



# Implication of chiral symmetry on the positive parity heavy-light meson spectroscopy

Meng-Lin Du

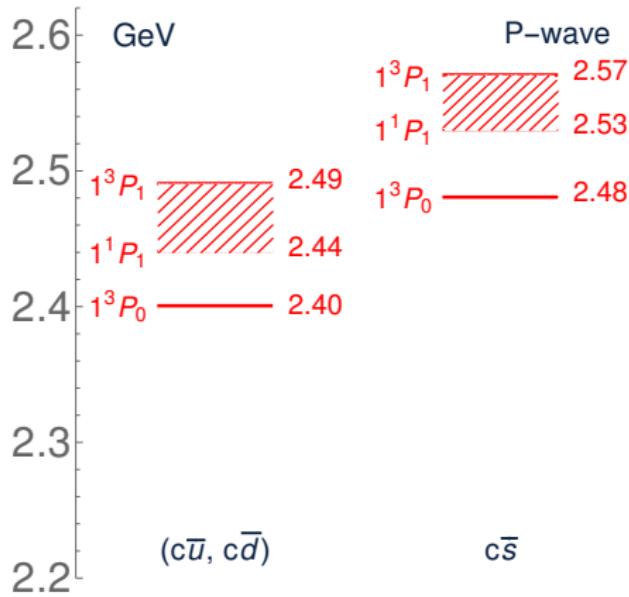
Helmholtz-Institut für Strahlen-und Kernphysik,  
Universität Bonn

*The 27th International Workshop on Weak Interactions and Neutrinos  
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# Outline

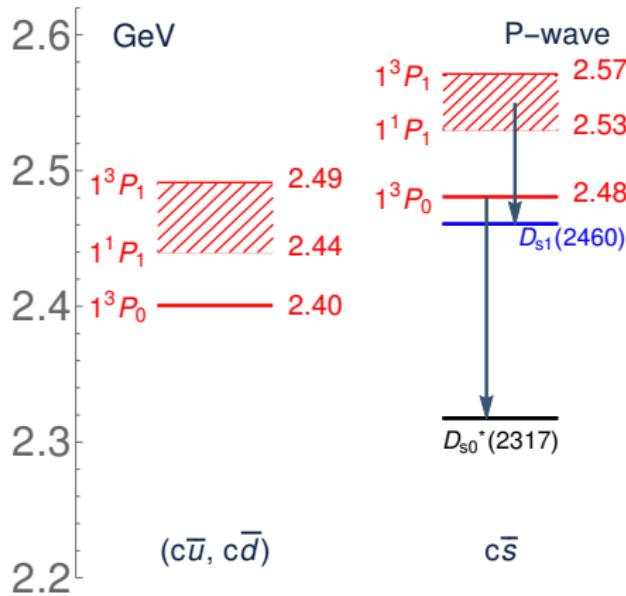
- 1 Introduction
- 2 Implication of chiral symmetry on Breit-Wigner resonances
- 3 Status of Positive-Parity Charmed Mesons (Lattice QCD + EFTs)
- 4 Analysis on the experimental data of  $B \rightarrow D\pi\pi$
- 5 Summary and outlook

# Positive parity ground state charm mesons



S. Godfrey and N. Isgur, PRD 32, 189 (1985)

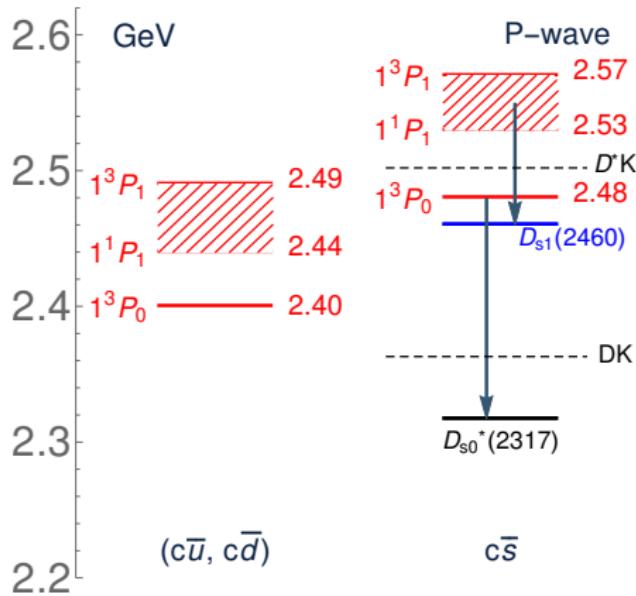
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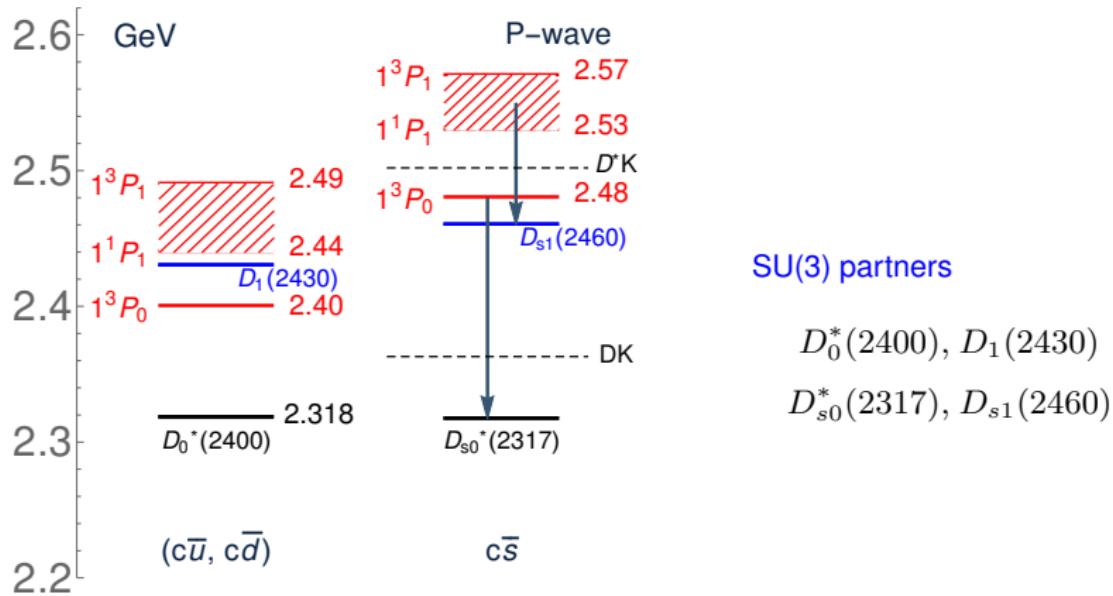
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BaBar (2003), CLEO (2003); Belle (2004)

