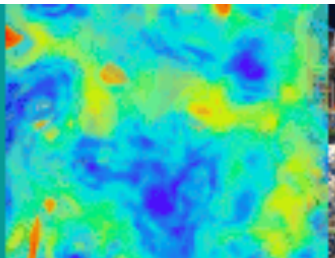
Leptonic flavour mixing in gauged SO(3)

Ye-Ling Zhou, Southampton U., 6 July 2019

Spontaneous breaking of $SO(3)$ to finite family symmetries with supersymmetry - an A_4 model
S.F. King, **YLZ**, arXiv:1809.10292

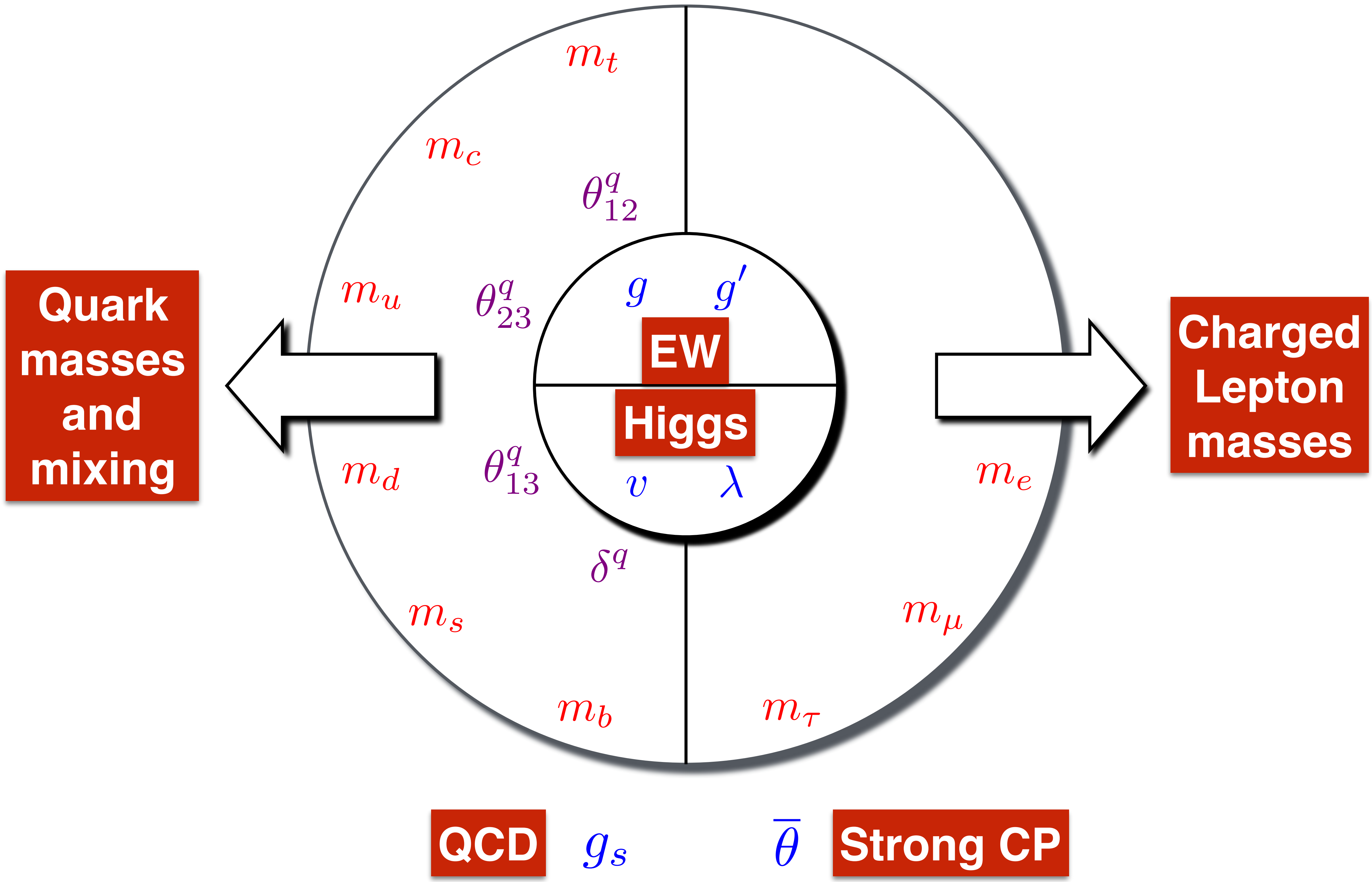
SHEP Southampton High Energy Physics



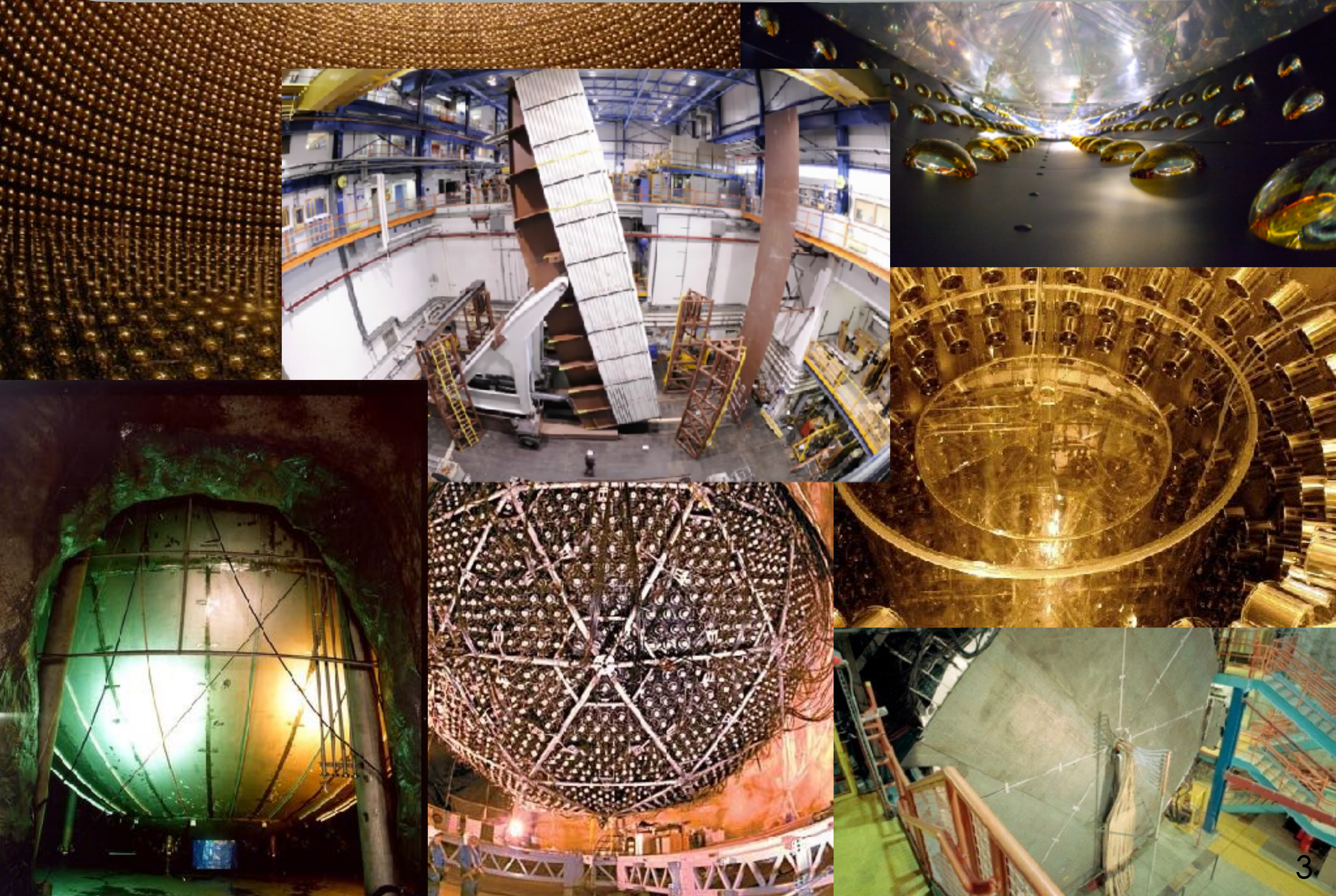
Science & Technology Facilities Council

The Stardard Model (SM)

17+2 free parameters



Neutrino oscillations confirmed neutrino masses and lepton mixing



Neutrino masses and lepton mixing

Flavour eigenstates

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Mass eigenstates

Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \begin{bmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{bmatrix} \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

