Beta-decay halflives and quasiparticle vibration coupling

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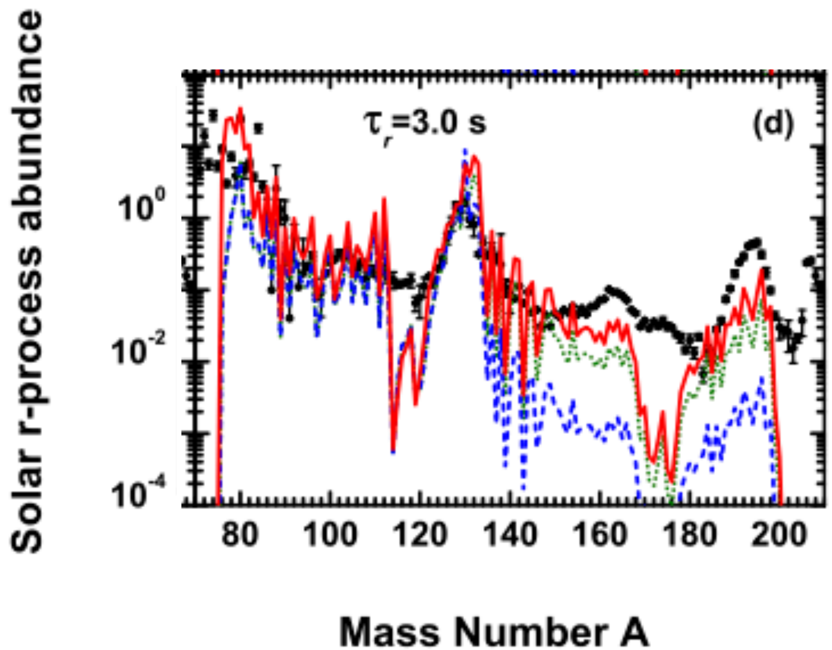
Influence of beta-decay lifetimes on the r-process

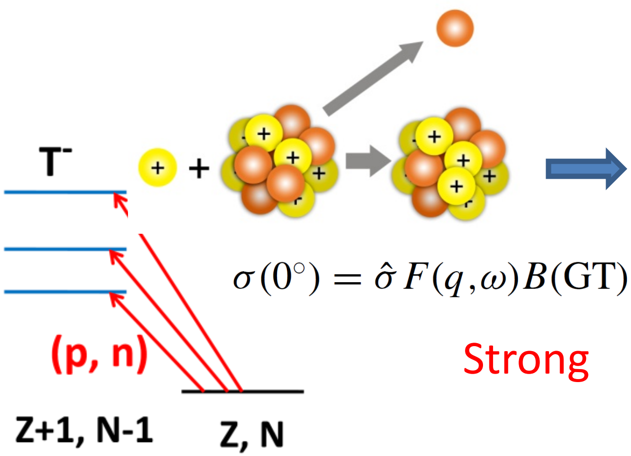


Z.M. Niu et al, PLB 723 (2013) 172

Gamow-Teller Resonance and β-decay

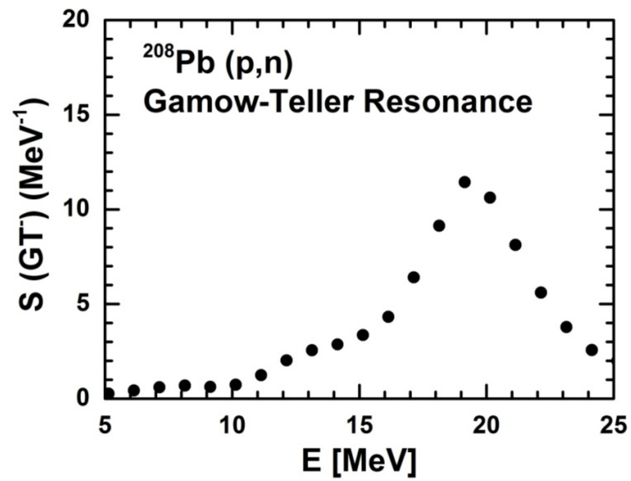
- **Gamow-Teller Resonance**

(p,n) reaction



$$\sigma(0^\circ) = \hat{\sigma}\, F(q,\omega) B(\mathrm{GT})$$

Wakasa, et al., PRC 85, 064606 (2012)



Gamow-Teller transition

ΔS=1 ΔL=0 ΔT=1

operator

$$\hat{O}_{\mathrm{GT-}} = \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \tau_{-}(i)$$

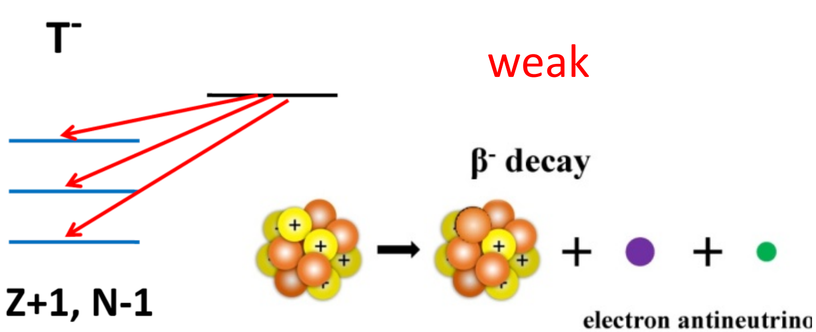
Transition probability

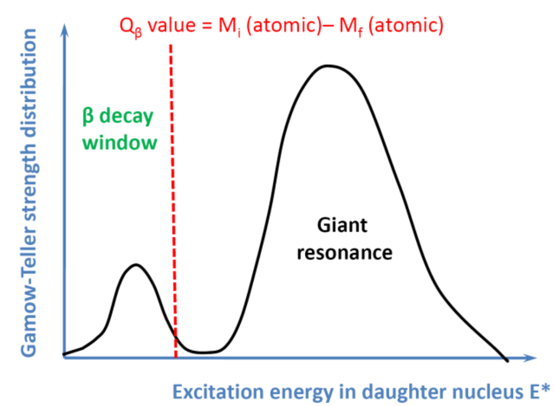
$$B(\mathrm{GT}^{-}) = \sum_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2$$