# Puzzles

## Puzzles in charm mesons:

- ➊ Why are the masses of the  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$  much lower than the quark model expectations for the lowest scalar and axial-vector charm-strange mesons?
- ➋ Why is the mass difference between the  $D_{s1}(2460)$  and the  $D_{s0}^*(2317)$  equal to that between the ground state vector meson and pseudoscalar meson within 2 MeV?

$$\underbrace{M_{D_{s1}(2460)\pm} - M_{D_{s0}^*(2317)\pm}}_{=(141.8 \pm 0.8) \text{ MeV}} \simeq \underbrace{M_{D^*\pm} - M_{D\pm}}_{=(140.67 \pm 0.08) \text{ MeV}}$$

- ➌ Why are the masses of the  $D_0^*(2400)$  and  $D_1(2430)$  almost equal to or even higher than their strange siblings?

Notice: all these experiments used a Breit–Wigner to extract the resonance

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# Implication of chiral symmetry on Breit-Wigner resonances

① Goldstone bosons: energy-dependent interactions

② The standard Breit-Wigner: constant coupling. ~~chiral symmetry~~

③ S-wave BW parameterization:  $F_0(s) \propto \frac{1}{s - m_0^2 + im_0\Gamma}$

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⑤ ?  $D_0^*(2400)$ ,  $D_1(2430)$

Coupled-channel  $\implies$  chiral EFT + unitarization

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# Lattice studies on the charmed scalar mesons: strange

- Early studies using only  $c\bar{s}$ -type interpolators  
→ give mass significantly larger than  $D_{s0}^*(2317)$  Bali (2003); UKQCD (2003); ...
- $c\bar{s} + DK$  interpolators: right mass  $M_\pi \approx 156$  MeV Mohler *et al.*, PRL111(2013)222001  
binding energy: 37 MeV,  $M_{D_{s0}^*} - \frac{1}{4}(M_{D_s} + 3M_{D_s^*})$ :

Mohler et al.	PDG
$(266 \pm 16)$ MeV	$(241.5 \pm 0.8)$ MeV

- New calculation:  $M_\pi = 150$  MeV Bali et al. [RQCD Col.], PRD96(2017)074501

	Energy [MeV]	Expt [MeV]
$m_{0-}$	1976.9(2)	1966.0(4)
$m_{1-}$	2094.9(7)	2111.3(6)
$m_{0+}$	2348(4)(+6)	2317.7(0.6)(2.0)
$m_{1+}$	2451(4)(+1)	2459.5(0.6)(2.0)

# Lattice studies on the charmed scalar mesons: nonstrange

$(S, I) = (0, 1/2)$ :

- $c\bar{q} + D\pi$  interpolator

Mohler *et al.*, PRD87(2013)034501

$$M_\pi \approx 266 \text{ MeV}, \quad M_D \approx 1558 \text{ MeV}, \quad M_{D^*} \approx 1690 \text{ MeV}$$

Lüscher's formula  $\Rightarrow D\pi$  phase shift

BW parameters of  $D_0^*(2400)$  consistent with PDG values

	Mohler et al.	PDG
$M_{D_0^*} - \frac{1}{4}(M_D + 3M_{D^*})$	$(351 \pm 21) \text{ MeV}$	$(347 \pm 29) \text{ MeV}$
$M_{D_1} - \frac{1}{4}(M_D + 3M_{D^*})$	$(380 \pm 21) \text{ MeV}$	$(456 \pm 40) \text{ MeV}$

- Coupled-channel:

$$\hookrightarrow c\bar{q} + D\pi + D\eta + D_s\bar{K} \quad \text{Moir } \textit{et al.} [\text{Hadron Spectrum Col.}], \text{JHEP1610}(2016)011$$

$$\hookrightarrow M_\pi \approx 391 \text{ MeV}, M_D \approx 1885 \text{ MeV}: D\pi \text{ threshold } (2276.4 \pm 0.9) \text{ MeV}$$

$K$ -matrix: a pole below threshold is found:  $2275.9 \pm 0.9 \text{ MeV}$  ?  $D_0^*(2400)$

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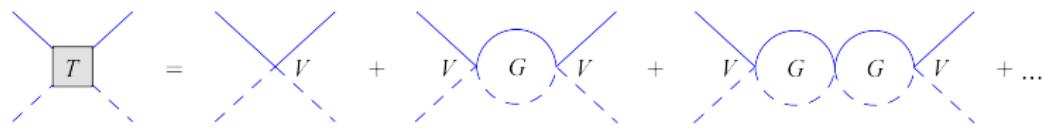
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# ChPT + unitarization

- Low-energy interactions between the charm and light pseudoscalar mesons:  
ChPT
- A nonperturbative treatment: unitarization Oller and Mei  ner, PLB500, 263 (2001)

$$T^{-1}(s) = V^{-1}(s) + G(s)$$



$V(s)$ : to be derived from SU(3) chiral effective Lagrangian

$G(s)$ : two-point scalar loop functions, regularized with a subtraction constant

- **NLO:** 5 free parameters are determined by fit to lattice data on scattering lengths in 5 channels (no disconnected contribution)  
 $D\bar{K}(I=1, I=0)$ ,  $D_s K$ ,  $D\pi(I=3/2)$ ,  $D_s \pi$

L. Liu, Oiginos, F.-K. Guo, Hanhart, Mei  ner, PRD86(2013)014508

# $D_{s0}^*(2317)$ and $D_{s1}(2460)$ as hadronic molecules

- Hadronic molecular model:  $D_{s0}^*(2317)[DK]$ ,  $D_{s1}(2460)[D^*K]$

Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); Guo et al. (2006); ...

