Weak decays of doubly heavy baryons

Zhen-Xing Zhao
Shanghai Jiao Tong University

WIN2019@Bari
Jun. 2019

arXiv:1902.01092
In collaboration with Yu-Ji Shi, Wei Wang

Outline

- Introduction
- QCD sum rules

  -- Hadronic side

  -- OPE side
- Numerical results
- Summary and outlook

Introduction

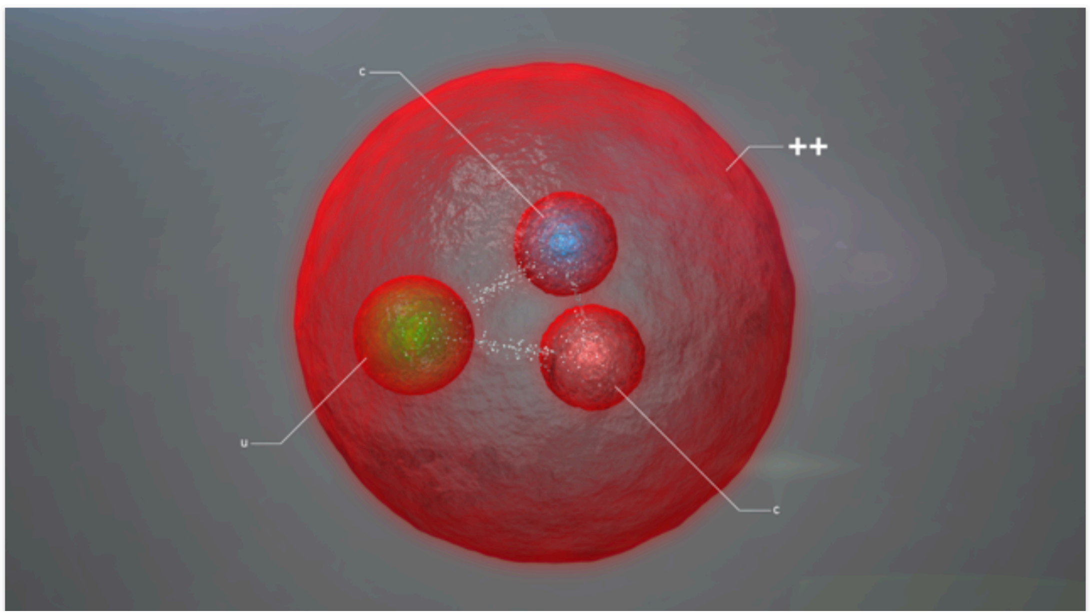
Discovery of Xi_cc^++

ABOUT NEWS SCIENCE

LHCb announces a charming new particle

The LHCb experiment has reported the observation of a baryon containing two charm quarks and one up quark

6 JULY, 2017



$$\Xi_{cc}^{++}$$

$$\Lambda_c^+ K^- \pi^+ \pi^+$$

PhysRevLett.119.112001

Other observations on Ξ_{cc}^{++}

✓ **Measurement of the lifetime of the doubly charmed baryon $\boldsymbol{\Xi_{cc}^{++}}$**

PhysRevLett.121.052002

$$0.256\,^{+0.024}_{-0.022}\,(\mathrm{stat})\;\pm 0.014\,(\mathrm{syst})\;\mathrm{ps}$$

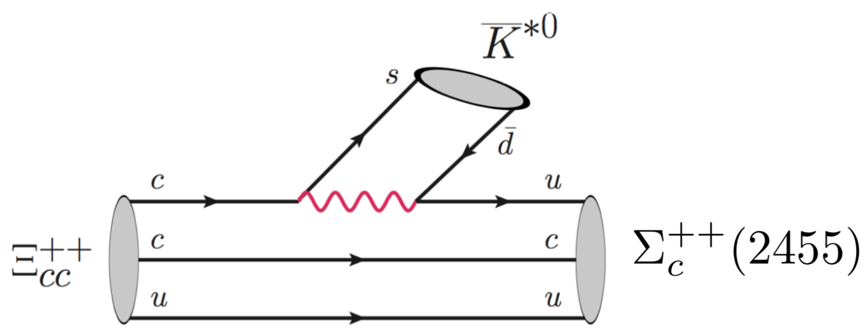
$$\Lambda_c^+ K^- \pi^+ \pi^+$$

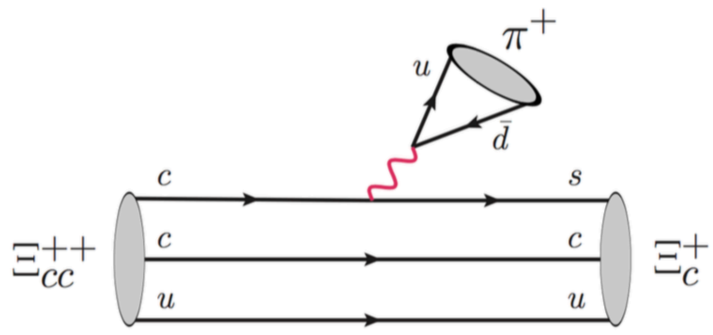
✓ **First observation of the doubly charmed baryon decay $\boldsymbol{\Xi_{cc}^{++} \to \Xi_c^+ \pi^+}$**

PhysRevLett.121.162002

Other studies on Ξ_{cc}^{++}

Two most promising channels!





$\Xi_{cc}^{++} \to \Sigma_c^{++}(2455)(\to \Lambda_c^+\pi^+)\overline{K}^{*0}(\to K^-\pi^+)$

$\Xi_{cc}^{++} \to \Xi_c^+\pi^+$

- Ξ_{cc}^{++} belongs to SU(3) triplet ($\Xi_{cc}^{++}, \Xi_{cc}^{+}, \Omega_{cc}^{+}$)
- Can decay into Λ_c or Ξ_c (SU(3) anti-triplet)
- Can decay into Σ_c , Ξ_c' , Ω_c (SU(3) sextet)

List of studies on weak decays

1. Doubly heavy baryon weak decays: $\Xi_{bc}^0 \to pK^-, \Xi_{cc}^+ \to \Sigma_c^{++}K^-$ 1701.03284
2. Discovery potentials of doubly charmed baryons 1703.09086
3. Weak decays of doubly heavy baryons: the 1/2 -> 1/2 case 1707.02834
4. Weak decays of doubly heavy baryons: SU(3) analysis 1707.06570
5. Weak decays of doubly heavy baryons: decay constant 1711.10289
6. Weak decays of doubly heavy baryons: Multi-body decays 1712.03830
7. Weak decays of doubly heavy baryons: the 1/2 -> 3/2 case 1805.10878
8. Weak decays of doubly heavy baryons: the FCNC processes 1807.03101
9. Weak decays of doubly heavy baryons: $B_{cc} \to B_c V$ 1810.00541
10. Weak decays of triply heavy baryons 1803.01476