Transition strength S

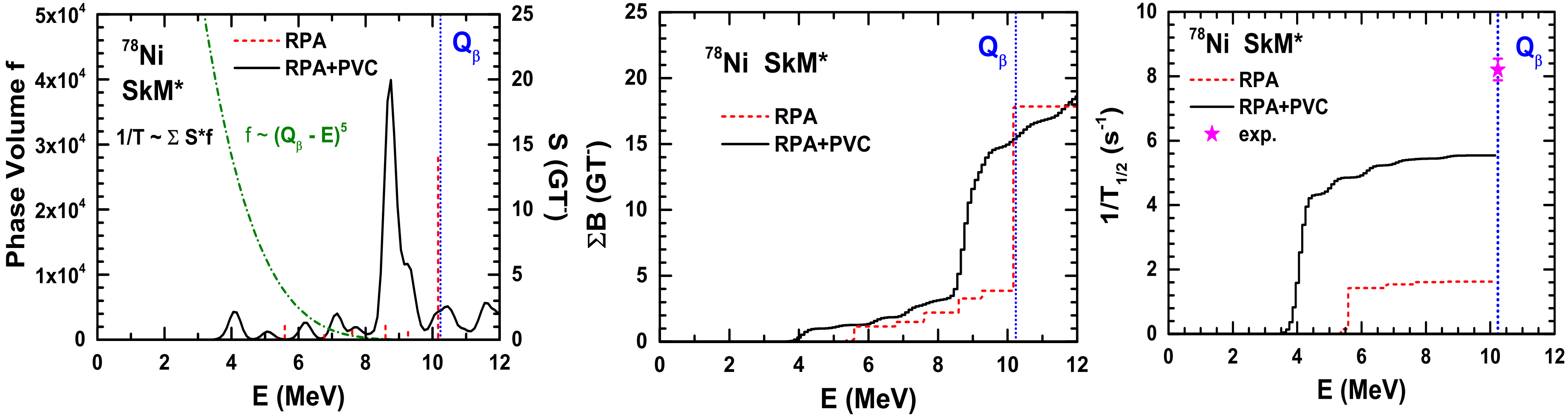
= smoothed B

- **β-decay** **dominated by low-energy GT transition**





Calculation of half-life



Exp.: Xu, et al., PRL 113, 032505, 2014

- Half-life

$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z,\omega) dE},$$

D= 6163 s

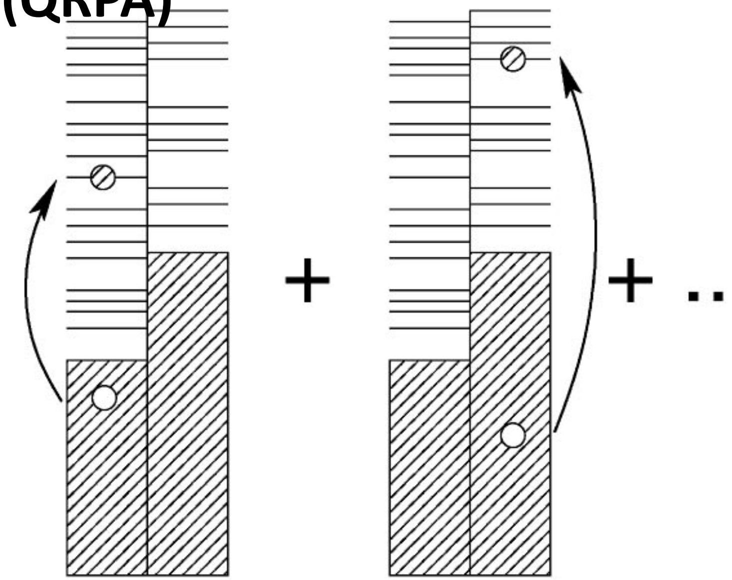
g_A = 1

- Phase Volume

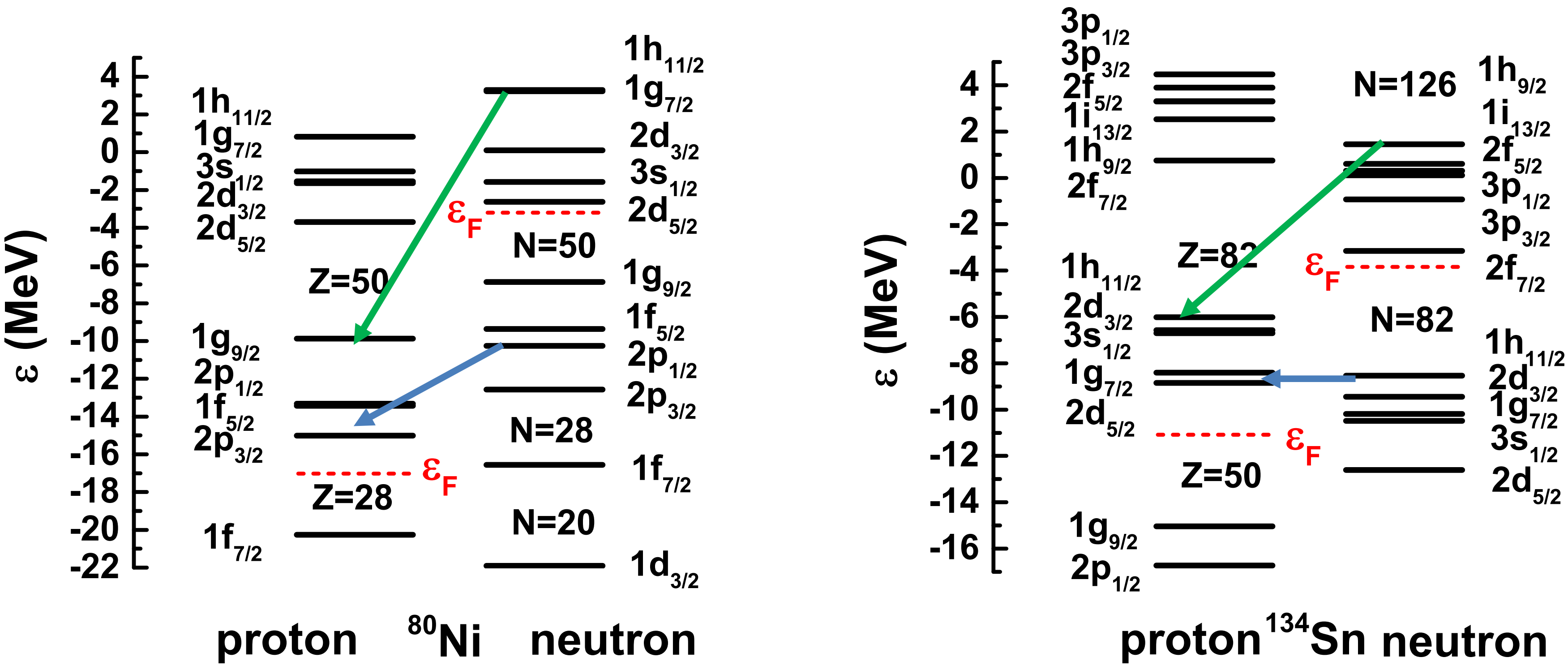
$$f(Z,\omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z+1, E_e) dE_e.$$

Theoretical Investigations

- **ab initio approach: for very light nuclei** *Barrett, et al., PPNP 69, 131 2013*
- **shell model: up to A=60 or around magic regions**
  Langanke et al., ADNDT 79, 1 2001; Suzuki, et al., PRC 85, 015802, 2012; Li and Ren, JPG 41, 105102, 2014; Zhi, et al., PRC 87, 025803 (2013)
- **Quasiparticle Random Phase Approximation (QRPA)**
  - ✓ Spherical
    - ○ Non-self-consistent
      - FRDM + Quasiparticle RPA (QRPA)
        Moller, et al., ADNDT 66, 131 1997
    - ○ Self-consistent
      - Skyrme QRPA
        Engel, et al., PRC 60, 014302, 1999
      - Relativistic QRPA
        Marketin, et al., PRC 93, 025805, 2016; Niu, et al., PLB 723, 172, 2013.
  - ✓ Quadrupole deformation
    - deformed QRPA with realistic force *Ni and Ren, PRC 89, 064320, 2014*
    - Skyrme deformed QRPA *Sarriguren, et al., PRC 81, 064314 (2013); Yoshida, JPS CP, 6, 020017, 2015; Mustonen and Engel, PRC 93, 014304 (2016)*



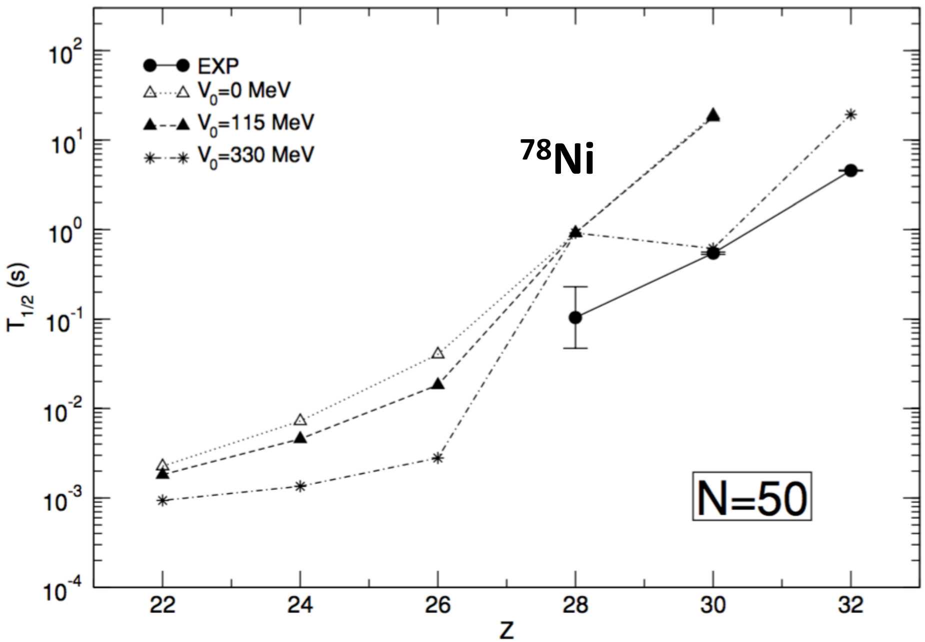
RPA



ε (MeV)
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
1h11/2
1g7/2
3s1/2
2d3/2
2d5/2
Z=50
1g9/2
2p1/2
1f5/2
2p3/2
εF
Z=28
1f7/2
1h11/2
1g7/2
2d3/2
3s1/2
2d5/2
εF
N=50
1g9/2
1f5/2
2p1/2
2p3/2
N=28
1f7/2
N=20
1d3/2
proton
80Ni
neutron
ε (MeV)
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
3p1/2
3p3/2
2f5/2
1i13/2
1h9/2
2f7/2
Z=82
1h11/2
2d3/2
3s1/2
1g7/2
2d5/2
εF
Z=50
1g9/2
2p1/2
N=126
1h9/2
1i13/2
2f5/2
3p1/2
3p3/2
εF
2f7/2
N=82
1h11/2
2d3/2
1g7/2
3s1/2
2d5/2
proton
134Sn
neutron

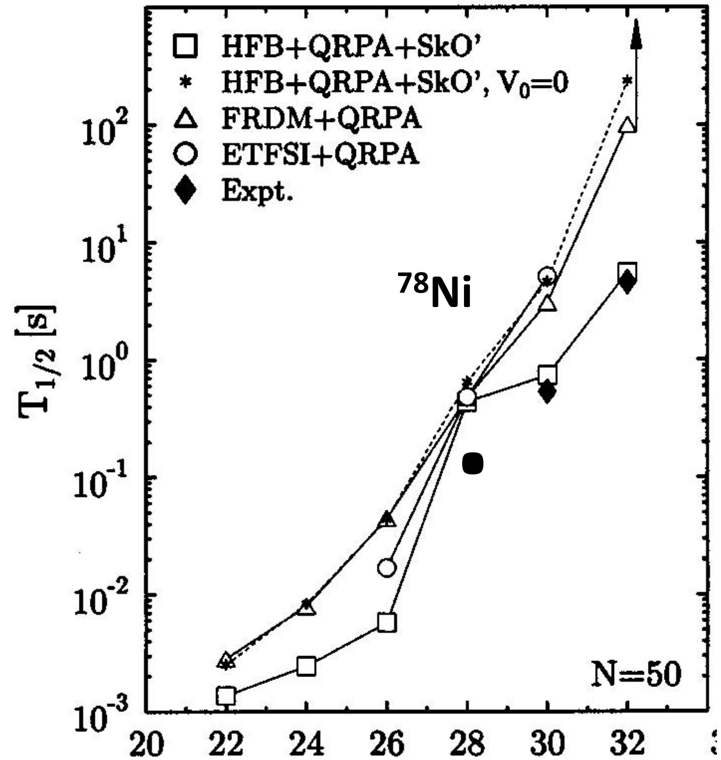
QRPA calculations tend to overestimate lifetimes