Atmospheric **Reactor** **Solar** **(Majorana)**

Neutrino masses and lepton mixing

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

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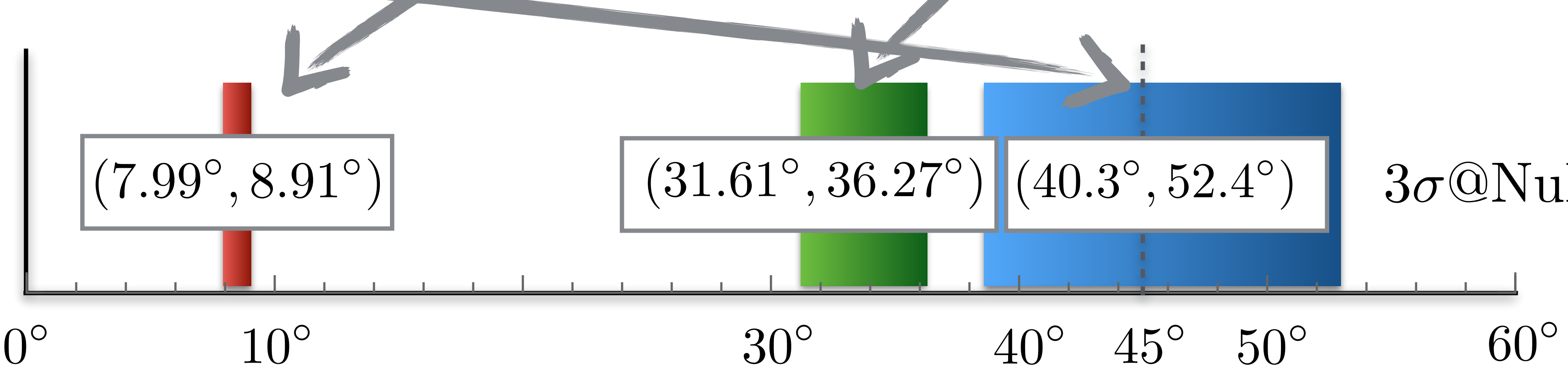
Atmospheric **Reactor** **Solar** **Majorana**

Global fits of oscillation data

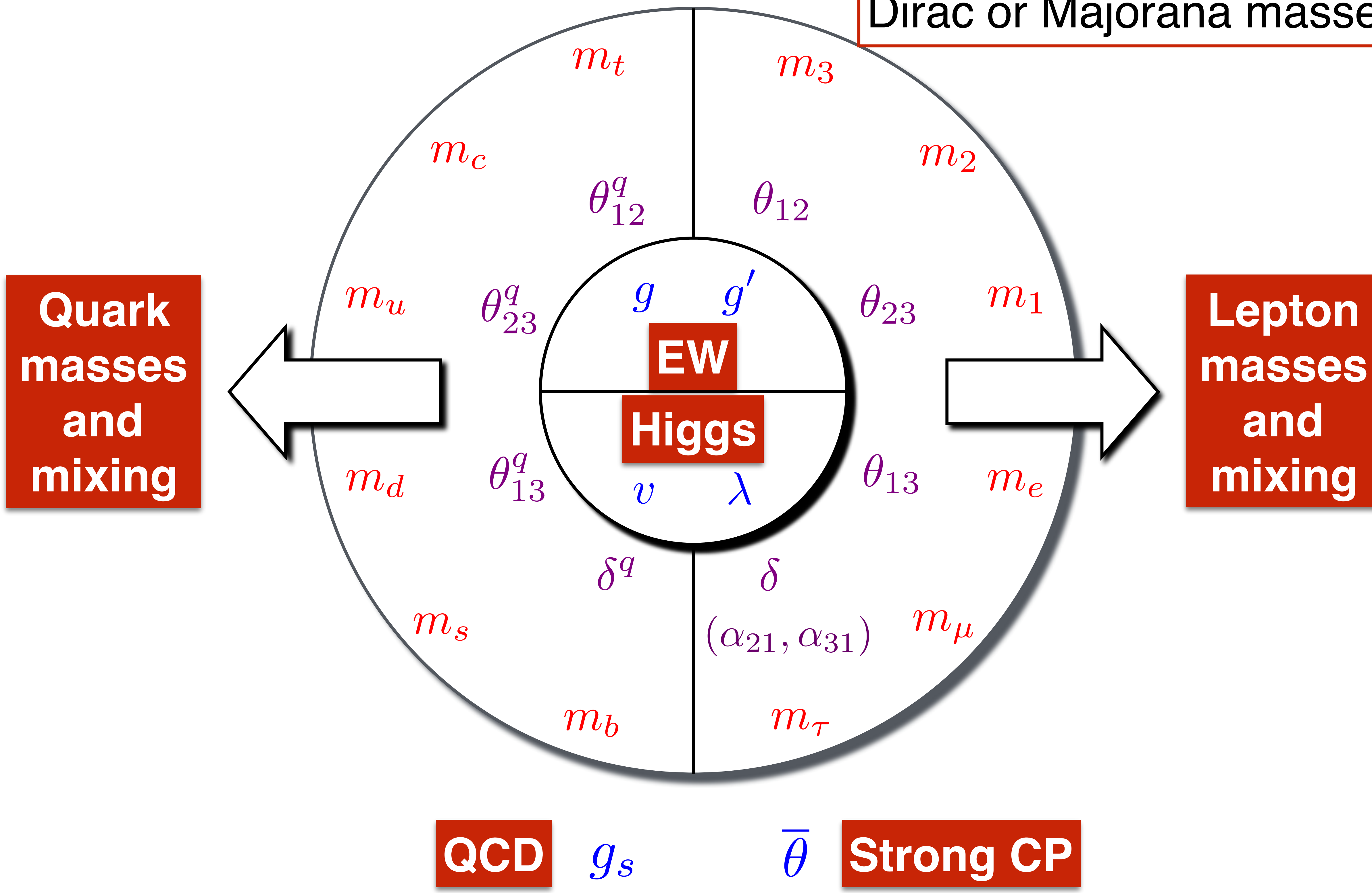


3σ@NuFIT

SM + massive neutrinos

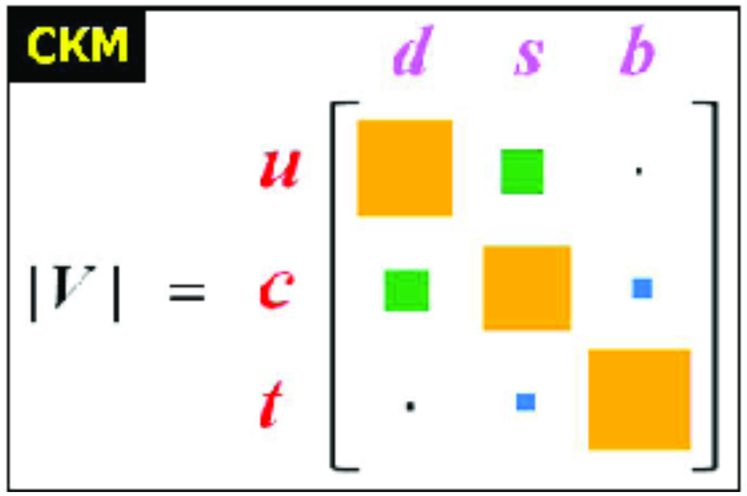
24(26)+2 free parameters

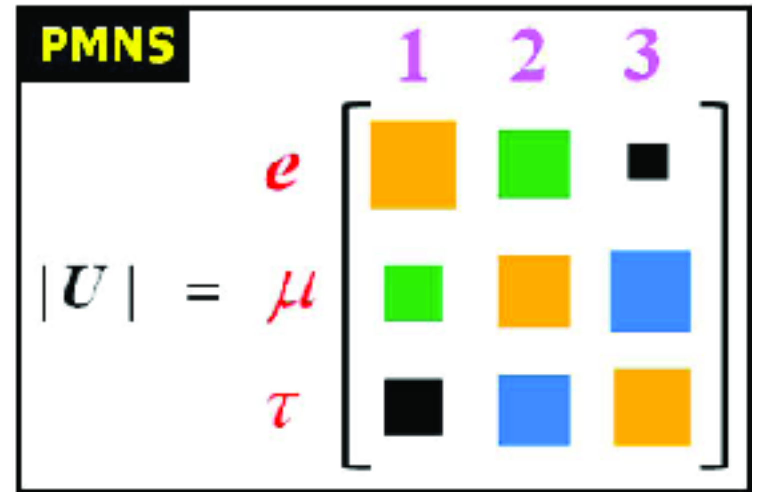
Neutrino may take
Dirac or Majorana masses



Why we need a flavour symmetry (FS)?