   ...... I am sorry I can not list all of them

- Most of them adopt QCD-based phenomenological model
- Model independent calculation is highly demanded

1. QCD Sum Rules Analysis of Weak Decays of Doubly-Heavy Baryons 1902.01092
2. Light-Cone Sum Rules Analysis of $\Xi_{QQ'q} \to \Lambda_{Q'}$ Weak Decays 1903.03921

QCD sum rules

Hadronic side

Form factors

Our *standard* parametrization:

$$\begin{aligned}\langle\mathcal{B}_2(p_2,s_2)|(V-A)_\mu|\mathcal{B}_1(p_1,s_1)\rangle &= \bar{u}(p_2,s_2)[\gamma_\mu f_1(q^2)+i\sigma_{\mu\nu}\frac{q^\nu}{M_2}f_2(q^2)+\frac{q_\mu}{M}f_3(q^2)]u(p_1,s_1)\\ &- \bar{u}(p_2,s_2)[\gamma_\mu g_1(q^2)+i\sigma_{\mu\nu}\frac{q^\nu}{M_2}g_2(q^2)+\frac{q_\mu}{M}g_3(q^2)]\gamma_5 u(p_1,s_1)\end{aligned}$$

Our *practical* parametrization:

$$\begin{aligned}\langle\mathcal{B}_2(p_2,s_2)|(V-A)_\mu|\mathcal{B}_1(p_1,s_1)\rangle &= \bar{u}(p_2,s_2)[\frac{p_{1\mu}}{M_1}F_1+\frac{p_{2\mu}}{M_2}F_2+\gamma_\mu F_3]u(p_1,s_1)\\ &- \bar{u}(p_2,s_2)[\frac{p_{1\mu}}{M_1}G_1+\frac{p_{2\mu}}{M_2}G_2+\gamma_\mu G_3]\gamma_5 u(p_1,s_1)\end{aligned}$$

$$\begin{pmatrix} f_1\\ f_2\\ f_3\end{pmatrix} = \begin{pmatrix} \frac{M_1+M_2}{2M_1} & \frac{M_1+M_2}{2M_2} & 1\\ \frac{1}{2} & \frac{M_1}{2M_2} & 0\\ \frac{1}{2} & -\frac{M_1}{2M_2} & 0\end{pmatrix}\begin{pmatrix} F_1\\ F_2\\ F_3\end{pmatrix}$$

Hadronic level: version 1

$$\Pi_\mu^V = i^2 \int d^4x d^4y e^{-ip_1\cdot x + ip_2\cdot y} \langle 0|\mathrm{T}\{J_f(y)V_\mu(0)\bar{J}_i(x)\}|0\rangle$$

$$I = \sum_{s_2}\int \frac{d^4p_2}{(2\pi)^4}\frac{i}{p_2^2 - M_2^2}|\Sigma_c^+(p_2)\rangle\langle\Sigma_c^+(p_2)| + \cdots$$

$$I = \sum_{s_1}\int \frac{d^4p_1}{(2\pi)^4}\frac{i}{p_1^2 - M_1^2}|\Xi_{cc}^{++}(p_1)\rangle\langle\Xi_{cc}^{++}(p_1)| + \cdots$$

$$\Pi_\mu^V \sim \langle 0|J_f(y)|\Sigma_c^+\rangle\langle\Sigma_c^+|V_\mu(0)|\Xi_{cc}^{++}\rangle\langle\Xi_{cc}^{++}|\bar{J}_i(x)|0\rangle$$

$$\langle 0|J_f(y)|\Sigma_c^+\rangle \propto u(p_2, s_2)$$
$$\langle 0|\bar{J}_i(x)|\Xi_{cc}^{++}\rangle \propto \bar{u}(p_1, s_1)$$

$$\langle\Sigma_c^+|V_\mu(0)|\Xi_{cc}^{++}\rangle = \bar{u}(p_2, s_2)(F_1\frac{p_{1\mu}}{M_1} + F_2\frac{p_{2\mu}}{M_2} + F_3\gamma_\mu)u(p_1, s_1)$$

$$\Pi_\mu^V = \lambda_i\lambda_f\frac{(\not{p}_2 + M_2)(F_1\frac{p_{1\mu}}{M_1} + F_2\frac{p_{2\mu}}{M_2} + F_3\gamma_\mu)(\not{p}_1 + M_1)}{(p_1^2 - M_1^2)(p_2^2 - M_2^2)}$$

12 Dirac structures

$$\Pi_\mu^V = \lambda_i \lambda_f \frac{(\not{p}_2 + M_2)(F_1 \frac{p_{1\mu}}{M_1} + F_2 \frac{p_{2\mu}}{M_2} + F_3 \gamma_\mu)(\not{p}_1 + M_1)}{(p_1^2 - M_1^2)(p_2^2 - M_2^2)}$$

$$\{\not{p}_2, 1\} \times \{p_{1\mu}, p_{2\mu}, \gamma_\mu\} \times \{\not{p}_1, 1\}$$

$$\begin{aligned} F_1 : &\qquad \{\not{p}_2, 1\} \times \{p_{1\mu}\} \times \{\not{p}_1, 1\} \\ F_2 : &\qquad \{\not{p}_2, 1\} \times \{p_{2\mu}\} \times \{\not{p}_1, 1\} \\ F_3 : &\qquad \{\not{p}_2, 1\} \times \{\gamma_\mu\} \times \{\not{p}_1, 1\} \end{aligned}$$

3 form factors, 12 Dirac structures!