- **RHB+QRPA**



Niksic, et al., PRC 71, 014308 (2005)

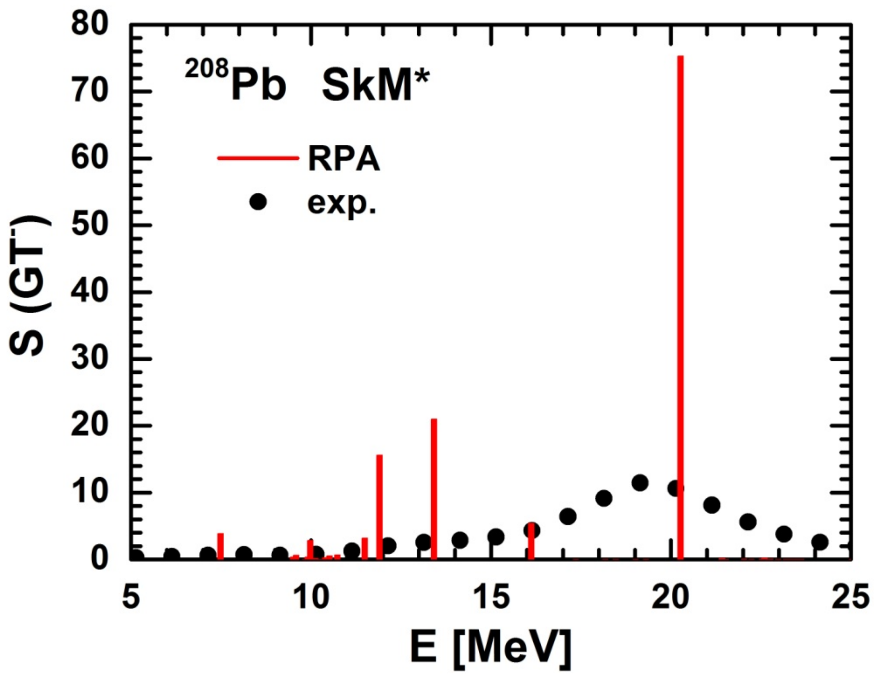
- **Skyrme HFB+QRPA**



Engel, et al., PRC 60, 014302 (1999)

Limits of (Q)RPA Description

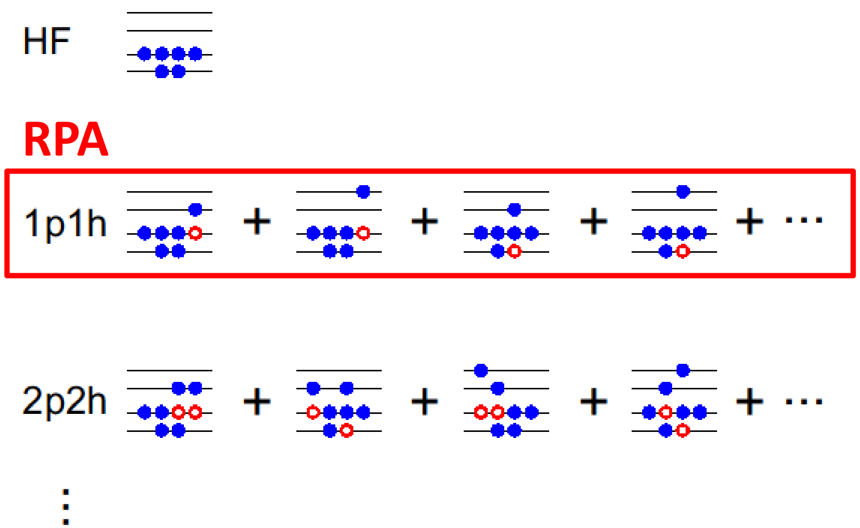
- **(Q)RPA cannot describe the spreading width**

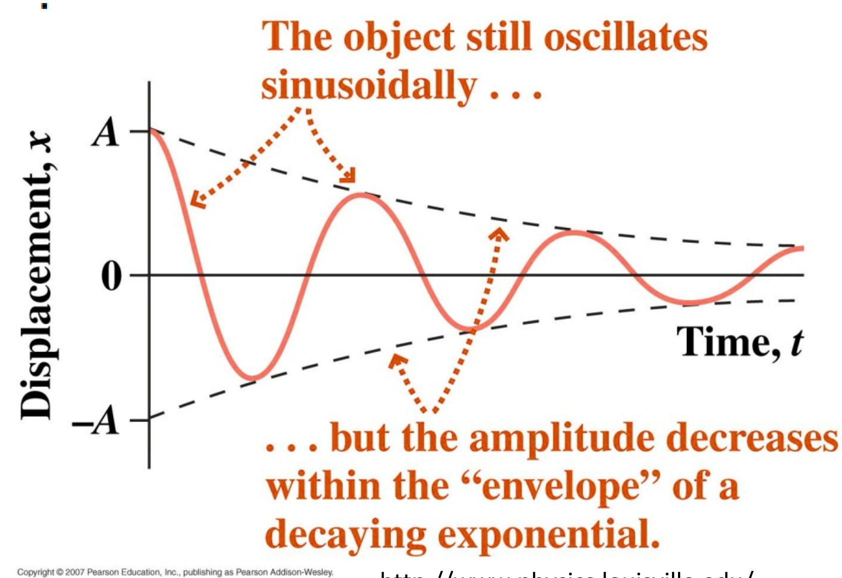


- **Spreading Width (Damping Width)**

  energy and angular momentum of coherent vibrations
  → more complicated states of 2p-2h, 3p-3h, … character

- **Correlations beyond RPA**



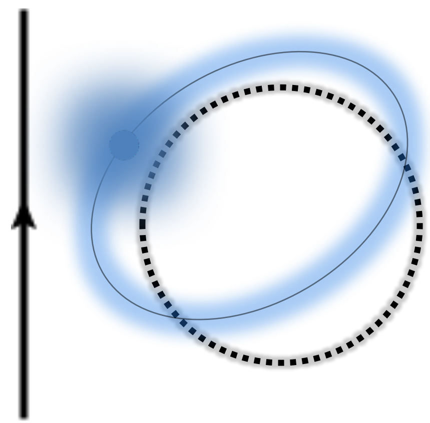


http://www.physics.louisville.edu/

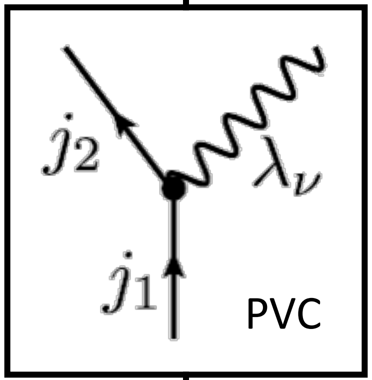
Elementary modes of excitation

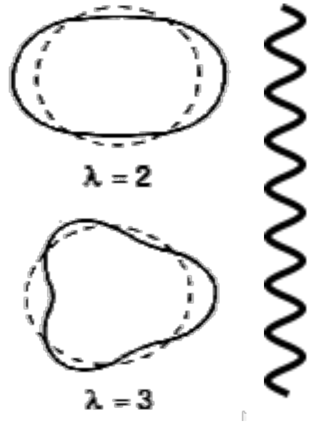
Independent Particle

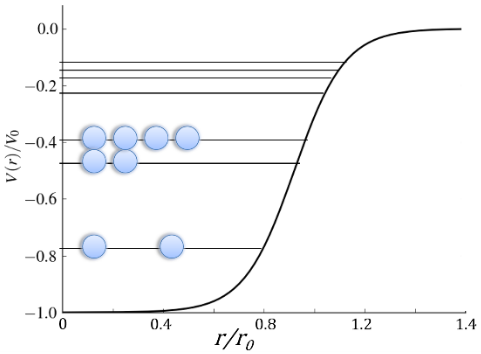
Collective Phonon

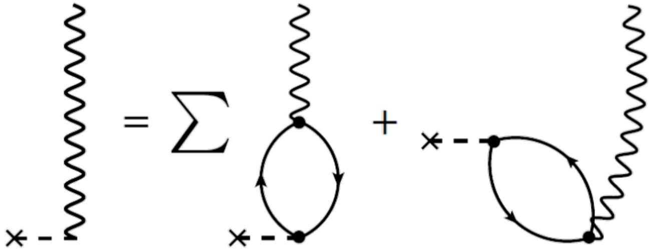


Particle-vibration coupling









Hartree-Fock mean Field

Random Phase Approximation

Coupling of single-particle states to surface modes

A reminder of basic concepts and notation

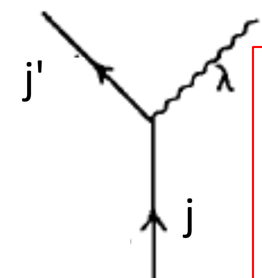
$$U(r) = -U_o/(1 + \exp(r-R_o)/a)$$ *Mean Field*

$$R(\theta,\phi) = R_o \left(1 + \sum \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)\right)$$ *Deformed Surface*

$$H_{PVC} = -R_o\, dU/dr \sum \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)$$ *Change in the mean field*

$$\alpha_{\lambda\mu} = \beta_\lambda (2\lambda+1)^{-1/2} (O^+_{\lambda\mu} + O_{\lambda\mu})$$ *Quantize surface oscillations*