- To explain large mixing in the lepton sector





- Tri-bimaximal (TBM) mixing

$$|U| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{13} = 0$$
$$\sin^2\theta_{12} = 1/3$$
$$\sin^2\theta_{23} = 1/2$$

Tri-bimaximal mixing and the neutrino oscillation data
P.F. Harrison (Queen Mary, U. of London), D.H. Perkins (Oxford U.), W.G. Scott (
Published in **Phys.Lett. B530 (2002) 167**
RAL-TR-2002-002
DOI: 10.1016/S0370-2693(02)01336-9
e-Print: **hep-ph/0202074** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 1414 records 1000+

Nearly tri bimaximal neutrino mixing and CP violation
Zhi-zhong Xing (Beijing, Inst. High Energy Phys.). Apr 2002. 10 pp.
Published in **Phys.Lett. B533 (2002) 85-93**
BIHEP-TH-2002-13
DOI: 10.1016/S0370-2693(02)01649-0
e-Print: **hep-ph/0204049** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
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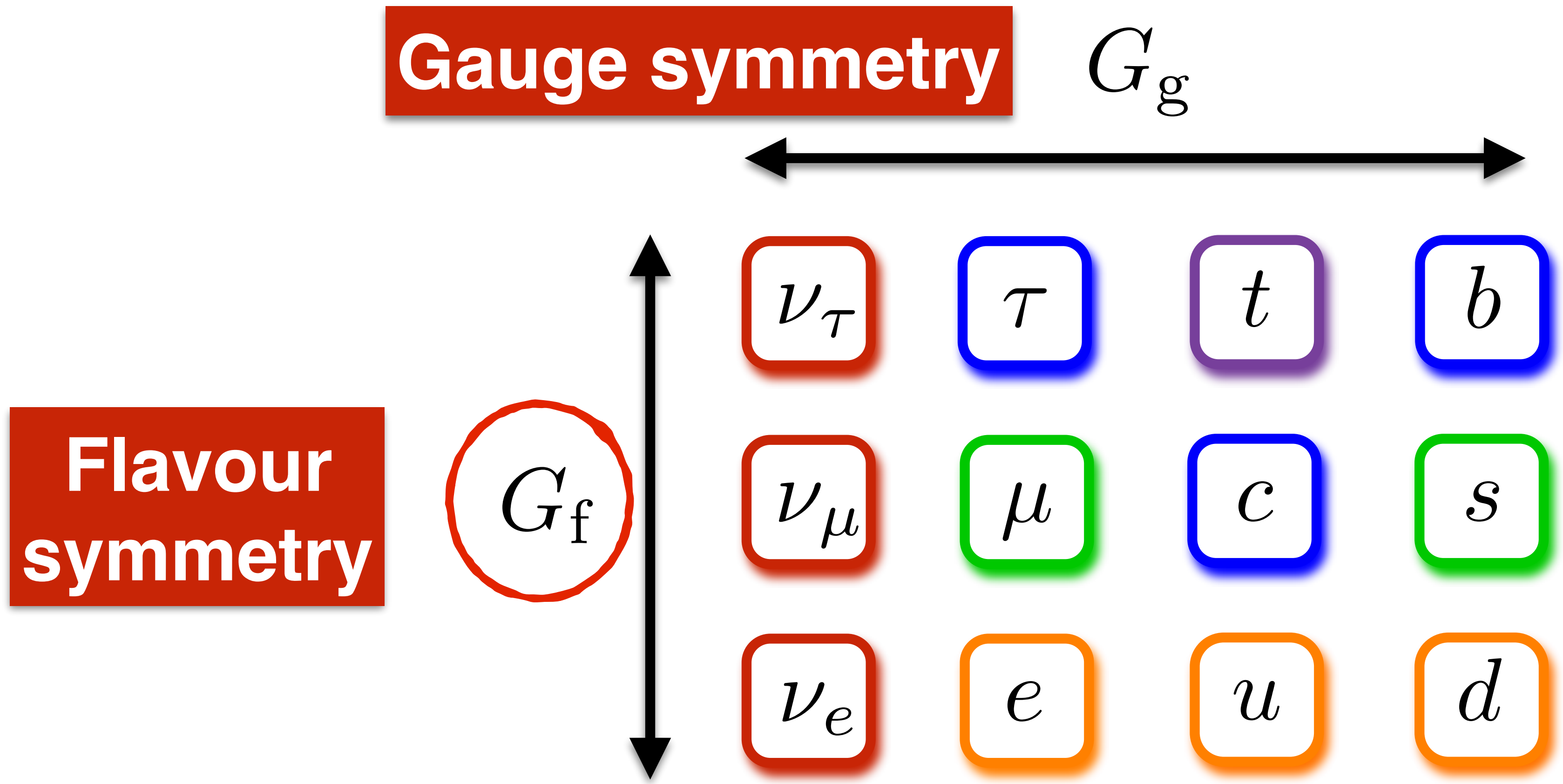
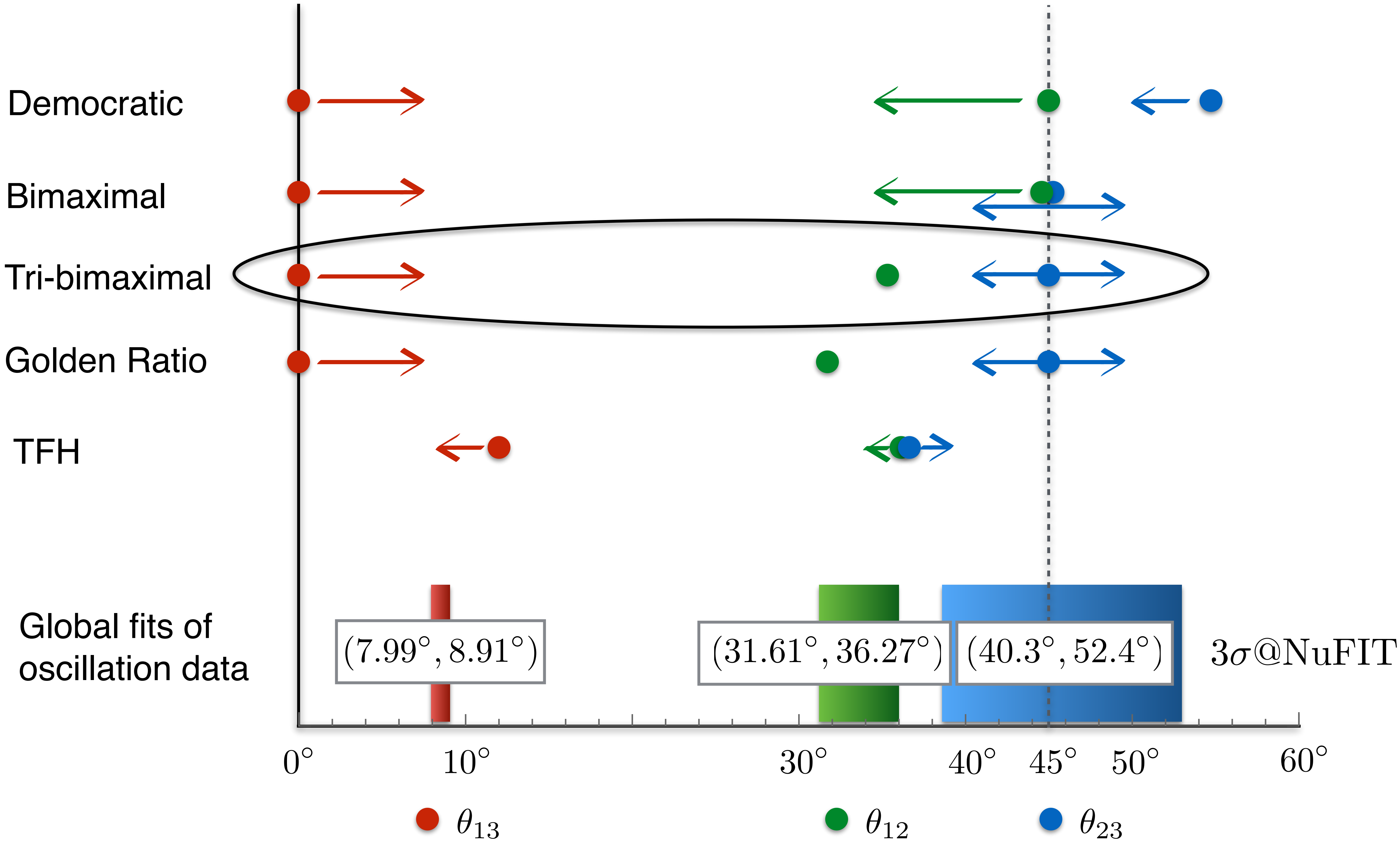
What is the flavour symmetry?



