

# Precision study of inclusive $B \rightarrow X d(s) \bar{d}$ decays

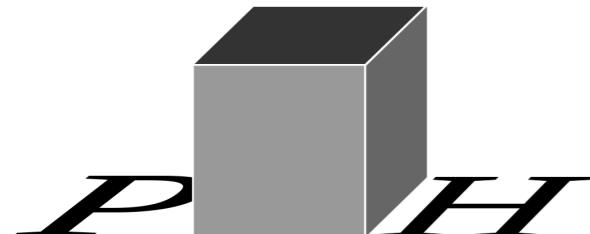
Precision study of inclusive  $B \rightarrow X q(s) \bar{q}$  decays

T. Huber, QQ, K. Vos, 1806.11521

T. Huber, T. Hurth, J. Jenkins, E. Lunghi, QQ, K. Vos, in preparation

Qin Qin  
University of Siegen

The 27th International Workshop on Weak Interactions and Neutrinos



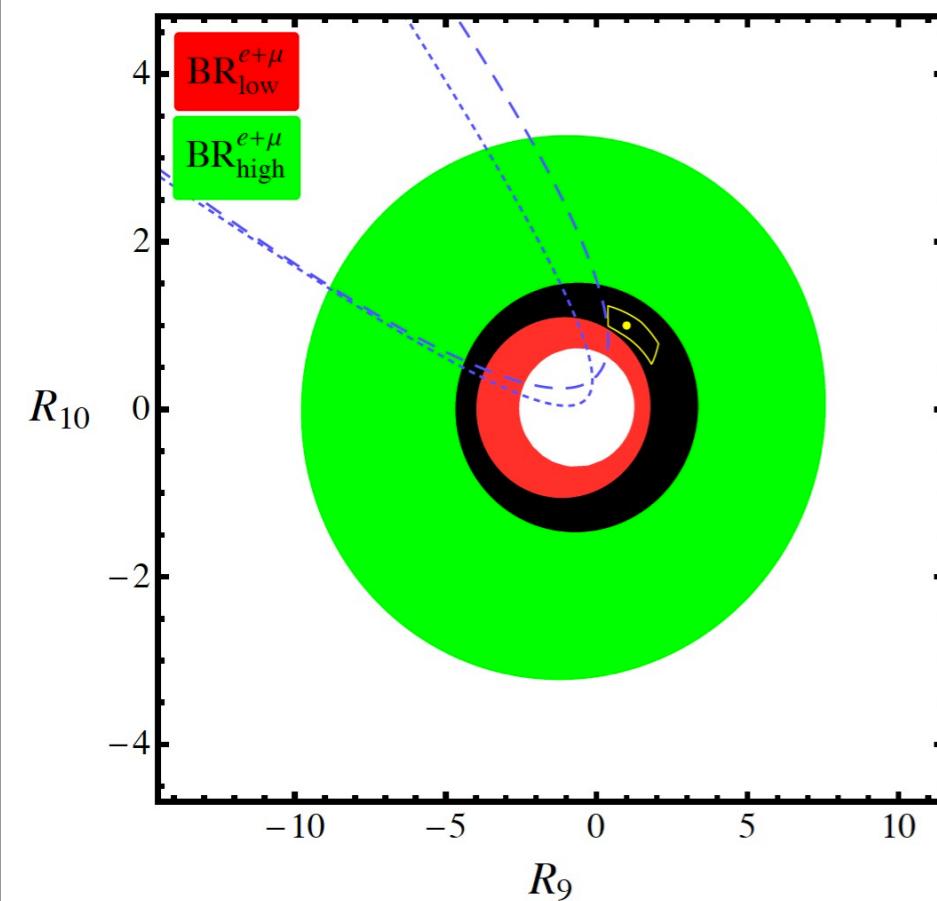
# Motivation

## ★ B anomalies in exclusive B decays might be hints for new physics

[BaBar,1205.5442;1303.0571;Belle,1507.03233;1604.04042;1612.05014;LHCb,1406.6482;1705.05802;1512.04442;1506.08614;  
ATLAS,1805.04000;CMS,1710.02846;1904.02440...] see Golob's,Ricciardi's,Matias's,Rozanska's,Pardinas',Meaux's,Palestini's talks

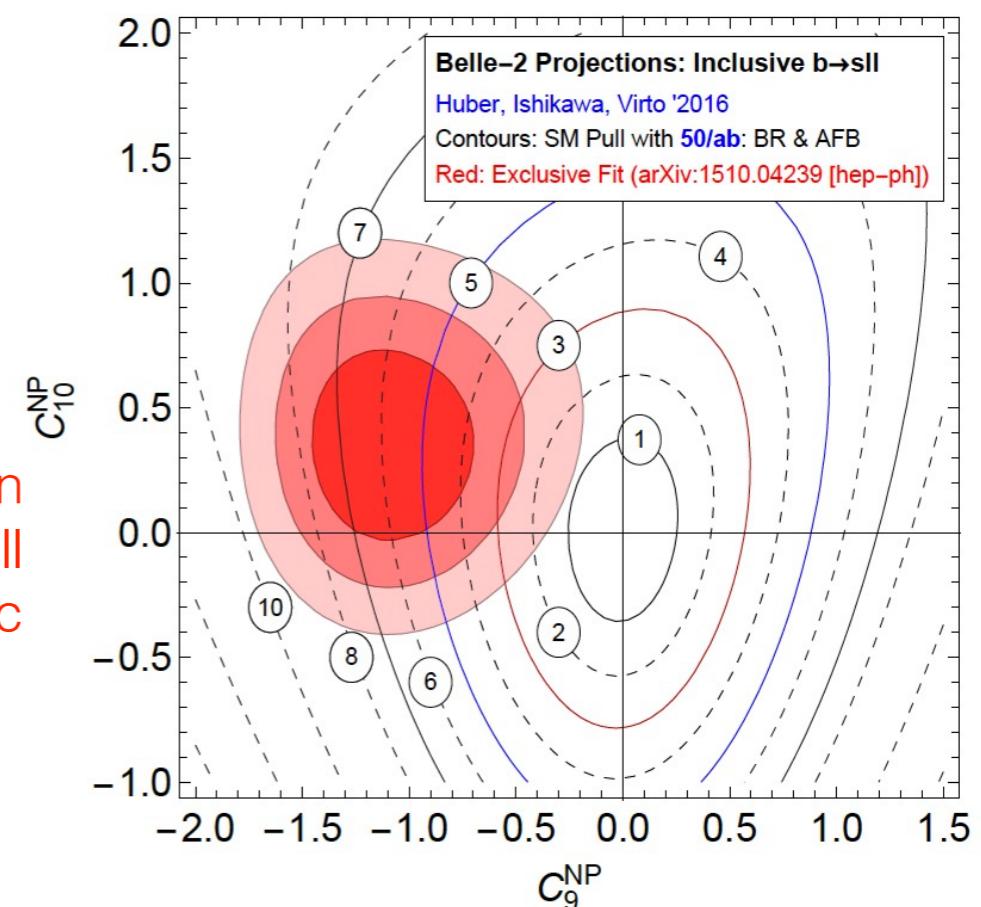
## ★ Precision measurements of inclusive $\bar{B} \rightarrow X_s \ell^+ \ell^-$ channel provide important complementary information

- \* Underlying hadronic uncertainties in inclusive mode are quite different and independent of those in exclusive transitions



