

# $CP$ violation in the charm sector within the Standard Model and beyond

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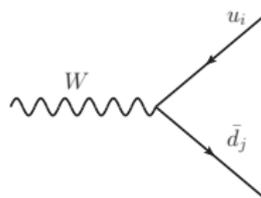


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based on [ArXiv:1903.10490](https://arxiv.org/abs/1903.10490)

# Introduction

- The quark flavour-changing weak interactions are described by:



A Feynman diagram showing a wavy line representing a W boson on the left. It splits into two straight lines on the right: an upper line with an arrow pointing right labeled  $u_i$ , and a lower line with an arrow pointing right labeled  $\bar{d}_j$ .

$$\mathcal{L}_{Wq} = \frac{g}{\sqrt{2}} (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{V_{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^\dagger + \text{h.c.}$$

- $V_{CKM}$  - quark-mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix
- In terms of three mixing angles  $\theta_{12}, \theta_{13}, \theta_{23}$  and one  $CP$ -violating phase  $\delta$

$$V_{CKM} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

$$s_{ij} = \sin \theta_{ij}, \quad c_{ij} = \cos \theta_{ij}.$$

# Direct CP asymmetry

- Definition of the **direct CP-asymmetry**:

$$A_{CP} = \frac{\Gamma(H \rightarrow f) - \Gamma(\bar{H} \rightarrow \bar{f})}{\Gamma(H \rightarrow f) + \Gamma(\bar{H} \rightarrow \bar{f})}$$

- Decomposing an amplitude

$$A(H \rightarrow f) = \underbrace{\lambda_1 e^{i\alpha_1}}_{\text{weak int.}} \underbrace{A_1 e^{i\phi_1}}_{\text{QCD}} + \underbrace{\lambda_2 e^{i\alpha_2}}_{\text{weak int.}} \underbrace{A_2 e^{i\phi_2}}_{\text{QCD}}$$

- The amplitude of the **CP-conjugated** mode

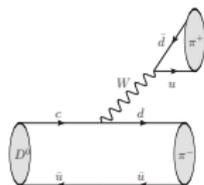
$$A(\bar{H} \rightarrow \bar{f}) = \underbrace{\lambda_1 e^{-i\alpha_1}}_{\text{weak int.}} \underbrace{A_1 e^{i\phi_1}}_{\text{QCD}} + \underbrace{\lambda_2 e^{-i\alpha_2}}_{\text{weak int.}} \underbrace{A_2 e^{i\phi_2}}_{\text{QCD}}$$

- Direct CP-asymmetry

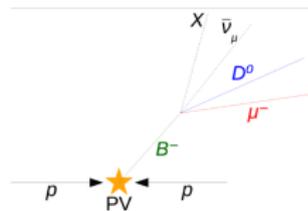
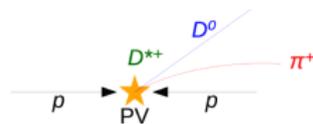
$$A_{CP} = \frac{-2 \left| \frac{\lambda_1}{\lambda_2} \right| \left| \sin(\alpha_1 - \alpha_2) \right| \left| \frac{A_1}{A_2} \right| \left| \sin(\phi_1 - \phi_2) \right|}{1 + 2 \left| \frac{\lambda_1}{\lambda_2} \right| \left| \cos(\alpha_1 - \alpha_2) \right| \left| \frac{A_1}{A_2} \right| \left| \cos(\phi_1 - \phi_2) \right| + \left| \frac{\lambda_1}{\lambda_2} \right|^2 \left| \frac{A_1}{A_2} \right|^2}$$

# $\Delta A_{CP}$ : the history of measurements

- Consider non-leptonic  $D^0 \rightarrow PP$  decays ( $P = \pi, K$ )
- Definition:  $\Delta A_{CP} = A_{CP}(K^- K^+) - A_{CP}(\pi^- \pi^+)$