	Continuous	Discrete
Abelian	U(1)	Z_n
Non-Abelian	SU(3), SO(3),…	A_4, S_4, T’, A_5, Δ(48), …

Powerful predictions of non-Abelian discrete flavour symmetries

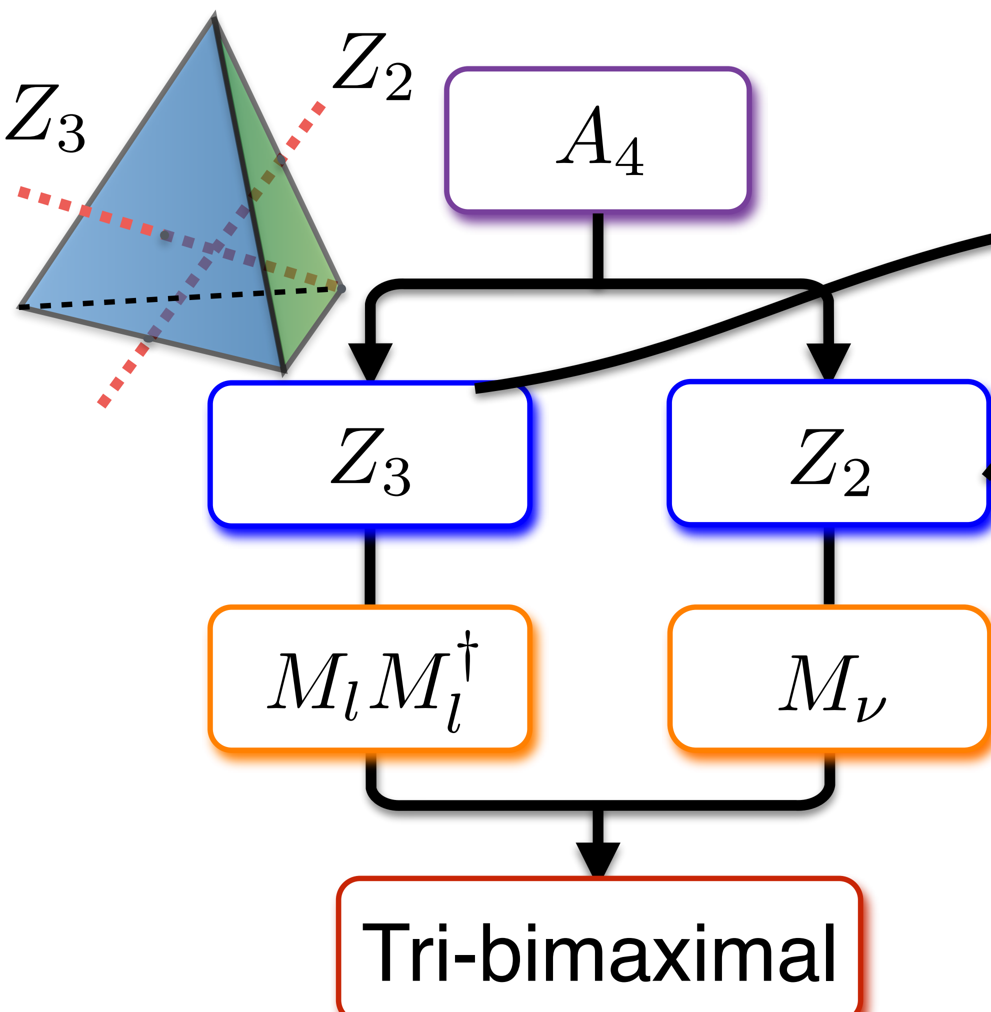


Symmetries vs mixing patterns

Sketch between A_4 and TBM

$$|U_{\rm TBM}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin\theta_{13} = 0$$
$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$
$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

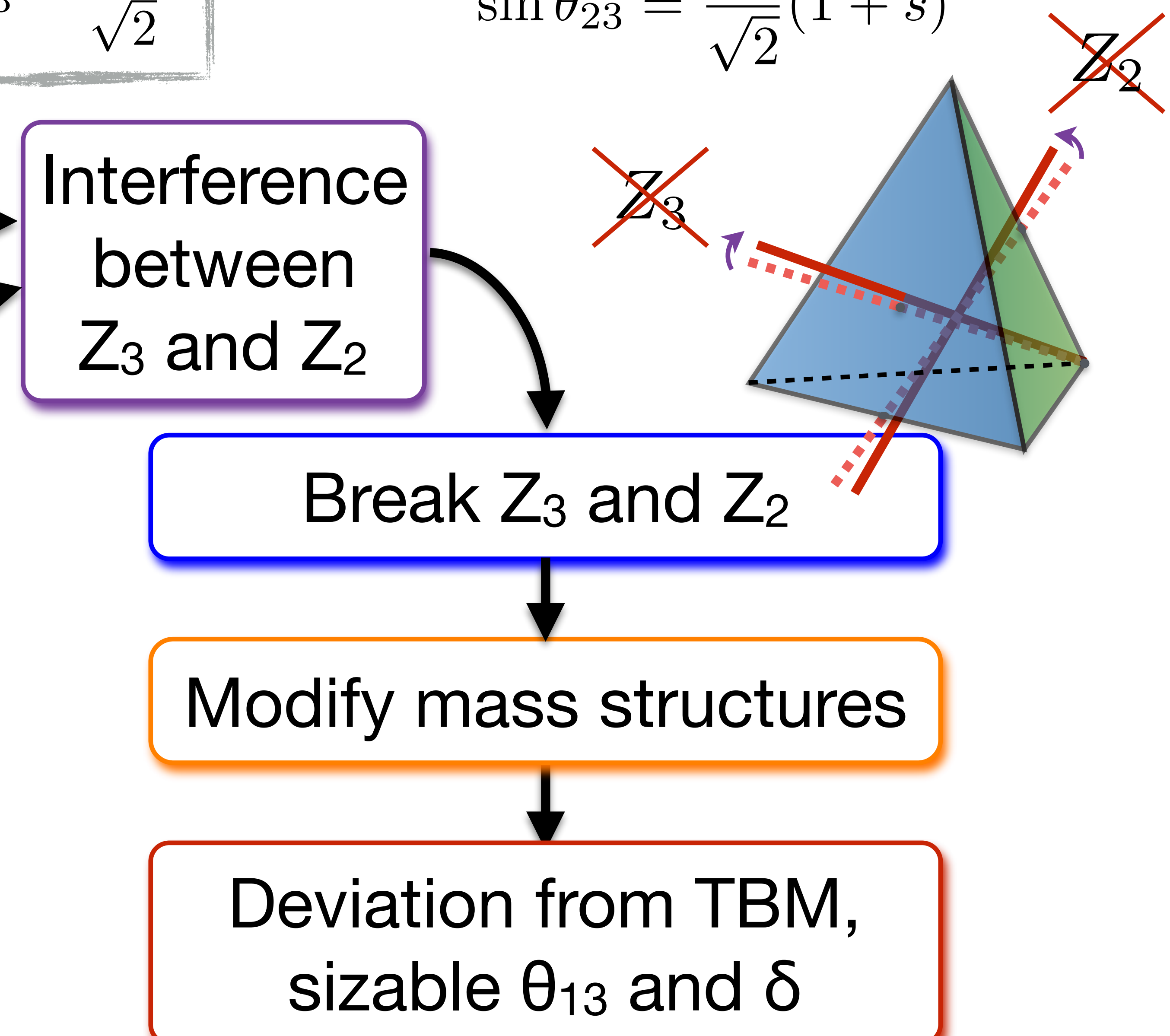


Altarelli, Feruglio,
hep-ph/0504165; 0512103

Necessary corrections

$$U = U_{\rm TBM} + \delta U$$

$$\sin\theta_{13} = \frac{r}{\sqrt{2}} \qquad \sin\theta_{12} = \frac{1}{\sqrt{3}}(1+a)$$
$$\sin\theta_{23} = \frac{1}{\sqrt{2}}(1+s)$$