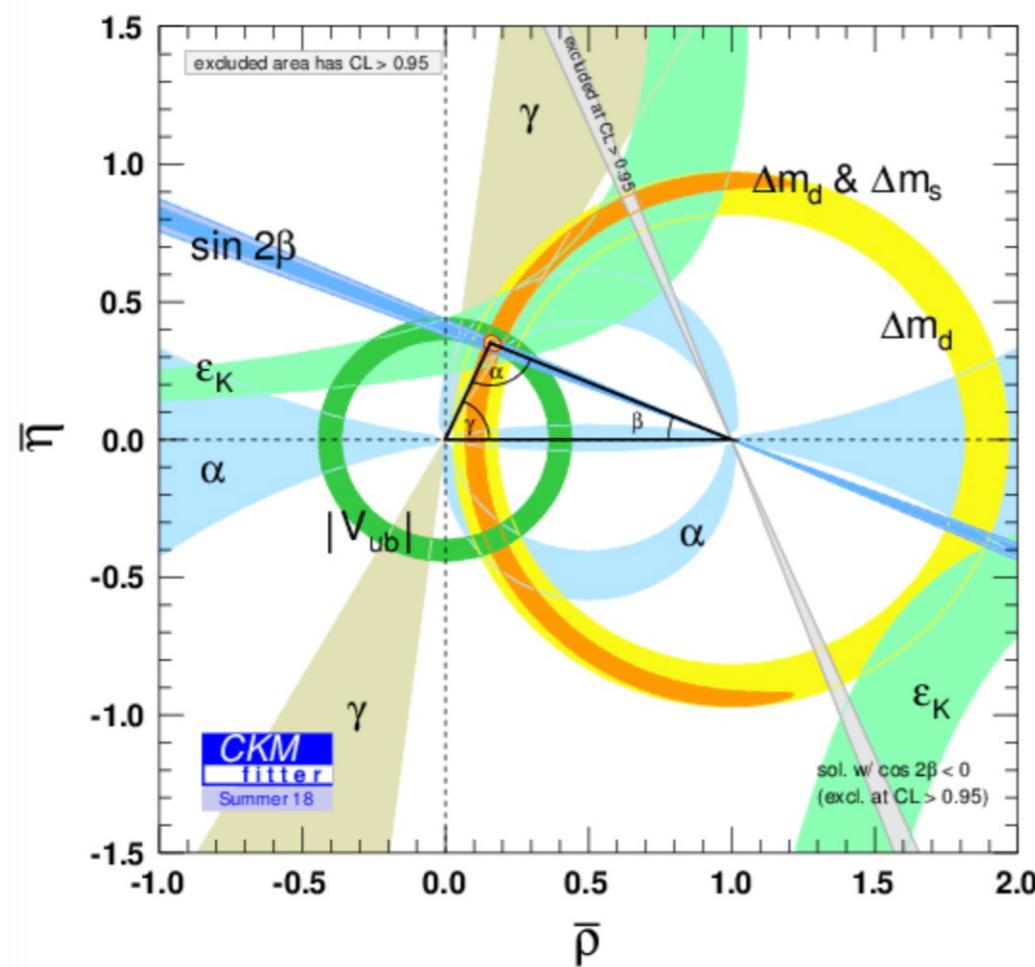
(left) Constraints to Wilson coefficients from inclusive b-sll observables;

(right) Comparison between inclusive and exclusive b-sll constraints to Wilson coefficients



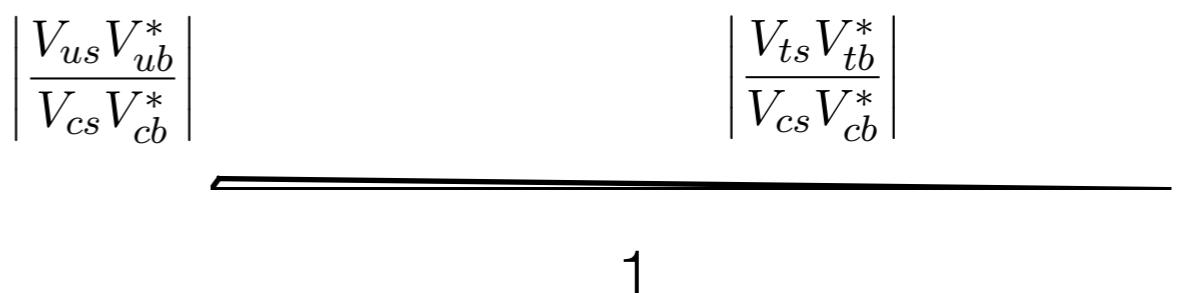
# Motivation

- ★ Inclusive  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  decay provides more observables
  - \* possible sizeable CP asymmetry



CKM triangle for b-d transition

[CKM fitter, 18']



CKM triangle for b-s transition

# Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}}(u, d, s, c, b, e, \mu, \tau)$$

$$-\frac{4G_F}{\sqrt{2}} \sum_{q=u,c} V_{qd}^* V_{qb} (C_1^q P_1^q + C_2^q P_2^q) + \frac{4G_F}{\sqrt{2}} V_{td}^* V_{tb} \sum_{i=3}^{10} C_i(\mu) P_i$$

$$P_1^q = (\bar{d}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a b_L),$$

$$P_6 = (\bar{d}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q),$$

$$P_2^q = (\bar{d}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu b_L),$$

$$P_7 = \frac{e}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} b_R) F_{\mu\nu},$$

$$P_3 = (\bar{d}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q),$$

$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a,$$

$$P_4 = (\bar{d}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q),$$

$$P_9 = (\bar{d}_L \gamma_\mu b_L) \sum_l (\bar{l} \gamma^\mu l),$$

$$P_5 = (\bar{d}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q),$$

$$P_{10} = (\bar{d}_L \gamma_\mu b_L) \sum_l (\bar{l} \gamma^\mu \gamma_5 l).$$

[for b -> d]

# Methodology

## ★ Nonperturbative dynamics involved in hadronic matrix elements

$$\langle X_{d,s} | \mathcal{O} | \bar{B} \rangle \quad \text{hadrons}$$

## ★ How to solve this? OPE (operator product expansion).

$$\sum_X \langle \bar{B} | \mathcal{O}^\dagger | X_{d,s} \rangle \langle X_{d,s} | \mathcal{O} | \bar{B} \rangle$$

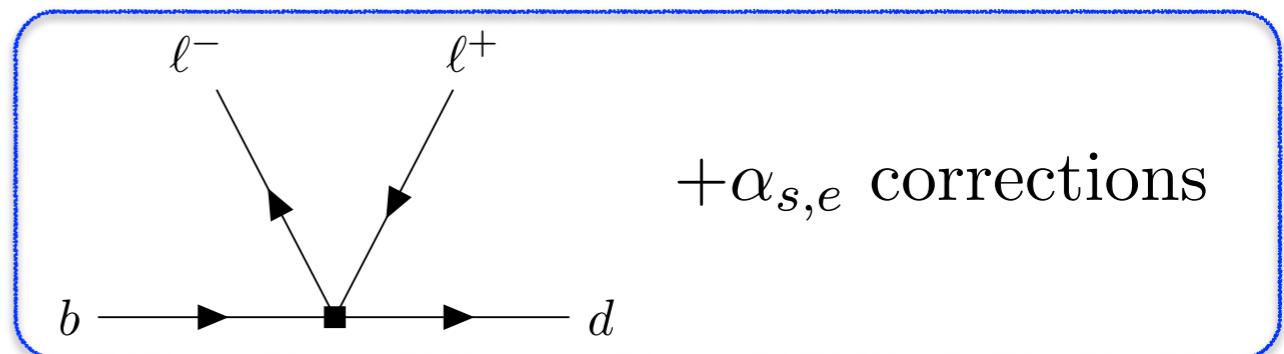
[Falk,Luke,Savage,93']

$$(\text{optical theorem}) \propto \text{Im} \langle \bar{B} | T\{\mathcal{O}^\dagger, \mathcal{O}\} | \bar{B} \rangle$$

1/m<sub>b</sub> power corrections,  
extracted from measurements

$$(\text{OPE}) = \text{Im} \langle \bar{B} | \mathcal{O}_0 + \frac{1}{m_b} \mathcal{O}_1 + \frac{1}{m_b^2} \mathcal{O}_2 + \dots | \bar{B} \rangle$$

