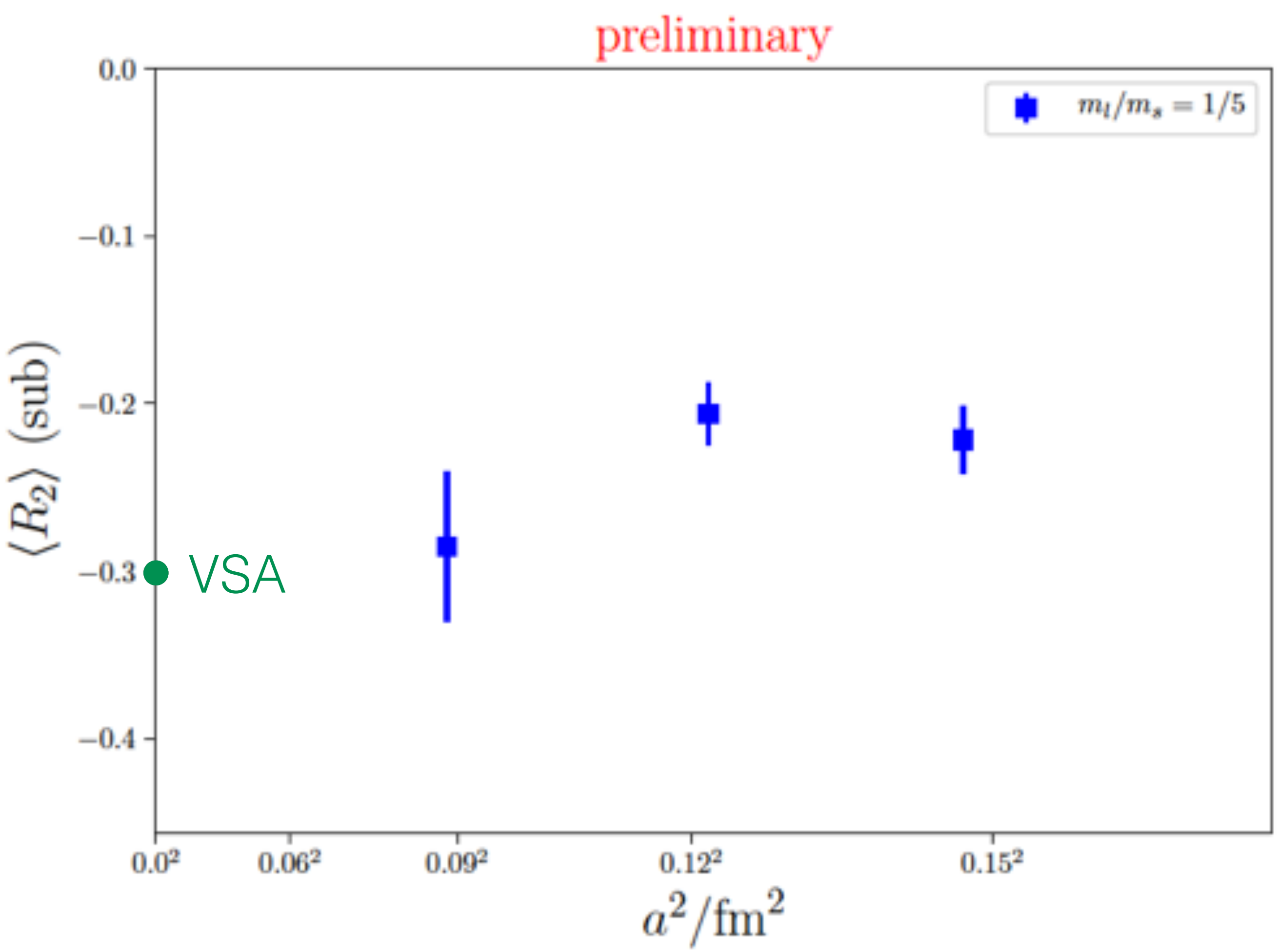

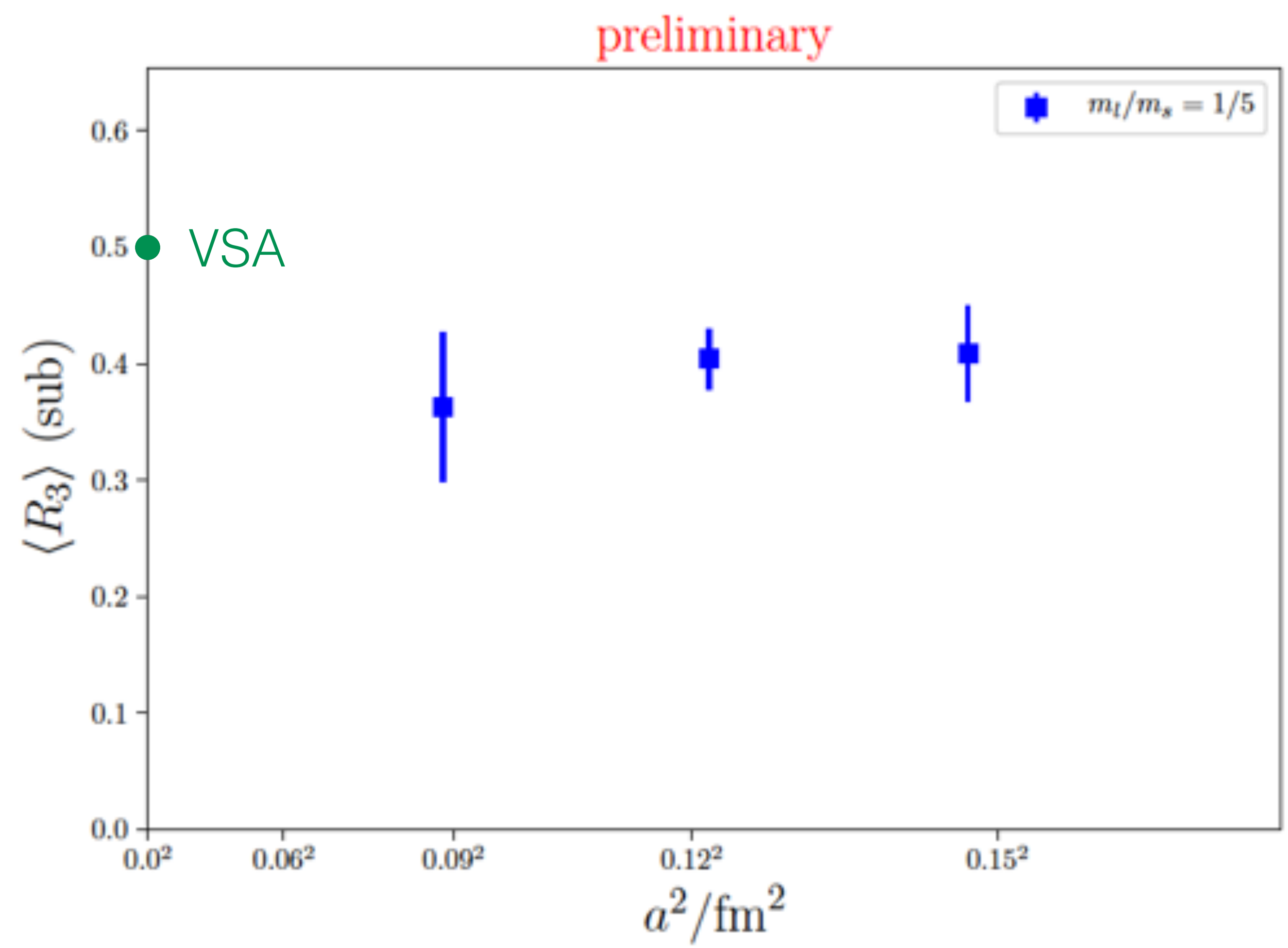

- valence b,s quark masses are physical
- they also have results at two different values of ml/ms to guide the subsequent extrapolation in sea quark mass

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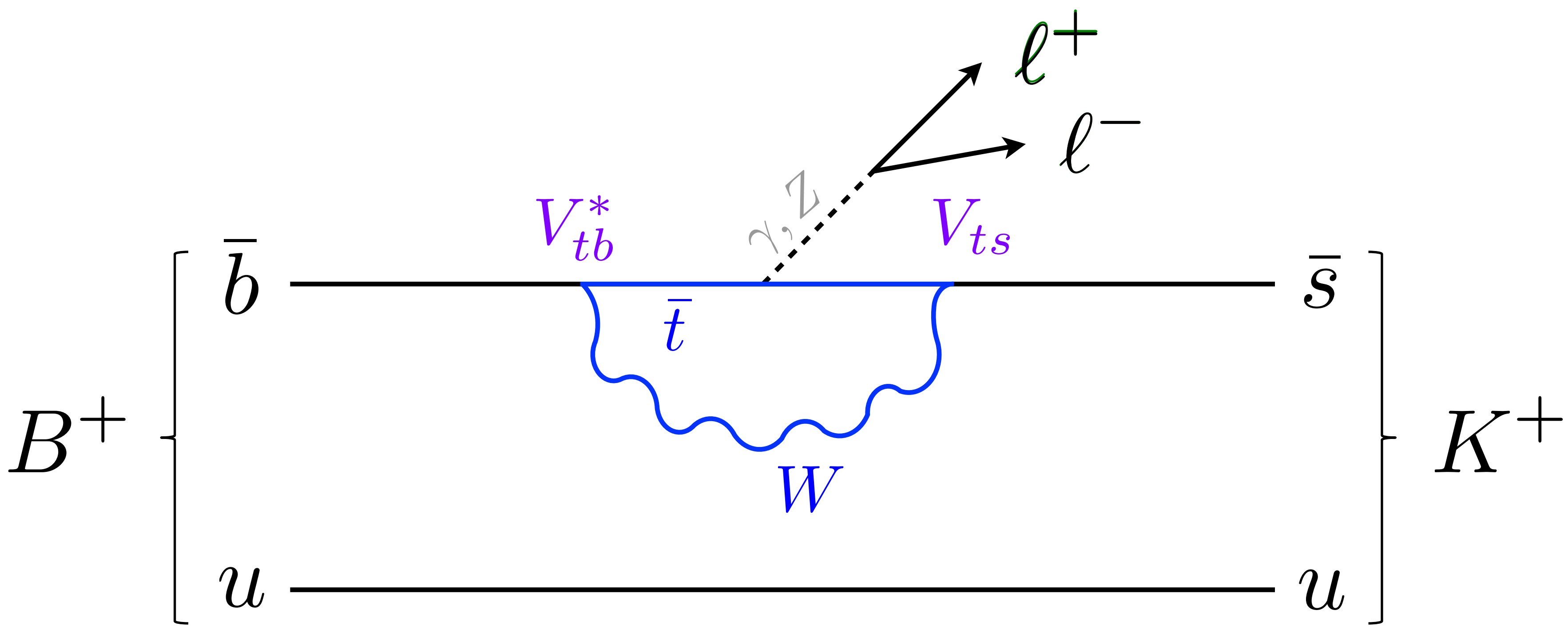
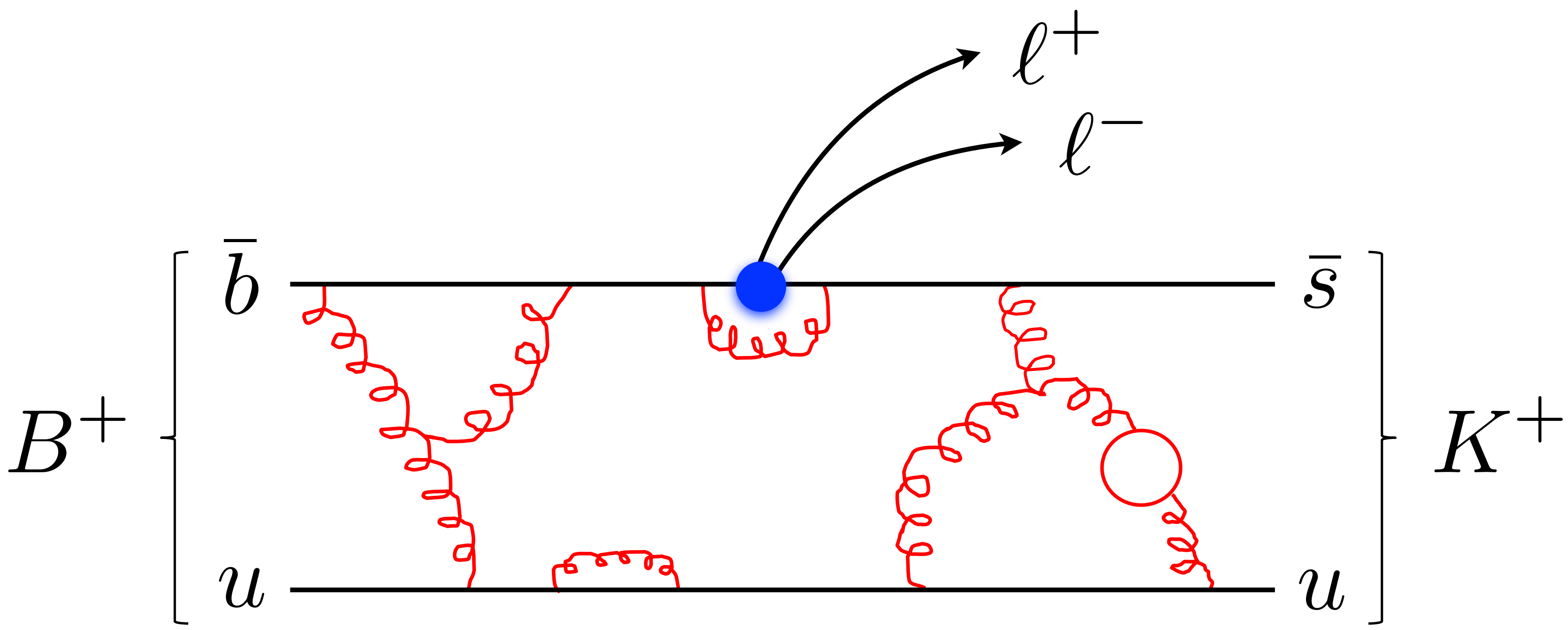