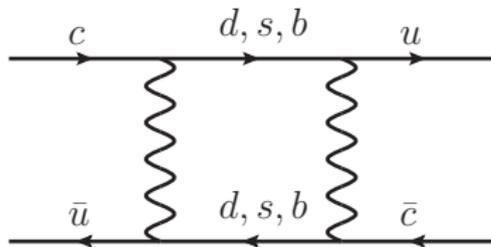
Experiment	$\Delta A_{CP} \times 10^4$	Tag	arXiv
BaBar	$+24 \pm 62 \pm 26$	pion	0709.2715
LHCb	$-82 \pm 21 \pm 11$	pion	1112.0938
CDF	$-62 \pm 21 \pm 10$	pion	1207.2158
Belle	$-87 \pm 41 \pm 6$	pion	1212.1975
LHCb	$+49 \pm 30 \pm 14$	muon	1303.2614
LHCb	$+14 \pm 16 \pm 8$	muon	1405.2797
LHCb	$-10 \pm 8 \pm 3$	pion	1602.03160
LHCb	$-18.2 \pm 3.2 \pm 0.9$	pion	1903.08726
LHCb	$-9 \pm 8 \pm 5$	muon	1903.08726



Federico Betti's talk  
Moriond'19

- LHCb, Moriond 2019:  $\Delta A_{CP}^{\text{Exp}} = (-15.4 \pm 2.9) \times 10^{-4}$  ( $5.3\sigma$  from 0)

## Mixing in the charm sector



- Diagonalisation of the  $2 \times 2$  matrix yields:

$$\Delta M_D^2 - \frac{1}{4} \Delta \Gamma_D^2 = 4 |M_{12}^D|^2 - |\Gamma_{12}^D|^2, \quad \Delta M_D \Delta \Gamma_D = 4 |M_{12}^D| |\Gamma_{12}^D| \cos(\phi_{12}^D)$$

- ▷  $\Delta M_D = M_1 - M_2$  - mass difference
- ▷  $\Delta \Gamma_D = \Gamma_2 - \Gamma_1$  - decay widths difference of the mass eigenstates
- ▷  $\Gamma_{12}^D$  - absorptive part of box diagram (on-shell)
- ▷  $M_{12}^D$  - dispersive part of box diagram (off-shell)
- ▷  $\phi_{12}^D = -\arg(-M_{12}^D/\Gamma_{12}^D)$  - relative phase

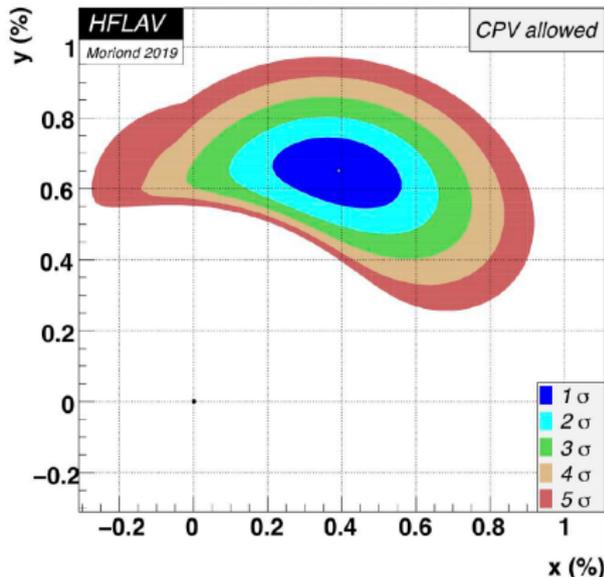
# Mixing in the charm sector

- Experimental data on

$$x \equiv \Delta M_D / \Gamma_D = (0.39^{+0.11}_{-0.12}) \%$$

$$y \equiv \Delta \Gamma_D / (2\Gamma_D) = (0.651^{+0.063}_{-0.069}) \%$$

- Future experimental sensitivity for  $x$  and  $y$  will reach the order of  $0.005 \%$



## Mixing in the charm sector

- The on-shell contribution from box diagrams yields

$$\Gamma_{12}^D = -\lambda_s^2 (\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D) + 2\lambda_s\lambda_b (\Gamma_{sd}^D - \Gamma_{dd}^D) - \lambda_b^2 \Gamma_{dd}^D$$

$$\lambda_x = V_{cx} V_{ux}^*, \quad \lambda_d \sim \lambda_s \sim \lambda, \quad \lambda_b \sim \lambda^5, \quad \lambda_d + \lambda_s + \lambda_b = 0$$

- The off-shell contribution from box diagrams yields

$$M_{12}^D = \lambda_s^2 [M_{ss}^D - 2M_{sd}^D + M_{dd}^D] \\ + 2\lambda_s\lambda_b [M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D] + \lambda_b^2 [M_{bb}^D - 2M_{bd}^D + M_{dd}^D]$$