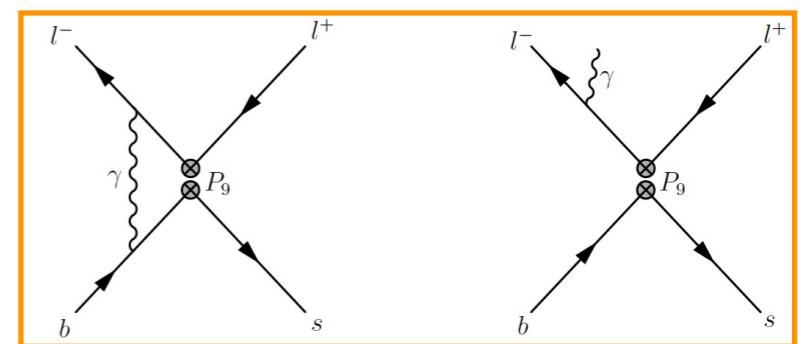
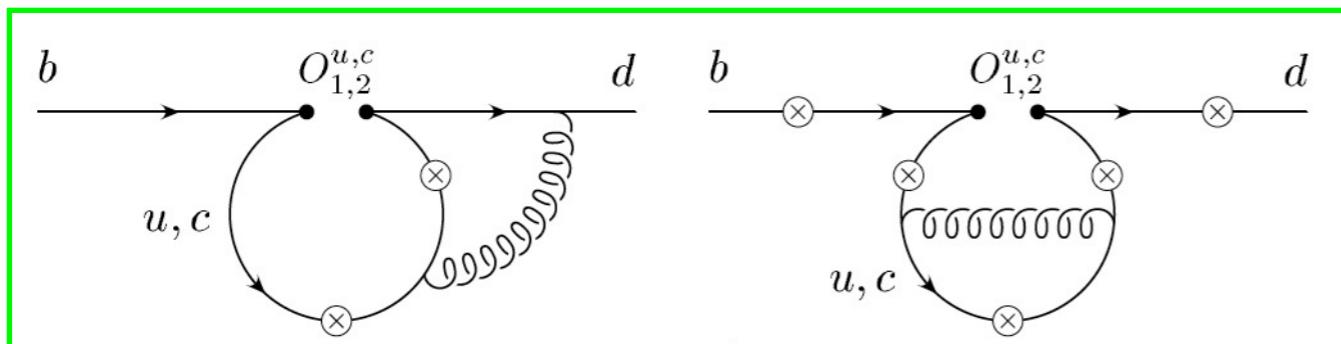
leading power:  
free quark interactions



# Previous studies

- ★ Nonperturbative power corrections, up to  $1/m_b^2$  &  $1/m_b^3$  (in specific kinetic regions)  
[Ali,Hiller,Handoko,Morozumi,97';Ligeti,Tackmann,07']

- ★ Perturbative  $\alpha_{s,e}$  corrections, including
  - (N)NLO QCD corrections [Misiak et al,92',99';Greub et al,01',02',03',04';Seidel,04']
  - log-enhanced NLO QED corrections [Huber et al,05',07']



- ★ Handling of long-distance subtleties (additional nonperturbative effects)  
[Kruger,Sehgal,96',96';Buchalla,Isidori,Rey,97';Benzke,Hurth,Turczyk,17']

# Five-particle Contributions

---

## Five-particle contributions

[Huber, QQ, Vos, Eur.Phys.J.C78(2018)748]

# Five-particle contributions

T. Huber, QQ, K. Vos, 1806.11521

- ★ Perturbative corrections include not only loop contributions but also multi-particle final-state contributions as  $b \rightarrow d(s) \ell^+ \ell^- \bar{q} q$

- $q = u, d$  and  $s$ , but not  $c$

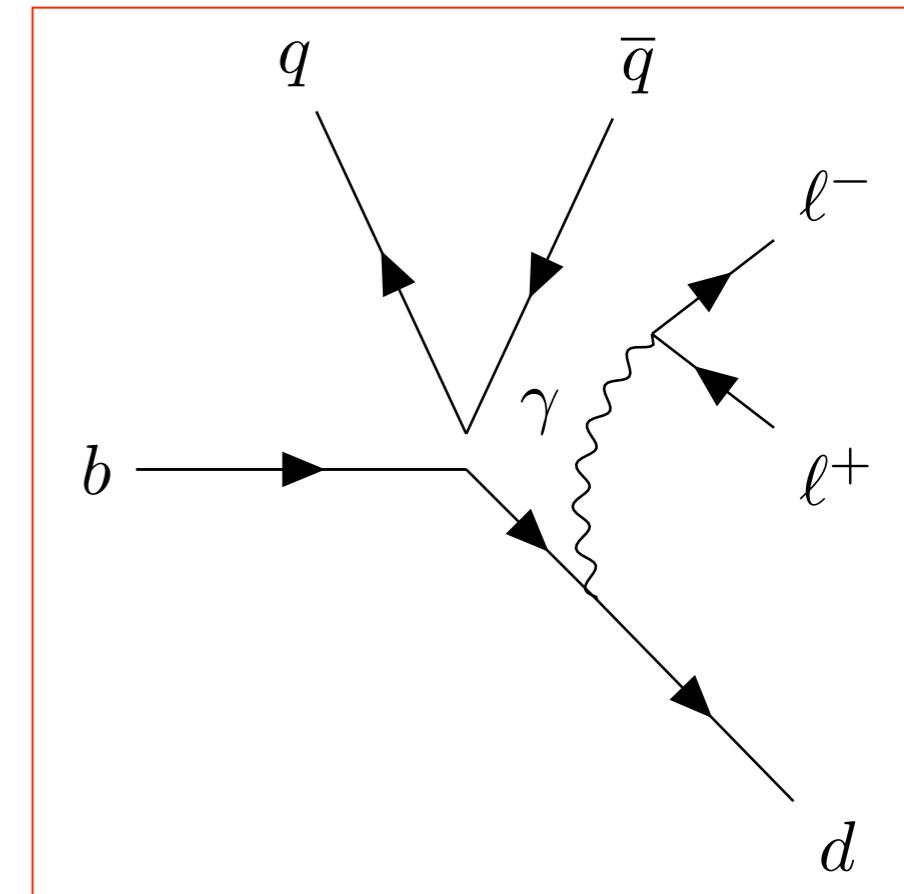
- ★ Expand in couplings up to  $\mathcal{O}(\tilde{\alpha}_s^3, \kappa^3)$

$$\left( \tilde{\alpha}_s \equiv \frac{\alpha_s}{4\pi}, \kappa \equiv \frac{\alpha_e}{\alpha_s} \right)$$