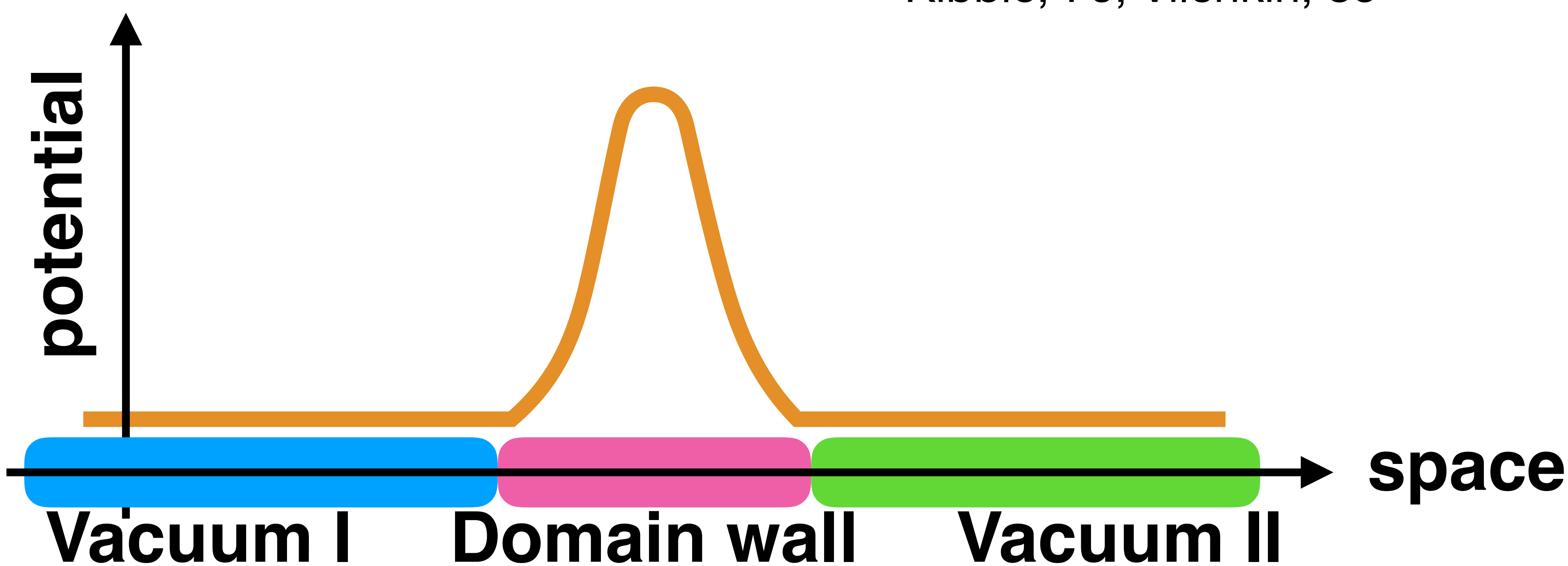


Origin of non-Abelian discrete symmetries

- A fundamental symmetry?
- A consequence of more fundamental physics?

Domain wall problem

Zeldovich, Kobzarev, Okun, 74; Kibble, 76; Vilenkin, 85



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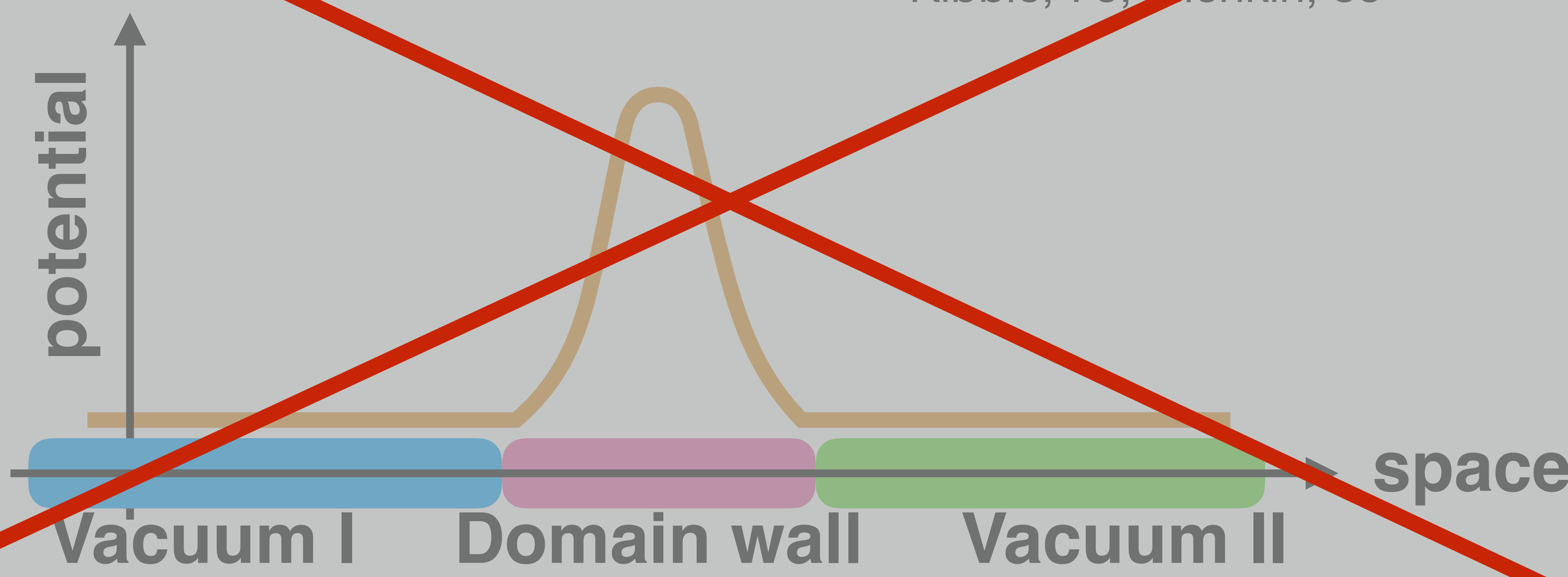
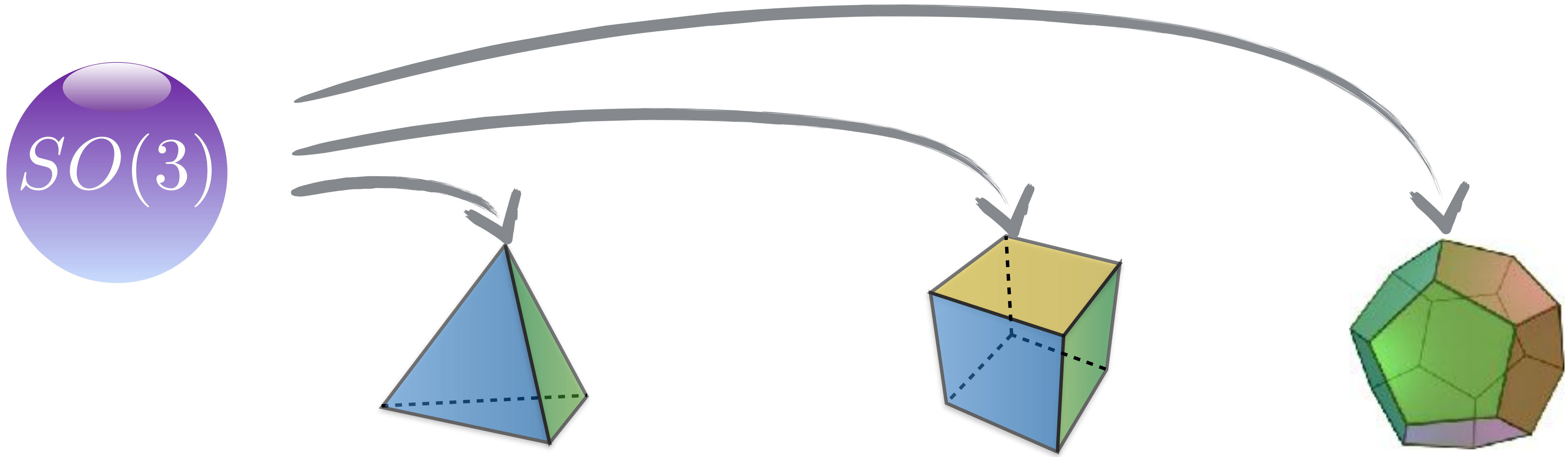


SO(3)→A_4, S_4 and A_5



$SO(3)$	A_4	S_4	A_5
$\underline{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\underline{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$\underline{5}$	$\mathbf{1'} + \mathbf{1''} + \mathbf{3}$	$\mathbf{2} + \mathbf{3'}$	$\mathbf{5}$
$\underline{7}$	$\mathbf{1} + \mathbf{3} + \mathbf{3}$	$\mathbf{1'} + \mathbf{3} + \mathbf{3'}$	$\mathbf{3'} + \mathbf{4}$
$\underline{9}$	$\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3}$	$\mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3'}$	$\mathbf{4} + \mathbf{5}$
$\underline{11}$	$\mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3} + \mathbf{3}$	$\mathbf{2} + \mathbf{3} + \mathbf{3} + \mathbf{3'}$	$\mathbf{3} + \mathbf{3'} + \mathbf{5}$
$\underline{13}$	$\mathbf{1} + \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3} + \mathbf{3}$	$\mathbf{1} + \mathbf{1'} + \mathbf{2} + \mathbf{3} + \mathbf{3'} + \mathbf{3'}$	$\mathbf{1} + \mathbf{3} + \mathbf{4} + \mathbf{5}$