- Computing a single diagram

$$\lambda_s^2 \Gamma_{12}^{ss} / \Gamma_D \approx 3.7 \times 10^{-2} \approx 5.7 y_D^{\text{exp}}$$

- But combining everything together

$$y_D^{\text{HQE}} \approx \lambda_s^2 (\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}) \approx 1.7 \times 10^{-4} y_D^{\text{exp}} \rightarrow \text{severe GIM cancellation !}$$

- To accommodate the data phase space dependent violations of duality of order 20 % is sufficient  $\rightarrow$  no need in large non-perturbative effects

# Lifetimes of the charm hadrons

- Experimental value of the  $D$ -meson lifetimes ratio

$$\left. \frac{\tau(D^+)}{\tau(D^0)} \right|_{\text{Exp.}} = \frac{(1040 \pm 7) \text{ fs}}{(410.1 \pm 1.5) \text{ fs}} = 2.536 \pm 0.019$$

- Heavy quark expansion (HQE)

$$\frac{1}{\tau} = \Gamma = \Gamma_0 + \frac{\Lambda^2}{m_Q^2} \Gamma_2 + \frac{\Lambda^3}{m_Q^3} \Gamma_3 + \frac{\Lambda^4}{m_Q^4} \Gamma_4 + \dots$$

- Each of  $\Gamma_i$  is presented as

$$\Gamma_i = \underbrace{\left[ \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_i^{(1)} + \frac{\alpha_s^2}{(4\pi)^2} \Gamma_i^{(2)} + \dots \right]}_{\text{Perturbatively calculable part}} \times \underbrace{\langle O^{d=i+3} \rangle}_{\text{Nonperturbative matrix element}}$$

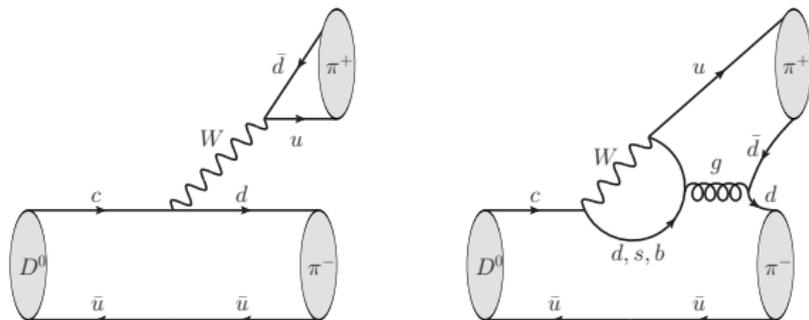
# Lifetimes of the charm hadrons

- Including  $\Gamma_3^{(1)}, \Gamma_4^{(0)}$  [Lenz, Rauh, 1305.3588] and  $\langle O^{d=6} \rangle$  from 3-loop HQET Sum Rules [Kirk, Lenz, Rauh, 1711.02100]

$$\left. \frac{\tau(D^+)}{\tau(D^0)} \right|_{\text{HQE}} = 2.70_{-0.82}^{+0.74} = [1 + 16\pi^2(0.25)^3(1 - 0.34)]_{-0.82}^{+0.74}$$

- ▷  $16\pi^2 \rightarrow$  loop enhancement factor
- ▷  $(0.25)^3 \rightarrow$  HQE
- ▷ 1 from dim-6
- ▷ 0.34 from dim-7 (in the vacuum insertion approximation)
- Indicates an expansion parameter  $\Lambda/m_c \approx 0.25 \dots 0.34$
- HQE works reasonably well in computation of charm-hadron lifetimes!
- More prediction for lifetimes to come, next one is  $\tau(D_s^+)/\tau(D^0)$

# Theory of $\Delta A_{CP}$



- The amplitude of the  $D^0 \rightarrow \pi^+ \pi^-$  decay

$$A(D^0 \rightarrow \pi^+ \pi^-) = \lambda_d (A_{\text{Tree}} + A_{\text{Penguin}}^d) + \lambda_s A_{\text{Penguin}}^s + \lambda_b A_{\text{Penguin}}^b$$