- ★ Contributions to observables

- branching ratio
- forward-backward asymmetry

- ★ Almost all results are analytical

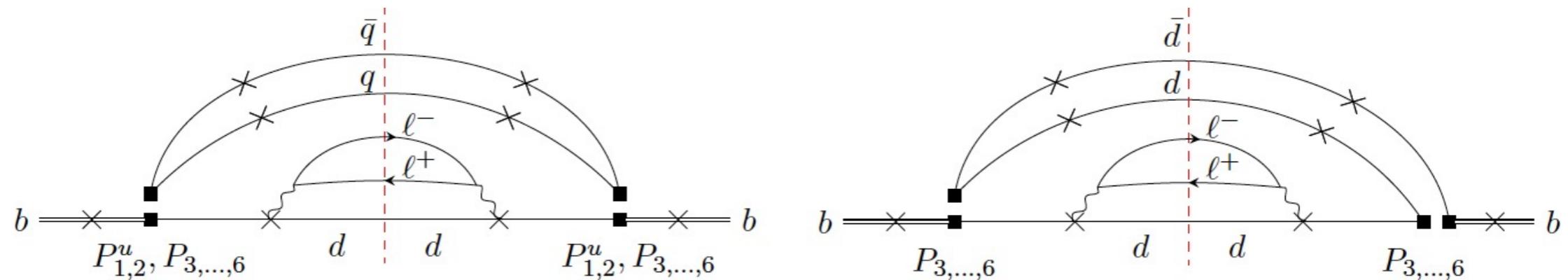


# Five-particle contributions

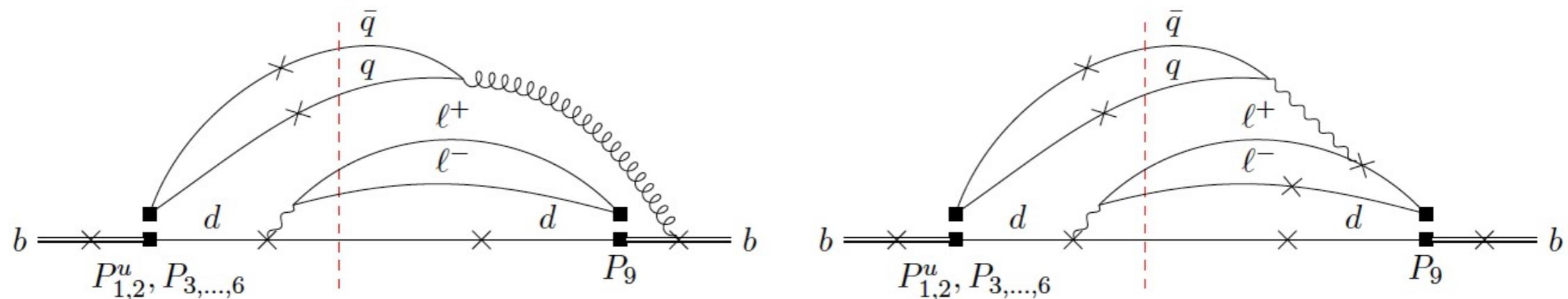
T. Huber, QQ, K. Vos, 1806.11521

## ★ Include tree-level interferences of

- $(P_{1,2}^u, P_{3,\dots,6})$  with  $(P_{1,2}^u, P_{3,\dots,6})$



- $(P_{1,2}^u, P_{3,\dots,6})$  with  $(P_{7,\dots,10})$

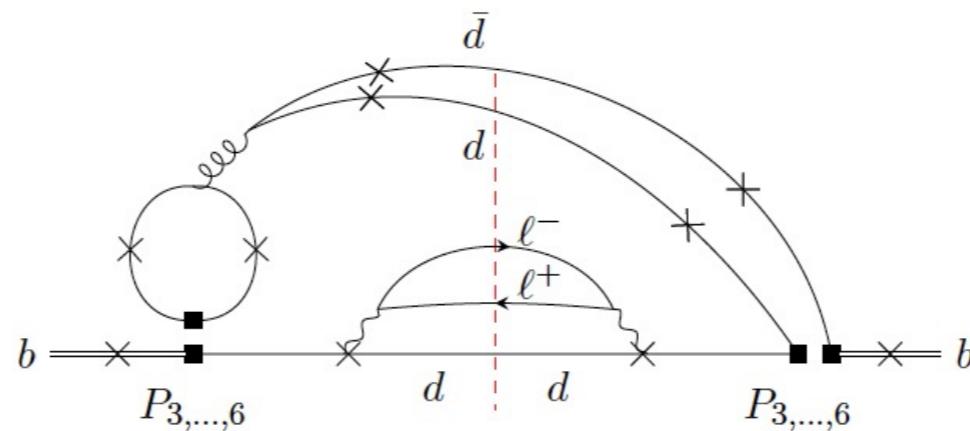


# Five-particle contributions

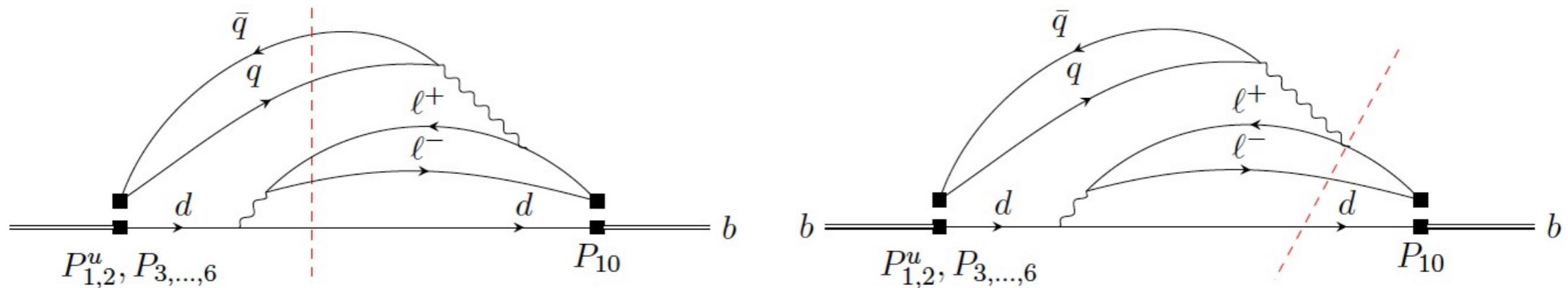
T. Huber, QQ, K. Vos, 1806.11521

## ★ Discard

- one-loop interferences of  $(P_{1,2}^u, P_{3,\dots,6})$  with  $(P_{1,2}^u, P_{3,\dots,6})$



- and interferences that require fewer-particle cuts involving loops for renormalisation



# Five-particle contributions

T. Huber, QQ, K. Vos, 1806.11521

★ Adopt two different phase-space parametrizations [Kumar,69';Heinrich,06']

$$\frac{d\Phi_5}{ds} |\mathcal{M}|^2 = \frac{\pi^2 m_b^6}{16(2\pi)^{11}} \int_s^1 ds_1 \int_s^{s_1} ds_2 \int_{s_2/s_1}^{1-s_1+s_2} du_1 \int_{u_2^-}^{u_2^+} du_2 \int_{u_3^-}^{u_3^+} du_3 \int_{t_2^-}^{t_2^+} dt_2 \int_{t_3^-}^{t_3^+} dt_3 \\ \times \frac{(s_2 - s)}{s_2 (u_2^+ - u_2^-) (u_3^+ - u_3^-) \sqrt{(t_2^+ - t_2)(t_2 - t_2^-)} \sqrt{(t_3^+ - t_3)(t_3 - t_3^-)}} \left( \frac{1}{2} |\mathcal{M}|^2 \Big|_{s_{34} \rightarrow s_{34}^+} + \frac{1}{2} |\mathcal{M}|^2 \Big|_{s_{34} \rightarrow s_{34}^-} \right)$$

$$\int d\Phi_5 |\mathcal{M}|^2 = \frac{m_b^6}{4^8 \pi^9} \int_0^1 dt_2 \dots dt_4 dt_6 \dots dt_{10} \frac{t_2 t_6 t_7^2 (1-t_6)(1-t_7) \delta(t_2 t_4 t_6 t_7 - s_{14})}{\sqrt{t_8} \sqrt{1-t_8} \sqrt{t_{10}} \sqrt{1-t_{10}}} \left( |\mathcal{M}|^2 \Big|_{t_5=0} + |\mathcal{M}|^2 \Big|_{t_5=1} \right)$$

★ Staying differential in  $q^2 \equiv (p_{\ell^+} + p_{\ell^-})^2$  allows to perform the calculation in  $D = 4$  dimensions. Seven integrations are required.

$$f_1(\hat{s}) = \frac{\pi}{6} - \arctan(x)$$

$$f_7(\hat{s}) = 2i f_1(\hat{s}) \left[ \text{Li}_2 \left( \frac{1-ix}{1+ix} \right) - c.c. \right] - \left[ \text{Li}_3 \left( \frac{1-ix}{1+ix} \right) + c.c. \right] + \frac{2\zeta(3)}{3}, \quad x = \sqrt{\frac{\hat{s}}{4-\hat{s}}}$$

# Five-particle contributions

T. Huber, QQ, K. Vos, 1806.11521

## ★ Differential branching ratio and forward-backward asymmetry

$$\frac{d\mathcal{B}(b \rightarrow d\ell^+\ell^-q\bar{q})}{d\hat{s}} = \mathcal{B}(\bar{B} \rightarrow X_c e\bar{\nu})_{\text{exp}} \left| \frac{V_{td}^* V_{tb}}{V_{cb}} \right|^2 \frac{4}{C\Phi_u} \left( \sum_{i,j=1}^{10} \mathcal{R}_{\text{CKM}}^{ij} C_i^* C_j \mathcal{F}_{ij}(\hat{s}) \right)$$

$$\frac{dA_{\text{FB}}}{d\hat{s}} = \mathcal{B}(\bar{B} \rightarrow X_c e\bar{\nu})_{\text{exp}} \left| \frac{V_{td}^* V_{tb}}{V_{cb}} \right|^2 \frac{4}{C\Phi_u} \sum_{i=1}^6 (-\mathcal{R}_{\text{CKM}}^{i10} C_i^* C_{10} \mathcal{A}_i(\hat{s}) + c.c.)$$