- NLO prediction for  $D_{s0}^*(2317)$ :  $2315^{+18}_{-28}$  MeV

L. Liu, Orginos, F.-K. Guo, Hanhart, Mei $\beta$ ner, PRD86(2013)014508

→ one possible solution to the 1st puzzle

- Solution to the 2nd puzzle: heavy quark spin symmetry

$DK$  and  $D^*K$  interaction almost same  $\Rightarrow$  similar binding energies

$$M_D + M_K - M_{D_{s0}^*(2317)} \simeq M_{D^*} + M_K - M_{D_{s1}(2460)} \pm 4 \text{ MeV}$$

Uncertainty: binding energy (45 MeV)  $\times \frac{\Lambda_{QCD}}{m_c} \frac{M_K}{\Lambda_\chi}$

$$\Rightarrow M_{D_{s1}(2460)\pm} - M_{D_{s0}^*(2317)\pm} \simeq M_{D^{*\pm}} - M_{D^\pm} \text{ is understood}$$

F.-K. Guo, C. Hanhart and U.-G. Mei $\beta$ ner, PRL102(2009)242004

## $DK$ component from lattice QCD

- Compositeness ( $1 - Z$ ) related to the  $S$ -wave scattering length: Weinberg (1965)

$$a \simeq -2 \frac{1-Z}{2-Z} \frac{1}{\sqrt{2\mu E_B}}$$

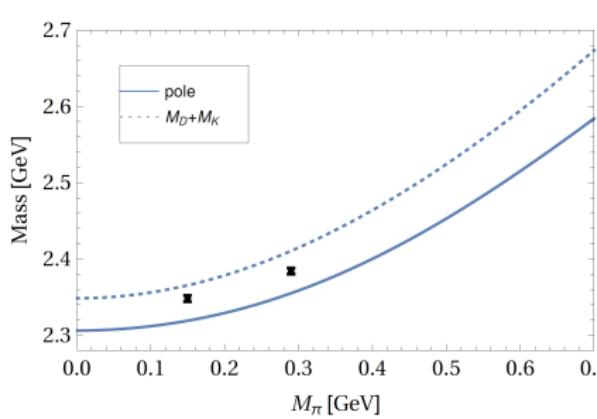
- From the lattice energy levels in C. Lang et al., PRD90(2014)034510  
 $D_{s0}^*(2317)$  contains  $\sim 70\%$   $DK$  Martínez Torres, Oset, Prelovsek, Ramos, JHEP1505,053
- Latest lattice results in G. Bali et al., PRD96(2017)074501

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$$1 - Z = 1.04(0.08)(+0.30)$$

$M_\pi$ [MeV]	150	290
$M_{D_{s0}^*(2317)}$ [MeV]	$2348 \pm 4$	$2384 \pm 3$
$M_{D_s}$ [MeV]	$1977 \pm 1$	$1980 \pm 1$

strong  $M_\pi$  dependence!

curves: prediction in

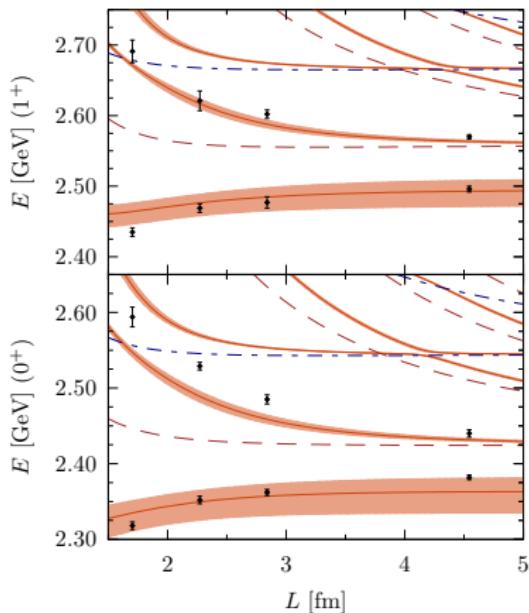
M. L. Du, F. K. Guo,

U. G. Meißner and D. L. Yao, EPJC77(2017)728

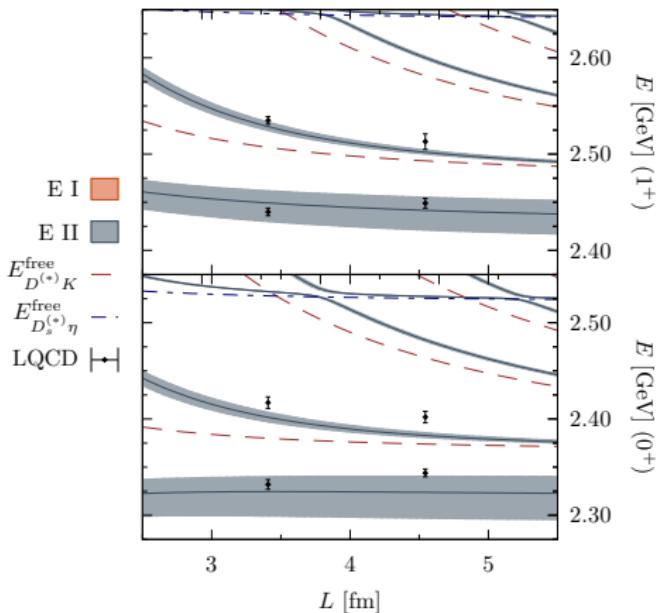
# Predictions versus recent lattice results: charm-strange

- Postdicted finite volume energy levels for  $(S, I) = (1, 0)$ ,  $J^P = 1^+$  &  $0^+$  versus lattice results by G. Bali, S. Collins, A. Cox, A. Schäfer, PRD96(2017)074501