- Using unitarity of the CKM matrix

$$A \equiv \frac{G_F}{\sqrt{2}} \lambda_d T \left[ 1 + \frac{\lambda_b P}{\lambda_d T} \right]$$

- Note that  $\lambda_b/\lambda_d \sim \lambda^4 \ll 1 \Rightarrow$  small **direct CP-asymmetry**

# Theory of $\Delta A_{CP}$

- Branching fraction

$$\text{BR} \propto \frac{G_F^2}{2} |\lambda_d|^2 |T|^2 \left| 1 + \frac{\lambda_b}{\lambda_d} \frac{P}{T} \right|^2$$

mostly given by the tree level amplitude  $T$

- Direct  $CP$ -asymmetry

$$a_{CP}^{\text{dir}} = \frac{-2 \left| \frac{\lambda_b}{\lambda_d} \right| \sin \gamma \left| \frac{P}{T} \right| \sin \phi}{1 - 2 \left| \frac{\lambda_b}{\lambda_d} \right| \cos \gamma \left| \frac{P}{T} \right| \cos \phi + \left| \frac{\lambda_b}{\lambda_d} \right|^2 \left| \frac{P}{T} \right|^2} \approx -13 \times 10^{-4} \left| \frac{P}{T} \right| \sin \phi$$

sensitive to the differences of both **strong** and **weak** phases as well as  $P/T$ -ratio

# Theory of $\Delta A_{CP}$

- In the SM:

$$|\Delta A_{CP}| \approx 13 \times 10^{-4} \left| \left| \frac{P}{T} \right|_{K^+K^-} \sin \phi_{K^+K^-} + \left| \frac{P}{T} \right|_{\pi^+\pi^-} \sin \phi_{\pi^+\pi^-} \right|$$

- From naive estimate of  $P/T \sim 0.1$ :

$$|\Delta A_{CP}| \leq 2.6 \times 10^{-4}$$

- To be compared with the measurement

$$\Delta A_{CP}^{\text{Exp}} = (-15.4 \pm 2.9) \times 10^{-4}$$

- To accommodate the experimental value one needs (at least)

$$\frac{P}{T} \sim 0.6$$

- Strong penguins? Or New Physics effects?

# Light-cone sum rule (LCSR) prediction

[Khodjamirian, Petrov, 1706.07780]

- Tree level amplitudes  $T$  fixed from the data on

$$\text{Br}(D^0 \rightarrow K^+ K^-) = (3.97 \pm 0.07) \times 10^{-3}$$

$$\text{Br}(D^0 \rightarrow \pi^+ \pi^-) = (1.407 \pm 0.025) \times 10^{-3}$$

- From LCSR for penguin amplitudes  $P$  (also their phases)

$$\left| \frac{P}{T} \right|_{\pi^+ \pi^-} = 0.093 \pm 0.056, \quad \left| \frac{P}{T} \right|_{K^+ K^-} = 0.075 \pm 0.048$$

- Compatible with the naive estimate  $P/T \sim 0.1$
- Relative strong phases **not predicted** (remain as arbitrary parameters)

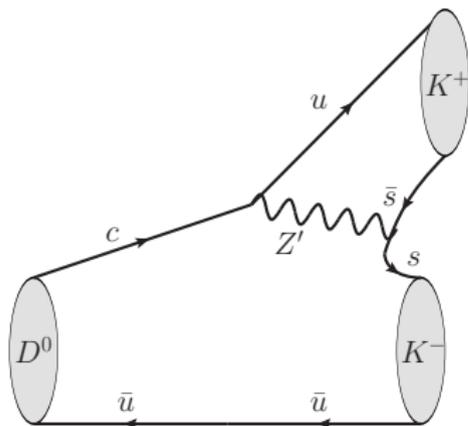
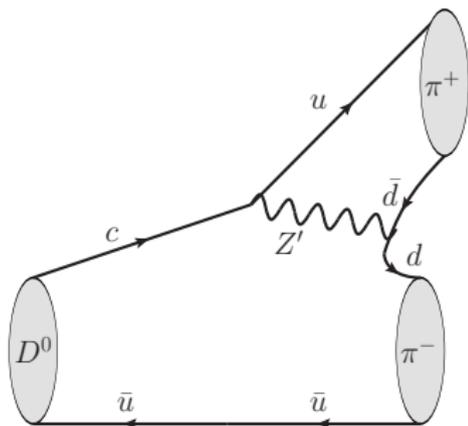
$$|\Delta A_{CP}| \leq (2.2 \pm 1.4) \times 10^{-4} \leq 3.6 \times 10^{-4}$$

- Uncertainty above is not just pure parametric one but also includes estimates of the higher twist effects, perturbative  $\alpha_s$  corrections, etc.