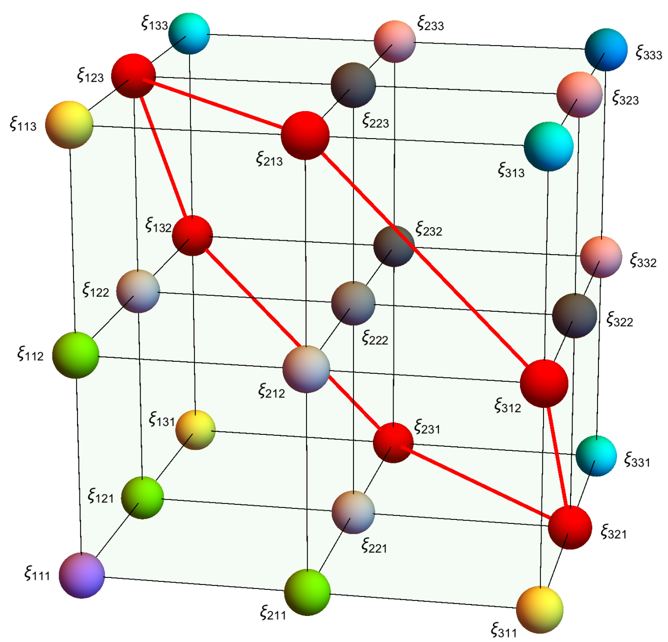
Ovrut, 77; Etesi, 9706029; Berger and Grossman, 0910.4392

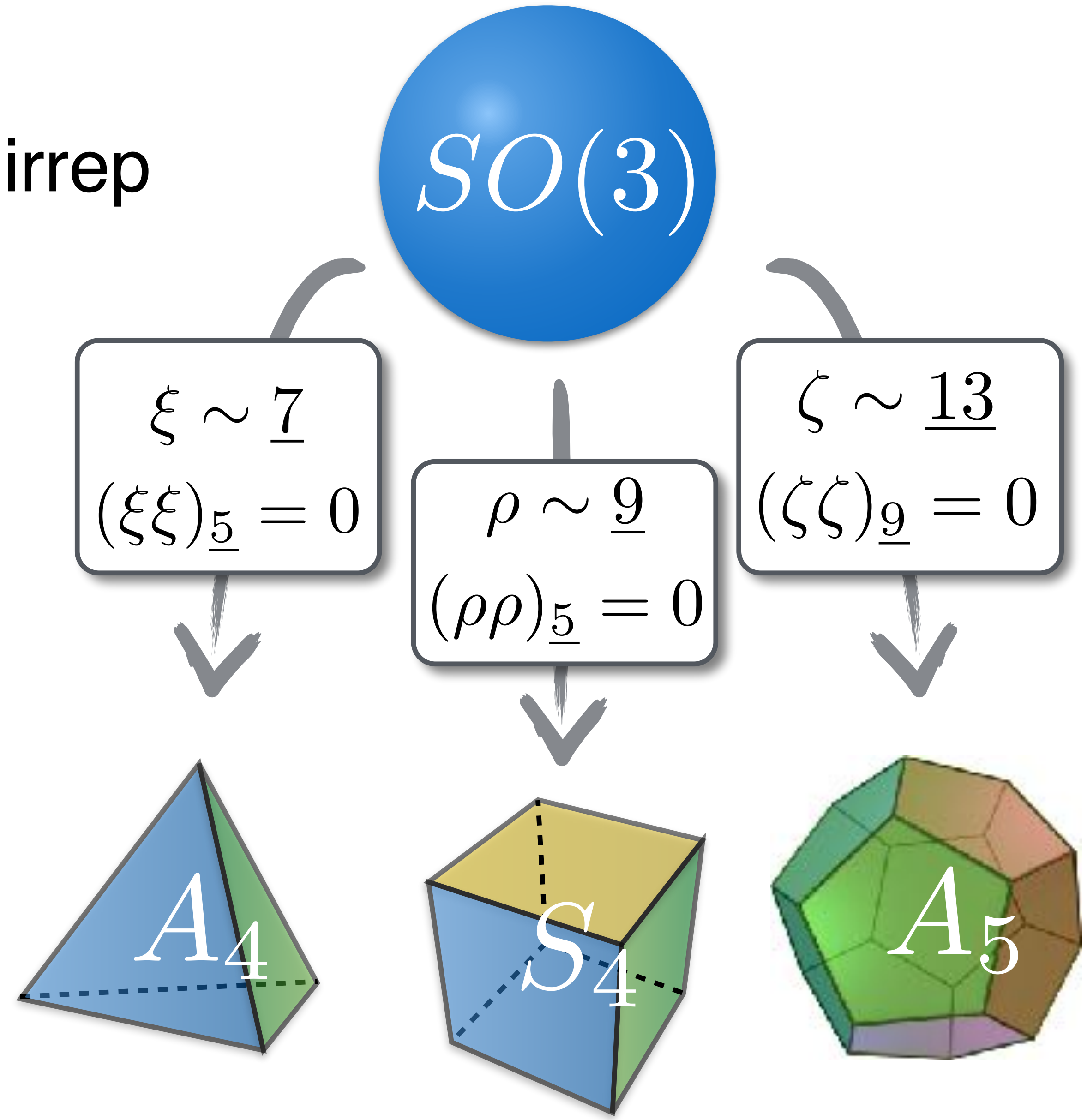
SU(3)→A_4 e.g., Luhn, 1101.2417; Merle, Zwicky, 1110.4891

SO(3) as origin of discrete symmetries

- **How to realise it?**
  ——using high dimensional irrep

$$\xi_{ijk} \sim \underline{7} =$$





- **For the first time, we realised it in SUSY with the help of flat direction**

King, **YLZ**, 1809.10292

A_4 breaking to Z_3 and Z_2

- One way (not unique) to breaking A_4 to Z_3 and Z_2

$$\varphi \sim \underline{3} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \qquad \chi_{ij} \sim \underline{5} = \begin{pmatrix} \frac{1}{\sqrt{3}}(\chi' + \chi'') & \frac{1}{\sqrt{2}}\chi_3 & \frac{1}{\sqrt{2}}\chi_2 \\ \frac{1}{\sqrt{2}}\chi_3 & \frac{1}{\sqrt{3}}(\omega\chi' + \omega^2\chi'') & \frac{1}{\sqrt{2}}\chi_1 \\ \frac{1}{\sqrt{2}}\chi_2 & \frac{1}{\sqrt{2}}\chi_1 & \frac{1}{\sqrt{3}}(\omega^2\chi' + \omega\chi'') \end{pmatrix}$$

$A_4 \rightarrow Z_3$

$$(\xi(\varphi\varphi)_{\underline{5}})_{\underline{5}} = 0$$

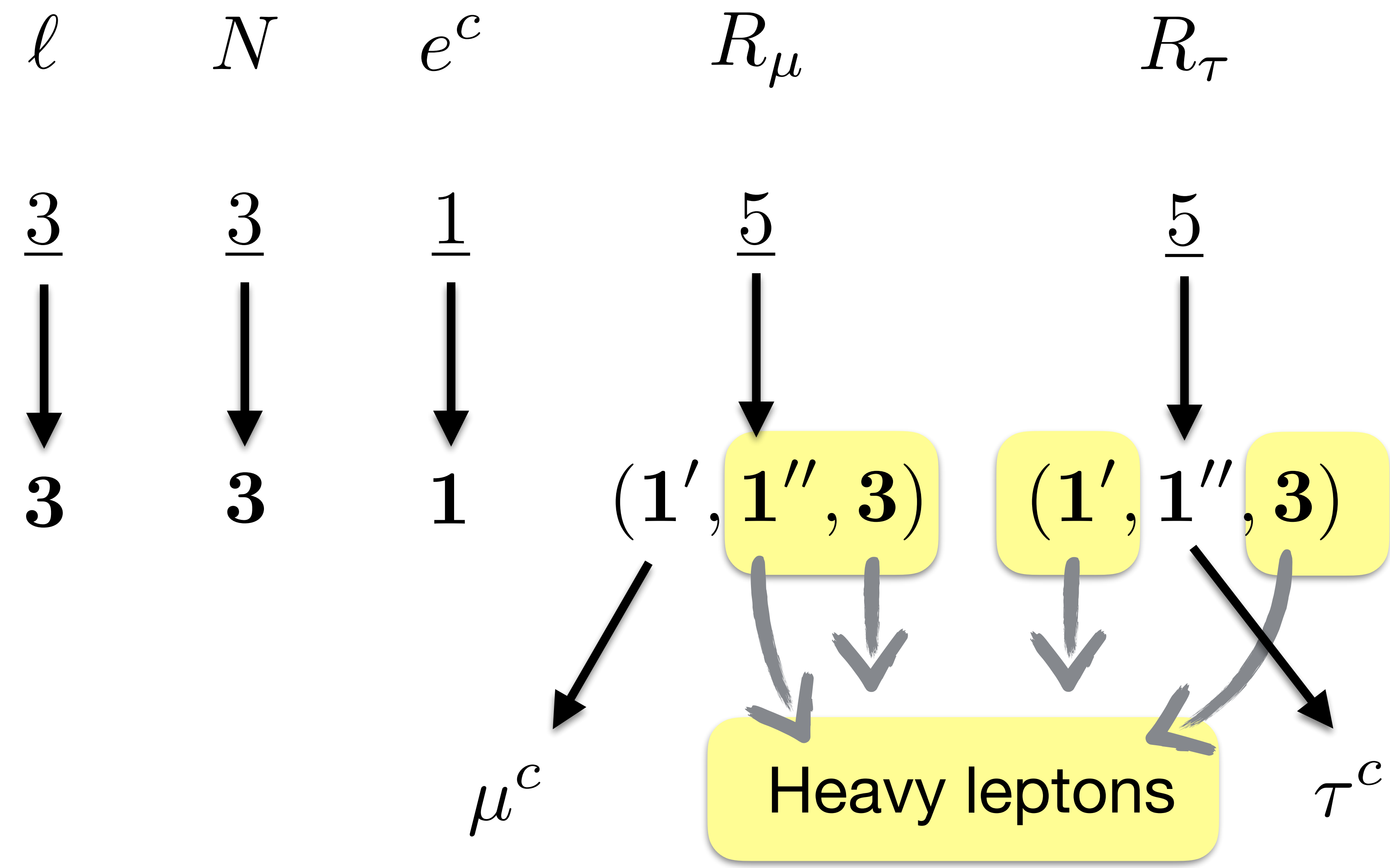
$$\begin{pmatrix} \langle\varphi_1\rangle \\ \langle\varphi_2\rangle \\ \langle\varphi_3\rangle \end{pmatrix} = \pm v_\varphi \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