Hadronic level: version 2

$$\Pi^V_\mu = i^2 \int d^4x d^4y e^{-ip_1\cdot x + ip_2\cdot y} \langle 0|\mathrm{T}\{J_f(y) V_\mu(0) \bar{J}_i(x)\}|0\rangle$$

$$I = \sum_{s_2} \int \frac{d^4p_2}{(2\pi)^4} \frac{i}{p_2^2 - M_2^2} |\mathcal{B}_{f+}(p_2)\rangle\langle\mathcal{B}_{f+}(p_2)| + \sum_{s_2} \int \frac{d^4p_2}{(2\pi)^4} \frac{i}{p_2^2 - M_2^2} |\mathcal{B}_{f-}(p_2)\rangle\langle\mathcal{B}_{f-}(p_2)| + \cdots$$

$$I = \sum_{s_1} \int \frac{d^4p_1}{(2\pi)^4} \frac{i}{p_1^2 - M_1^2} |\mathcal{B}_{i+}(p_1)\rangle\langle\mathcal{B}_{i+}(p_1)| + \sum_{s_1} \int \frac{d^4p_1}{(2\pi)^4} \frac{i}{p_1^2 - M_1^2} |\mathcal{B}_{i-}(p_1)\rangle\langle\mathcal{B}_{i-}(p_1)| + \cdots$$

$$\langle 0|J_+|\mathcal{B}_-(p,s)\rangle = (i\gamma_5)\lambda_- u(p,s)$$

$$\begin{aligned}
\Pi^{V,\mathrm{pole}}_\mu &= \lambda_f^+\lambda_i^+ \frac{(\not{p}_2 + M_2^+)(\frac{p_{1\mu}}{M_1^+}F_1^{++} + \frac{p_{2\mu}}{M_2^+}F_2^{++} + \gamma_\mu F_3^{++})(\not{p}_1 + M_1^+)}{(p_2^2 - M_2^{+2})(p_1^2 - M_1^{+2})} \\
&+ \lambda_f^+\lambda_i^- \frac{(\not{p}_2 + M_2^+)(\frac{p_{1\mu}}{M_1^-}F_1^{+-} + \frac{p_{2\mu}}{M_2^+}F_2^{+-} + \gamma_\mu F_3^{+-})(\not{p}_1 - M_1^-)}{(p_2^2 - M_2^{+2})(p_1^2 - M_1^{-2})} \\
&+ \lambda_f^-\lambda_i^+ \frac{(\not{p}_2 - M_2^-)(\frac{p_{1\mu}}{M_1^+}F_1^{-+} + \frac{p_{2\mu}}{M_2^-}F_2^{-+} - \gamma_\mu F_3^{-+})(\not{p}_1 + M_1^+)}{(p_2^2 - M_2^{-2})(p_1^2 - M_1^{+2})} \\
&+ \lambda_f^-\lambda_i^- \frac{(\not{p}_2 - M_2^-)(\frac{p_{1\mu}}{M_1^-}F_1^{--} + \frac{p_{2\mu}}{M_2^-}F_2^{--} - \gamma_\mu F_3^{--})(-\not{p}_1 + M_1^-)}{(p_2^2 - M_2^{-2})(p_1^2 - M_1^{-2})}
\end{aligned}$$

Hadronic level: version 2

$$
\begin{aligned}
\Pi^{V,\mathrm{pole}}_\mu &= \lambda_f^+\lambda_i^+ \frac{(\not{p}_2+M_2^+)(\frac{p_{1\mu}}{M_1^+}F_1^{++}+\frac{p_{2\mu}}{M_2^+}F_2^{++}+\gamma_\mu F_3^{++})(\not{p}_1+M_1^+)}{(p_2^2-M_2^{+2})(p_1^2-M_1^{+2})} \\
&+ \lambda_f^+\lambda_i^- \frac{(\not{p}_2+M_2^+)(\frac{p_{1\mu}}{M_1^-}F_1^{+-}+\frac{p_{2\mu}}{M_2^+}F_2^{+-}+\gamma_\mu F_3^{+-})(\not{p}_1-M_1^-)}{(p_2^2-M_2^{+2})(p_1^2-M_1^{-2})} \\
&+ \lambda_f^-\lambda_i^+ \frac{(\not{p}_2-M_2^-)(\frac{p_{1\mu}}{M_1^+}F_1^{-+}+\frac{p_{2\mu}}{M_2^-}F_2^{-+}-\gamma_\mu F_3^{-+})(\not{p}_1+M_1^+)}{(p_2^2-M_2^{-2})(p_1^2-M_1^{+2})} \\
&+ \lambda_f^-\lambda_i^- \frac{(\not{p}_2-M_2^-)(\frac{p_{1\mu}}{M_1^-}F_1^{--}+\frac{p_{2\mu}}{M_2^-}F_2^{--}-\gamma_\mu F_3^{--})(-\not{p}_1+M_1^-)}{(p_2^2-M_2^{-2})(p_1^2-M_1^{-2})}
\end{aligned}
$$

12 equations, 12 unknowns

$$
\Pi^{V,\mathrm{OPE}}_\mu = \sum_{i=1}^{12} A_i e_{i\mu}
$$

$$
\begin{aligned}
e_{1,2,3,4} &= \{\not{p}_2, 1\} \times \{p_{1\mu}\} \times \{\not{p}_2, 1\} \\
e_{5,6,7,8} &= \{\not{p}_2, 1\} \times \{p_{2\mu}\} \times \{\not{p}_2, 1\} \\
e_{9,10,11,12} &= \{\not{p}_2, 1\} \times \{\gamma_\mu\} \times \{\not{p}_2, 1\}
\end{aligned}
$$

Hadronic level: version 2

$$\frac{\lambda_i^+\lambda_f^+(F_1^{++}/M_1^+)}{(p_1^2-M_1^{+2})(p_2^2-M_2^{+2})} = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{A_1, A_2, A_3, A_4\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

$$\frac{\lambda_i^+\lambda_f^+(F_2^{++}/M_2^+)}{(p_1^2-M_1^{+2})(p_2^2-M_2^{+2})} = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{A_5, A_6, A_7, A_8\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

$$\frac{\lambda_i^+\lambda_f^+F_3^{++}}{(p_1^2-M_1^{+2})(p_2^2-M_2^{+2})} = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{A_9, A_{10}, A_{11}, A_{12}\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

$$\lambda_i^+\lambda_f^+(F_1^{++}/M_1^+)\exp\left(-\frac{M_1^{+2}}{T_1^2}-\frac{M_2^{+2}}{T_2^2}\right) = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{\mathcal{B}_{T_1^2,T_2^2}A_1, \mathcal{B}_{T_1^2,T_2^2}A_2, \mathcal{B}_{T_1^2,T_2^2}A_3, \mathcal{B}_{T_1^2,T_2^2}A_4\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

$$\lambda_i^+\lambda_f^+(F_2^{++}/M_2^+)\exp\left(-\frac{M_1^{+2}}{T_1^2}-\frac{M_2^{+2}}{T_2^2}\right) = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{\mathcal{B}_{T_1^2,T_2^2}A_5, \mathcal{B}_{T_1^2,T_2^2}A_6, \mathcal{B}_{T_1^2,T_2^2}A_7, \mathcal{B}_{T_1^2,T_2^2}A_8\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

$$\lambda_i^+\lambda_f^+F_3^{++}\exp\left(-\frac{M_1^{+2}}{T_1^2}-\frac{M_2^{+2}}{T_2^2}\right) = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{\mathcal{B}_{T_1^2,T_2^2}A_9, \mathcal{B}_{T_1^2,T_2^2}A_{10}, \mathcal{B}_{T_1^2,T_2^2}A_{11}, \mathcal{B}_{T_1^2,T_2^2}A_{12}\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