Forward scattering vertex



Vjj'λ : $\langle j || H_{PVC} || j'\lambda \rangle =$

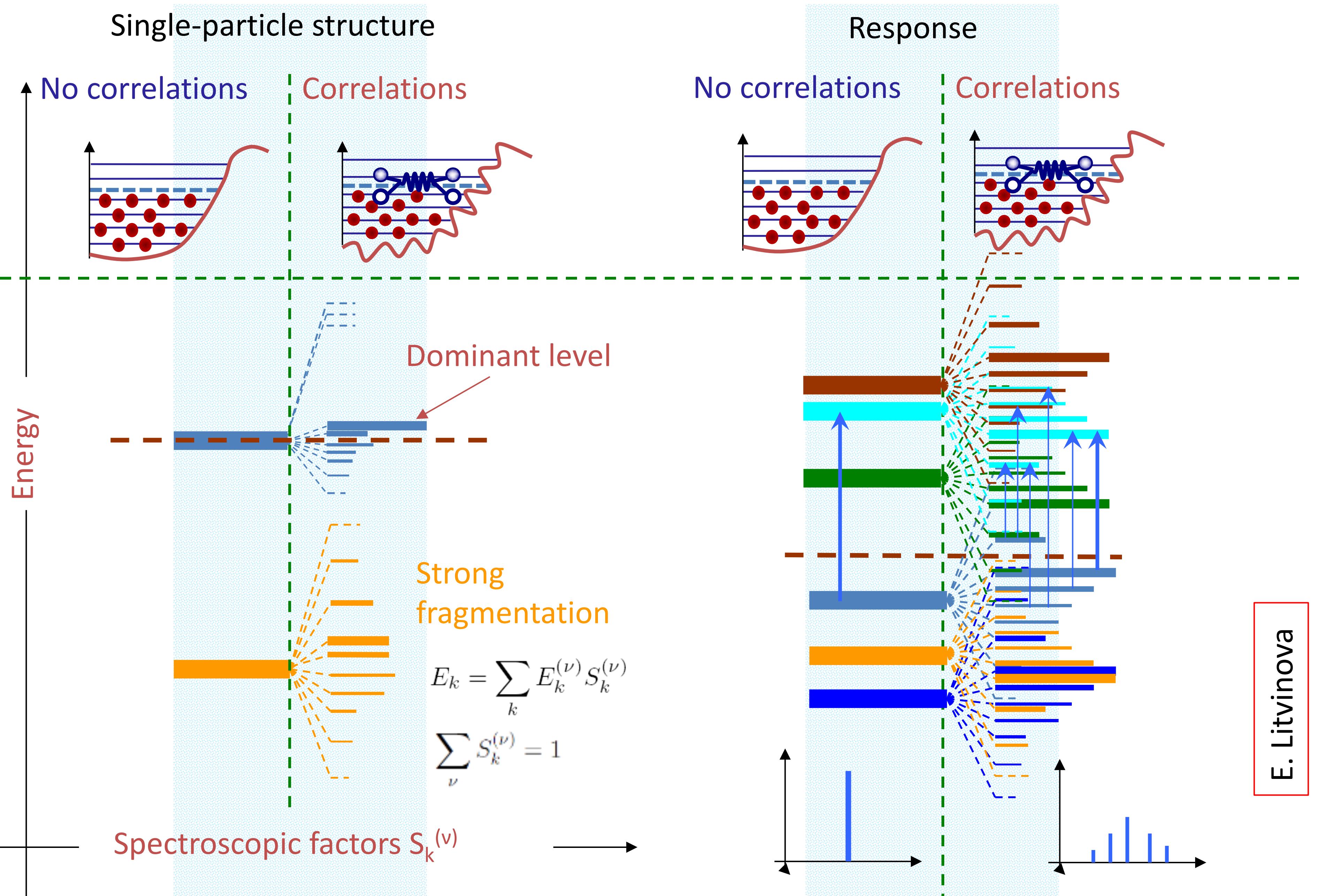
$$\beta_\lambda (2\lambda+1)^{-1/2} \langle j || \boldsymbol{R_o\, dU/dr}\, Y_\lambda || j' \rangle\, \boldsymbol{(u_j u_{j'} - v_j v_{j'})} (2j+1)^{-1/2}$$

βλ is extracted from experimental B(Eλ), inelastic cross sections, …

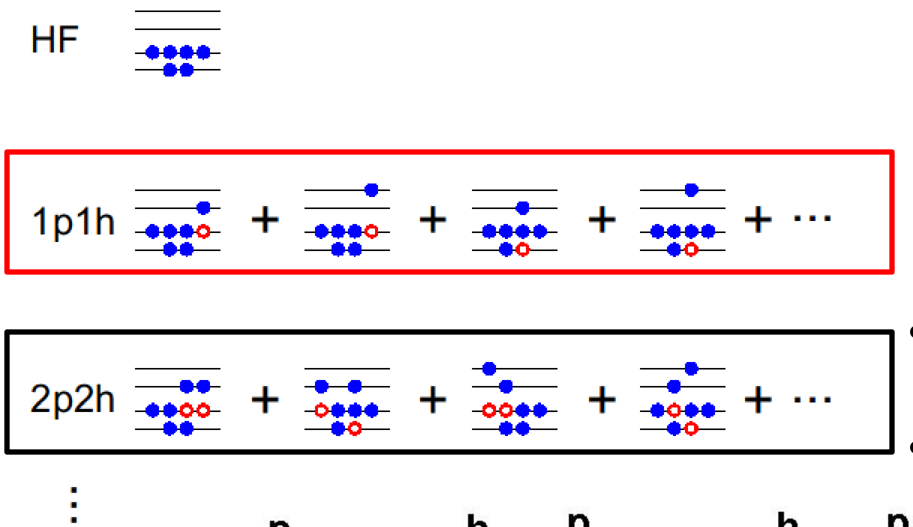
Backward scattering vertex

Wjj'λ : $\langle j(j'\lambda)_j || HPVC || gs \rangle = \beta_\lambda (2\lambda+1)^{-1/2} \langle j || R_o\, dU/dr\, Y_\lambda || j' \rangle\, \boldsymbol{(u_j v_{j'} + u_j v_{j'})} (2j+1)^{-1/2}$

Fragmentation of states in odd and even systems (schematic)

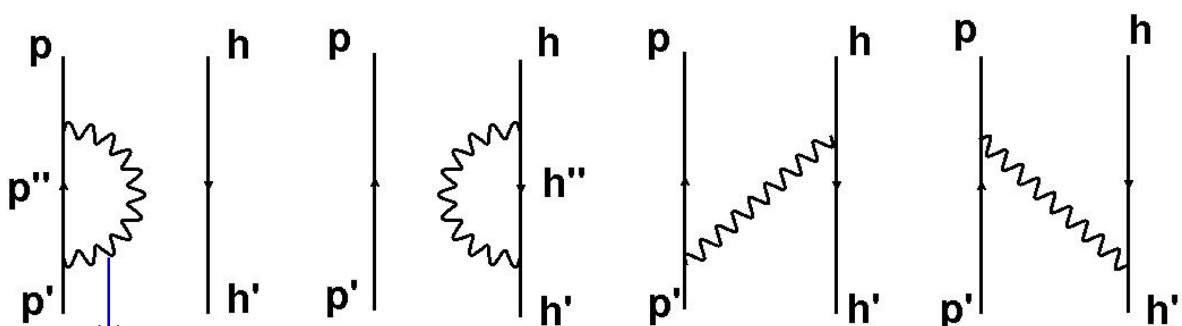


Solution: RPA + PVC



RPA

- **Second RPA** drozdz et al., PR 197, 1 (1990)
  Gambacurta et al., PRC 81, 054312 (2010)
- **RPA + PVC (particle vibration coupling)**



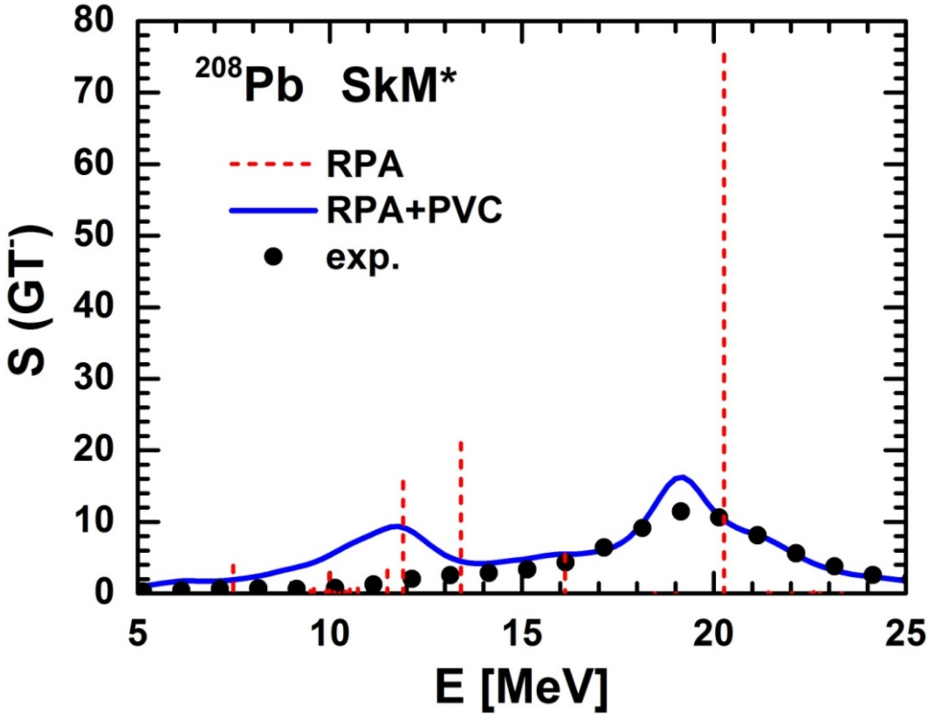
Low-lying vibration phonons |N>

$$W^{\downarrow}_{ph,p'h'}(\omega) = \sum_N \frac{\langle ph|V|N\rangle\langle N|V|p'h'\rangle}{\omega - \omega_N}$$