E I:  $M_\pi = 290$  MeV



E II:  $M_\pi = 150$  MeV

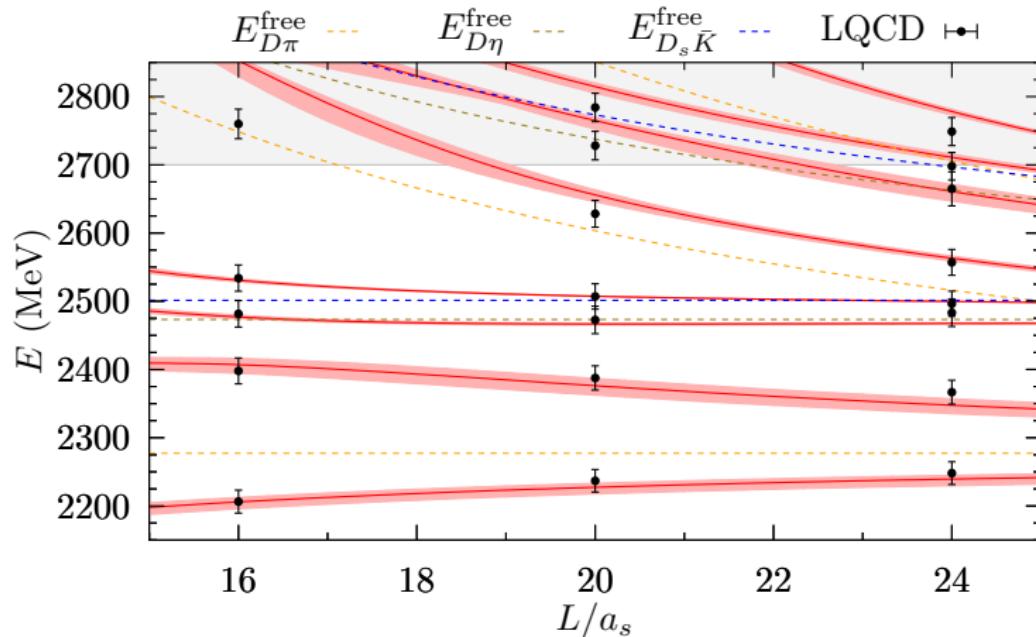


M. Albaladejo *et al.*, EPJC78(2018)722

# Predictions versus recent lattice results: charm-nonstrange

- Postdicted finite volume energy levels for  $I = 1/2$  agree very well with lattice results by  
G. Moir *et al.* [Hadron Spectrum Collaboration], JHEP1610(2016)011  
**NOT a fit!**

$$M_\pi = 391 \text{ MeV}$$



## Two states in $I = 1/2$ sector

- The amplitudes are based on QCD
- Two states in  $I = 1/2$  sector were found in Kolomeitsev, Lutz (2004); Guo, Shen, Chiang, Ping, Zou (2006); F.-K. Guo, Hanhart, Meißner (2009); Z.-H. Guo, Meißner, D.-L. Yao (2015)
- remarkable agreement with lattice data  $\Rightarrow$  a strong support
- two states also in heavy meson sectors ( $M, \Gamma/2$ ) in MeV:

	lower pole	higher pole	RPP
$D_0^*$	$\left\{ 2105^{+6}_{-8}, 102^{+10}_{-11} \right\}$	$\left\{ 2451^{+35}_{-26}, 134^{+7}_{-8} \right\}$	$(2318 \pm 29, 134 \pm 20)$
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$B_0^*$	$\left\{ 5535^{+9}_{-11}, 113^{+15}_{-17} \right\}$	$\left\{ 5852^{+16}_{-19}, 36 \pm 5 \right\}$	—
$B_1$	$\left\{ 5584^{+9}_{-11}, 119^{+14}_{-17} \right\}$	$\left\{ 5912^{+15}_{-18}, 42^{+5}_{-4} \right\}$	—

↪ solution to the third puzzle

- But is there any experimental support?

to compare with the most precise measurement of  $B^- \rightarrow D^+ \pi^- \pi^-$  by LHCb  
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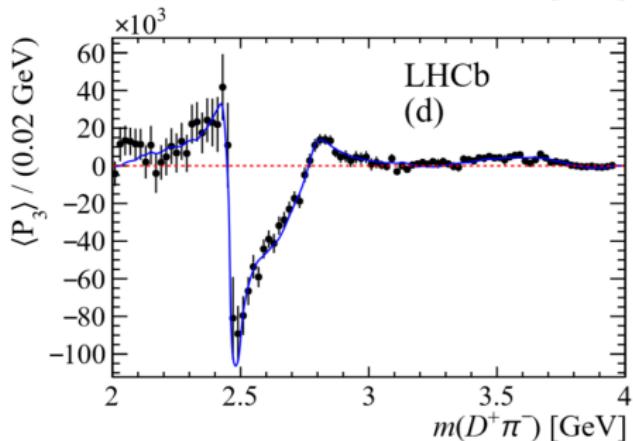
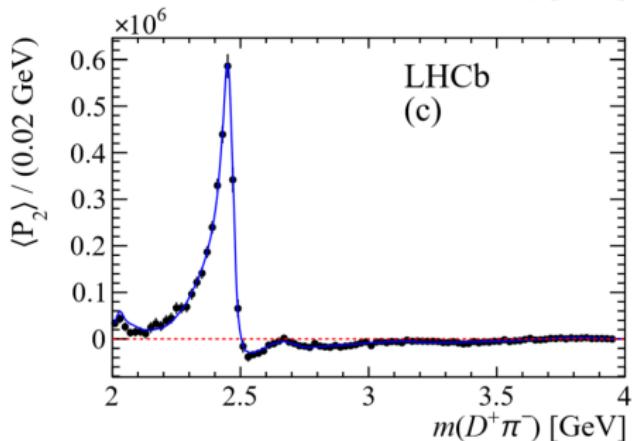
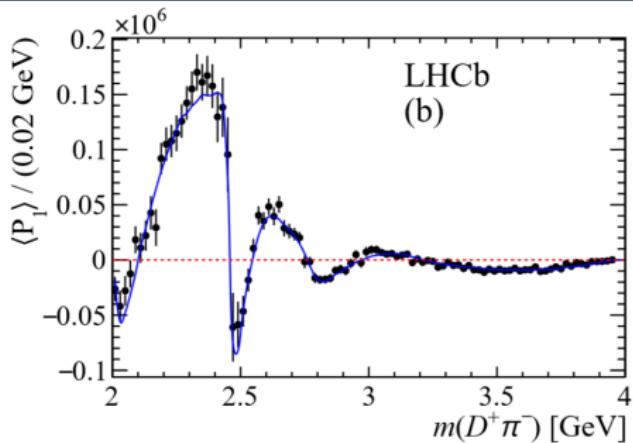
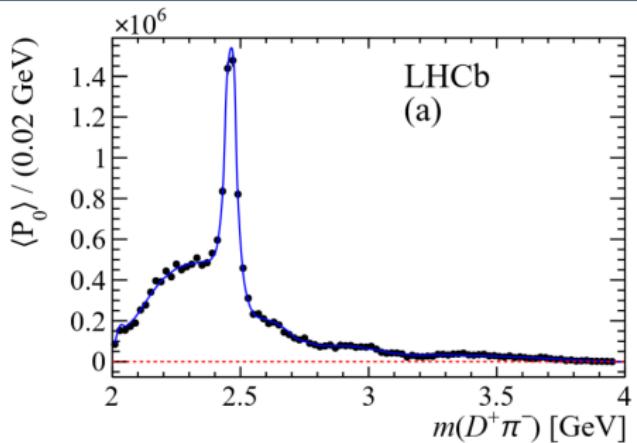
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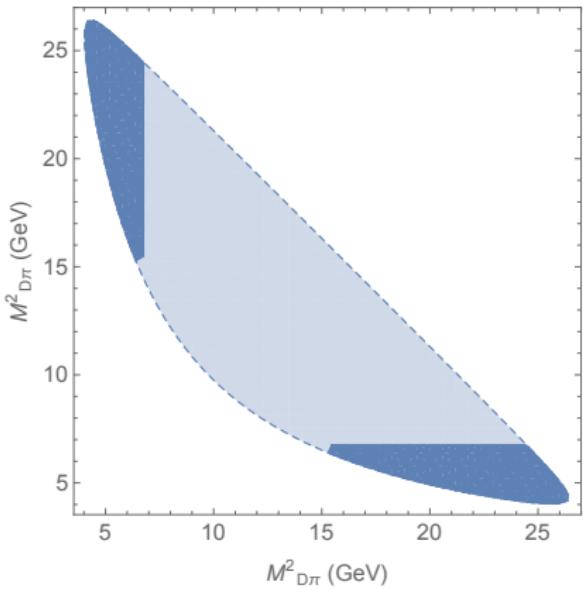
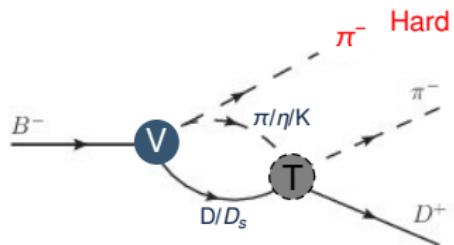
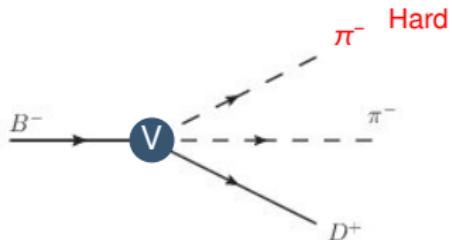
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PRD94(2016)072001