T_1^2, T_2^2: Borel parameters

Hadronic level: version 2

$$\lambda_i^+\lambda_f^+(F_1^{++}/M_1^+)\exp\left(-\frac{M_1^{+2}}{T_1^2}-\frac{M_2^{+2}}{T_2^2}\right) = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{\mathcal{B}_{T_1^2,T_2^2}A_1, \mathcal{B}_{T_1^2,T_2^2}A_2, \mathcal{B}_{T_1^2,T_2^2}A_3, \mathcal{B}_{T_1^2,T_2^2}A_4\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

$$\lambda_i^+\lambda_f^+(F_2^{++}/M_2^+)\exp\left(-\frac{M_1^{+2}}{T_1^2}-\frac{M_2^{+2}}{T_2^2}\right) = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{\mathcal{B}_{T_1^2,T_2^2}A_5, \mathcal{B}_{T_1^2,T_2^2}A_6, \mathcal{B}_{T_1^2,T_2^2}A_7, \mathcal{B}_{T_1^2,T_2^2}A_8\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

$$\lambda_i^+\lambda_f^+F_3^{++}\exp\left(-\frac{M_1^{+2}}{T_1^2}-\frac{M_2^{+2}}{T_2^2}\right) = \frac{\{M_1^-M_2^-, M_2^-, M_1^-, 1\}.\{\mathcal{B}_{T_1^2,T_2^2}A_9, \mathcal{B}_{T_1^2,T_2^2}A_{10}, \mathcal{B}_{T_1^2,T_2^2}A_{11}, \mathcal{B}_{T_1^2,T_2^2}A_{12}\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}$$

How can we get these A_i?

$$\Pi_\mu^{\rm OPE} = \sum_{i=1}^{12} A_i e_{i\mu}$$

$$A_i\left(p_1^2, p_2^2, q^2\right) = \int ds_1 ds_2 \frac{\rho_i^{\rm OPE}\left(s_1, s_2, q^2\right)}{\left(s_1 - p_1^2\right)\left(s_2 - p_2^2\right)}$$

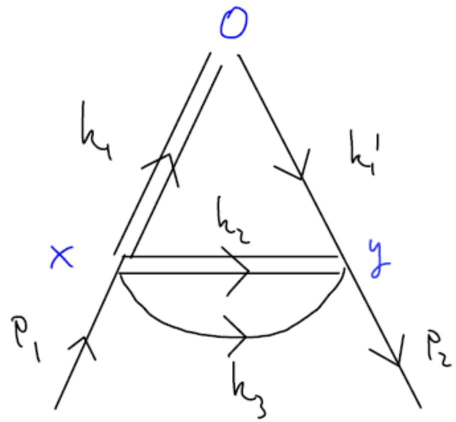
$$B_j \equiv {\rm Tr}\left[\Pi_\mu^{\rm OPE}.e_j^\mu\right] = {\rm Tr}\left[\left(\sum_{i=1}^{12} A_i e_{i\mu}\right).e_j^\mu\right]$$

Cutkosky rules

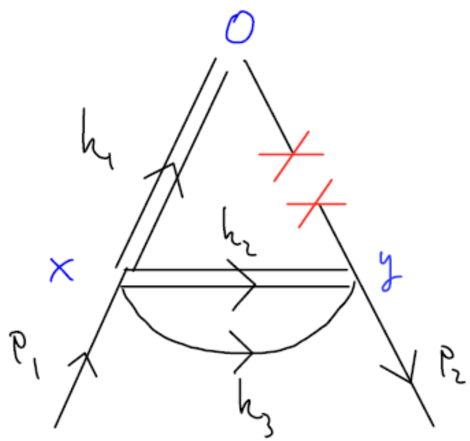
QCD sum rules

OPE side

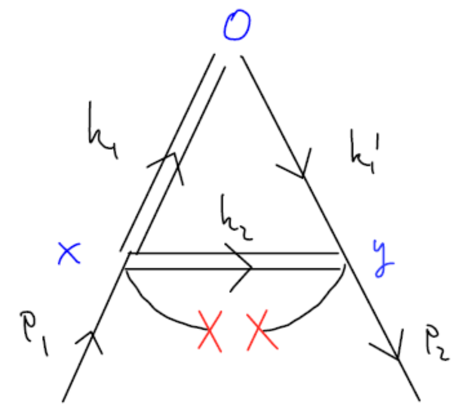
Dim-0 and dim-3



pert



qq1

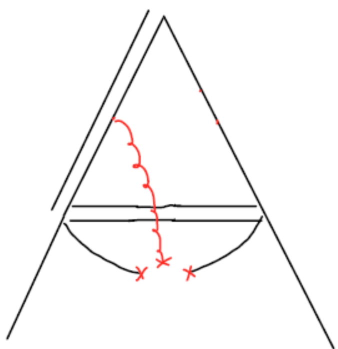


qq2

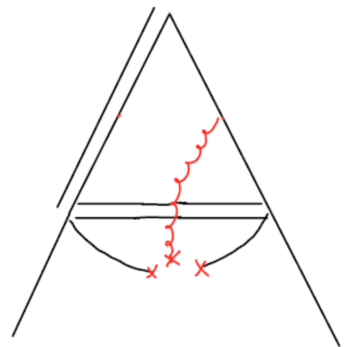
$$\sim \int \frac{d^4k_2}{(2\pi)^4} \frac{N_\mu^{V,\langle\bar{q}q\rangle,b}}{(q^2-m_1^2)(k_2^2-m_2^2)((p_2-k_2)^2-m_3^2)}$$