## ★ $\mathcal{O}(1\%)$ correction in the case of $\bar{B} \rightarrow X_d \ell^+ \ell^-$

$$q^2 \equiv (p_{\ell^+} + p_{\ell^-})^2$$

	[1, 3.5] GeV <sup>2</sup>	[3.5, 6] GeV <sup>2</sup>	[1, 6] GeV <sup>2</sup>
$\mathcal{B}(b \rightarrow d\ell^+\ell^-q\bar{q}) (\times 10^{-10})$	9.22	0.30	9.52
$A_{\text{FB}}(b \rightarrow d\ell^+\ell^-q\bar{q}) (\times 10^{-12})$	1.48	0.49	1.97

## ★ $\mathcal{O}(0.01\%)$ correction in the case of $\bar{B} \rightarrow X_s \ell^+ \ell^-$ (CKM suppression)

	[1, 3.5] GeV <sup>2</sup>	[3.5, 6] GeV <sup>2</sup>	[1, 6] GeV <sup>2</sup>
$\mathcal{B}(b \rightarrow s\ell^+\ell^-q\bar{q}) (\times 10^{-10})$	2.18	0.05	2.23
$A_{\text{FB}}(b \rightarrow s\ell^+\ell^-q\bar{q}) (\times 10^{-11})$	1.57	0.52	2.10

# **Long-distance Effects**

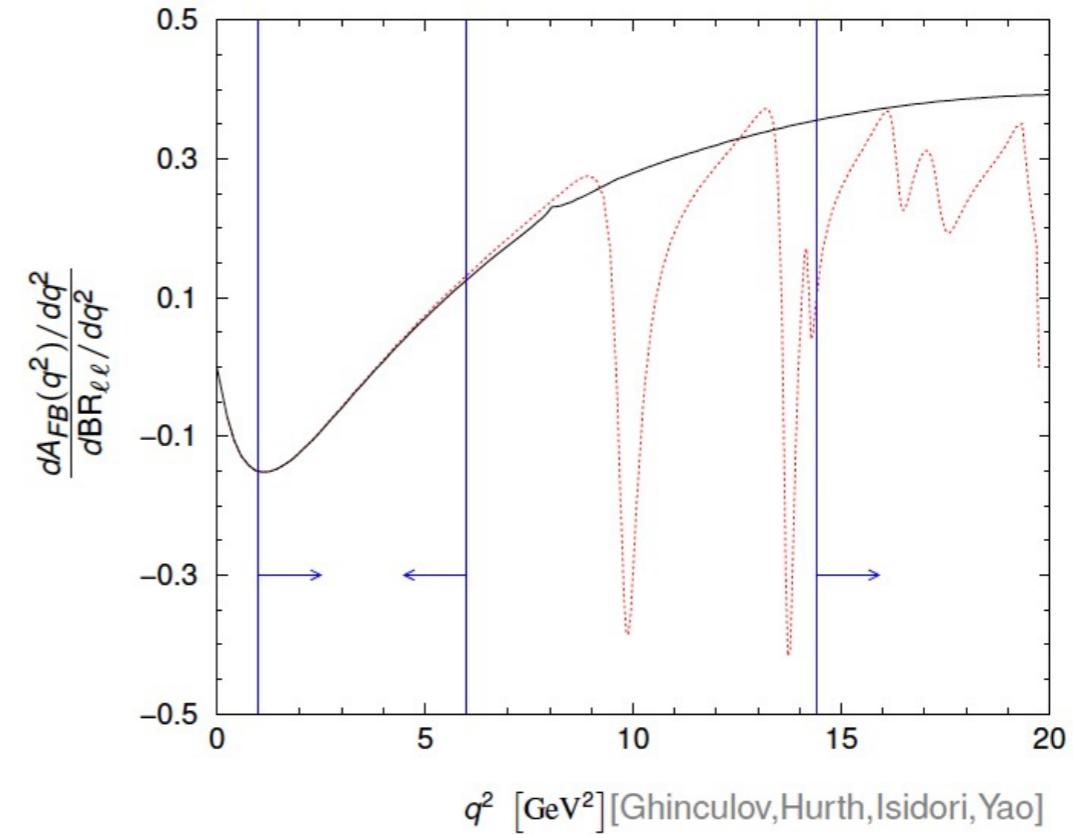
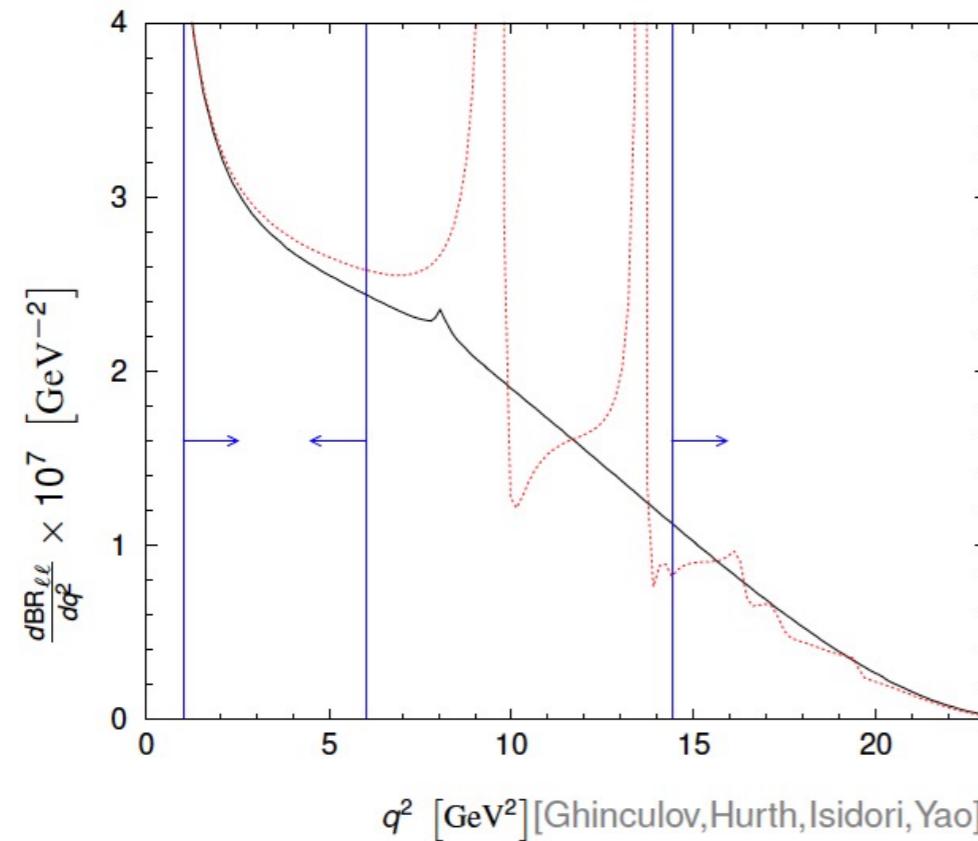
---

## Long-distance Effects

[T. Huber, T. Hurth, J. Jenkins, E. Lunghi, QQ, K. Vos, in preparation]