# Angular moments of $B^- \rightarrow D^+ \pi^- \pi^-$

LHCb, PRD94(2016)072001



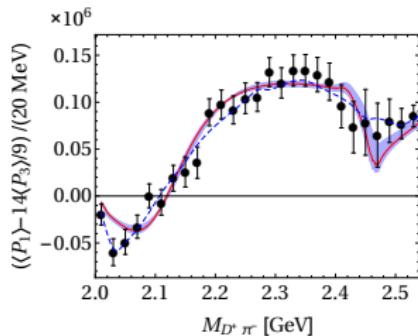
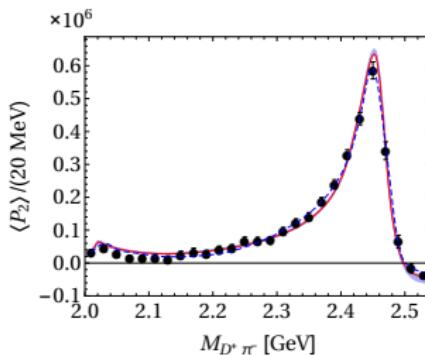
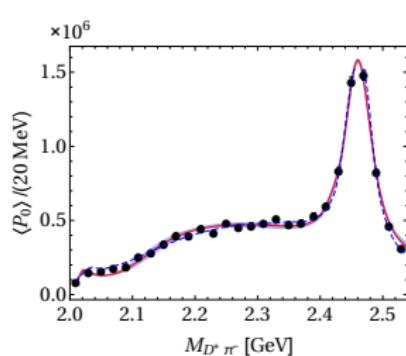
# $B^+ \rightarrow D^+ \pi^- \pi^-$ kinematics



$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2,$$

$$\langle P_2 \rangle \propto \frac{2}{5} |\mathcal{A}_1|^2 + \frac{2}{7} |\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}} |\mathcal{A}_0| |\mathcal{A}_2| \cos(\delta_0 - \delta_2),$$

$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}} |\mathcal{A}_0| |\mathcal{A}_1| \cos(\delta_0 - \delta_1)$$

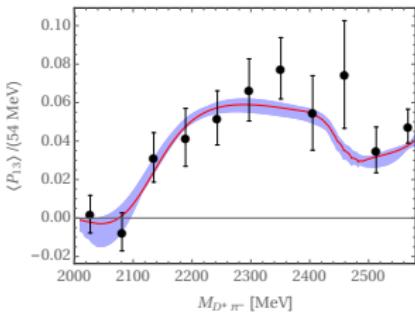
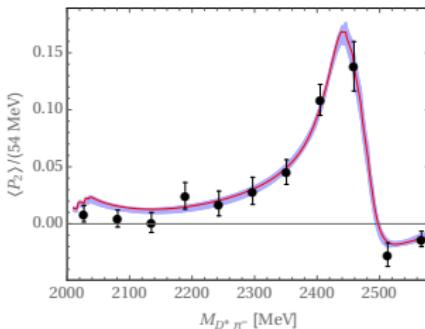
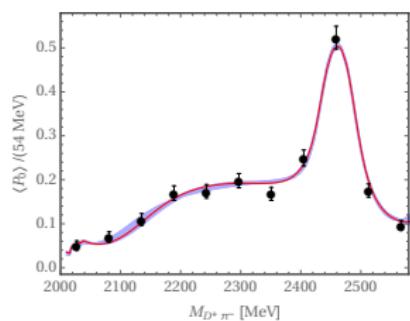