no contribution

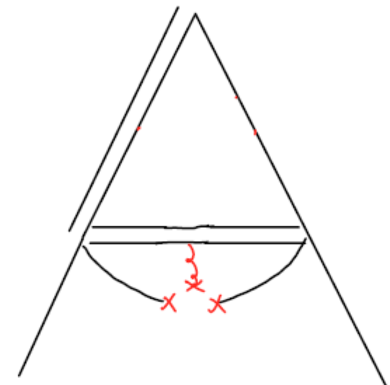
Dim-5



qgq1

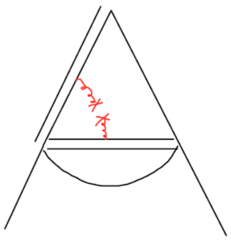


qgq2

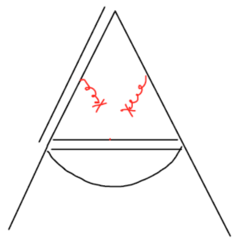


qgq3

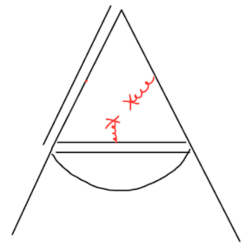
Dim-4



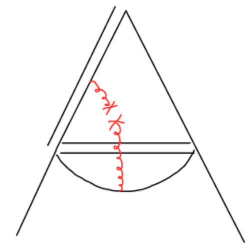
gg1



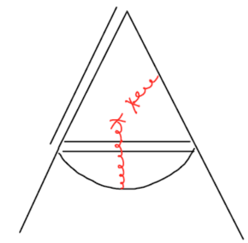
gg2



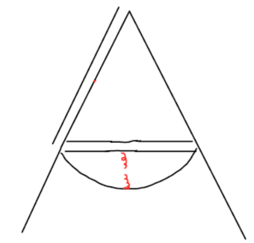
gg3



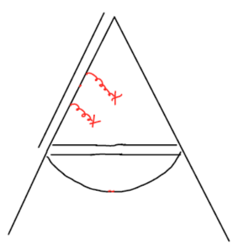
gg4



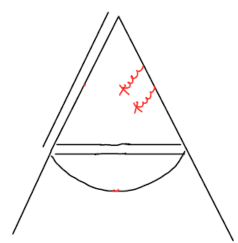
gg5



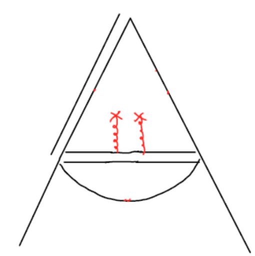
gg6



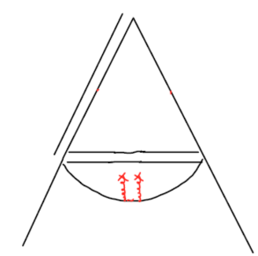
gg7



gg8

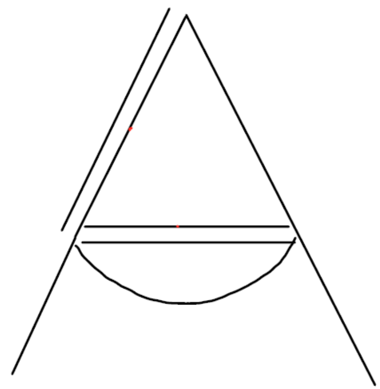


gg9

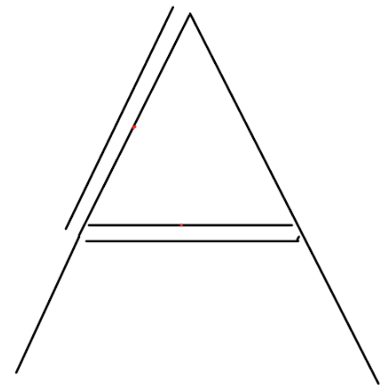


gg10

2 topologies



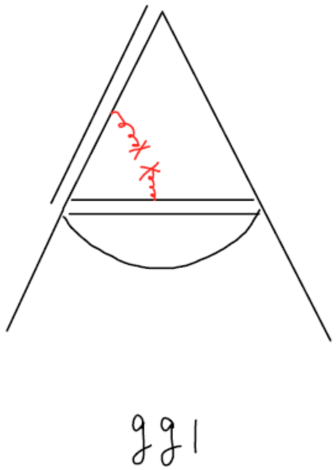
pert, gg



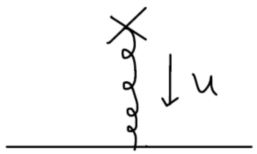
qbarq, qgq

cutting rule

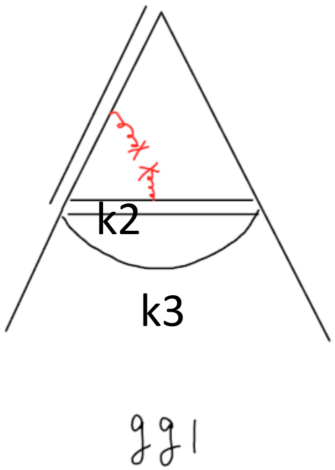
How to calculate



?



$$= (igT^a\gamma^\alpha)\left(\int \frac{d^4u}{(2\pi)^4}\frac{-i}{2}(2\pi)^4\frac{\partial}{\partial u_\rho}\delta^4(u)G^a_{\rho\alpha}(0)\right)$$



$$= 2\sqrt{2}i^2\epsilon_{a'b'c'}\epsilon_{abc}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_3}{(2\pi)^4}\int\frac{d^4u}{(2\pi)^4}\int\frac{d^4v}{(2\pi)^4}$$

$$* \Bigg[\gamma_{\nu'}\gamma_5$$

$$\left(\frac{i}{\not k_2+\not v-m_2}\delta_{c'j'}(ig\mathrm{T}^b_{j'j}\gamma^\beta)\left(\frac{-i}{2}(2\pi)^4\frac{\partial}{\partial v_\sigma}\delta^4(v)G^b_{\sigma\beta}(0)\right)\frac{i}{\not k_2-m_2}\delta_{jb}\right)$$

$$\gamma^\nu$$

$$\left(\frac{i}{\not p_1-\not k_2-\not k_3+m_1}\delta_{ia}(ig\mathrm{T}^a_{i'i}\gamma^\alpha)\left(\frac{-i}{2}(2\pi)^4\frac{\partial}{\partial u_\rho}\delta^4(u)G^a_{\rho\alpha}(0)\right)\frac{i}{\not p_1-\not k_2-\not k_3+\not u+m_1}\delta_{a''i'}\right)$$

$$\gamma_\mu$$

$$\frac{i}{\not p_2-\not k_2-\not k_3-\not v+m_1'}\delta_{b'a''}$$

$$\gamma^{\nu'}$$

$$\frac{i}{\not k_3-m_3}\delta_{a'c}$$

$$\gamma_\nu\gamma_5\Bigg]$$

15 gamma's
→
19 gamma's

1. integrate out u and v
2. write the amp in terms of 12 Dirac structures
3. integrate out k2 and k3