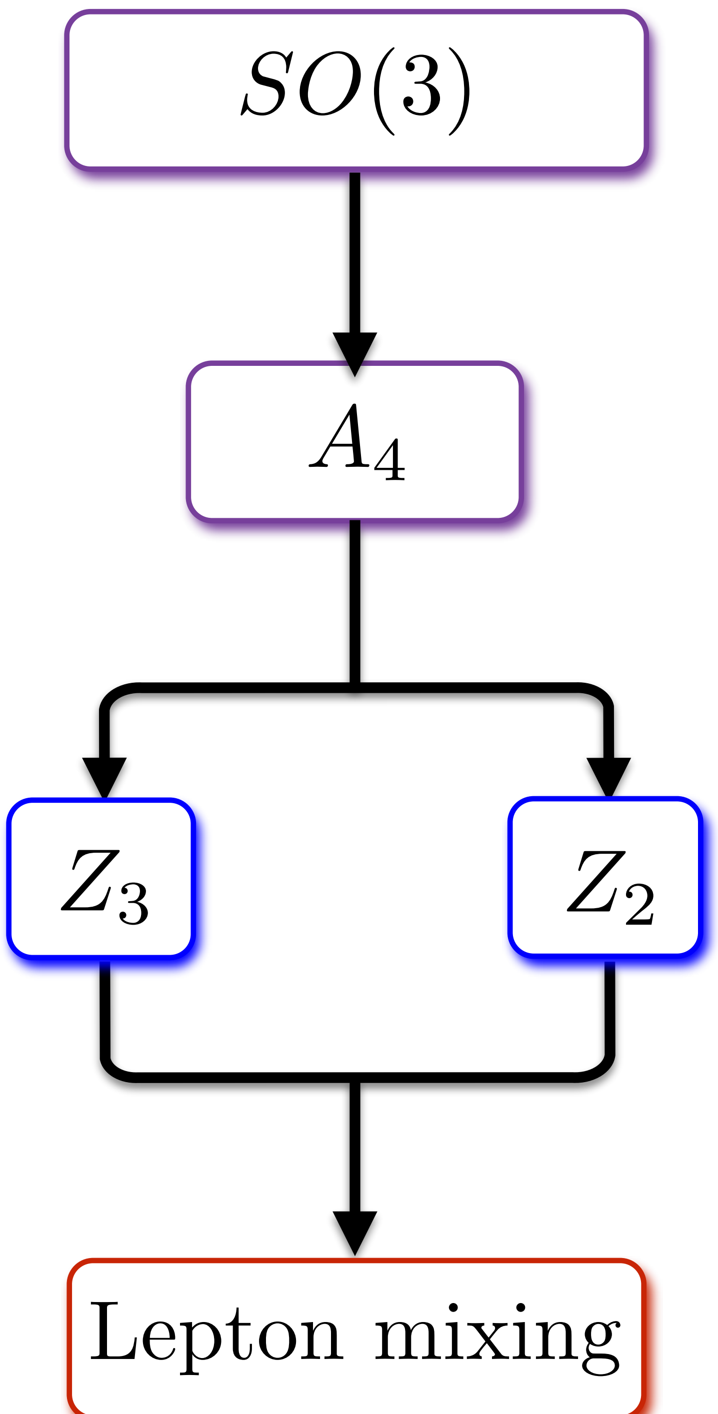
$A_4 \rightarrow Z_2$

$$(\xi\chi)_{\underline{5}} = (\xi(\chi\chi)_{\underline{5}})_{\underline{3}} = 0$$

$$\begin{pmatrix} \langle\chi'\rangle \sim \mathbf{1}' \\ \langle\chi''\rangle \sim \mathbf{1}'' \\ \begin{pmatrix} \langle\chi_1\rangle \\ \langle\chi_2\rangle \\ \langle\chi_3\rangle \end{pmatrix} \sim \mathbf{3} \end{pmatrix} = \left\{ \begin{pmatrix} 0 \\ 0 \\ \begin{pmatrix} \pm v_\chi \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \begin{pmatrix} 0 \\ \pm v_\chi \\ 0 \end{pmatrix} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \begin{pmatrix} 0 \\ 0 \\ \pm v_\chi \end{pmatrix} \end{pmatrix} \right\}$$

Framework of model building





$$\langle \zeta \rangle \sim \{v_\zeta,\ 0,\ (0,0,0)\}$$
$$\underline{5} \sim (\mathbf{1}',\ \mathbf{1}'',\ \ \mathbf{3})$$

Lepton masses and mixing

- Charged lepton mass matrices

$$w_e^{\rm eff} = y_e \frac{v_\varphi^3}{\Lambda^3} \ \ell^T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^c H_d$$

$\underline{3}$ $\underline{1}$

$$SO(3) \times U(1) \simeq SU(2) \times U(1)$$

How to extract the 1' and 1" of A_4 from the irrep of SO(3)?

$$w_{R_\mu}^{\rm eff} = (\ell^T, L_{\mu 0}, L_{\mu \mathbf{3}}^T) \begin{pmatrix} y_{\mu 1} \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} V_\omega H_d & y_{\mu 1} \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} V_\omega^* H_d & 2\sqrt{3} Y_{\mu 3} \frac{v_\xi}{\Lambda} \ \mathbb{1}_{3\times 3} \ H_d \\ 0 & Y_{\mu 1} v_\zeta & \mathbb{0}_{1\times 3} \\ y_{\mu 2} \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda} V_\omega & y_{\mu 2} \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda} V_\omega^* & 2\sqrt{3} Y_{\mu 2} v_\xi \ \mathbb{1}_{3\times 3} \end{pmatrix} \begin{pmatrix} \mu^c \\ R''_\mu \\ R_{\mu \mathbf{3}} \end{pmatrix} \begin{matrix} \mathbf{1}' \\ \mathbf{1}'' \\ \mathbf{3} \end{matrix}$$

$(\underline{3},\ \underline{1},\ \underline{3})$ $\underline{5}$

$$w_{R_\tau}^{\rm eff} = (\ell^T, L_{\tau 0}, L_{\tau \mathbf{3}}^T) \begin{pmatrix} y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} V_\omega^* H_d & y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} V_\omega H_d & \mathcal{O}(y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda}) H_d \\ 0 & Y_{\tau 1} \frac{2 v_\zeta^2}{\sqrt{3}\Lambda} & \mathbb{0}_{1\times 3} \\ \mathbb{0}_{3\times 1} & \mathbb{0}_{3\times 1} & 2\sqrt{3} Y_{\tau 2} v_\xi \ \mathbb{1}_{3\times 3} \end{pmatrix} \begin{pmatrix} \tau^c \\ R'_\tau \\ R_{\tau \mathbf{3}} \end{pmatrix} \begin{matrix} \mathbf{1}' \\ \mathbf{1}'' \\ \mathbf{3} \end{matrix}$$

$(\underline{3},\ \underline{1},\ \underline{3})$ $\underline{5}$

After heavy leptons decouple,

$$M_l = \begin{pmatrix} y_e \frac{v_\varphi^3}{\Lambda^3} & y_\mu \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} & y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} \\ y_e \frac{v_\varphi^3}{\Lambda^3} & \omega y_\mu \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} & \omega^2 y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} \\ y_e \frac{v_\varphi^3}{\Lambda^3} & \omega^2 y_\mu \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} & \omega y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} \end{pmatrix} \frac{v_d}{\sqrt{2}} \qquad \delta M_l = \frac{v_\eta v_\chi v_\varphi}{\Lambda^3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & c\omega + d\omega^2 & 0 \\ 0 & c\omega^2 + d\omega & 0 \end{pmatrix} \frac{v_d}{\sqrt{2}}$$

Lepton masses and mixing

- Neutrino mass matrix

$$w_N = y_N(\ell N)_{\underline{1}} H_u + \frac{\lambda_\eta}{\Lambda}\bar{\eta}^2 (NN)_{\underline{1}} + \lambda_\chi\big(\chi(NN)_{\underline{5}}\big)_{\underline{1}}$$

$$M_D = \frac{y_D v_u}{\sqrt{2}} \mathbb{1}_{3\times 3}\,, \qquad M_M = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}$$

Nothing special, but the Z_2-preserving one.