- The  $S$ -wave  $D\pi$  can be well described using our amplitudes with pre-fixed LECs (the same as before)
- Fast variation** in [2.4, 2.5] GeV in  $\langle P_{13} \rangle$ : **cusps** at  $D\eta$  and  $D_s\bar{K}$  thresholds

# Angular moments: $B^- \rightarrow D^+ \pi^- K^-$ and $B^0 \rightarrow \bar{D}^0 \pi^- \pi^+$

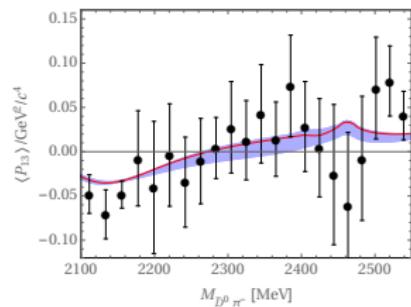
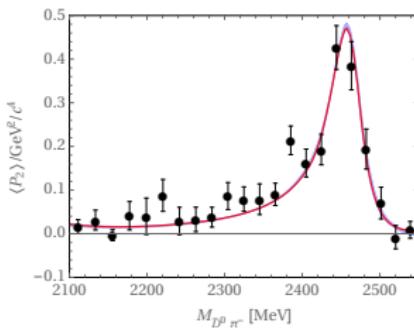
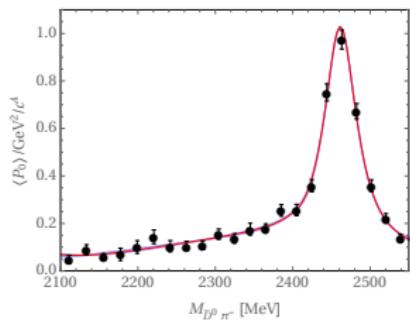
$B^- \rightarrow D^+ \pi^- K^-$

LHCb, PRD91(2015)092002; Du et al., arXiv:1903.08516



$B^0 \rightarrow \bar{D}^0 \pi^- \pi^+$

LHCb, PRD92(2015)032002; Du et al., arXiv:1903.08516



## Summary and outlook (I)

Thanks to the recent experiment, lattice and EFT developments

⇒ likely resolution to all 3 puzzles of positive-parity charm mesons:

- Q: Why are the masses of  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$  much lower than quark model predictions for  $c\bar{s}$  mesons ?  
A: They are dominantly  $DK$  and  $D^*K$  molecular states, respectively.
- Q: Why  $M_{D_{s1}(2460)^\pm} - M_{D_{s0}^*(2317)^\pm} \simeq M_{D^{*\pm}} - M_{D^\pm}$  within 2 MeV ?  
A: Consequence of HQSS as dominantly  $DK$  and  $D^*K$  molecules.
- Q: Why are the masses of the  $D_0^*(2400)$  and  $D_1(2430)$  almost equal to or even higher than their strange siblings?  
A: There are two  $D_0^*$  and two  $D_1$ , and the lower ones have smaller masses.

## Summary and outlook (II)

- Chiral symmetry
  - a shift of the BW peak
- Two-pole structures of  $D_0^*(2400)$  and  $D_1(2430)$
- Fully consistent with the high quality LHCb data on  $B$  decays
- Call for a change of paradigm for the positive-parity mesons:
  - dynamically generated for ground states
  - already have been established for the scalars made from light quarks
- More data with accuracy for the  $B \rightarrow D^{(*)}\pi\pi$  and  $B \rightarrow D_s^{(*)}\bar{K}\pi$
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- Searching for the bottom cousins

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Experiments

Lattice

Thank you very much for your attention!

# Predictions for heavy-strange mesons

- Predictions:

Du et al., PRD98(2018)094018

meson	$J^P$	prediction	PDG	lattice
$D_{s0}^*$	$0^+$	$2315^{+18}_{-28}$	$2317.7 \pm 0.6$	$2348^{+7}_{-4}[1]$
$D_{s1}$	$1^+$	$2456^{+15}_{-21}$	$2459.5 \pm 0.6$	$2451 \pm 4[1]$
$B_{s0}^*$	$0^+$	$5720^{+16}_{-23}$	—	$5711 \pm 23[2]$
$B_{s1}$	$1^+$	$5772^{+15}_{-21}$	—	$5750 \pm 25[2]$

[1] Bali, Collins, Cox, Schäfer, PRD96(2017)074501

[2] Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

# Chiral effective Lagrangian

- ① Effective weak Hamiltonian  $H_{\text{eff}}$  for  $\Delta b = 1$  and  $\Delta c = 1$ :

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} (C_1 \mathcal{O}_1^d + C_2 \mathcal{O}_2^d) + (b \rightarrow s) + h.c.,$$

$$\mathcal{O}_1^d = (\bar{c}_a b_b)_L (\bar{d}_b u_a)_L, \quad \mathcal{O}_2^d = (\bar{c}_a b_a)_L (\bar{d}_b u_b)_L.$$

- ② Left: Symmetry under  $g_L \times g_R \in SU(3)_L \times SU(3)_R$ ,  $h \in SU(3)_V$

Goldstone fields:  $u \mapsto g_R u h^\dagger = h u g_L^\dagger$ ,  $u_\mu \mapsto h u_\mu h^\dagger$ ,

Matter fields:  $B \mapsto B h^\dagger$ ,  $D \mapsto D h^\dagger$ ,  $M \mapsto h M h^\dagger$

- ③ Introducing  $t = -v^2 T$

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & B \left( c_1 (u_\mu M + M u_\mu) + c_2 (u_\mu M + M u_\mu) t + c_3 t (u_\mu M + M u_\mu) \right. \\ & \left. + c_4 (u_\mu \langle M t \rangle + M \langle u_\mu t \rangle) + c_5 t \langle M u_\mu \rangle + c_6 \langle (M u_\mu + u_\mu M) t \rangle \right) \partial^\mu D^\dagger \end{aligned}$$

# Chiral effective Lagrangian

- ① Introducing a spurion  $H: H_i^j \mapsto H_{i'}^{j'} (g_L)^{i'}_i (g_L^\dagger)^{j'}_j$

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* H_i^j C (\bar{c} b)_L (\bar{q}^i q_j)_L, \quad H = \begin{pmatrix} 0 & 0 & 0 \\ V_{ud} & 0 & 0 \\ V_{us} & 0 & 0 \end{pmatrix}$$