1. Integrate out u and v

$$\int d^4u d^4v \frac{\partial}{\partial u_\alpha}\delta^4(u)\frac{\partial}{\partial v_\beta}\delta^4(v) f(u,v) = \frac{\partial}{\partial v_\beta}\left(\frac{\partial}{\partial u_\alpha} f(u,v)\Big|_{u=0}\right)\Bigg|_{v=0}$$

$$\frac{\partial}{\partial u_\alpha}\frac{1}{\not{p}+\not{u}-m}\Bigg|_{u=0} = (-1)\frac{1}{\not{p}-m}\gamma^\alpha\frac{1}{\not{p}-m}$$

2. Project onto 12 Dirac structures

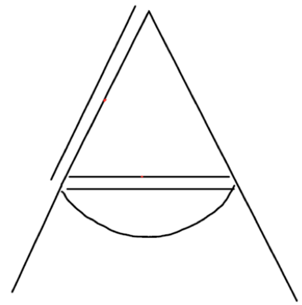
$$\Pi_\mu = \sum_{i=1}^{12} A_i e_{i\mu}, \quad e_{i\mu} : \text{12 Dirac structures}$$

$$B_j \equiv \mathrm{Tr}\left[\Pi_\mu . e_j^\mu\right] = \mathrm{Tr}\left[\left(\sum_{i=1}^{12} A_i e_{i\mu}\right) . e_j^\mu\right]$$

Advantages:

- A_i are all scalars.
- We need not calculate Π_μ! Calculate scalar $B_j \equiv \mathrm{Tr}\left[\Pi_\mu . e_j^\mu\right]$ instead!

3. Integrate out k2 and k3



$$
\begin{aligned}
&\int d^4k_2 d^4k_3 \frac{1}{(k_1^2-m_1^2)\delta(k_1'^2-m_1'^2)\delta(k_2^2-m_2^2)\delta(k_3^2-m_3^2)} \\
\to &\int d^4k_2 d^4k_3 \delta(k_1^2-m_1^2)\delta(k_1'^2-m_1'^2)\delta(k_2^2-m_2^2)\delta(k_3^2-m_3^2) \\
= &\int dm_{23}^2 \int_\triangle \int_2
\end{aligned}
$$

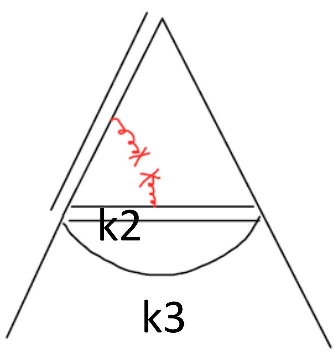
$$
\int_\triangle = \int d^4k_1 d^4k_1' d^4k_{23} \delta(k_1^2-m_1^2)\delta(k_1'^2-m_1'^2)\delta(k_{23}^2-m_{23}^2)\delta^4(p_1-k_1-k_{23})\delta^4(p_2-k_1'-k_{23})
$$

$$
\int_2 = \int d^4k_2 d^4k_3 \delta(k_2^2-m_2^2)\delta(k_3^2-m_3^2)\delta^4(k_{23}-k_2-k_3)
$$

3. Integrate out k2 and k3

$$\int d^4k_2 d^4k_3 \delta(k_1^2 - m_1^2)\delta(k_1'^2 - m_1'^2)\delta(k_2^2 - m_2^2)\delta(k_3^2 - m_3^2) = \int dm_{23}^2 \int_\triangle \int_2$$

$$\begin{array}{ll}
\text{rank} & \displaystyle\int dm_{23}^2 \int_\triangle \int_2 (\ldots) \\
0: & 1 \\
1: & k_2 \cdot p_1, \quad k_2 \cdot p_2 \\
2: & (k_2 \cdot p_1)(k_2 \cdot p_1), \quad (k_2 \cdot p_1)(k_2 \cdot p_2), \quad (k_2 \cdot p_2)(k_2 \cdot p_2) \\
 & \vdots
\end{array}$$



gg1

Building blocks: k_2

Numerical results

Inputs

$$\Xi_{cc}^{++}(ccu) \to \Sigma_c^{+}(dcu):$$

$$\begin{gathered}
m_c = (1.35 \pm 0.10)\ \mathrm{GeV}, \quad m_u = m_d = 0, \\
s_1^0 = (4.1\ \mathrm{GeV})^2, \quad s_2^0 = (3.2\ \mathrm{GeV})^2, \quad \underline{T_1^2 \in [4.8, 6.8]\ \mathrm{GeV}^2}, \\
\lambda_{\Xi_{cc}^{++}} = 0.109 \pm 0.020\ \mathrm{GeV}^3, \quad \lambda_{\Sigma_c^+} = \sqrt{2}(0.046 \pm 0.006)\ \mathrm{GeV}^3, \\
M_{\Xi_{cc}^{++}(\frac{1}{2}^+)} = 3.621\ \mathrm{GeV}, \quad M_{\Xi_{cc}^{++}(\frac{1}{2}^-)} = (3.77 \pm 0.18)\ \mathrm{GeV}, \\
M_{\Sigma_c^{+}(\frac{1}{2}^+)} = 2.453\ \mathrm{GeV}, \quad M_{\Sigma_c^{+}(\frac{1}{2}^-)} = (2.74 \pm 0.20)\ \mathrm{GeV}, \\
\langle \bar{q}q \rangle = ((-0.24 \pm 0.01)\ \mathrm{GeV})^3, \quad \langle \bar{q} g_s \sigma G q \rangle = m_0^2 \langle \bar{q}q \rangle, \quad m_0^2 = (0.8 \pm 0.2)\ \mathrm{GeV}^2
\end{gathered}$$

$$\frac{T_1^2}{T_2^2} \approx \frac{M_1^2 - m_1^2}{M_2^2 - m_1'^2}$$

P. Ball, V. M. Braun and H. G. Dosch, Phys. Rev. D 44, 3567 (1991)

2 criteria are satisfied

TABLE I: $\Xi_{cc}^{++} \to \Sigma_c^+$, $T_1^2 \in [4.8, 6.8]$ GeV^2, $q^2 = -0.75$ GeV^2, pole contribution dominated

F_1	F_2	F_3	G_1	G_2	G_3
(58 – 82)%	(55 – 80)%	(50 – 77)%	(55 – 82)%	(54 – 82)%	(56 – 76)%

TABLE II: $\Xi_{cc}^{++} \to \Sigma_c^+$, $T_1^2 = 5.8$ GeV^2, $q^2 = -0.75$ GeV^2, contributions from dim-0, 3, 5

	F_1	F_2	F_3	G_1	G_2	G_3
dim-0	0.321	0.135	−0.548	−0.692	0.373	0.157
dim-3	0.481	0.327	−0.938	−1.197	0.829	0.211
dim-5	0.036	0.047	−0.097	−0.214	0.160	0.016