- RPA+PVC model based on Skyrme DFT
  Colo et al., PRC 50, 1496 (1994); Niu et al., PRC 85, 034314 (2012)
- RPA+PVC model based on relativistic DFT Litvinova et al., PRC 75,064308 (2007)

RPA+PVC: Gamow-Teller Resonance

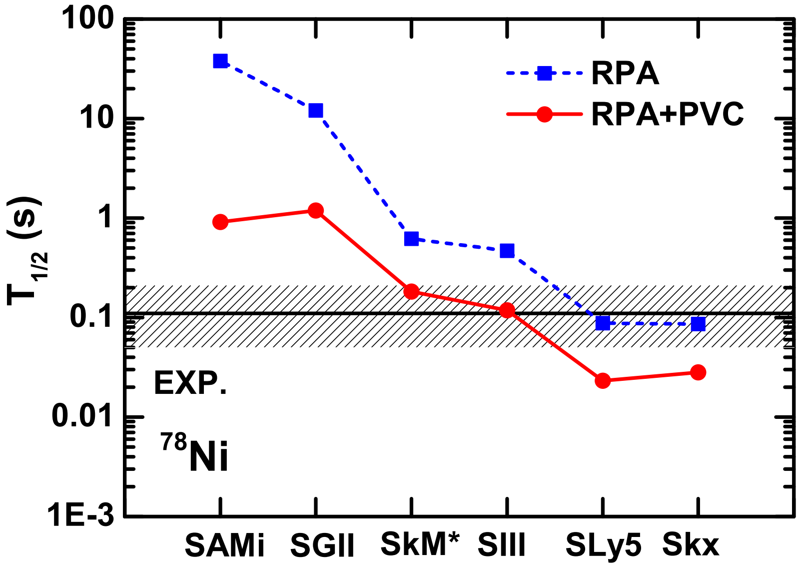
- **Improved description of GT resonance in ^{208}Pb**

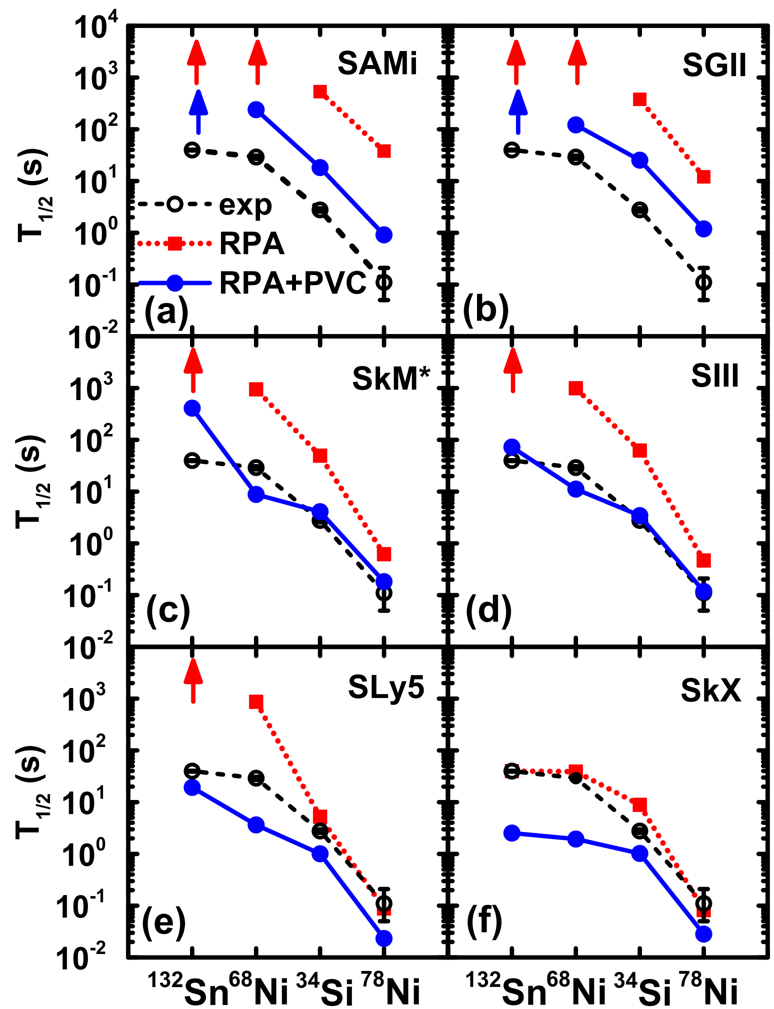


✓ **Develop a spreading width**
✓ **Reproduce resonance lineshape**

Y. F. Niu, G. Colo, and E. Vigezzi, **PRC** 90, 054328 (2014)

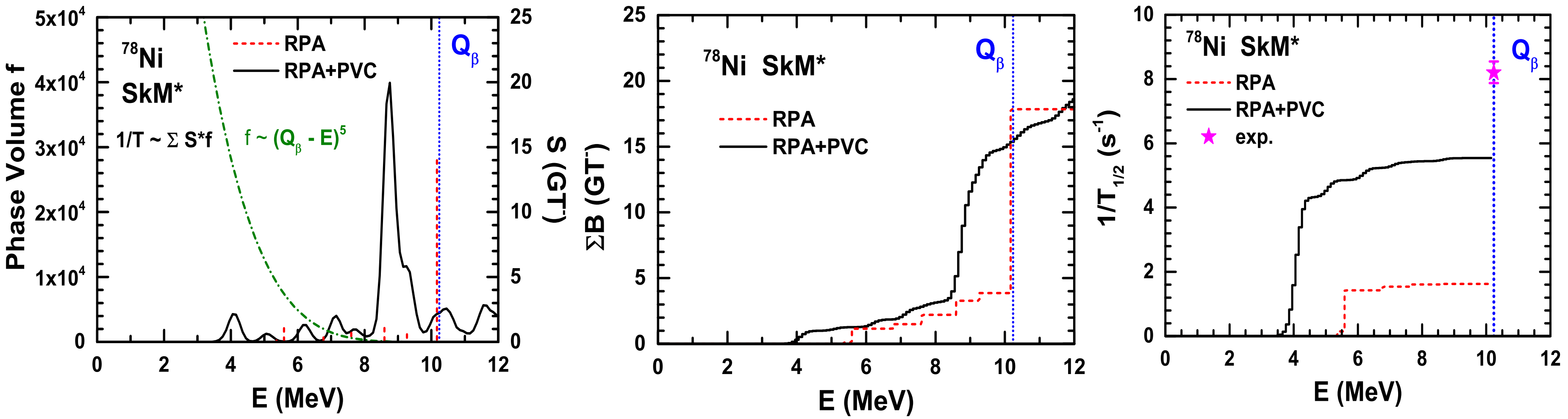
RPA+PVC: β-Decay Half-Lives





Y.F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, **PRL** 114, 142501 (2015).

How PVC reduces half-lives?



Exp.: Xu, et al., PRL 113, 032505, 2014

- Half-life

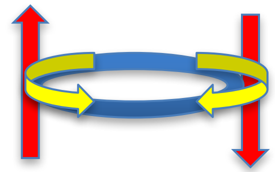
$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z, \omega) dE},$$

- Phase Volume

$$f(Z, \omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z+1, E_e) dE_e.$$

Two particle system

T=1, S=0 pair

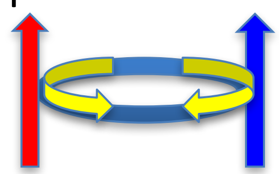


p(n) p(n)

$$\left|(L=S=0)J=0,T=1\right\rangle \Rightarrow \left|(j=j^{'})J=0,T=1\right\rangle$$

T=0, S=1 pair

p n



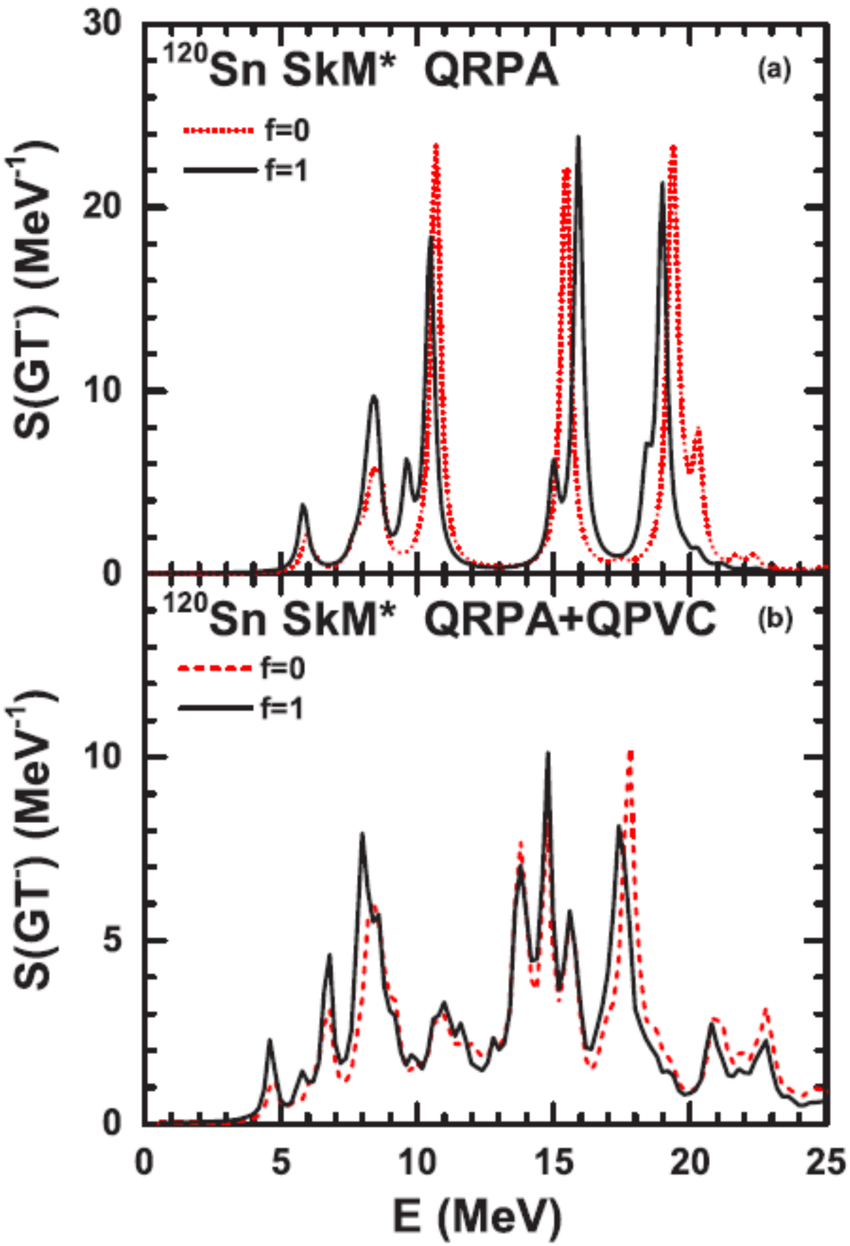
$$\left|(L=0,S=1)J=1,T=0\right\rangle \Rightarrow$$