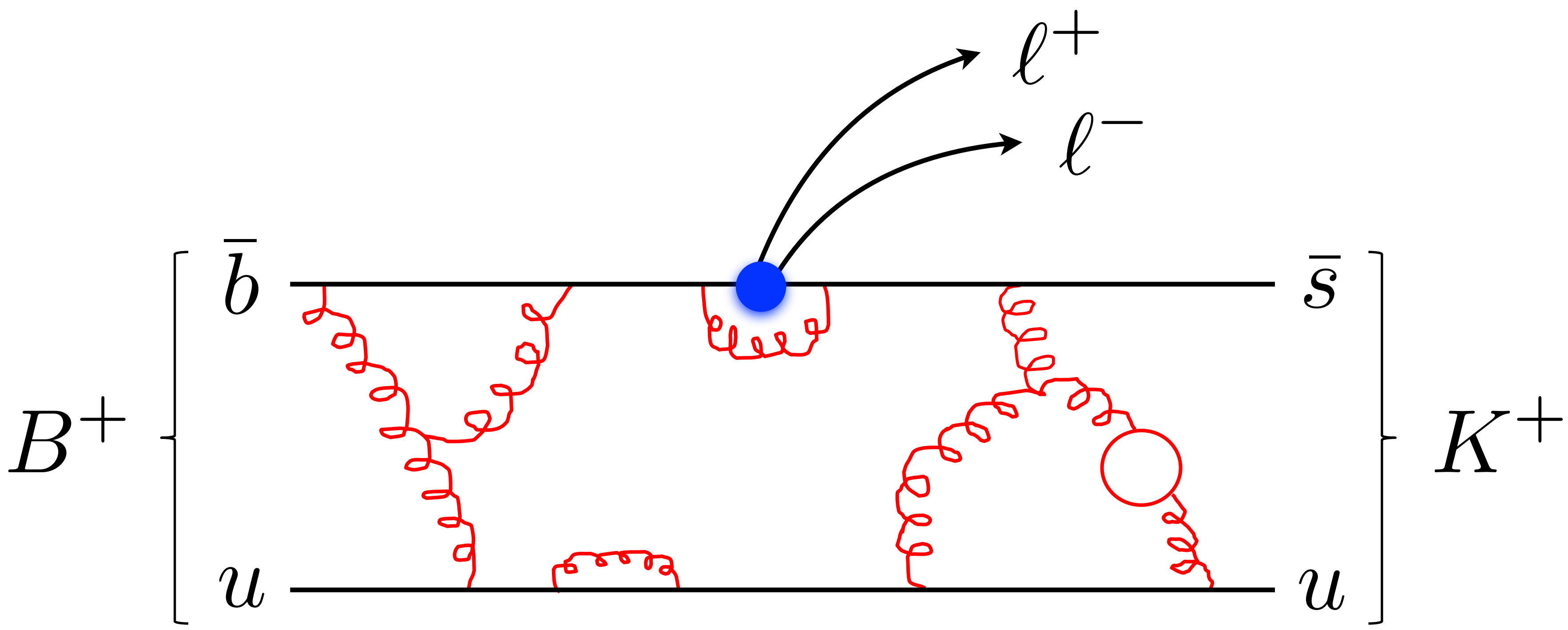
- momentum transfer (q^2) dependence
- improbable in the SM
  - proportional to $V_{tb}^* V_{ts}$
  - requires quantum fluctuations



Physics at disparate scales factorizes

$$\frac{d\Gamma}{dq^2} = \left(\sum_i C_i \langle K | J_i | B \rangle \right)^2 + \quad \ldots$$

- Wilson coefficients: short distance, *perturbative*
- hadronic matrix elements: long distance, *nonperturbative*

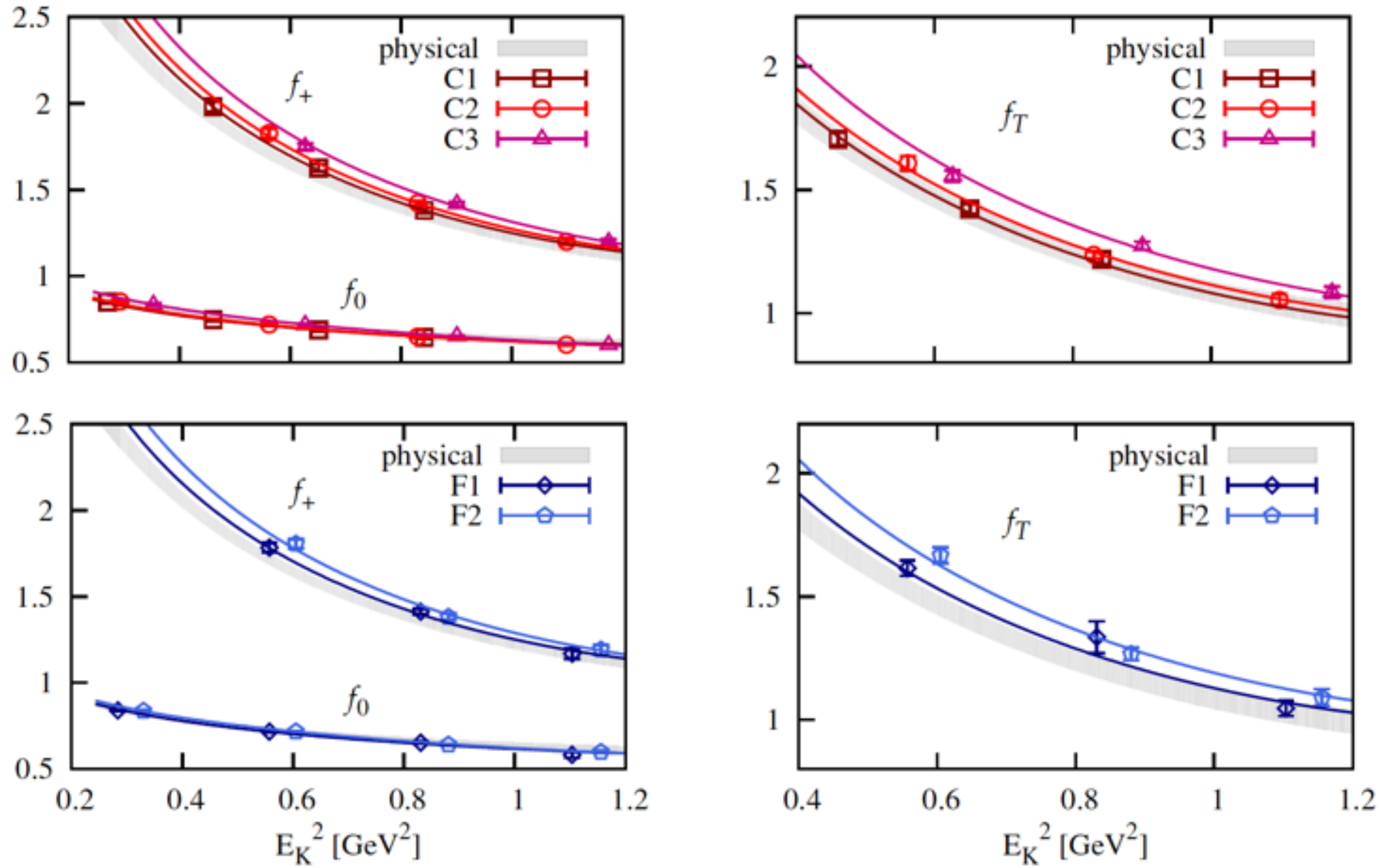


Physics at disparate scales factorizes

$$\frac{d\Gamma}{dq^2} = \left(\sum_i C_i \, \langle K | J_i | B \rangle \right)^2 + \quad \ldots$$

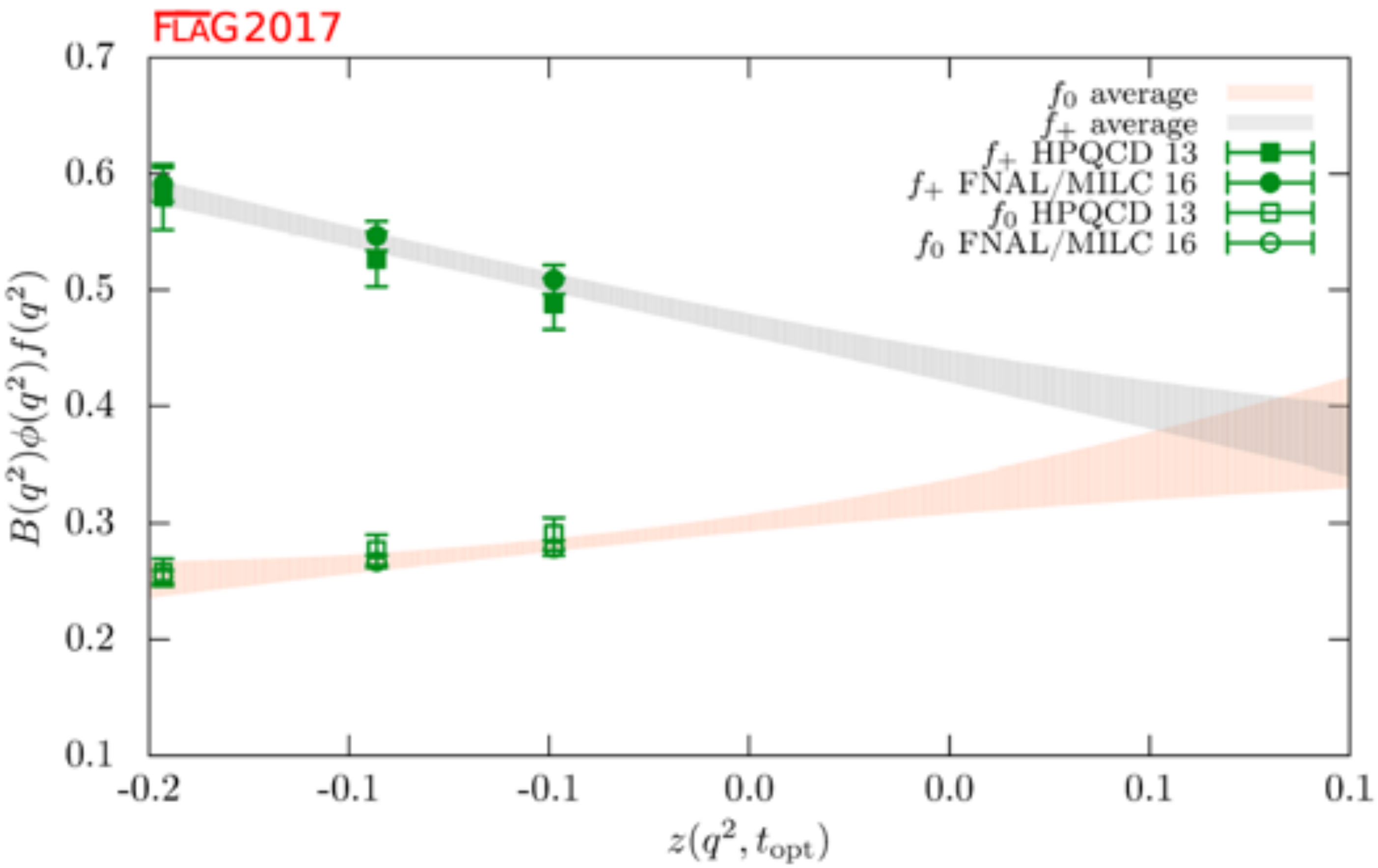
Form factors parameterize hadronic matrix elements. E.g., in SM...