Reminder: Form factors

Our *standard* parametrization:

$$\begin{aligned}\langle \mathcal{B}_2(p_2,s_2)|(V-A)_\mu|\mathcal{B}_1(p_1,s_1)\rangle &= \bar{u}(p_2,s_2)[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_2}f_2(q^2) + \frac{q_\mu}{M}f_3(q^2)]u(p_1,s_1) \\ &- \bar{u}(p_2,s_2)[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_2}g_2(q^2) + \frac{q_\mu}{M}g_3(q^2)]\gamma_5 u(p_1,s_1)\end{aligned}$$

Our *practical* parametrization:

$$\begin{aligned}\langle \mathcal{B}_2(p_2,s_2)|(V-A)_\mu|\mathcal{B}_1(p_1,s_1)\rangle &= \bar{u}(p_2,s_2)[\frac{p_{1\mu}}{M_1}F_1 + \frac{p_{2\mu}}{M_2}F_2 + \gamma_\mu F_3]u(p_1,s_1) \\ &- \bar{u}(p_2,s_2)[\frac{p_{1\mu}}{M_1}G_1 + \frac{p_{2\mu}}{M_2}G_2 + \gamma_\mu G_3]\gamma_5 u(p_1,s_1)\end{aligned}$$

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} \frac{M_1+M_2}{2M_1} & \frac{M_1+M_2}{2M_2} & 1 \\ \frac{1}{2} & \frac{M_1}{2M_2} & 0 \\ \frac{1}{2} & -\frac{M_1}{2M_2} & 0 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$$

Error estimate

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} \frac{M_1+M_2}{2M_1} & \frac{M_1+M_2}{2M_2} & 1 \\ \frac{1}{2} & \frac{M_1}{2M_2} & 0 \\ \frac{1}{2} & -\frac{M_1}{2M_2} & 0 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$$

TABLE III: error estimate

		Central value	m_c	s_1^0	s_2^0	T_1^2	λ_i	λ_f	M_1^-	M_2^-	$\langle\bar{q}q\rangle$	$\langle\bar{q}g_s\sigma Gq\rangle$
f_1	$f_1(0)$	-0.30	0.07	0.00	0.02	0.02	0.05	0.03	0.01	0.00	0.02	0.00
	m_{pole}	1.76	0.04	0.50	0.43	0.02	0.00	0.00	0.00	0.01	0.05	0.02
	δ	-0.65	0.38	1.12	1.06	0.28	0.00	0.00	0.01	0.00	0.03	0.03

$$F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{\text{fit}}^2} + \delta\left(\frac{q^2}{m_{\text{fit}}^2}\right)^2}$$

when $m_{\text{fit}} \to m_{\text{pole}}$ and $\delta \to 0$

$$F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{\text{pole}}^2}}$$

Results and comparison

TABLE IV: $\Xi_{cc}^{++} \to \Sigma_c^{+}$, Comparison with other works

	This work	LFQM	NRQM	MBM
$f_1(0)$	-0.30 ± 0.10	-0.46	-0.28	-0.30
$f_2(0)$	1.04 ± 0.44	1.04	0.14	0.91
$f_3(0)$	0.10 ± 0.06	- -	-0.10	0.07
$g_1(0)$	0.47 ± 0.19	-0.62	-0.70	-0.56
$g_2(0)$	-0.09 ± 0.07	0.04	-0.02	0.05
$g_3(0)$	-2.99 ± 1.32	- -	0.10	2.59

$$\Xi_{cc}^{++} \to \Sigma_c^{+} l^{+} \nu_l$$

$$\Gamma = (5.58 \pm 4.42) \times 10^{-15}$$

$$\mathcal{B} = (2.17 \pm 1.72) \times 10^{-3}$$

$$\Gamma_L/\Gamma_T = 1.12 \pm 0.24$$

TABLE V: Comparison with other works (in units of 10^{-15} GeV)

	This work	LFQM	HQSS	NRQM	MBM
$\Xi_{cc}^{++} \to \Sigma_c^{+} l^{+} \nu_l$	5.58 ± 4.42	9.60	5.22	6.58	2.63

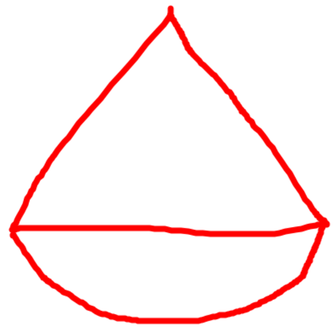
Summary and outlook

Summary

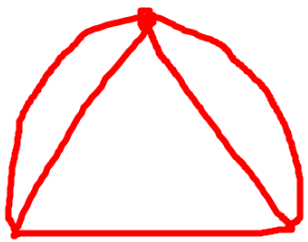
- We have adopted QCDSR to investigate the weak decays of spin-1/2 doubly-heavy baryon to spin-1/2 singly-heavy baryon
- On the OPE side, dim-0, 3, 5 are considered, and dim-4 can be neglected
- We have also considered the contribution from the negative parity baryons to eliminate the ambiguousness on the choice of the form factors
- No model-dependent parameters are introduced
- Our results are comparable to other works

Outlook

- Go ahead to investigate
  -- 1/2 -> 3/2 cases
  -- FCNC processes



- More important, the lifetime of doubly-heavy baryon!



$$\tau_{\Xi_{cc}^{++}} \gg \tau_{\Xi_{cc}^{+}}$$

B. Guberina, B. Melic and H. Stefancic, hep-ph/9911241
H. Y. Cheng and Y. L. Shi, Phys. Rev. D 98, no. 11, 113005 (2018)

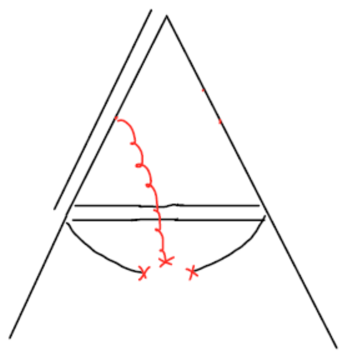
Thank you for your attention!

backup

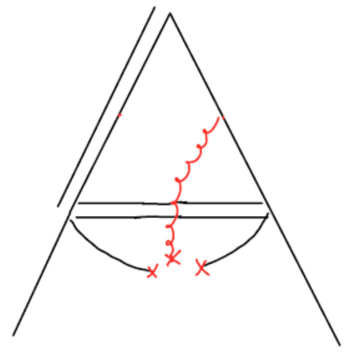
Outline

- Hadronic level
- QCD level
  - Dimension 0: perturbative
  - Dimension 3: $\langle \bar{q}q \rangle$ condensate
  - Dimension 4: $\langle GG \rangle$ condensate
  - Dimension 5: $\langle \bar{q}Gq \rangle$ condensate