# New Physics?

- Consider an extension of the SM with  $Z'$ -boson

$$\mathcal{L}_{BSM} = \frac{1}{2} m_{Z'}^2 Z'_\mu Z'^\mu + Z'_\mu \left[ g_{dd} \bar{d}_L \gamma^\mu d_L + g_{ss} \bar{s}_L \gamma^\mu s_L + (g_{cu} \bar{u}_L \gamma^\mu c_L + \text{h.c.}) \right]$$

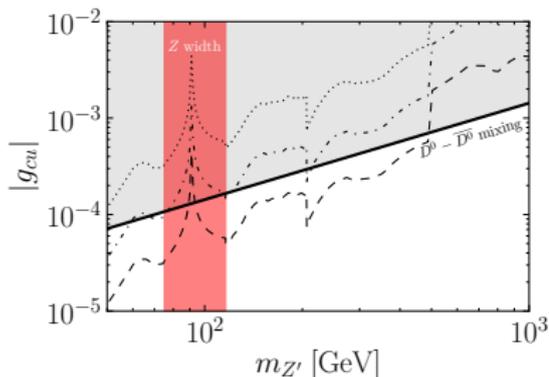


# New Physics?

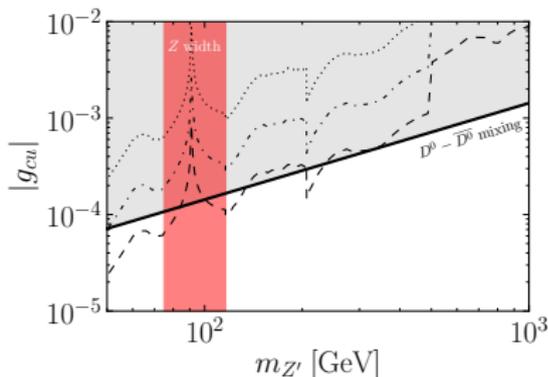
- Assuming that the whole effect originates in the  $K^+K^-$  final state

$$|g_{cu}| = \Delta_{NP} \frac{\sqrt{2} G_F \lambda_s m_{Z'}^2}{g_{ss}} \left( \left| \frac{A_{BSM}^s}{T} \right| \sin \delta_{BSM}^s \sin \phi_{BSM}^s \right)^{-1}$$

$$\Delta_{NP} = \Delta A_{CP}^{\text{exp}} - \Delta A_{CP}^{\text{SM}}$$



$$\Delta_{NP} = -11.8 \times 10^{-4}$$



$$\Delta_{NP} = -6.0 \times 10^{-4}$$

# Conclusion

- Theory of charm decay is extremely complicated due to large (approx. 0.3) expansion parameters  $\Lambda/m_c$  and  $\alpha_s(m_c)/\pi$  in combination with severe numerical cancellations
- Nevertheless, HQE looks promising in computation of charm-hadron lifetimes (albeit large uncertainties)
- To accommodate large exp. value one needs enhanced ( $\mathcal{O}(10)$ ) penguins contributions in the SM
- A complete (including computation of both tree and penguin contributions and their relative phase) Standard Model prediction for  $\Delta A_{CP}$  (based on non-perturbative method e.g. LCSR) is needed

# Backup

## $\Delta I = 1/2$ rule in $K \rightarrow \pi\pi$

- In the  $K \rightarrow \pi\pi$  decays

$$\frac{A_0(K \rightarrow \pi\pi)}{A_2(K \rightarrow \pi\pi)} \approx 22.5$$

$A_I$  – amplitude of the  $K$ -meson decay to two pions with isospin  $I = 0, 2$

- Strong penguin contributions enhancing  $A_0$ ?
- Lattice simulation: no enhancement of penguins contributions seen, but there are severe cancellations of tree level contributions  
[RBC and UKQCD Collab., 1212.1474, 1505.07863]
- The  $\Delta I = 1/2$  rule does not motivate for penguin enhancement