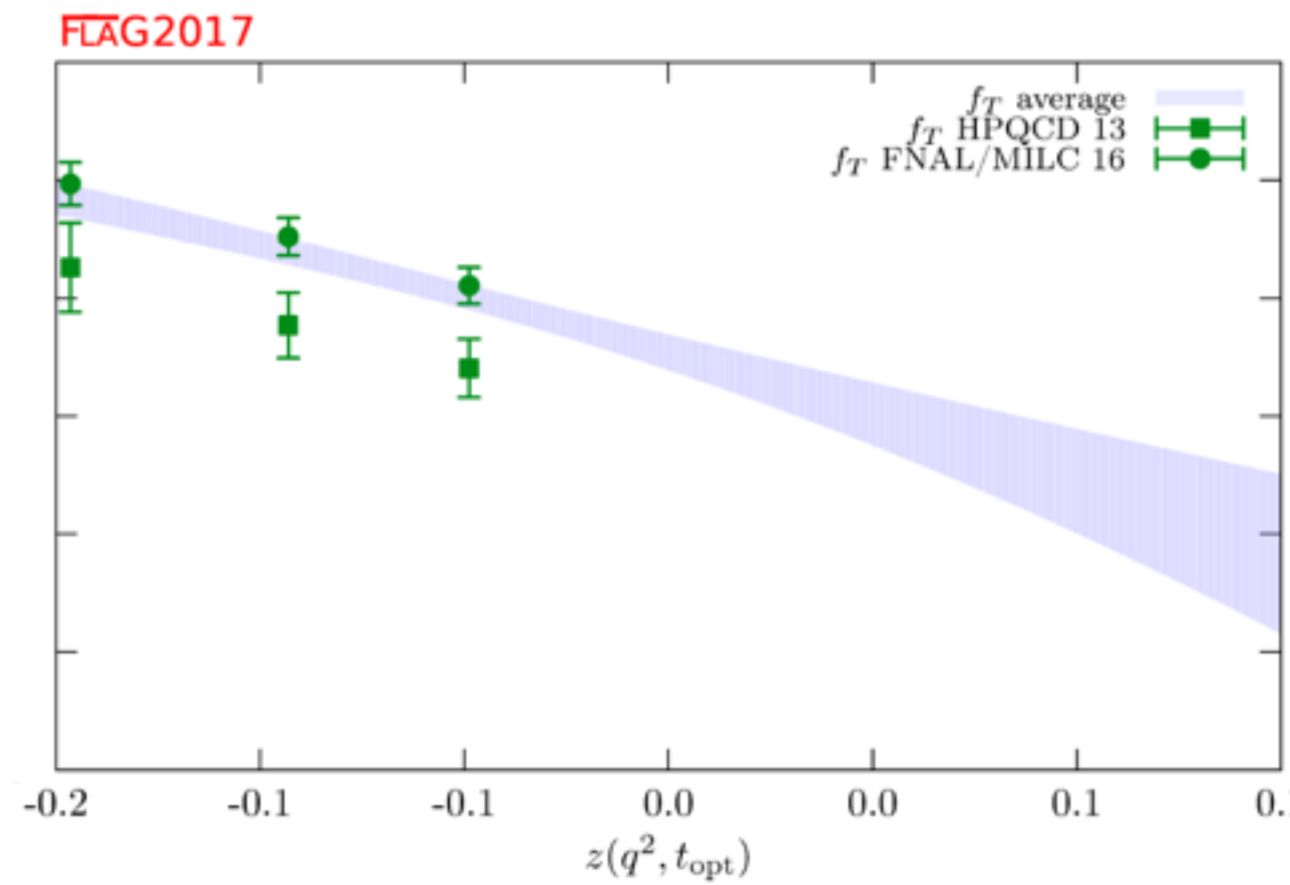
$$\frac{d\Gamma}{dq^2} = \mathcal{K}(q^2) \left[c_1 \, f_+^2(q^2) + c_2 \, f_T^2(q^2) + c_3 \, f_+(q^2) f_T(q^2) \right]$$



HPQCD, PRL 111 (2013) 162002

- Matrix elements calculated at various lattice spacings and quark masses
  - coarse lattices with spacing 0.12 fm, fine lattices with spacing 0.09 fm
  - light quark masses down to $M_\pi = 250 \text{ MeV}$
- results extrapolated to continuum and physical quark masses via ChPT





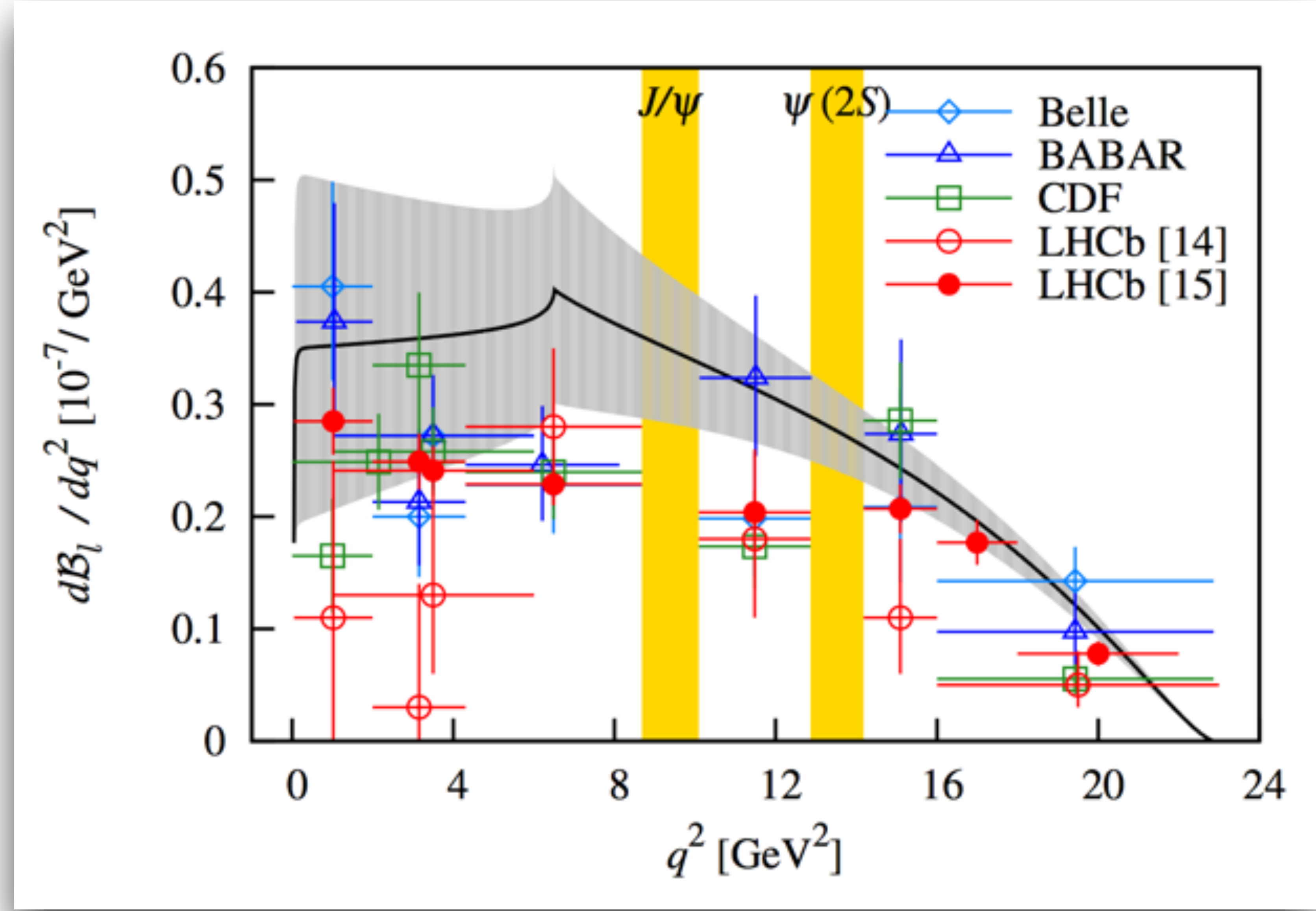
HPQCD, PRL 111 (2013) 162002

FNAL/MILC, PRD93 (2016) 025026

- FLAG provides averaged lattice QCD results
  - z-expansion (Bourrely, Caprini, Lellouch)
  - covariance matrix among all coefficients
- Lattice calculations simulate with small momenta (large q^2, small z)
  - discretization errors $O(|a\vec{p}_K|^2)$
  - chiral perturbation theory requires $E_K < \Lambda_\chi$

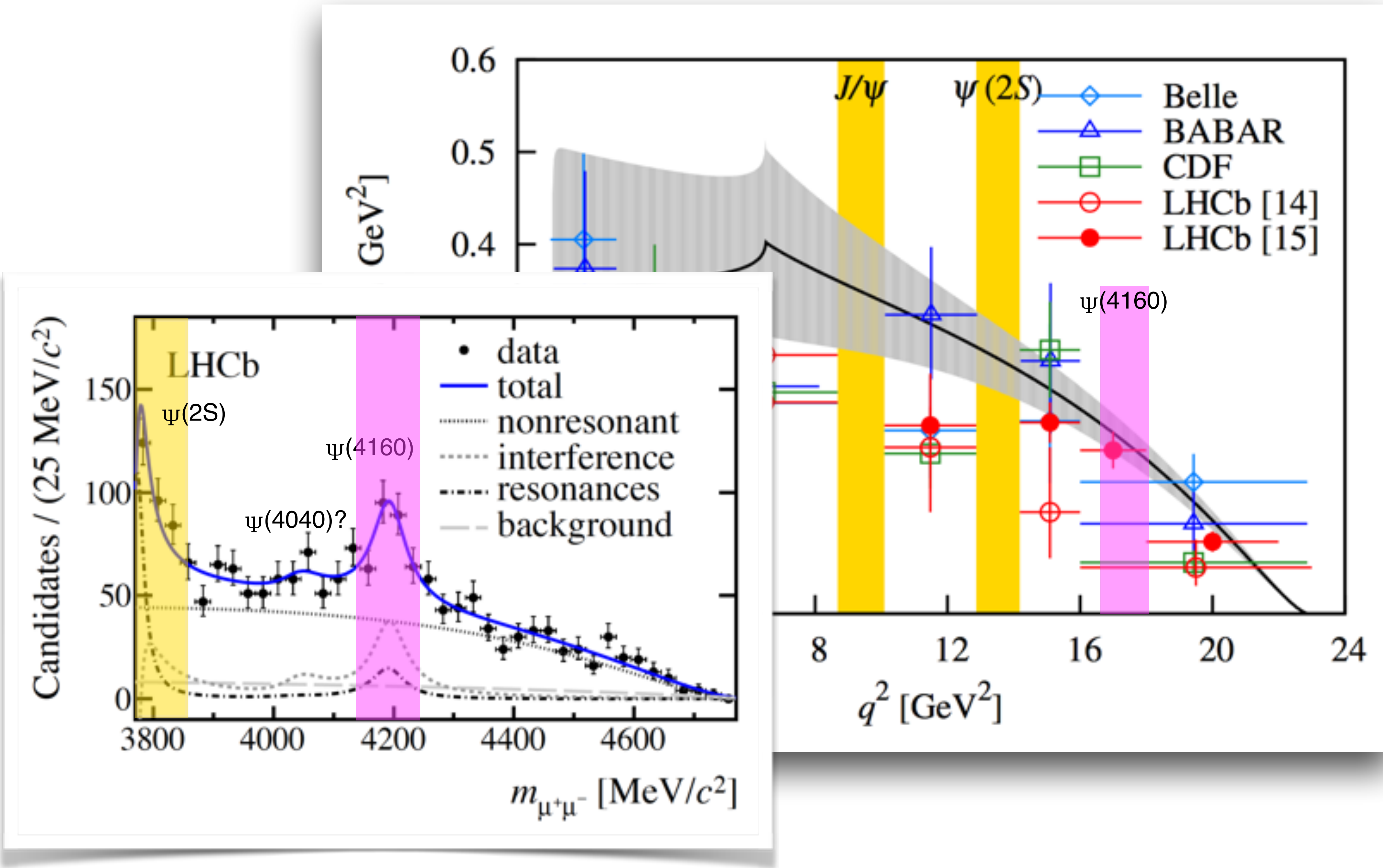
Comparison with experiment.

with HPQCD, PRL 111 (2013) 162002; PRD 88 (2013) 05409



Properly handling resonances an open issue...

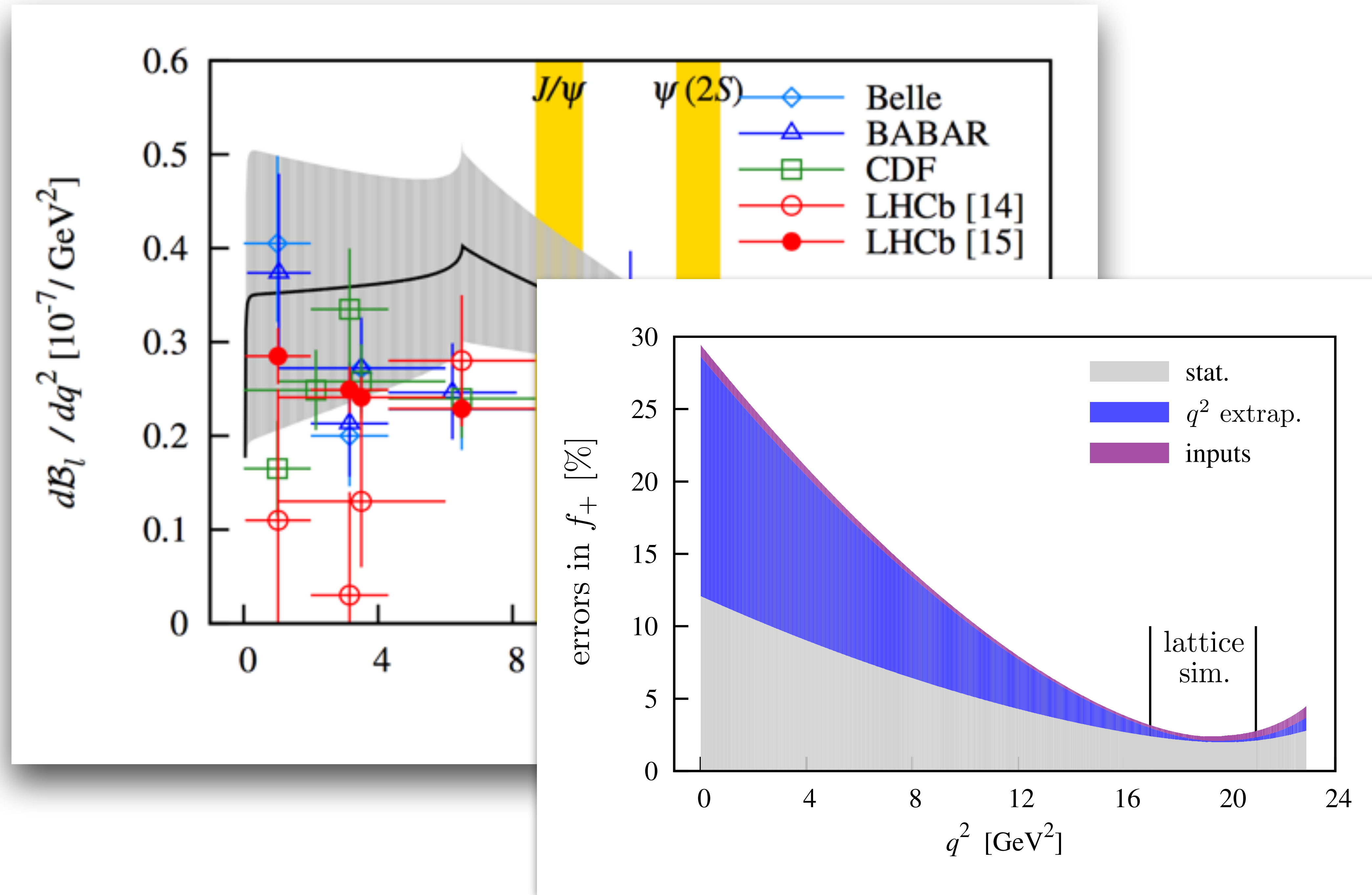
with HPQCD, PRL 111 (2013) 162002; PRD 88 (2013) 05409