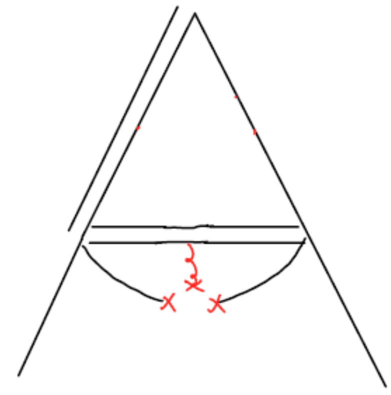
Dim-5



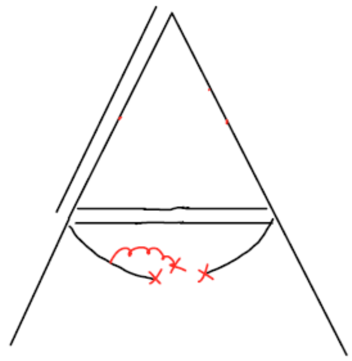
qgq1



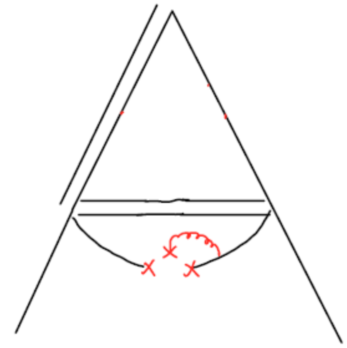
qgq2



qgq3



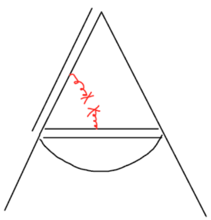
qgq4



qgq5

Calculation: the 1st step

```
(amp["1"] = {2 √2 Hold[ⅈ²] ϵ[ap, bp, cp] ϵ[a, b, c], int[4][u]/Hold[(2 π)⁴], int[4][v]/Hold[(2 π)⁴],
    int[4][k2]/Hold[(2 π)⁴], int[4][k3]/Hold[(2 π)⁴], Dot[
    ga[vp].ga[5],
    prop["f", "1g", "n"][{k2, v}, m2, {cp, b}][B, β, σ],
    ga[v],
    prop["f", "1g", "a"][{p1 - k2 - k3, u}, m1, {app, a}][A, α, ρ],
    ga[μ],
    prop["f", "a"][p2 - k2 - k3 - v, m1p, {bp, app}],
    ga[vp],
    prop["f", "n"][k3, m3, {ap, c}],
    ga[v].ga[5]
   ]}) // StandardForm
```



gg1

$$
= 2\sqrt{2}i^2\epsilon_{a'b'c'}\epsilon_{abc}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_3}{(2\pi)^4}\int\frac{d^4u}{(2\pi)^4}\int\frac{d^4v}{(2\pi)^4}
$$

$$
* \Bigg[\gamma_{\nu'}\gamma_5
\left(\frac{i}{\not{k}_2+\not{v}-m_2}\delta_{c'j'}(igT^b_{j'j}\gamma^\beta)\left(\frac{-i}{2}(2\pi)^4\frac{\partial}{\partial v_\sigma}\delta^4(v)G^b_{\sigma\beta}(0)\right)\frac{i}{\not{k}_2-m_2}\delta_{jb}\right)
\gamma^\nu
\left(\frac{i}{\not{p}_1-\not{k}_2-\not{k}_3+m_1}\delta_{ia}(igT^a_{i'i}\gamma^\alpha)\left(\frac{-i}{2}(2\pi)^4\frac{\partial}{\partial u_\rho}\delta^4(u)G^a_{\rho\alpha}(0)\right)\frac{i}{\not{p}_1-\not{k}_2-\not{k}_3+\not{u}+m_1}\delta_{a''i'}\right)
\gamma_\mu
\frac{i}{\not{p}_2-\not{k}_2-\not{k}_3-\not{v}+m_1'}\delta_{b'a''}
\gamma^{\nu'}
\frac{i}{\not{k}_3-m_3}\delta_{a'c}
\gamma_\nu\gamma_5\Bigg]
$$

Calculation: intermediate results

$$
\Big\{-\frac{1}{96\sqrt{2}\ \text{Hold}\big[(2\pi)^4\big]^2}\, g^2\ \text{cond}[GG]\ \text{Hold}\big[i^2\big]\ \delta[A,B]\ \delta[ap,c]\ \delta[app,bp]
$$

$$
\epsilon[a,b,c]\ \epsilon[ap,bp,cp]\ \text{int}[4][k2]\ \text{int}[4][k3]\ T[A][app,a]\ T[B][cp,b],
$$

$$
\Big(-\text{ga}[\nu p].\text{ga}[5].\frac{1}{-m2+\text{slash}[k2]}.\text{ga}[\beta].\frac{1}{-m2+\text{slash}[k2]}.\text{ga}[\nu].\frac{1}{m1+\text{slash}[k1]}.
$$

$$
\text{ga}[\alpha].\frac{1}{m1+\text{slash}[k1]}.\text{ga}[\rho].\frac{1}{m1+\text{slash}[k1]}.\text{ga}[\mu].\frac{1}{m1p+\text{slash}[k1p]}.
$$

$$
\text{ga}[\sigma].\frac{1}{m1p+\text{slash}[k1p]}.\text{ga}[\nu p].\frac{1}{-m3+\text{slash}[k3]}.\text{ga}[\nu].\text{ga}[5]\ +
$$

$$
\text{ga}[\nu p].\text{ga}[5].\frac{1}{-m2+\text{slash}[k2]}.\text{ga}[\sigma].\frac{1}{-m2+\text{slash}[k2]}.\text{ga}[\beta].\frac{1}{-m2+\text{slash}[k2]}.\text{ga}[\nu].
$$

$$
\frac{1}{m1+\text{slash}[k1]}.\text{ga}[\alpha].\frac{1}{m1+\text{slash}[k1]}.\text{ga}[\rho].\frac{1}{m1+\text{slash}[k1]}.\text{ga}[\mu].\frac{1}{m1p+\text{slash}[k1p]}.
$$

$$
\text{ga}[\nu p].\frac{1}{-m3+\text{slash}[k3]}.\text{ga}[\nu].\text{ga}[5]\Big)\ \text{lor}[-g[\beta,\rho]\ g[\sigma,\alpha]+g[\beta,\alpha]\ g[\sigma,\rho]]\Big\}
$$