$$a\left|(l=l^{'} j=j^{'})J=1,T=0\right\rangle + b\left|((l=l^{'})j,j^{'}=j\pm 1)J=1,T=0\right\rangle$$

Single-particle wave function $\vec{j}=\vec{l}+\vec{s}$

The total wave function should be anti-symmetric in spin-isospin-relative angular momentum quantum space.

Isoscalar Pairing



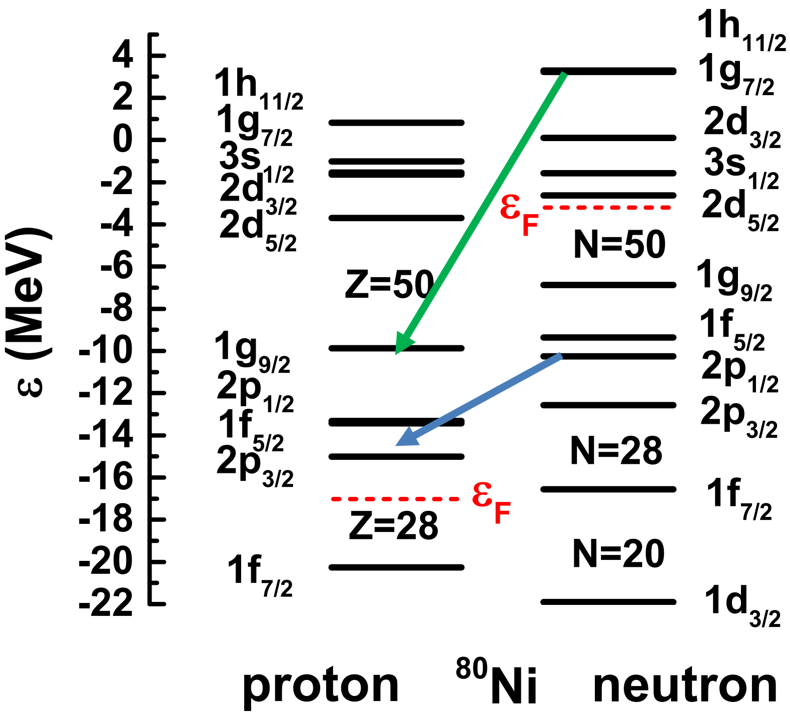
$$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2) = V_0 \frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

$$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2) = f\,V_0 \frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

f = 0 : without isoscalar pairing
f = 1 : with isoscalar pairing

Effect of isoscalar pairing

- QRPA level
  - Increase the low-lying strength
  - Decrease the splitting between two high-lying states
- QRPA+QPVC level
  - Increase the low-lying strength
  - Similar profile


ε (MeV)
4
2
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-20
-22
1h11/2
1g7/2
3s1/2
2d3/2
2d5/2
Z=50
1g9/2
2p1/2
1f5/2
2p3/2
εF
Z=28
1f7/2
1h11/2
1g7/2
2d3/2
3s1/2
2d5/2
εF
N=50
1g9/2
1f5/2
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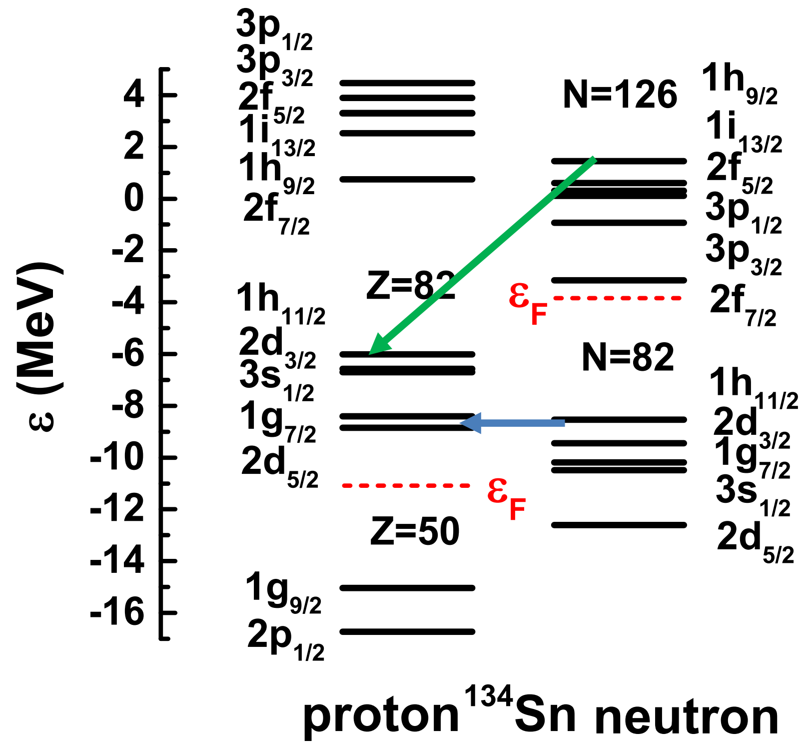
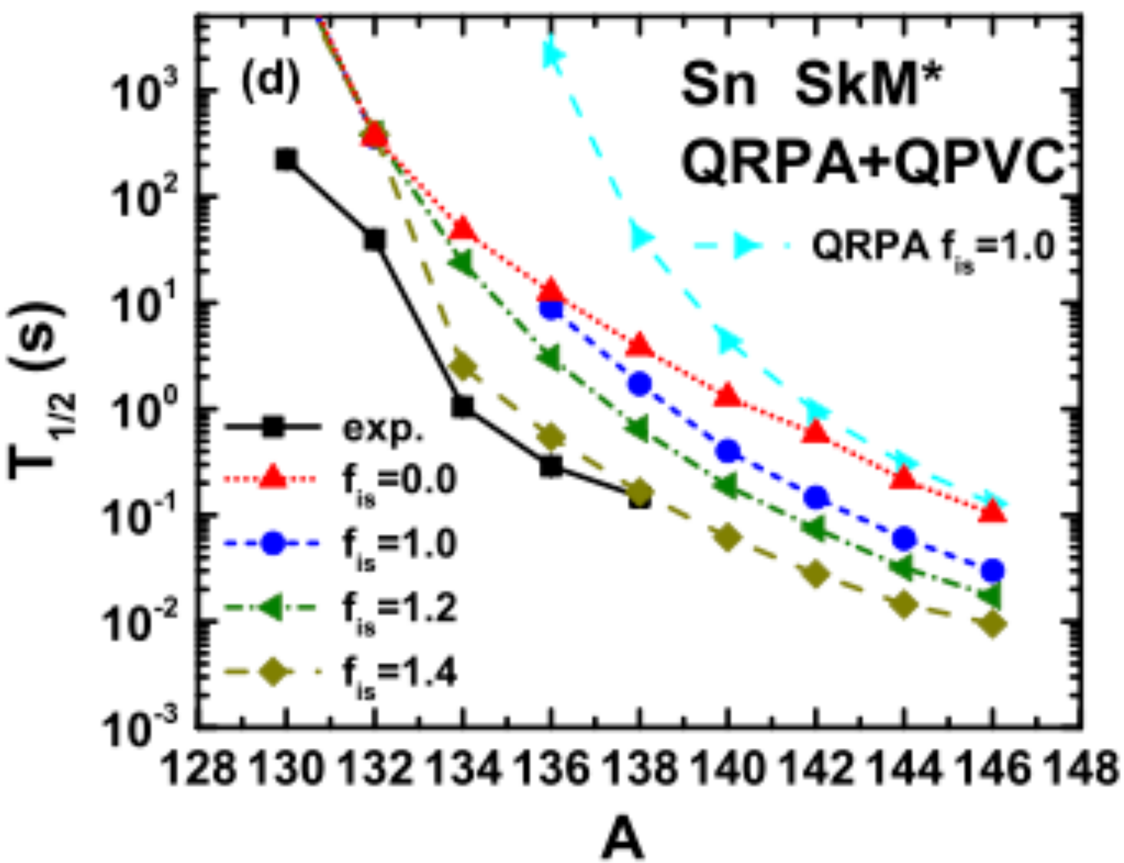
1f7/2
N=20
1d3/2
proton
80Ni
neutron