LHCb, PRL 111 (2013) 112003

Large errors from lattice QCD at low q^2

with HPQCD, PRL 111 (2013) 162002; PRD 88 (2013) 05409



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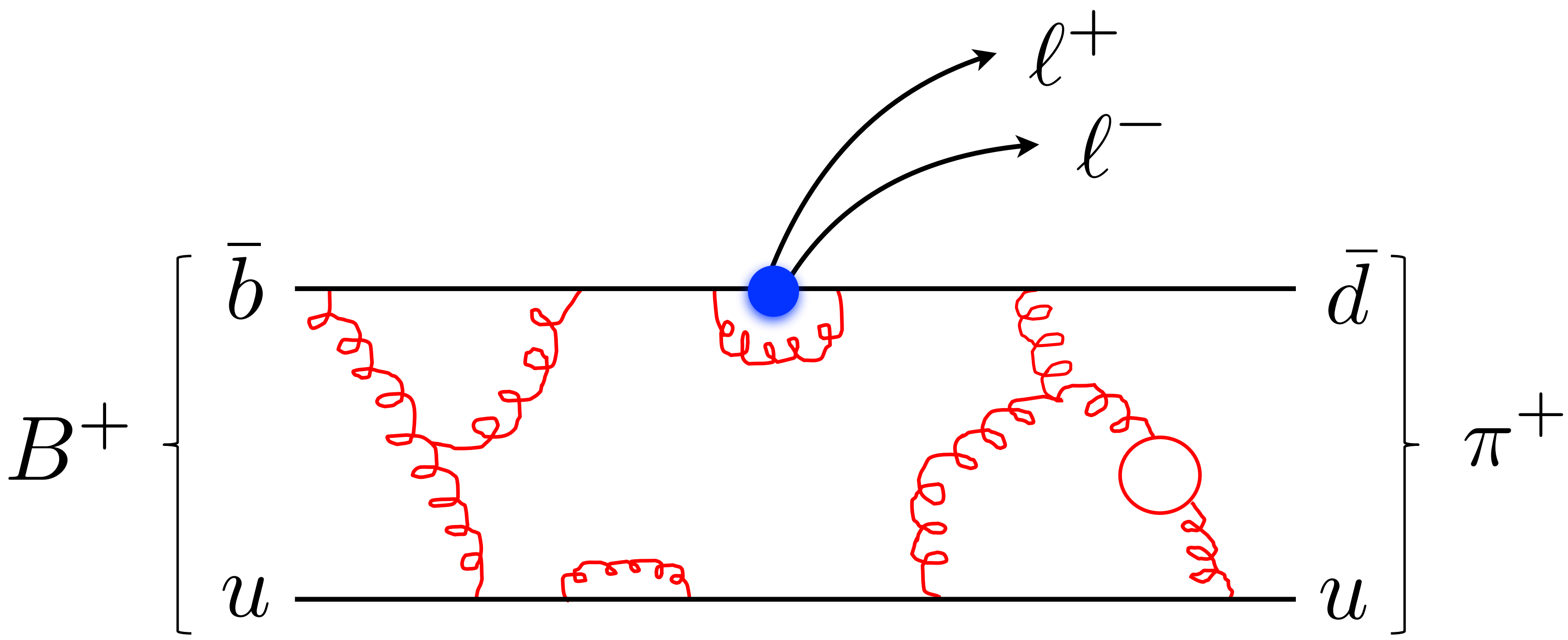
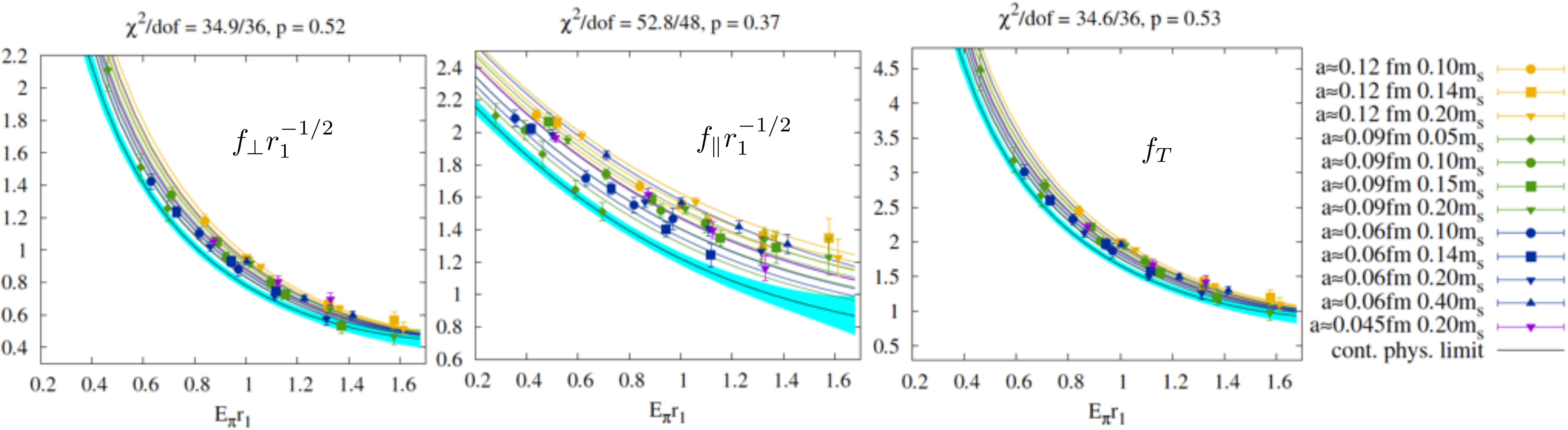


Same story ... physics at disparate scales factorizes

$$\frac{d\Gamma}{dq^2} = \left(\sum_i C_i \, \langle \pi | J_i | B \rangle \right)^2 + \quad \ldots$$

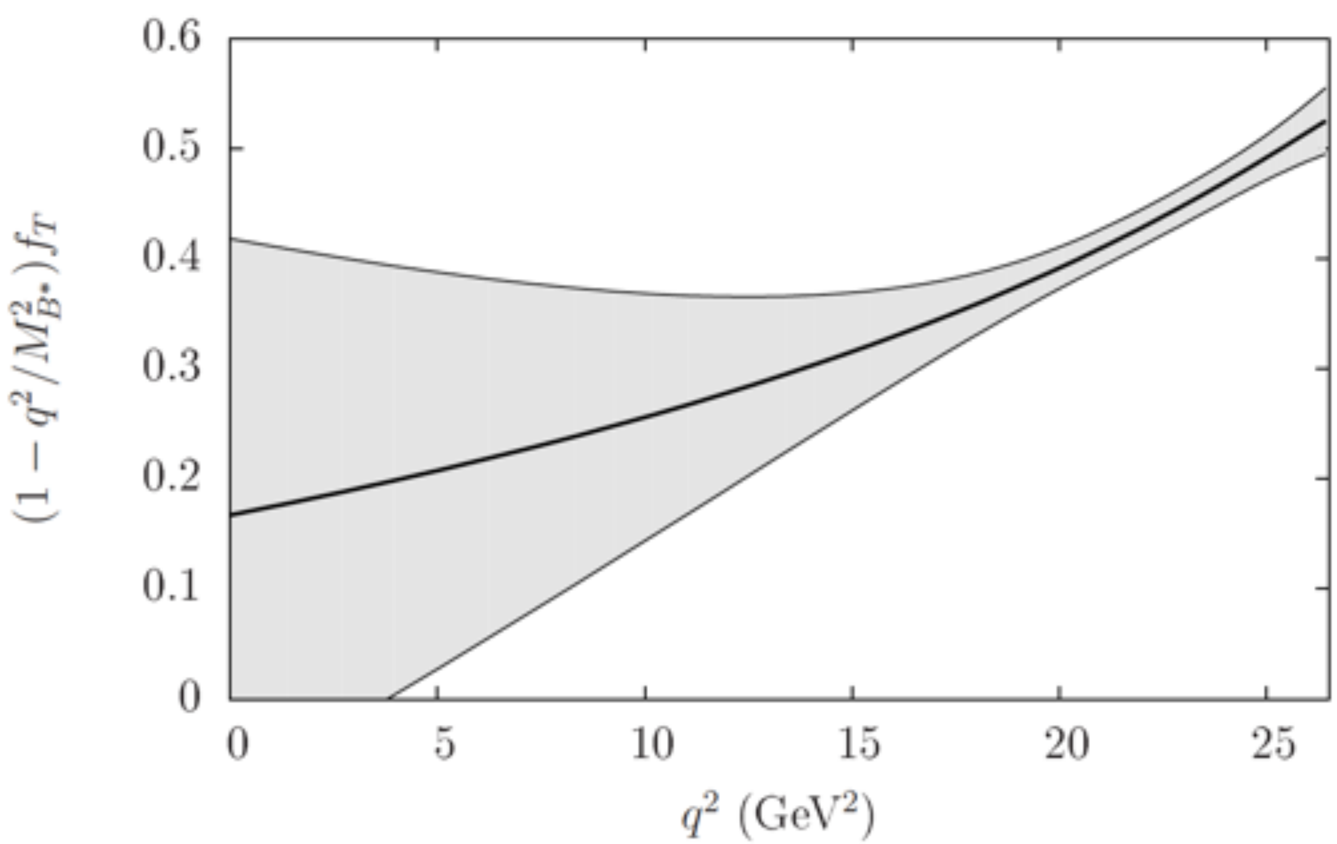
Form factors parameterize hadronic matrix elements. E.g., in SM...

$$\frac{d\Gamma}{dq^2} = \mathcal{K}(q^2) \left[c_1 \, f_+^2(q^2) + c_2 \, f_T^2(q^2) + c_3 \, f_+(q^2) f_T(q^2) \right]$$