# Long-distance effects

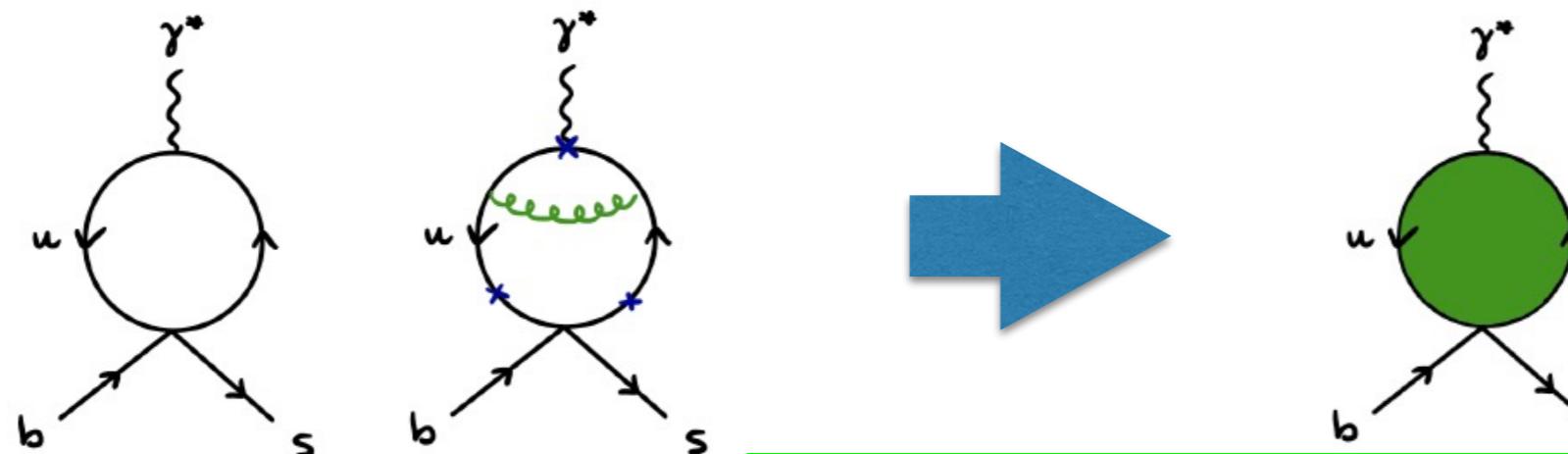
- ★ In the  $q^2 \equiv (p_{\ell^+} + p_{\ell^-})^2$  spectrum of  $\bar{B} \rightarrow X_s \ell^+ \ell^-$ ,  $c\bar{c}$  resonances show up as large peaks
  - The  $\bar{B} \rightarrow X_s \psi \rightarrow X_s \ell^+ \ell^-$  BR exceeds the short-distance BR by 2 orders



- ★ In  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  also light-quark resonances are relevant

# Long-distance effects

- ★ Despite of cuts on  $q^2$ , we need to capture the tail effects of the resonances following the idea of Kruger and Sehgal [Kruger,Sehgal,96']
  - Replace the perturbative amplitudes (factorizable) by data extracted ones



$$(q^\mu q^\nu - q^2 g^{\mu\nu}) h_q^{\text{KS}}(q^2) = \frac{16\pi^2}{9Q_q} i \int d^4x e^{iqx} \langle 0 | T J_q^\mu(0) J_{\text{em}}^\nu(x) | 0 \rangle$$

- Very similar currents contribute to  $R_{\text{had}}(s) \equiv \frac{\sigma_{\text{had}}(s)}{4\pi\alpha^2/3s} = 12\pi \text{ Im}[\Pi_\gamma(s)]$  (and hadronic tau decays)

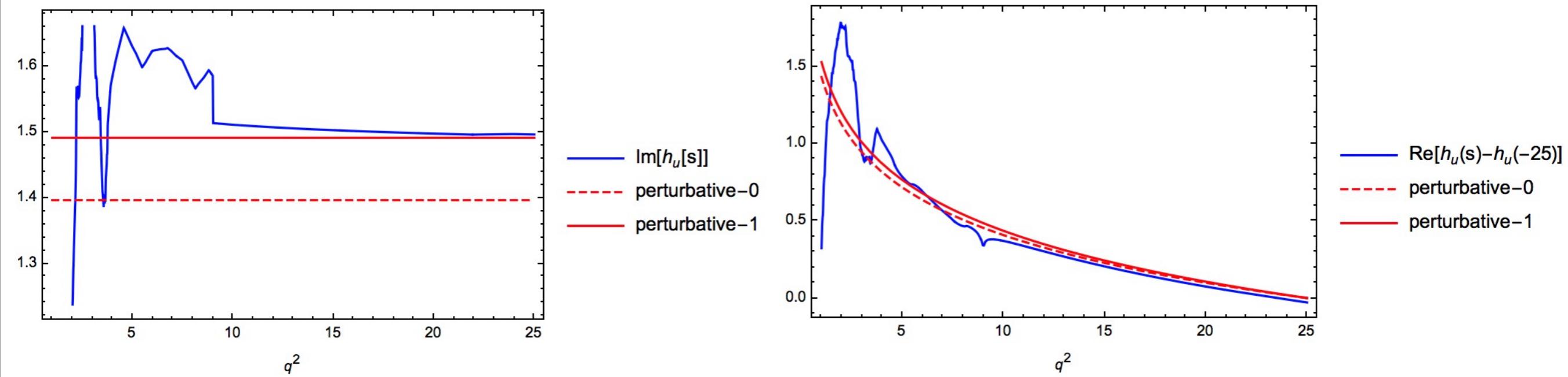
$$(q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_\gamma(q^2) = i \int d^4x e^{iqx} \langle 0 | T J_{\text{em}}^\mu(0) J_{\text{em}}^\nu(x) | 0 \rangle$$

- Data tells us the imaginary part, and the real part is obtained by the dispersion relation

$$\text{Re}[h_q^{\text{KS}}(s)] = \text{Re}[h_q^{\text{KS}}(s_0)] + \frac{s - s_0}{\pi} \int_0^\infty dt \frac{\text{Im}[h_q^{\text{KS}}(t + i\epsilon)]}{(t - s_0)(t - s - i\epsilon)}.$$

# Long-distance effects

- ★ Use latest data (including ee collision and tau decays) from BESIII, BaBar, ALEPH in the resonance regions [see Keshavarzi,Nomura,Teubner,18']
- ★ In the replacement of the perturbative function we take into account all factorizable pieces up to two loops. Separation of factorizable two-loop perturbative function are provided by St. de Boer. [de Boer,17']
- ★ Choose as subtraction point  $s_0 = -(5\text{GeV})^2$