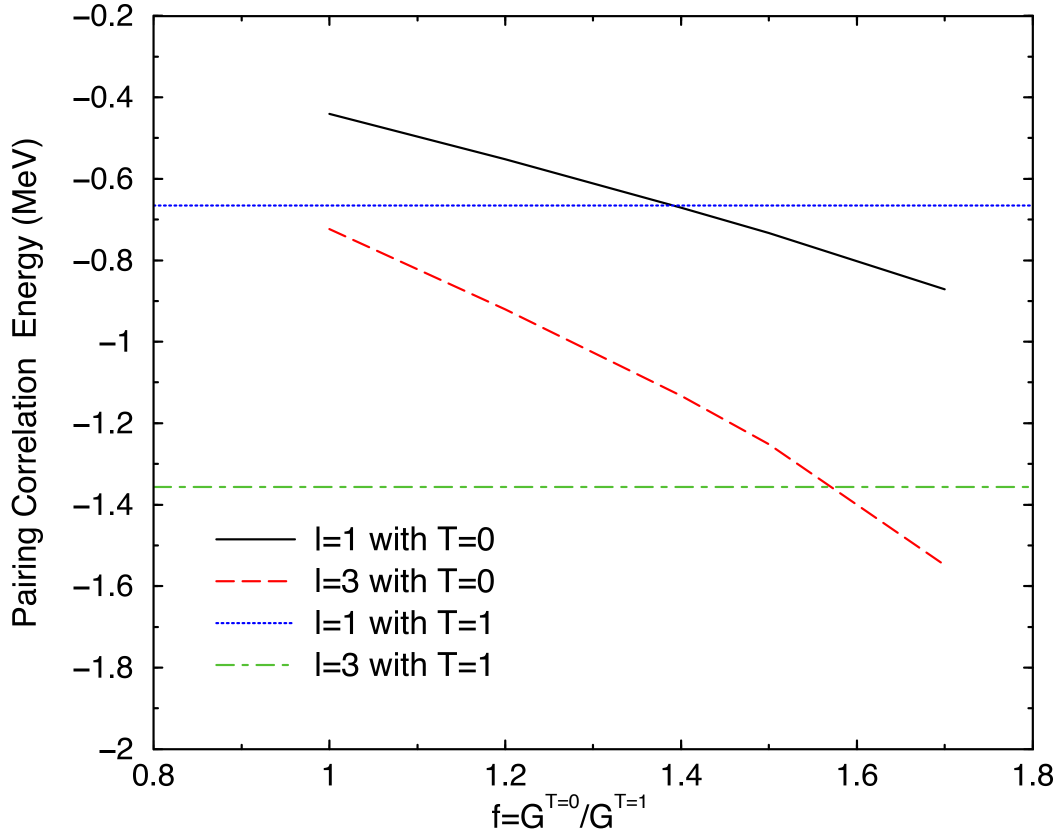

ε (MeV)
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
3p1/2
3p3/2
2f5/2
1i13/2
1h9/2
2f7/2
Z=82
1h11/2
2d3/2
3s1/2
1g7/2
2d5/2
εF
Z=50
1g9/2
2p1/2
N=126
1h9/2
1i13/2
2f5/2
3p1/2
3p3/2
εF
2f7/2
N=82
1h11/2
2d3/2
1g7/2
3s1/2
2d5/2
proton
134Sn
neutron

Effect of T=0 pairing on β-decay half-lives in Sn isotopes

$$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2) = V_0\frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

$$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2) = f\,V_0\frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$





Pairing correlation energy of (J,T)=(0,1) and (1,0) states in *pf* shell

Even with large spin-orbit splitting for *f* -orbits, the spin-triplet correlations will be larger than the spin-singlet one for f>1.5

HS, Y. Tanimura and K. Hagino, PRC87, 034310 (2013)

Summary and Perspectives

Summary

We have developed self-consistent Quasiparticle RPA (**QRPA**) + quasiparticle vibration coupling (**QPVC**) based on Skyrme density functional

- We studied the Gamow-Teller resonance in ^{120}Sn
  - ✓ the effect of isoscalar pairing
  - ✓ QPVC develops a spreading width
- We studied the β-decay half-lives in Ni and Sn isotopes
  - ✓ QPVC reduces half-lives in Ni isotopes
  - ✓ QPVC + isoscalar pairing reduce half-lives in Sn isotopes

Perspectives

- QRPA+QPVC: systematic study of isotopic chain
- QRPA+QPVC: other weak interaction processes
- QRPA+QPVC: giant dipole resonances, pygmy dipole resonances

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Extreme Light Infrastructure – Nuclear Physics (ELI-NP)

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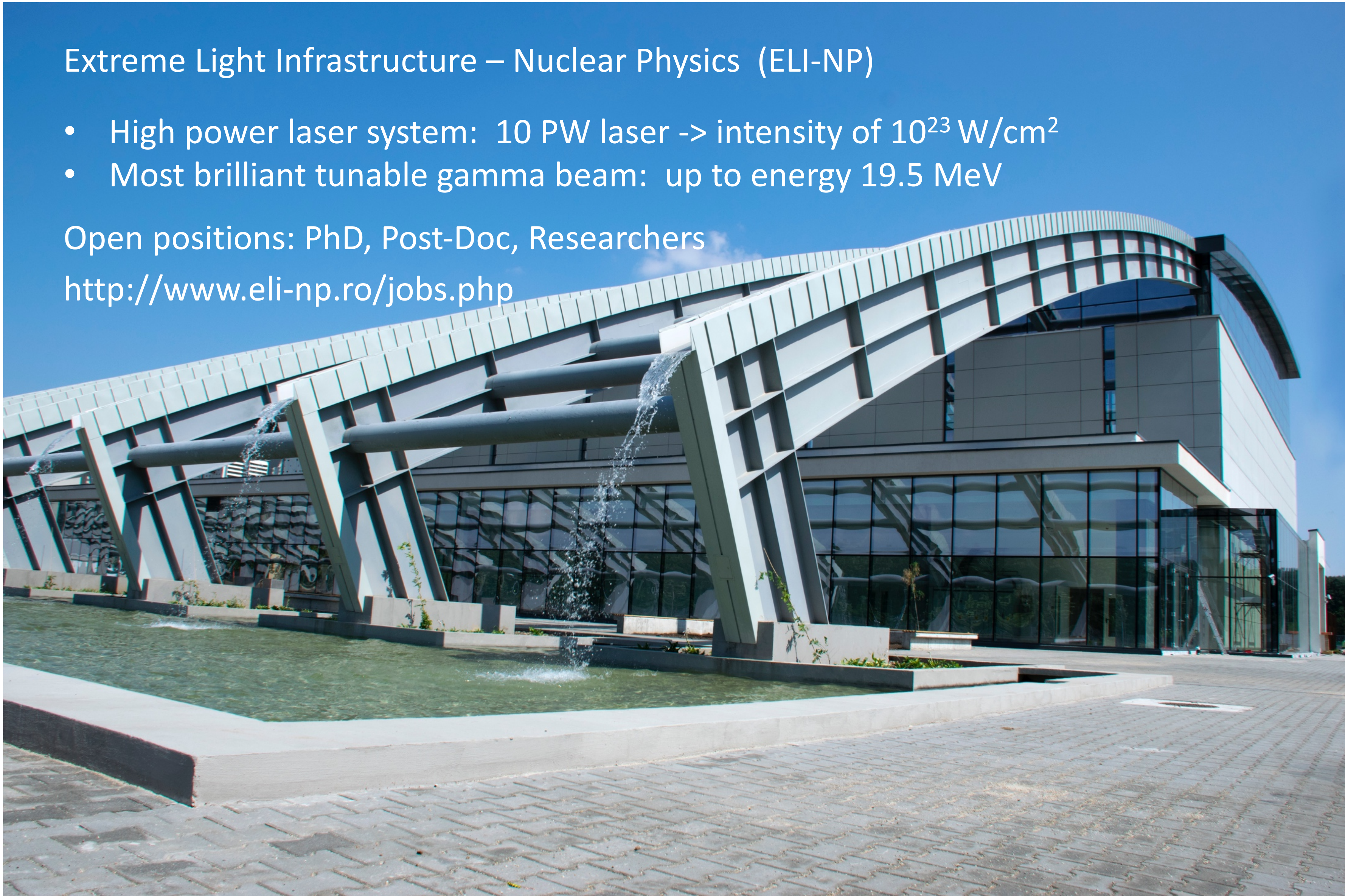
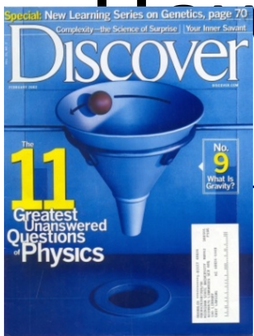


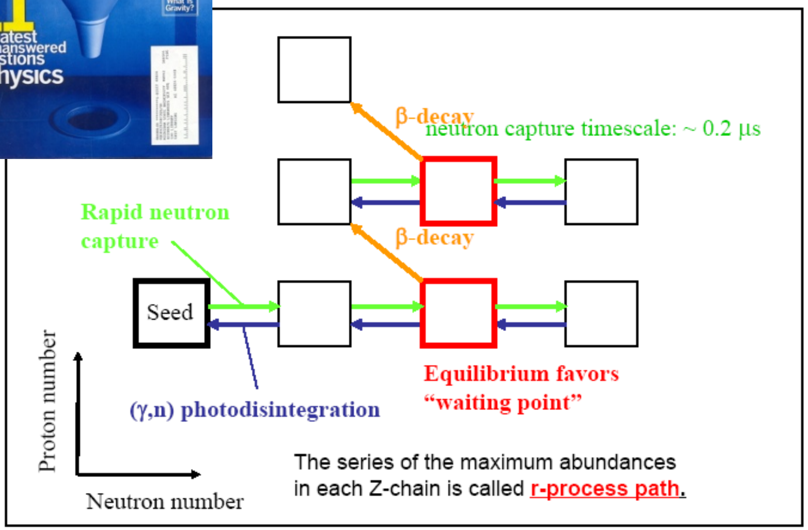
Thank you!