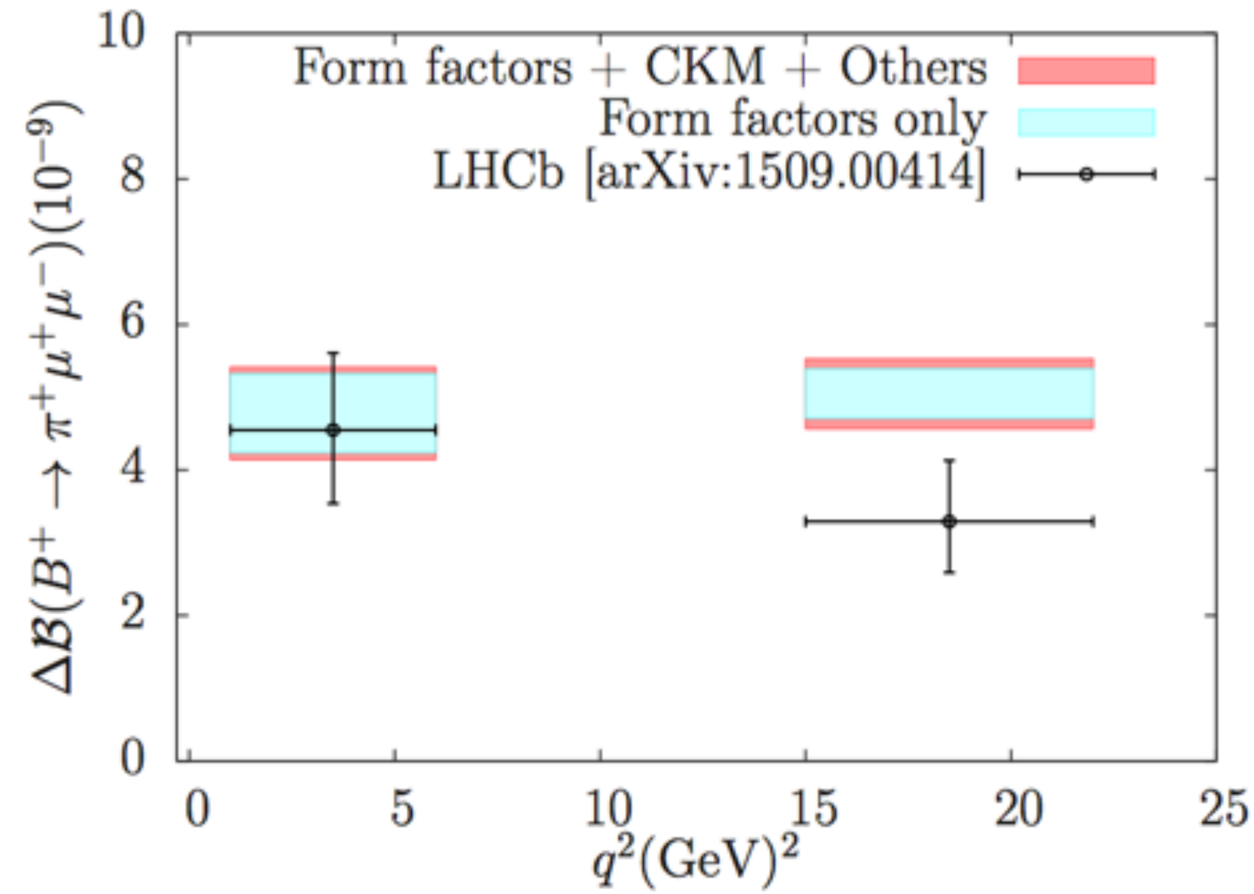


FNAL/MILC, PRD92 (2015) 014024

- Matrix elements calculated at 4 lattice spacings: 0.12, 0.09, 0.06, 0.045 fm
- light quark masses down to $M_\pi = 177 \text{ MeV}$
- results extrapolated to continuum and physical quark masses via ChPT
- $f_{\perp,\parallel}$ are linear combinations of $f_{0,+}$, and $r_1 = 0.3117(22) \text{ fm}$



Bailey et al. (FNAL/MILC), PRL 115 (2015) 152002

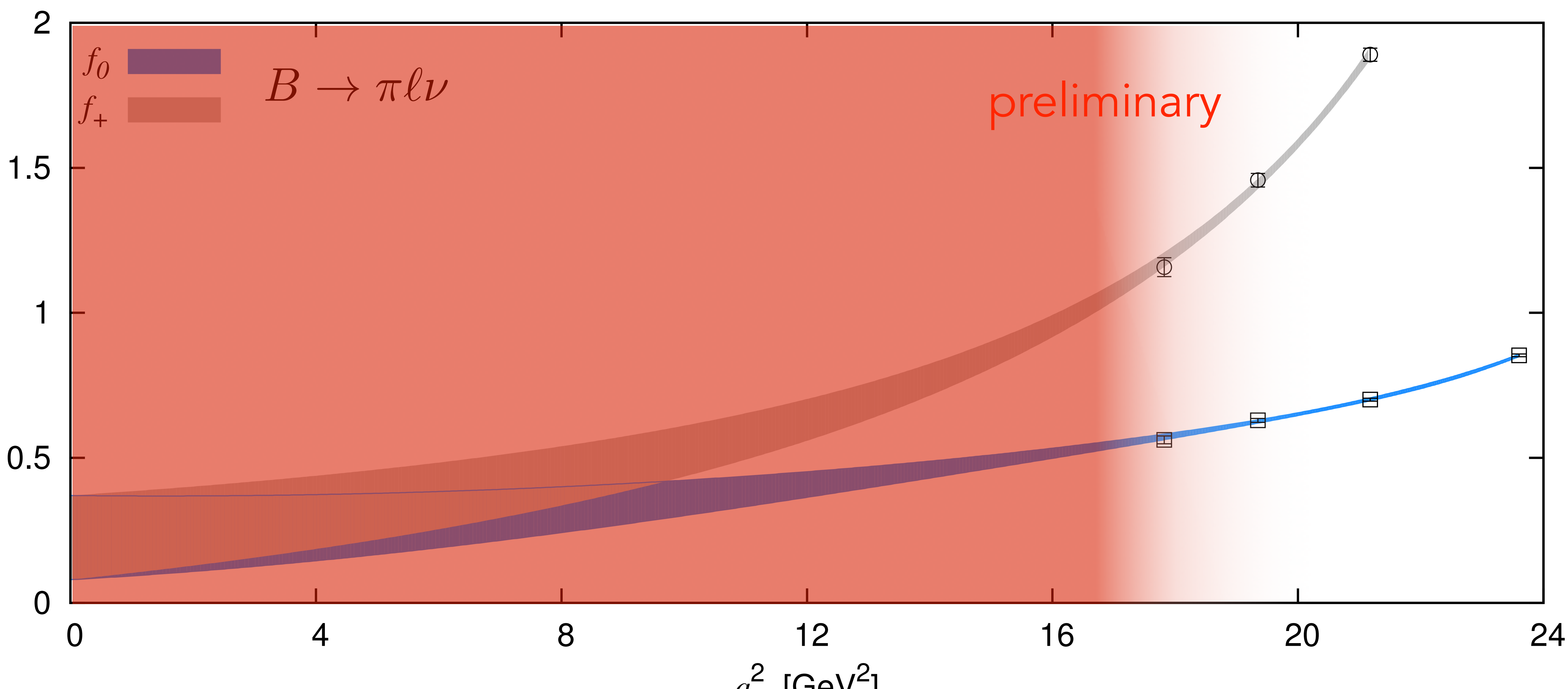


Du et al. (FNAL/MILC), PRD93 (2016) 034005

- Lattice uncertainties greatest at low q^2 from kinematic extrapolation
- on par with experiment for now, but would be good to improve this...

Problems:

- Growth in discretization errors with momentum
- Chiral Perturbation Theory valid only for $q^2 \gtrsim 17\ \mathrm{GeV}^2$



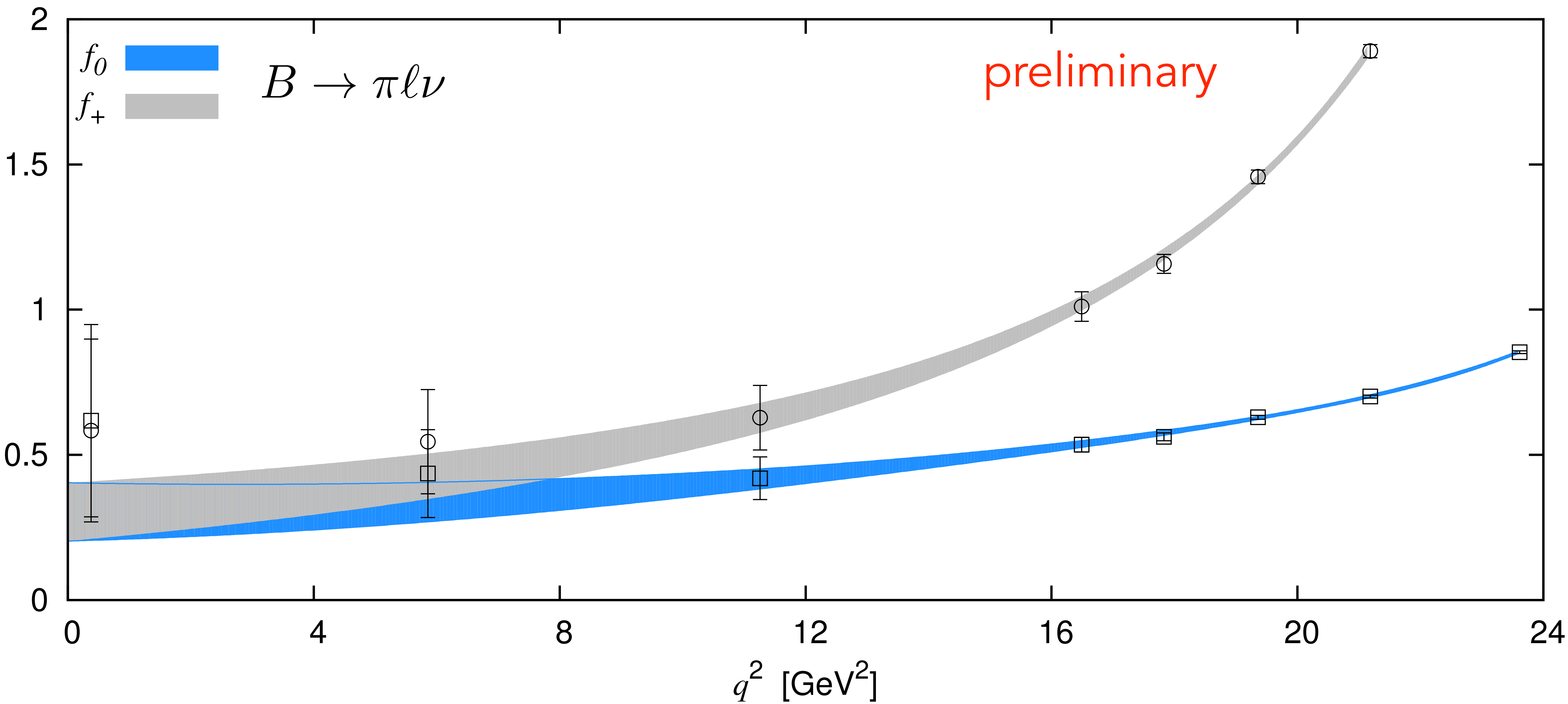
with HPQCD [underway]

Problems:

- Growth in discretization errors with momentum
- Chiral Perturbation Theory valid only for $q^2 \gtrsim 17\ \mathrm{GeV}^2$

Solutions:

- with HISQ quark action, observe $\sigma \sim 0.1 \times |a\vec{p}_\pi|^2$
- *Hard Pion* Chiral Perturbation Theory   Bijnens and Jemos, NPB 846 (2011) 145
- chiral physics and kinematics factorize
- HPChPT with z-expansion   with HPQCD, PRD 90 (2014) 054506



- improved uncertainty at low q^2
- greater confidence in kinematic extrapolation

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Look forward to...

- $B_{(s)} - \bar{B}_{(s)}$ matrix elements
  - next generation calculation underway by FNAL/MILC
  - HPQCD calculating Bs mixing, dim-7 operators for

- $B \to \pi$ form factors
  - HPQCD calculation underway, extending to $q^2=0$

- Ongoing work, including charm decays, by other collaborations
  - ETMC, RBC/UKQCD, ALPHA, ...

Thank you!

backup slides...

Basics of lattice QCD…

QCD correlation function: continuum PI, Euclidean, Berezin integration done

$$\langle X \rangle = \frac{\int dG \;X[G,(\not{D}+m)^{-1}]\; e^{-S[G]+\ln\det(\not{D}+m)}}{\int dG \; e^{-S[G]+\ln\det(\not{D}+m)}}$$

QCD correlation function: continuum PI, Euclidean, Berezin integration done

$$\langle X \rangle = \frac{\int dG \ X[G, (\not{D} + m)^{-1}] \ e^{-S[G] + \ln\det(\not{D}+m)}}{\int dG \ e^{-S[G]+\ln\det(\not{D}+m)}}$$

X a composite operator...

$$X = (\text{final state}) \times (\text{interaction}) \times (\text{initial state})^\dagger$$

$$= B(T) \times O_i(t) \times \bar{B}(0)^\dagger$$ B interpolating & 4-quark mixing operators

$$= (\bar{d}\gamma_5 b)_T \ (\bar{b}\Gamma_1 d \ \bar{b}\Gamma_2 d)_t \ (\bar{d}\gamma_5 b)_0$$ Wick contractions

$$= Tr[\gamma_5(\not{D} + m_b)^{-1}_{t,T} \ \Gamma_1 \ (\not{D} + m_d)^{-1}_{T,t}] \ Tr[\gamma_5(\not{D} + m_b)^{-1}_{t,0} \ \Gamma_2 \ (\not{D} + m_d)^{-1}_{0,t}]$$