## Experiments Lattice

- ② Transformation under  $g_L \times g_R \in SU(3)_L \times SU(3)_R, h \in SU(3)_V$

Goldstone fields:  $u \mapsto g_R u h^\dagger = h u_L^\dagger, \quad u_\mu \mapsto h u_\mu h^\dagger,$

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- ③ Introducing  $t = u H u^\dagger, t \mapsto h t h^\dagger$

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# Amplitudes

Amplitudes up to  $D$ -wave:

$$\mathcal{A}(B^- \rightarrow D^+ \pi^- \pi^-) = \mathcal{A}_0(s) + \sqrt{3}\mathcal{A}_1(s)P_1(\cos\theta) + \sqrt{5}\mathcal{A}_2(s)P_2(\cos\theta)$$

- $S$ -wave: ( $C = (c_2 + c_6)/(c_1 + c_4)$ ),

$$\begin{aligned} \mathcal{A}_0(s) \propto & \left\{ E_\pi \left[ 2 + G_1(s) \left( \frac{5}{3}T_{11}^{1/2}(s) + \frac{1}{3}T^{3/2}(s) \right) \right] + \frac{1}{3}E_\eta G_2(s)T_{21}^{1/2}(s) \right. \\ & \left. + \sqrt{\frac{2}{3}}E_K G_3(s)T_{31}^{1/2}(s) \right\} + \textcolor{red}{C}E_\eta G_2(s)T_{21}^{1/2}(s) \end{aligned}$$

- Charge symmetry ✓
- Unitarity ✓

$$\text{Im}\mathcal{A}(s) = -T^\dagger(s)\rho(s)\mathcal{A}(s)$$

$$\text{Im}G(s) = -\rho(s)$$

- $P$ - and  $D$ -wave: Breit-Wigner

$$\mathcal{A}_i = |\mathcal{A}_i|e^{i\delta_i}$$

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# Chiral Lagrangian (I)

- The leading order Lagrangian:

$$\mathcal{L}_{\phi P}^{(1)} = D_\mu P D^\mu P^\dagger - m^2 P P^\dagger$$

with  $P = (D^0, D^+, D_s^+)$  denoting the  $D$ -mesons, and the covariant derivative being

$$\begin{aligned} D_\mu P &= \partial_\mu P + P \Gamma_\mu^\dagger, & D_\mu P^\dagger &= (\partial_\mu + \Gamma_\mu) P^\dagger, \\ \Gamma_\mu &= \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \end{aligned}$$

where  $u_\mu = i [u^\dagger (\partial_\mu - ir_\mu) u + u (\partial_\mu - il_\mu) u^\dagger]$ ,  $u = e^{i \lambda_a \phi_a / (2F_0)}$

Burdman, Donoghue (1992); Wise (1992); Yan et al. (1992)

- this gives the Weinberg–Tomozawa term for  $P\phi$  scattering

## Chiral Lagrangian (II)

- At the next-to-leading order  $\mathcal{O}(p^2)$ : Guo, Hanhart, Krewald, Meißner, PLB666(2008)251

$$\begin{aligned}\mathcal{L}_{\phi P}^{(2)} = & P [-h_0 \langle \chi_+ \rangle - h_1 \chi_+ + h_2 \langle u_\mu u^\mu \rangle - h_3 u_\mu u^\mu] P^\dagger \\ & + D_\mu P [h_4 \langle u_\mu u^\nu \rangle - h_5 \{u^\mu, u^\nu\}] D_\nu P^\dagger,\end{aligned}$$

$$\chi_{\pm} = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B_0 \text{diag}(m_u, m_d, m_s)$$

- LECs:  $h_{1,3,5} = \mathcal{O}(N_c^0)$ ,  $h_{2,4,6} = \mathcal{O}(N_c^{-1})$

$$M_{D_s} - M_D \Rightarrow h_1 = 0.42$$

$h_0$ : can be fixed from lattice results of charmed meson masses

$h_{2,3,4,5}$ : to be fixed from lattice results on scattering lengths

- Extensions to  $\mathcal{O}(p^4)$ : see Y.-R. Liu, X. Liu, S.-L. Zhu, PRD79(2009)094026; L.-S. Geng et al., PRD82(2010)054022; D.-L. Yao, M.-L. Du, F.-K. Guo, J.-C. Leilei, JHEP1511(2015)058; M.-L. Du, F.-K. Guo, U.-G. Meißner, D.-L. Yao, EPJC77(2017)728

renormalization:

PCB-term subtraction in EOMS scheme using path integral:

$$(M.-L. Du, F.-K. Guo, D.-G. Meißner, JHEP1610(2016)122)$$

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M.-L. Du, F.-K. Guo, U.-G. Meißner, JPG44(2017)014001

PCB-term subtraction in EOMS scheme using path integral:

M.-L. Du, F.-K. Guo, U.-G. Meißner, JHEP1610(2016)122

# Energy levels in a finite volume

- Goal: predict finite volume (FV) energy levels for  $I = 1/2$ , and compare with recent lattice data by the Hadron Spectrum Col. in JHEP1610(2016)011  
⇒ insights into  $D_0^*(2400)$
- In a FV, momentum gets quantized:  $\vec{q} = \frac{2\pi}{L} \vec{n}, \vec{n} \in \mathbb{Z}^3$
- Loop integral  $G(s)$  gets modified:  $\int d^3\vec{q} \rightarrow \frac{1}{L^3} \sum_{\vec{q}}$ , and one gets

M. Döring, U.-G. Meißner, E. Oset, A. Rusetsky, EPJA47(2011)139

$$\tilde{G}(s, L) = G(s) + \lim_{\Lambda \rightarrow +\infty} \underbrace{\left[ \frac{1}{L^3} \sum_{\vec{n}}_{|\vec{q}| < \Lambda} I(\vec{q}) - \int_0^\Lambda \frac{q^2 dq}{2\pi^2} I(\vec{q}) \right]}_{\text{finite volume effect}}$$