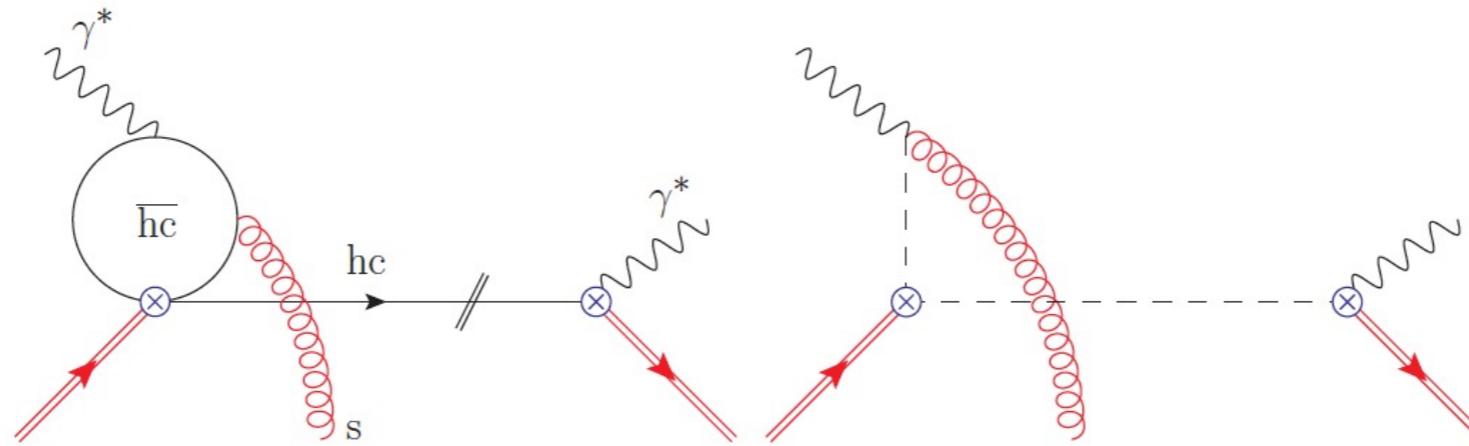


Asymptotic behaviours of perturbative and nonperturbative functions

# Long-distance effects

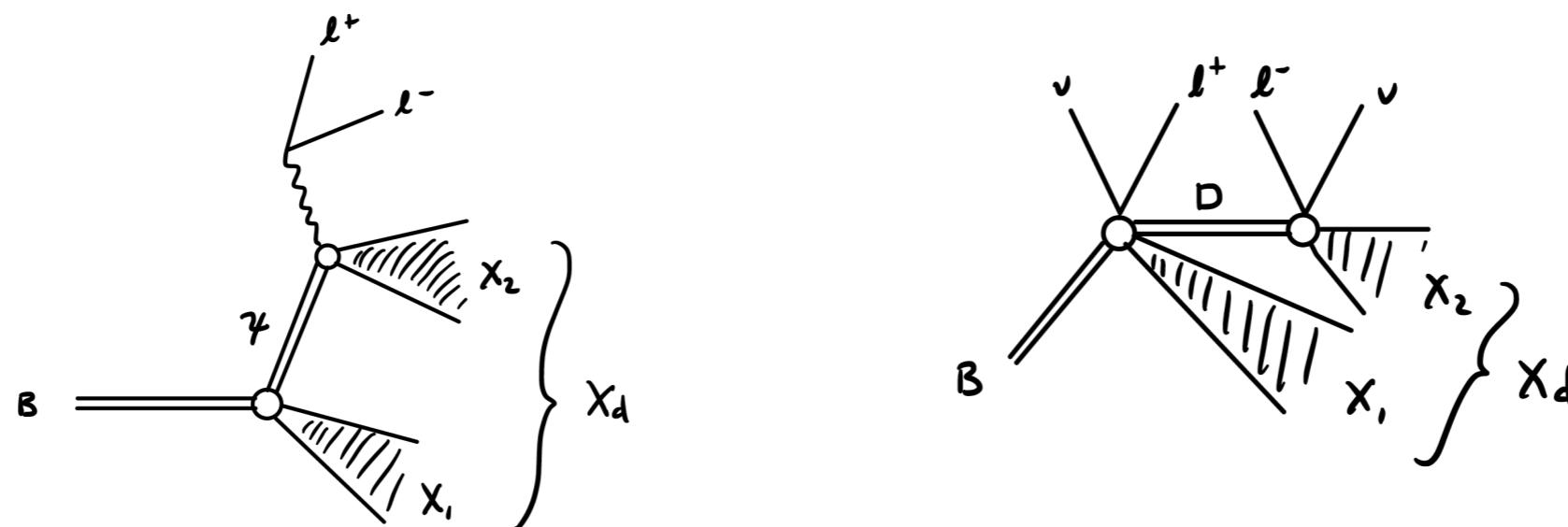
- ★ Resolved photon effects cause additional uncertainty of ~5%

[See Voloshin,96'; Buchalla,Isidori,Rey,97'; Benzke,Hurth,Turczyk,17']



- ★ Cascade decays not covered by either local OPE or KS

[see Buchalla,Isidori,Rey,97'; Beneke,Buchalla,Neubert,Sachrajda,09']



- Q: More investigation from theory or cuts from experiments?

# **Power Corrections**

---

## POWER CORRECTIONS

[T. Huber, T. Hurth, J. Jenkins, E. Lunghi, QQ, K. Vos, in preparation]

# Power corrections

- ★ Computing power-corrections requires HQET matrix elements of dimension-five and -six operators

$$\lambda_1 \equiv \langle B | \bar{h}_\nu (iD)^2 h_\nu | B \rangle$$

$$\lambda_2 \equiv \langle B | \bar{h}_\nu g \sigma_{\mu\nu} G^{\mu\nu} h_\nu | B \rangle$$

$$\frac{1}{3} \rho_1 (g_{\alpha\beta} - v_\alpha v_\beta) v_\mu \equiv \frac{1}{2m_B} \langle B | \bar{h}_\nu iD_\alpha iD_\mu iD_\beta h_\nu | B \rangle$$

$$\frac{1}{2} \rho_2 i \epsilon_{\nu\alpha\beta\delta} v^\nu v_\mu \equiv \frac{1}{2m_B} \langle B | \bar{h}_\nu iD_\alpha iD_\mu iD_\beta \gamma_\delta \gamma_5 h_\nu | B \rangle$$

$$f_q^{0,\pm} \equiv \frac{1}{2m_B} \langle B^{0,\pm} | Q_2^q - Q_1^q | B^{0,\pm} \rangle$$

- $\lambda_i, \rho_i$  were extracted from moments of inclusive  $B \rightarrow X_c \ell \nu$  spectrum  
[Gambino,Healey,Turczyk,16']
- We extract the weak-annihilation matrix elements  $f_q^{\pm,0}$  from data of semileptonic D meson decays following [Gambino,Kamenik,10']

## ★ Flavor symmetries assumed for weak-annihilation matrix elements

$$f_V \equiv f_u^\pm \stackrel{SU(2)}{=} f_d^0$$

$$f_{NV} \equiv f_u^0 \stackrel{SU(2)}{=} f_d^\pm \stackrel{SU(3)}{=} f_s^0 \stackrel{SU(2)}{=} f_s^\pm$$

V/NV stands for valence and non-valence terms  
with respect to external B meson states

- (Largely) uncorrelated combinations are  $f_{NV}$  and  $f_V - f_{NV}$
- Introduce SU(2,3) breaking effects according to [Ligeti,Tackmann,07']

$$f_{NV} = (-0.02 \pm 0.16) \text{ GeV}^3$$

$$f_V - f_{NV} = (-0.041 \pm 0.052) \text{ GeV}^3$$

$$[\delta f]_{SU(3)} = (0 \pm 0.04) \text{ GeV}^3$$

$$[\delta f]_{SU(2)} = (0 \pm 0.004) \text{ GeV}^3$$

$$\lambda_2^{\text{eff}} = \lambda_2 - \frac{\rho_2}{m_b} = 0.130(21) \text{ GeV}^2$$

$$\lambda_1 = -0.267(90) \text{ GeV}^2$$

$$\rho_1 = 0.038(70) \text{ GeV}^3$$

$$(f_V + f_{NV})/2 = (-0.04 \pm 0.17) \text{ GeV}^3$$

**Q: Can lattice QCD compute these matrix elements in analogy to the ones in B lifetime?**

# **Phenomenology**

Φενωμενολογία

[Huber, Hurth, Jenkins, Lunghi, QQ, Vos, in preparation]

# Phenomenology

$$q^2 \equiv (p_{\ell^+} + p_{\ell^-})^2$$

★ We consider the observables in two kinetic regions  $q^2 \in [1, 6]\text{GeV}^2$  and  $q^2 > 14.4\text{GeV}^2$ , to reduce the charm resonance effects, including

- The branching ratios in the low- and high- $q^2$  regions
- The forward-backward asymmetries in the low- $q^2$  region
- The CP asymmetries in both regions
- The ratio between  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  and  $B^0 \rightarrow X_u \ell \nu$  with the same kinematic cut in the high- $q^2$  region to reduce power-correction uncertainties [Ligeti,Tackmann,07']

$$\mathcal{R}(s_0) = \int_{\hat{s}_0}^1 d\hat{s} \frac{d\Gamma(\bar{B} \rightarrow X_d \ell^+ \ell^-)}{d\hat{s}} / \int_{\hat{s}_0}^1 d\hat{s} \frac{d\Gamma(B^0 \rightarrow X_u \ell \nu)}{d\hat{s}}$$

★ Branching ratios and the R-ratio of  $\bar{B} \rightarrow X_d \ell^+ \ell^-$ 

$$\begin{aligned}\mathcal{B}[1, 6]_{ee} &= (7.81 \pm 0.37_{\text{scale}} \pm 0.08_{m_t} \pm 0.17_{C, m_c} \pm 0.08_{m_b} \\ &\quad 0.04_{\alpha_s} \pm 0.15_{\text{CKM}} \pm 0.12_{\text{BR}_{\text{sl}}} \pm 0.05_{\lambda_2}) \times 10^{-8} = (7.81 \pm 0.47) \times 10^{-8} . \\ \mathcal{B}[1, 6]_{\mu\mu} &= (7.59 \pm 0.35_{\text{scale}} \pm 0.08_{m_t} \pm 0.17_{C, m_c} \pm 0.09_{m_b} \\ &\quad \pm 0.04_{\alpha_s} \pm 0.14_{\text{CKM}} \pm 0.11_{\text{BR}_{\text{sl}}} \pm 0.05_{\lambda_2}) \times 10^{-8} = (7.59 \pm 0.45) \times 10^{-8} .\end{aligned}$$