Numerically evaluate

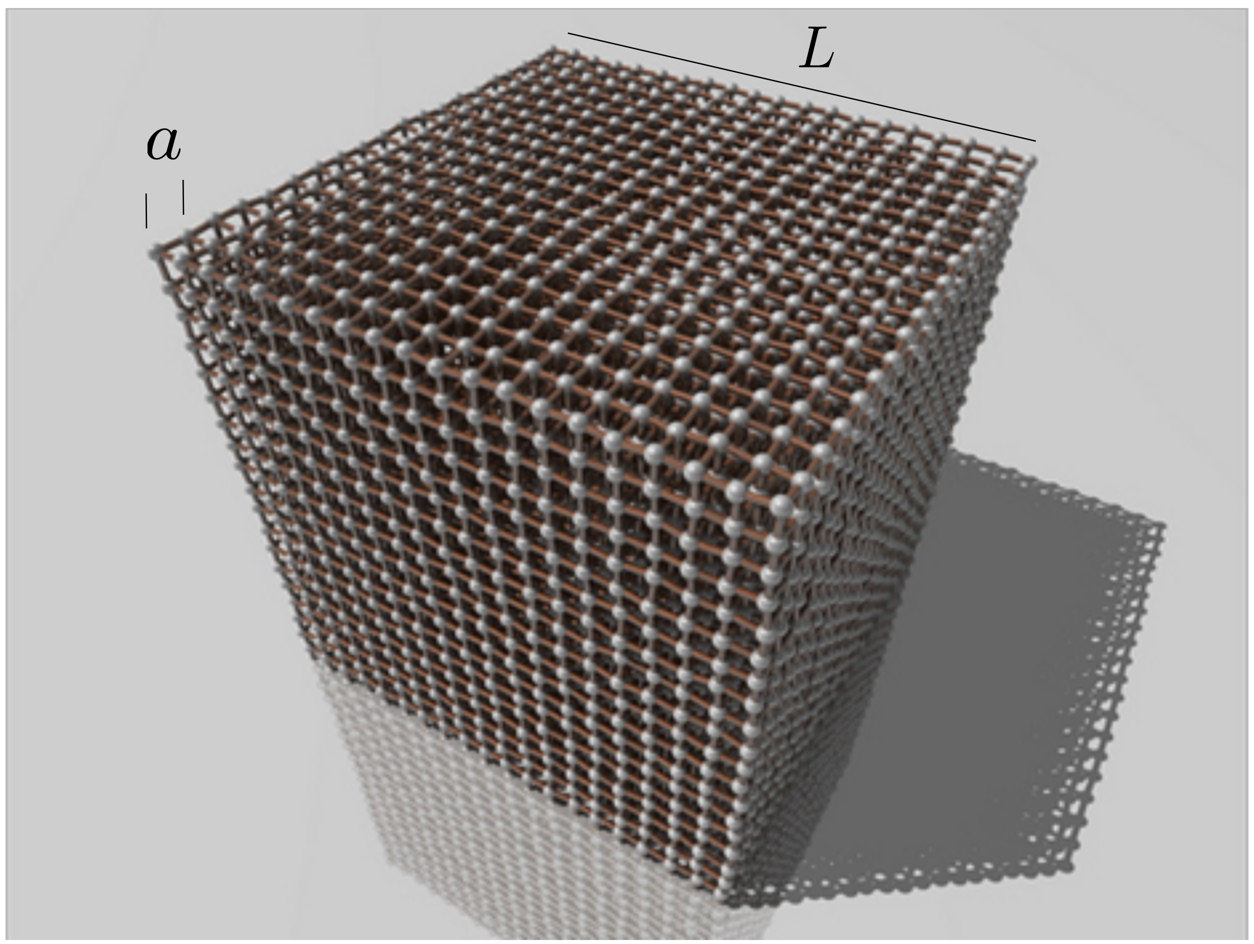
$$\langle X \rangle = \frac{\int dG \; X[G, (\not{D}+m)^{-1}] \; e^{-S[G]+\ln\det(\not{D}+m)}}{\int dG \; e^{-S[G]+\ln\det(\not{D}+m)}}$$

discretize S and $\not{D}$

- $\displaystyle\int d^4x \to \sum_{x,y,z,t}$
- $\partial_\mu f(x) = \dfrac{f(x+a) - f(x)}{a}$
- and much more...

Numerically evaluate

$$\langle X \rangle = \frac{\int dG \; X[G, (\not{D}+m)^{-1}] \; e^{-S[G]+\ln\det(\not{D}+m)}}{\int dG \; e^{-S[G]+\ln\det(\not{D}+m)}}$$



importance sampling

generate {Gn} with probability distribution

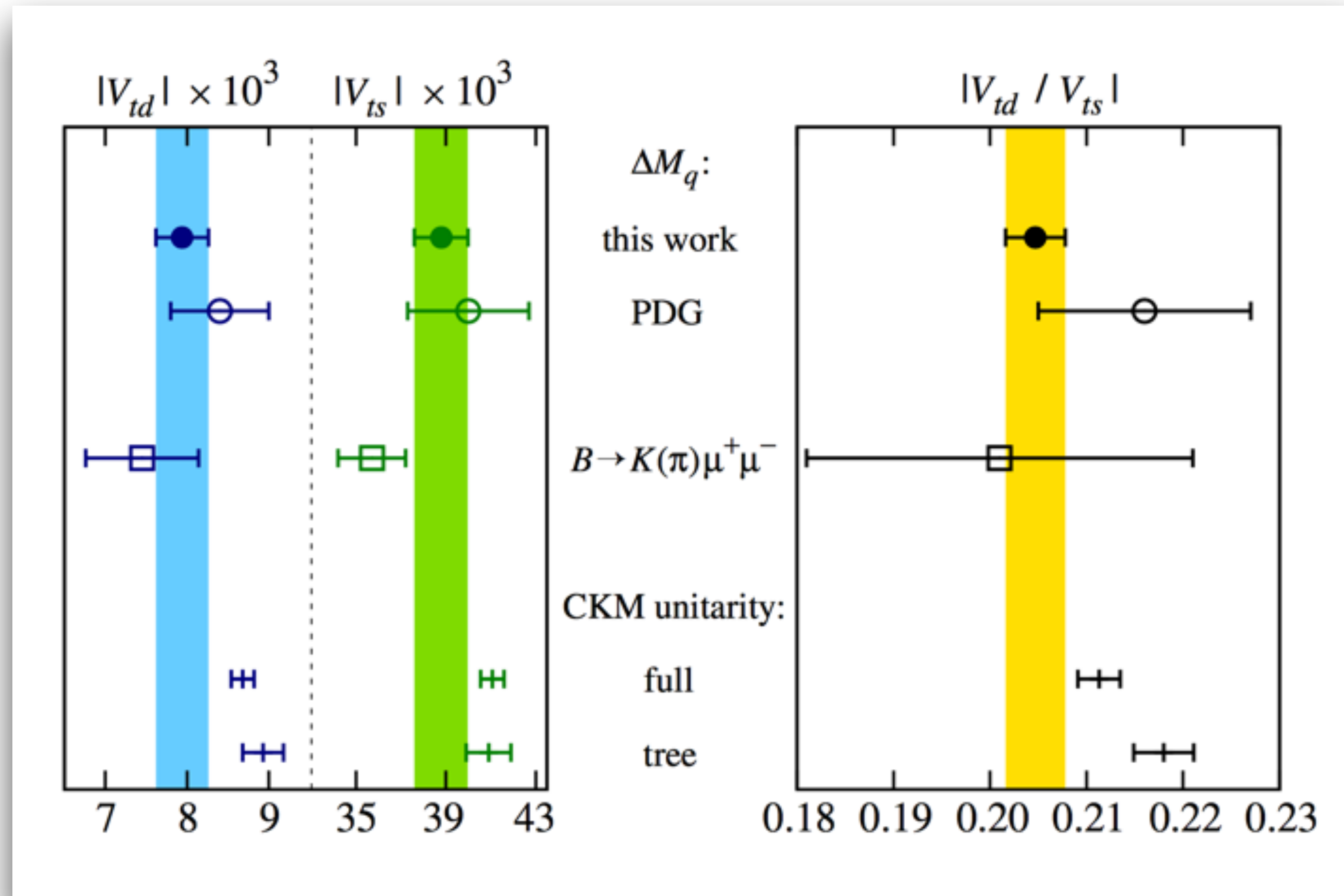
$$e^{-S[G]+\ln\det(\not{D}+m)}$$

$$\langle X \rangle = \frac{1}{N}\sum_{n=1}^{N} X[G_n, (\not{D}+m)_n^{-1}]$$

More on B mixing...

Extract V from combination with experiment.

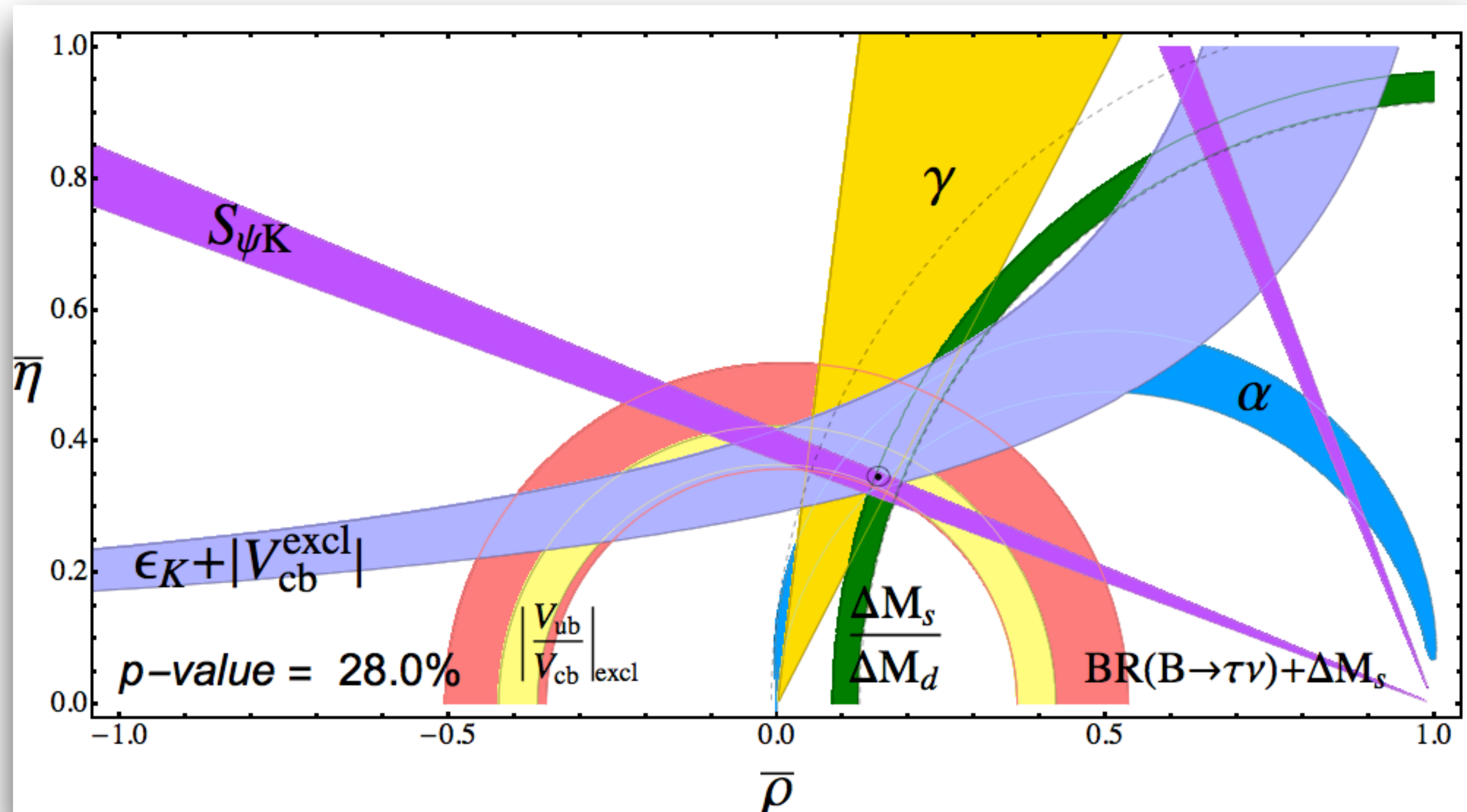
$$\Delta M = C(V) \langle B^0 | \mathcal{O} | \bar{B}^0 \rangle$$



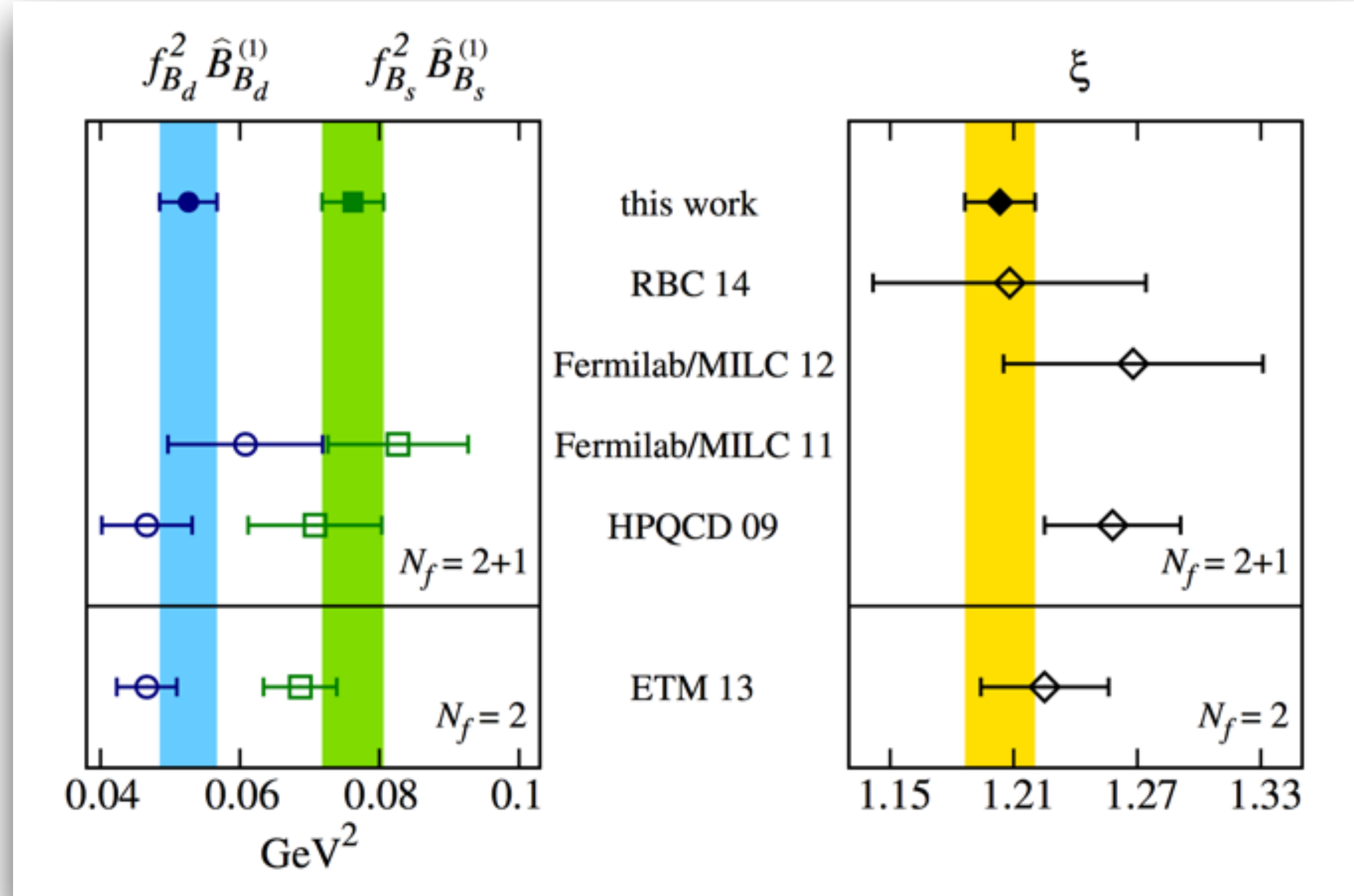
Bazavov et al. (FNAL/MILC), PRD93 (2016) 113016