- Mixing is given by TBM + eμ-mixing correction

$$\sin\theta_{13} = \frac{\sin\theta_{e\mu}}{\sqrt{2}}\,,$$

$$\sin\theta_{12} = \sqrt{\frac{2 - 2\sin 2\theta_{e\mu}\cos\phi_{e\mu}}{3(2-\sin^2\theta_{e\mu})}}\,,$$

$$\sin\theta_{23} = \frac{\cos\theta_{e\mu}}{\sqrt{2-\sin^2\theta_{e\mu}}}\,.$$

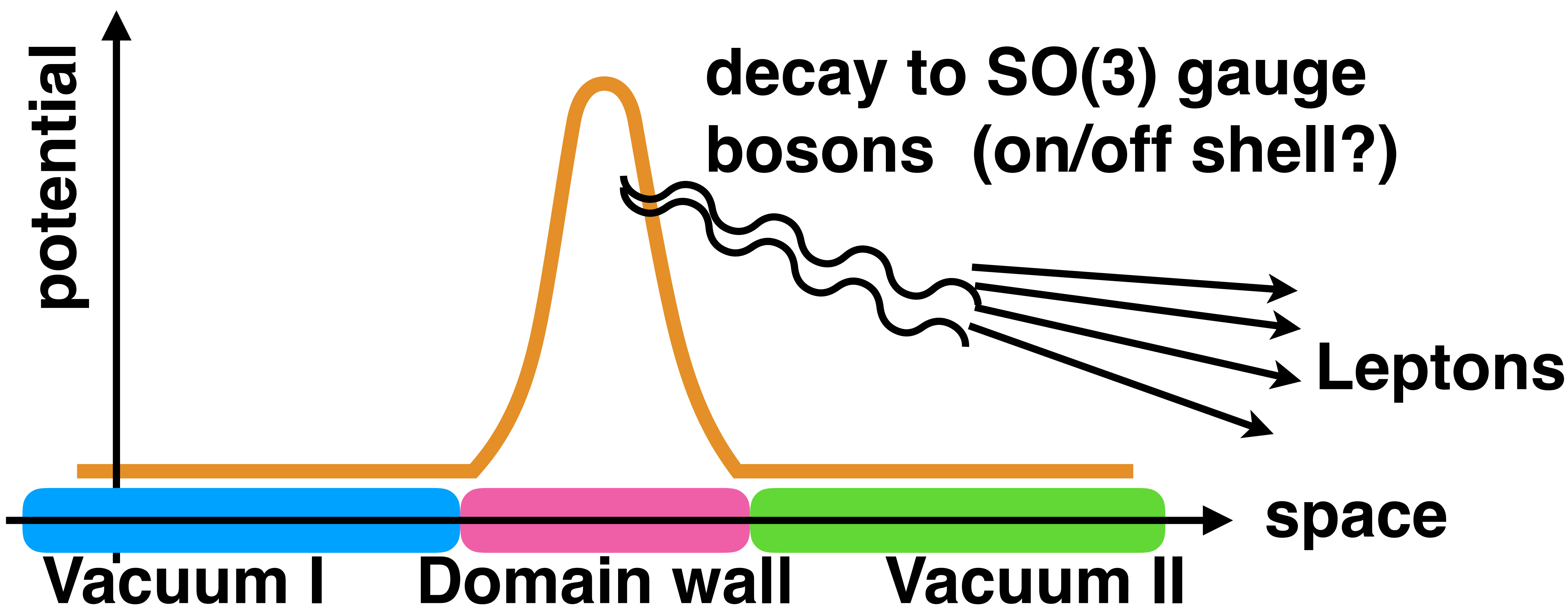
King,0506297;
Antusch and King,0508044;
King and Malinsky, 0608021;
Masina, 0508031;
Antusch, Huber, King, Schwetz, 0702286; Ballett, King, Luhn, Pascoli, Schmidt, 1410.7573;
Girardi, Petcov, Titov, 1410.8056.

$$\delta = \arg\left((3\cos 2\theta_{e\mu} + \cos 4\theta_{e\mu})\cos\phi_{e\mu} - i(\cos 2\theta_{e\mu} + 3)\sin\phi_{e\mu} + \sin 2\theta_{e\mu}\right)$$

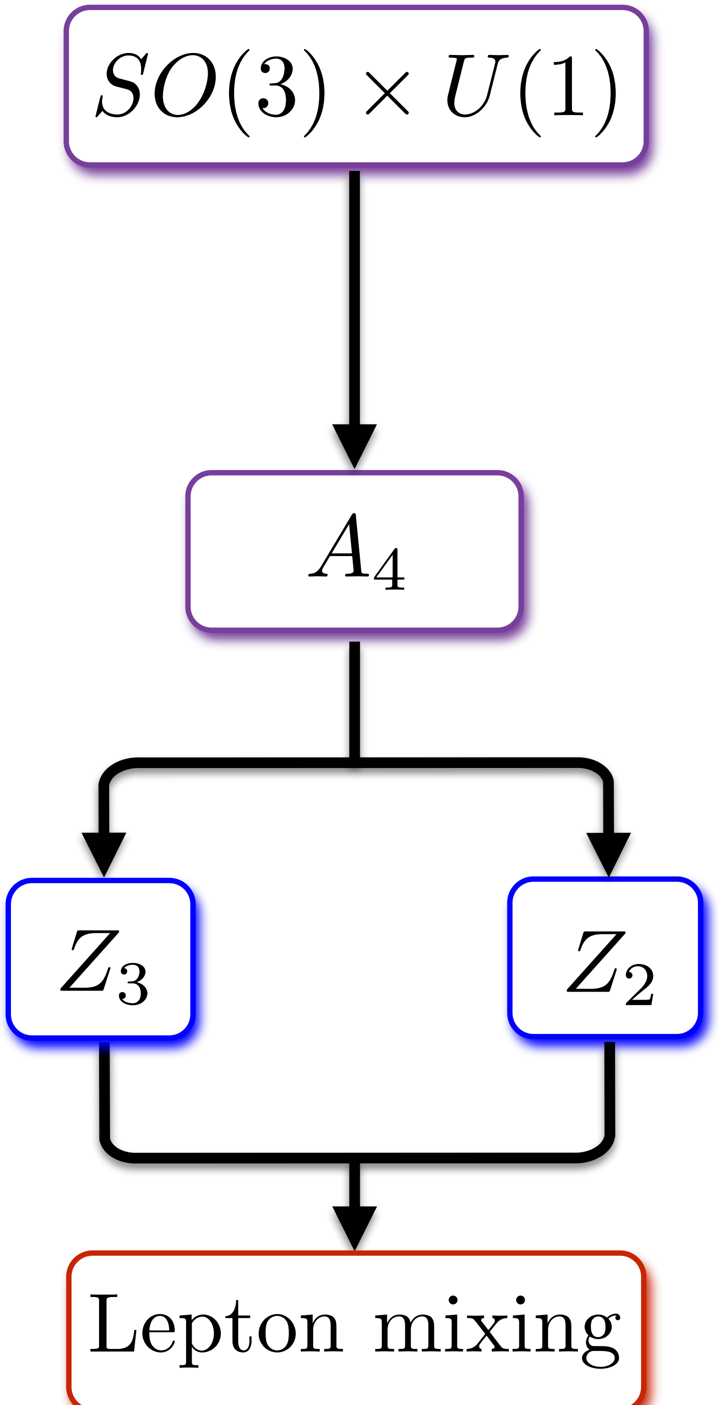
The absence of domain wall in our model

- $SO(3) \times U(1) \rightarrow A_4$, the breaking of gauge symmetry does not generate domain walls.

- $A_4 \rightarrow Z_2$ and Z_3 , if the energy gap between different vacuums is generated. it will decay to gauge bosons and finally to leptons. The two vacuums are finally identical with each other via gauge transformation.



A SUSY gauged SO(3)xU(1) flavour model



- ☑ No fundamental discrete symmetries introduced at the UV scale. No domain wall problems any more.
- ☑ Two-step symmetry breaking $SO(3) \to A_4 \to Z_3, Z_2$ achieved.
- ☑ Vacuum stable under sub-leading correction by imposing U(1).
- ☑ Tri-bimaximal mixing predicted for the first time in continuous symmetry.
- ☑ An extra eμ mixing induced by higher-dim operators, making the model fully compatible with oscillation data.

Thank you!