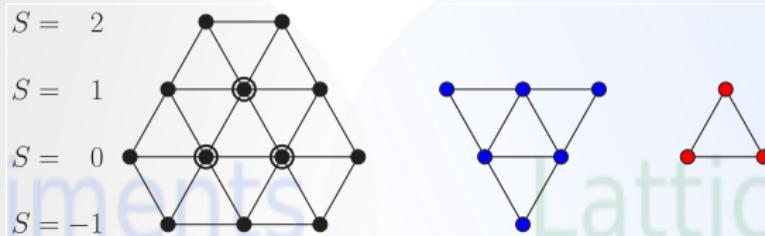
$I(\vec{q})$ : loop integrand

- FV energy levels obtained by as poles of  $\tilde{T}(s, L)$ :

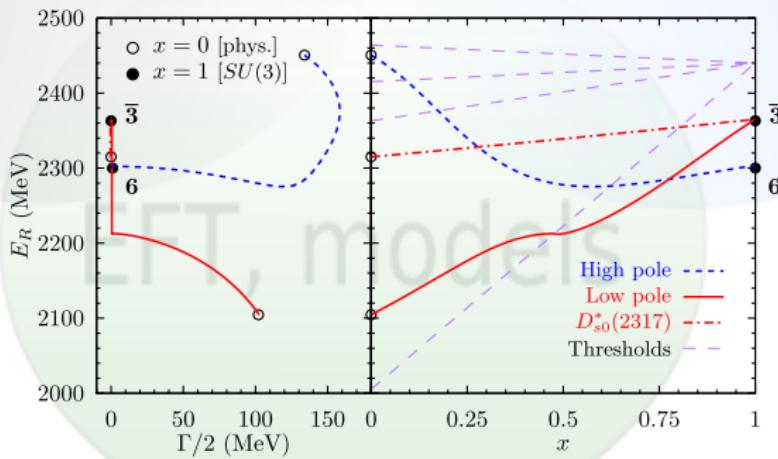
$$\tilde{T}^{-1}(s, L) = V^{-1}(s) - \tilde{G}(s, L)$$

## SU(3) analysis

- In the SU(3) limit, irreps:  $\bar{3} \otimes 8 = \bar{15} \oplus 6 \oplus \bar{3}$

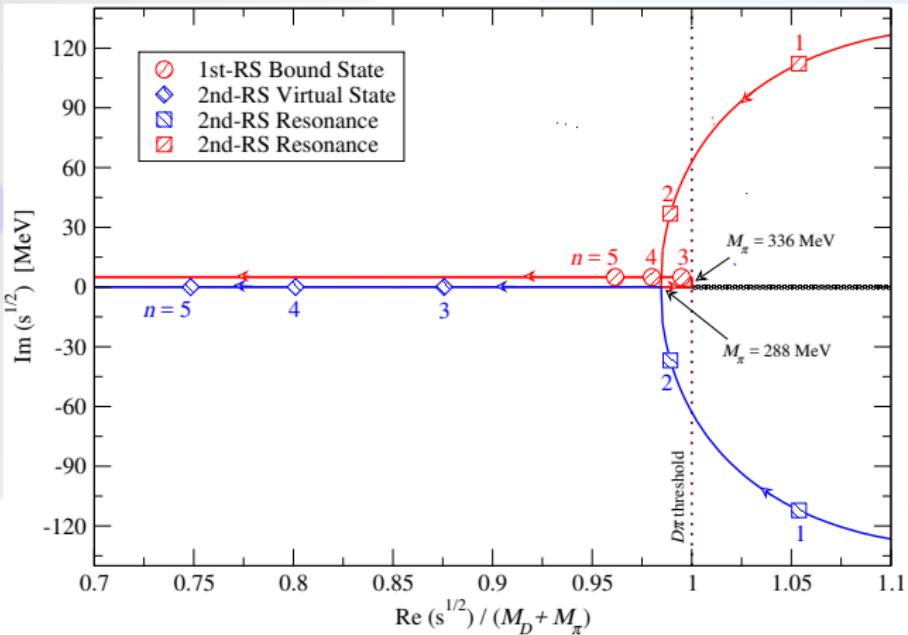


- Evolution of the two poles (LO) from the physical to the SU(3) symmetric case



Albaladejo et al., PLB767, 465 (2017)

# Trajectories of the ( $S=0, I=1/2$ ) resonance at around 2.1 GeV



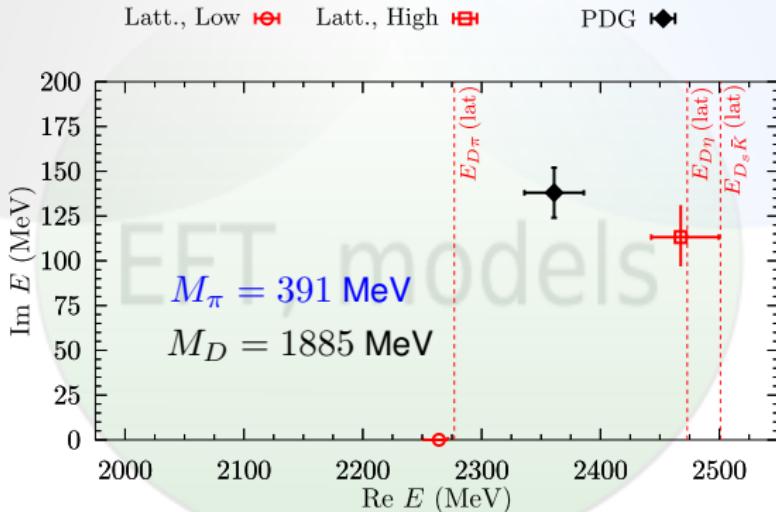
Trajectories of the ( $S = 0, I = 1/2$ ) resonance at around 2.1 GeV with varying  $M_\pi$ .  $n$  is defined by  $M_\pi = n M_\pi^{\text{phys}}$ .

Z. H. Guo *et al.*, PRD92 (2015) no.9, 094008

# There are two poles (states) !

Masses	$M$ (MeV)	$\Gamma/2$ (MeV)	RS	$ g_{D\pi} $	$ g_{D\eta} $	$ g_{D_s\bar{K}} $
lattice	$2264^{+8}_{-14}$	0	(000)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	$2468^{+32}_{-25}$	$113^{+18}_{-16}$	(110)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$

Experiments Lattice



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physical	$2105^{+6}_{-8}$	$102^{+10}_{-11}$	(100)	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	$2451^{+36}_{-26}$	$134^{+7}_{-8}$	(110)	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$