Extract V from combination with experiment.

$$\Delta M = C(V)\,\langle B^0|\mathcal{O}|\bar{B}^0\rangle$$



Bazavov et al. (FNAL/MILC), PRD93 (2016) 113016



Bazavov et al. (FNAL/MILC), PRD93 (2016) 113016

- $f^2\hat{B}$ parametrizes the matrix element
- ξ is proportional to V_{td} / V_{ts}

... match lattice results to continuum scheme and scale:

$$\langle \mathcal{O}_i \rangle(\mu) \;=\; Z_{ij}(a\mu)\,\langle O_i \rangle(1/a) \quad + \quad \mathrm{O}(\alpha_s^2, \alpha_s a \Lambda_{\mathrm{QCD}})$$

- continuum $\overline{\text{MS}}$-NDR, at scale μ : $\langle \mathcal{O}_i \rangle(\mu)$
- "mostly nonperturbative" matching coefficients:

$$Z_{ij} = Z_{V^4_{bb}} Z_{V^4_{qq}} [\delta_{ij} + \alpha_s \zeta^{[1]}_{ij} + \mathrm{O}(\alpha_s^2)]$$

- lattice, at scale $1/a$: $\langle O_i \rangle(1/a)$

… extrapolate to continuum and physical quark masses:

rooted, staggered, heavy meson SU(3) chiral perturbation theory

Bernard, PRD87 (2013) 114503

$$\langle \mathcal{O}_i \rangle = \beta_i + \beta_j [\chi \, \mathrm{logs}]_{ij} + [\mathrm{analytic}]_i$$

$$+ [m_b \text{ tuning}]_i + [am_b \text{ effects}]_i$$

$$+ \beta_j [\mathrm{O}(\alpha_s^2) \text{ matching effects}]_{ij}$$

- generic & light quark discretization and chiral extrapolation
- heavy quark tuning and discretization
- higher order matching effects

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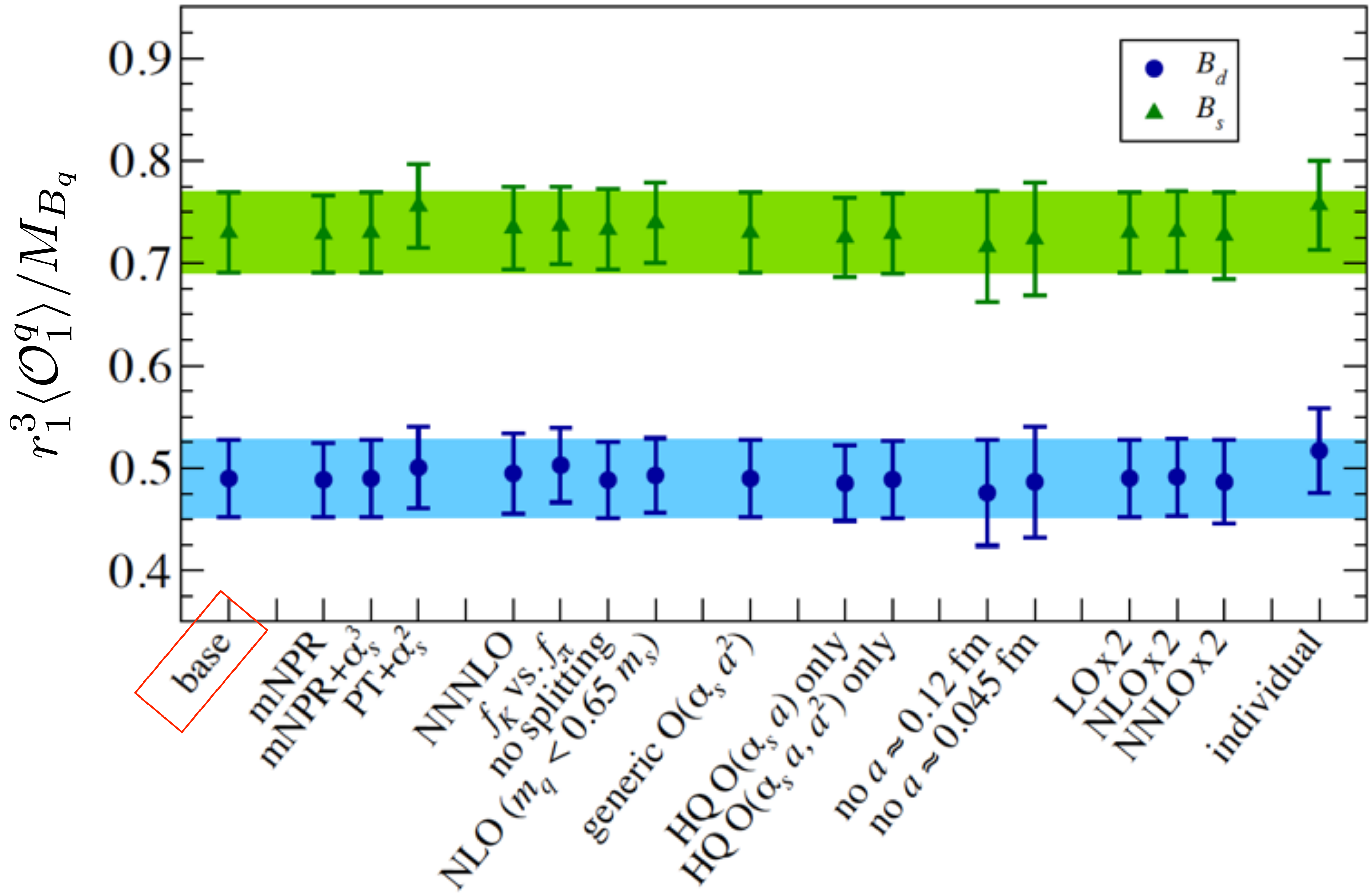
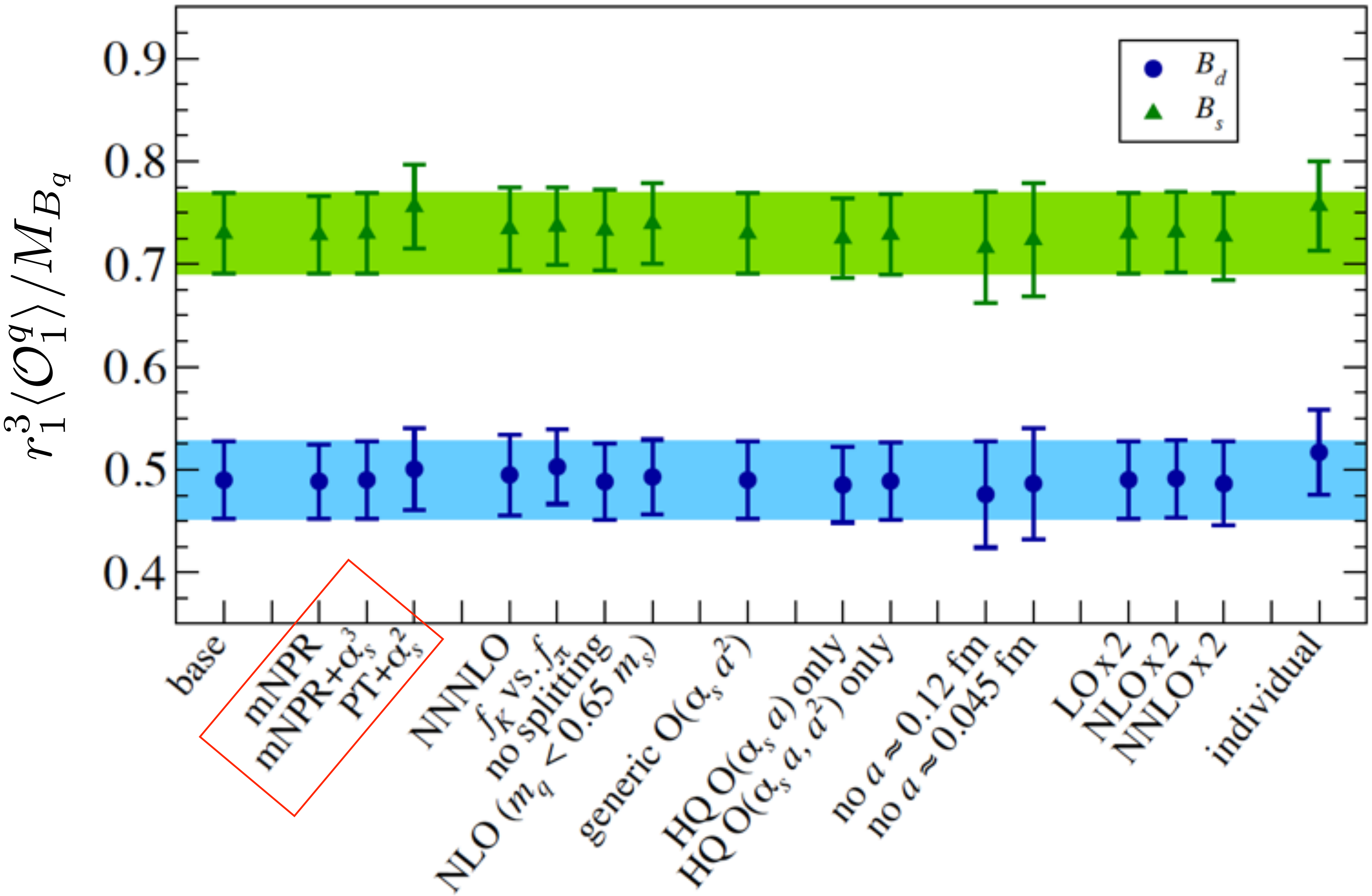
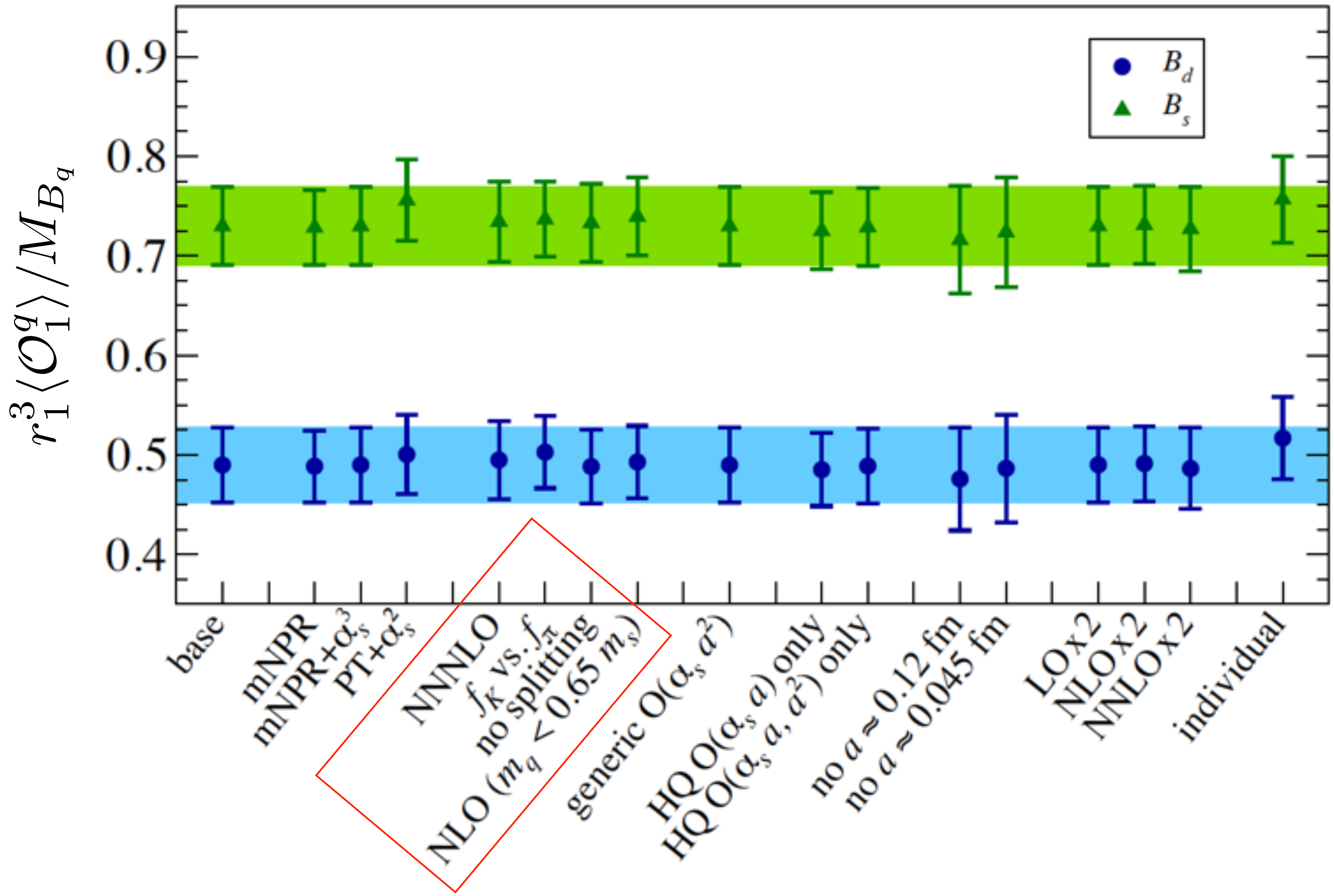