★ Branching ratios and the R-ratio of  $\bar{B} \rightarrow X_d \ell^+ \ell^-$ 

$$\mathcal{B}[> 14.4]_{ee} = (0.98 \pm 0.12_{\text{scale}} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.08_{m_b} \pm 0.02_{\text{CKM}}$$

$$\pm 0.02_{\text{BR}_{\text{sl}}} \pm 0.06_{\lambda_2} \pm 0.25_{\rho_1} \pm 0.25_{f_{u,d}}) \times 10^{-8} = (0.98 \pm 0.38) \times 10^{-8},$$

$$\mathcal{B}[> 14.4]_{\mu\mu} = (1.12 \pm 0.12_{\text{scale}} \pm 0.01_{m_t} \pm 0.02_{C,m_c} \pm 0.09_{m_b} \pm 0.02_{\text{CKM}}$$

$$\pm 0.02_{\text{BR}_{\text{sl}}} \pm 0.05_{\lambda_2} \pm 0.25_{\rho_1} \pm 0.25_{f_{u,d}}) \times 10^{-8} = (1.12 \pm 0.38) \times 10^{-8}.$$

$$\mathcal{R}(14.4)_{ee} = (0.96 \pm 0.02_{\text{scale}} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.002_{m_b} \pm 0.01_{\alpha_s} \pm 0.05_{\text{CKM}}$$

$$\pm 0.004_{\lambda_2} \pm 0.06_{\rho_1} \pm 0.05_{f_{u,d}}) \times 10^{-4} = (0.96 \pm 0.09) \times 10^{-4},$$

$$\mathcal{R}(14.4)_{\mu\mu} = (1.11 \pm 0.01_{\text{scale}} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.002_{m_b} \pm 0.01_{\alpha_s} \pm 0.05_{\text{CKM}}$$

$$\pm 0.002_{\lambda_2} \pm 0.04_{\rho_1} \pm 0.02_{f_{u,d}}) \times 10^{-4} = (1.11 \pm 0.07) \times 10^{-4}.$$

## ★ Forward-backward asymmetries of $\bar{B} \rightarrow X_d \ell^+ \ell^-$

$$\mathcal{A}_{\text{FB}}[1, 3.5]_{ee} = (-7.10 \pm 0.67_{\text{scale}} \pm 0.01_{m_t} \pm 0.11_{C, m_c} \pm 0.22_{m_b} \pm 0.19_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.04_{\lambda_2})\%$$

$$= (-7.11 \pm 0.74)\% ,$$

$$\mathcal{A}_{\text{FB}}[3.5, 6]_{ee} = (8.60 \pm 0.74_{\text{scale}} \pm 0.01_{m_t} \pm 0.13_{C, m_c} \pm 0.37_{m_b} \pm 0.18_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.11_{\lambda_2})\%$$

$$= (8.60 \pm 0.87)\% ,$$

$$\mathcal{A}_{\text{FB}}[1, 6]_{ee} = (-0.12 \pm 0.77_{\text{scale}} \pm 0.004_{m_t} \pm 0.13_{C, m_c} \pm 0.29_{m_b} \pm 0.20_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.02_{\lambda_2})\%$$

$$= (-0.12 \pm 0.86)\% .$$

$$\mathcal{A}_{\text{FB}}[1, 3.5]_{\mu\mu} = (-7.97 \pm 0.69_{\text{scale}} \pm 0.01_{m_t} \pm 0.11_{C, m_c} \pm 0.22_{m_b} \pm 0.20_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.05_{\lambda_2})\%$$

$$= (-7.97 \pm 0.76)\% ,$$

$$\mathcal{A}_{\text{FB}}[3.5, 6]_{\mu\mu} = (8.16 \pm 0.82_{\text{scale}} \pm 0.01_{m_t} \pm 0.13_{C, m_c} \pm 0.39_{m_b} \pm 0.19_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.11_{\lambda_2})\%$$

$$= (8.16 \pm 0.94)\% ,$$

$$\mathcal{A}_{\text{FB}}[1, 6]_{\mu\mu} = (-0.70 \pm 0.82_{\text{scale}} \pm 0.004_{m_t} \pm 0.13_{C, m_c} \pm 0.30_{m_b} \pm 0.21_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.02_{\lambda_2})\%$$

$$= (-0.70 \pm 0.91)\% .$$

★ The “zeros” of  $\bar{B} \rightarrow X_d \ell^+ \ell^-$ (  $q^2$  where  $A_{FB}$  is zero)

$$(q_0^2)_{ee} = 3.28 \pm 0.11_{\text{scale}} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.05_{m_b} \\ \pm 0.03_{\alpha_s} \pm 0.004_{\text{CKM}} \pm 0.002_{\lambda_1} \pm 0.001_{\lambda_2} = (3.28 \pm 0.13) \text{ GeV}^2$$

$$(q_0^2)_{\mu\mu} = 3.39 \pm 0.12_{\text{scale}} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.05_{m_b} \\ \pm 0.03_{\alpha_s} \pm 0.004_{\text{CKM}} \pm 0.002_{\lambda_1} \pm 0.002_{\lambda_2} = (3.39 \pm 0.13) \text{ GeV}^2$$

★ CP asymmetries of  $\bar{B} \rightarrow X_d \ell^+ \ell^-$ : order 1%

$$\mathcal{A}_{\text{CP}}[1, 6]_{ee} = (-1.45 \pm 0.75_{\text{scale}} \pm 0.015_{m_t} \pm 0.019_{C, m_c} \pm 0.05_{m_b} \\ \pm 0.15_{\alpha_s} \pm 0.03_{\text{CKM}} \pm 0.002_{\lambda_2}) \times 10^{-2} = (-1.45 \pm 0.77) \times 10^{-2}.$$

$$\mathcal{A}_{\text{CP}}[1, 6]_{\mu\mu} = (-1.32 \pm 0.71_{\text{scale}} \pm 0.014_{m_t} \pm 0.015_{C, m_c} \pm 0.05_{m_b} \\ \pm 0.15_{\alpha_s} \pm 0.03_{\text{CKM}} \pm 0.002_{\lambda_2}) \times 10^{-2} = (-1.32 \pm 0.72) \times 10^{-2}.$$

$$\mathcal{A}_{\text{CP}}[> 14.4]_{ee} = (-1.68 \pm 0.14_{\text{scale}} \pm 0.01_{m_t} \pm 0.12_{C, m_c} \pm 0.12_{m_b} \pm 0.05_{\alpha_s} \pm 0.05_{\text{CKM}} \\ \pm 0.28_{\lambda_2} \pm 0.34_{\rho_1} \pm 0.55_{f_{u,d}}) \times 10^{-2} = (-1.68 \pm 0.74) \times 10^{-2}.$$

$$\mathcal{A}_{\text{CP}}[> 14.4]_{\mu\mu} = (-1.64 \pm 0.21_{\text{scale}} \pm 0.01_{m_t} \pm 0.09_{C, m_c} \pm 0.11_{m_b} \pm 0.04_{\alpha_s} \pm 0.05_{\text{CKM}} \\ \pm 0.24_{\lambda_2} \pm 0.30_{\rho_1} \pm 0.45_{f_{u,d}}) \times 10^{-2} = (-1.64 \pm 0.64) \times 10^{-2}.$$

# **Summary and Outlook**

---

## Summary and Outlook

# Summary and Outlook

- ★ We update the SM theory predictions in inclusive  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  decay and  $\bar{B} \rightarrow X_s \ell^+ \ell^-$  is on the way
- ★ Further investigate the resolved photon effects and the Cascade decay effects...
- ★ Study effects of lepton-flavour universality violation, e.g. in the observable

$$R_{X_s} = \frac{\mathcal{B}(\bar{B} \rightarrow X_s \mu^+ \mu^-)}{\mathcal{B}(\bar{B} \rightarrow X_s e^+ e^-)}$$

Thank you very much!