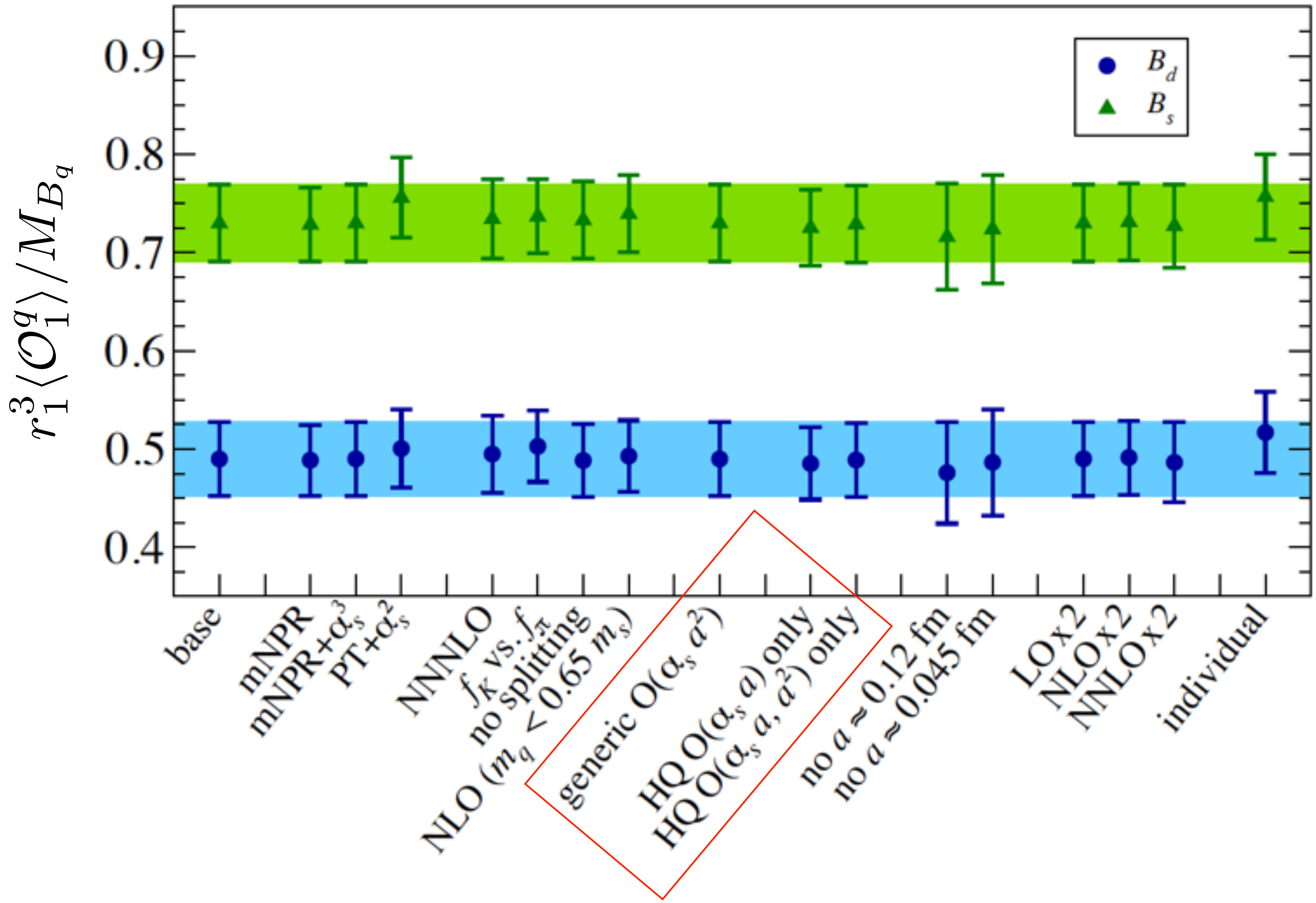
- mNPR (mostly nonperturbative renormalization) matching with O(α_s^2)
- chiral logs with analytic terms through NNLO
- HQ (heavy quark) discretization through O($\alpha_s a, a^2, a^3$)
- simultaneous fit to all 5 matrix elements



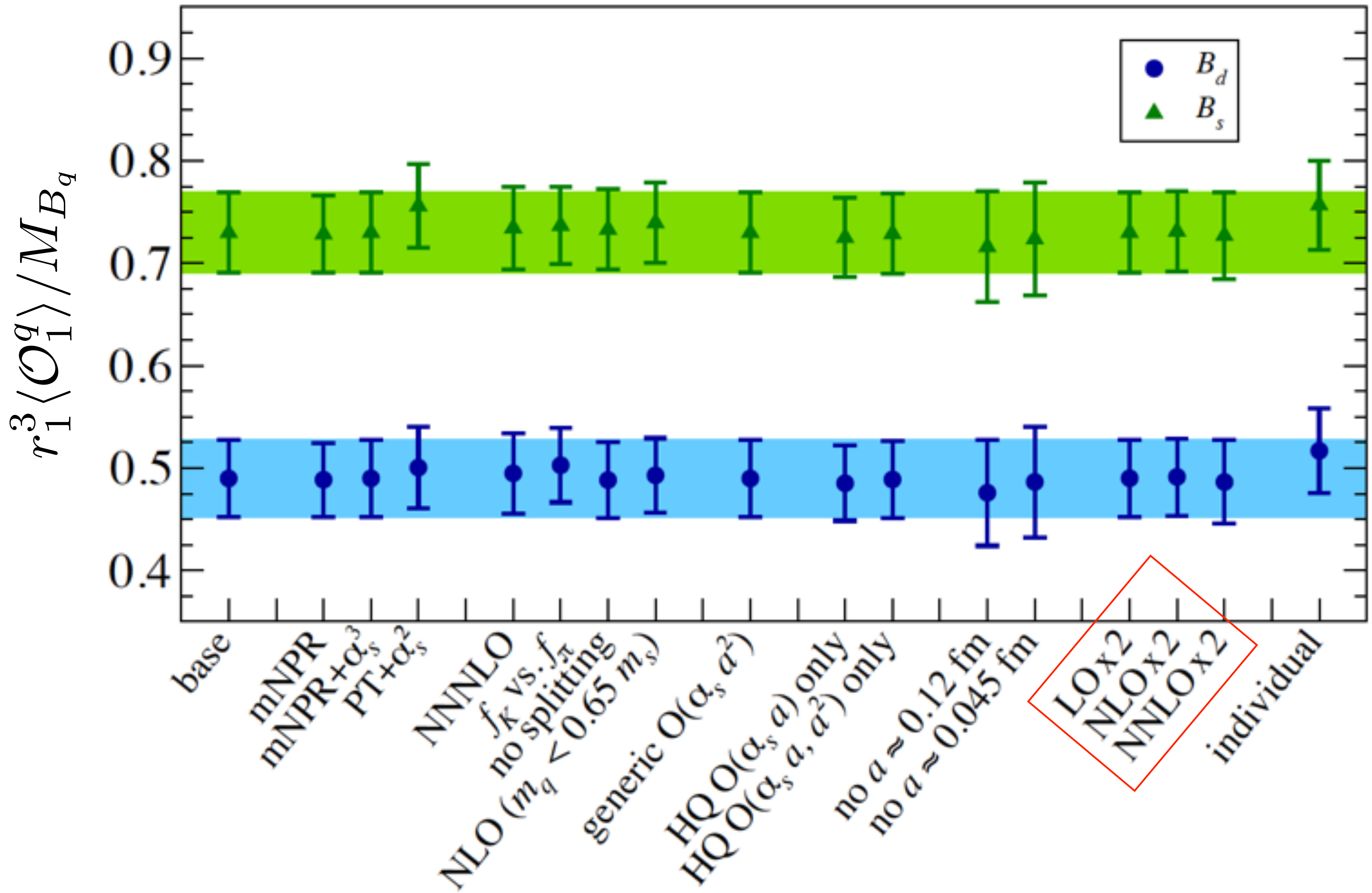
vary how the matching is done



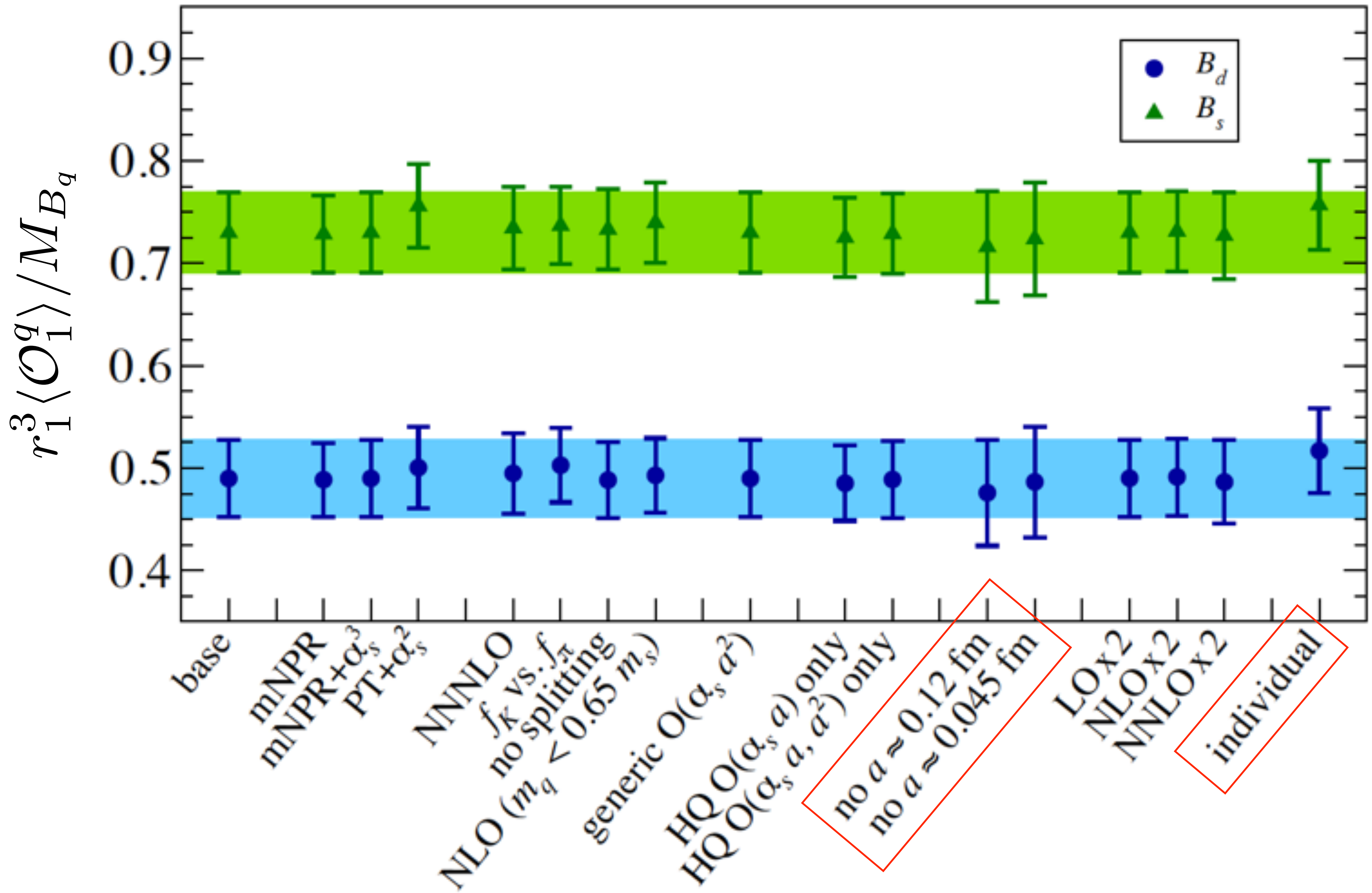
variations in chiral perturbation theory



vary accommodation of discretization effects



reduce precision of *a priori* information



study impact of

- data with largest lattice spacing
- data with finest lattice spacing
- simultaneous fit to all 5 operators