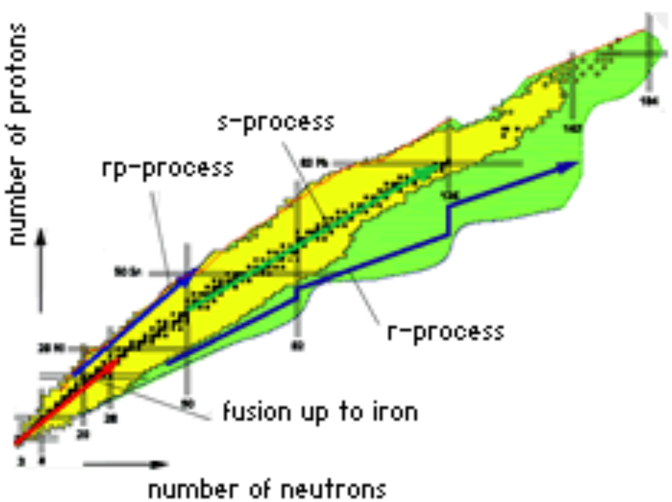
How were the heavy elements made?

The 11 greatest unanswered questions of physics



The r-process





Question 3

How were the heavy elements from iron to uranium made?

rapid neutron capture process (r-process)

↓

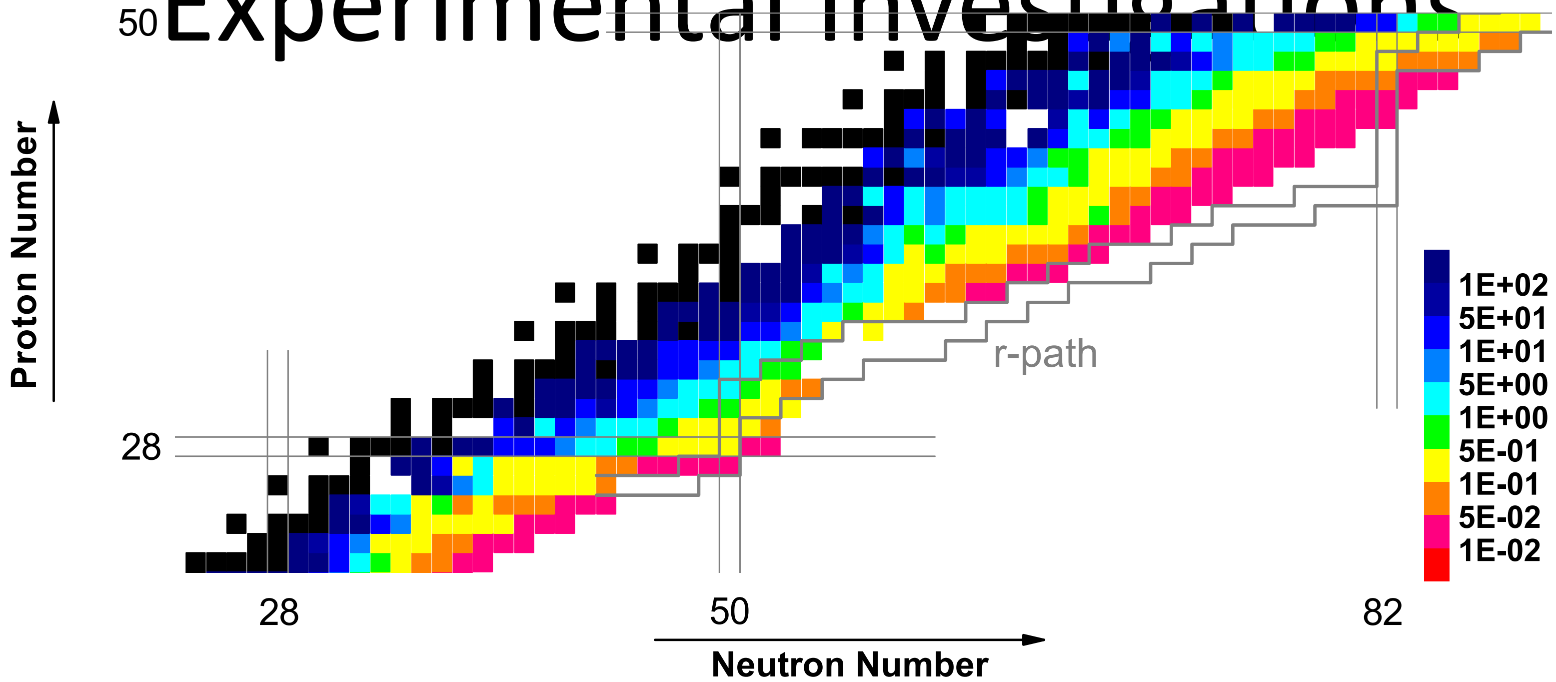
nuclear physics inputs:

mass, β-decay half-lives, …

↓

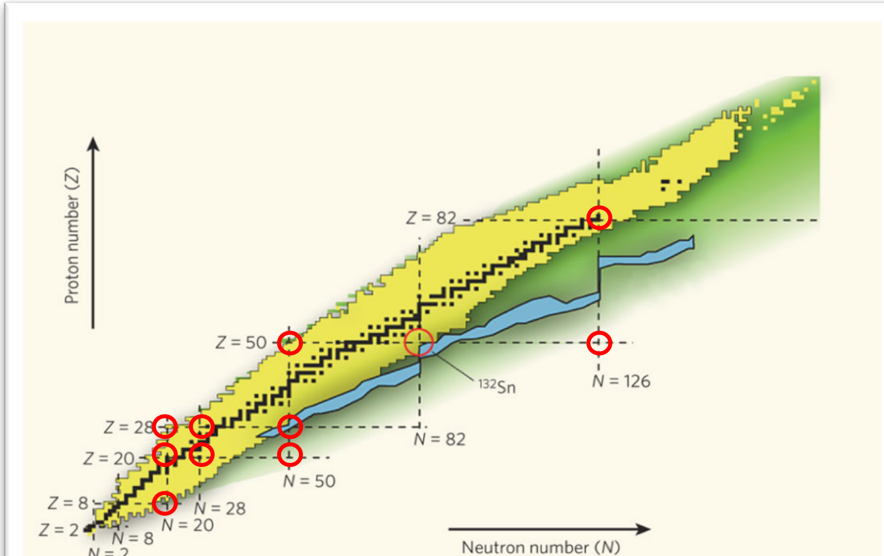
setting the time-scale of r-process

Experimental Investigations



- **Development of radioactive ion-beam facilities => important advances**
  - ^{78}Ni and around Hosmer, et al., PRL 94, 112501, 2005; Xu, et al., PRL 113, 032505, 2014
  - very neutron rich Kr to Tc isotopes Nishimura, et al., PRL 106, 052502, 2011
  - Zn Ga isotopes Madurga et al., PRL 109, 112501, 2012
  - 110 neutron-rich nuclei across N=82 shell gap Lorusso, et al., PRL 114, 192501,2015
  - 94 neutron-rich nuclei from Z=55-67 Wu, et al., PRL 118, 072701, 2017
  - N=126 r-process nuclei? Not reached yet. Important goal at ELI-NP laser driven nuclear physics
- **provide a good test ground for theoretical models**

RPA+PVC: only for magic nuclei…



Taken from Nature News & Views 465 2010

- To include pairing correlations for superfluid nuclei

Quasiparticle RPA + quasiparticle vibration coupling
(**QRPA**) + (**QPVC**)

- ✓ for the study of Gamow-Teller resonance in superfluid nuclei
- ✓ for the study of β-decay half-lives of the whole isotopic chain

QRPA+QPVC model

Step 1: HFB+QRPA calculation (charge-exchange) => Gamow-Teller response in QRPA level
HFB+QRPA calculation (non-charge-exchange) => vibrational phonons 1-, 2+, 3-, 4+, 5-
Particle-particle interaction

$$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2) = V_0 \frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

$$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2) = f\, V_0 \frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

The QRPA equation will give the energy E_n, and wavefunction $X^{(n)}, Y^{(n)}$

Step 2: QPVC calculation => Gamow-Teller response in QRPA+QPVC level

The QRPA+QPVC equation reads

$$\begin{pmatrix} \mathcal{D}+\mathcal{A}_1(\omega) & \mathcal{A}_2(\omega) \\ -\mathcal{A}_3(\omega) & -\mathcal{D}-\mathcal{A}_4(\omega) \end{pmatrix} \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix} = (\Omega_\nu - i\frac{\Gamma_\nu}{2}) \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix},$$

where $\mathcal{D} = E_n$, and the $\mathcal{A}_i$ matrices contain the spreading contributions, e.g.,

$$(\mathcal{A}_1)_{mn} = \sum_{ab,a'b'} W^{\downarrow}_{ab,a'b'}(E) X^{(m)}_{ab} X^{(n)}_{a'b'} + W^{\downarrow *}_{ab,a'b'}(-E) Y^{(m)}_{ab} Y^{(n)}_{a'b'},$$

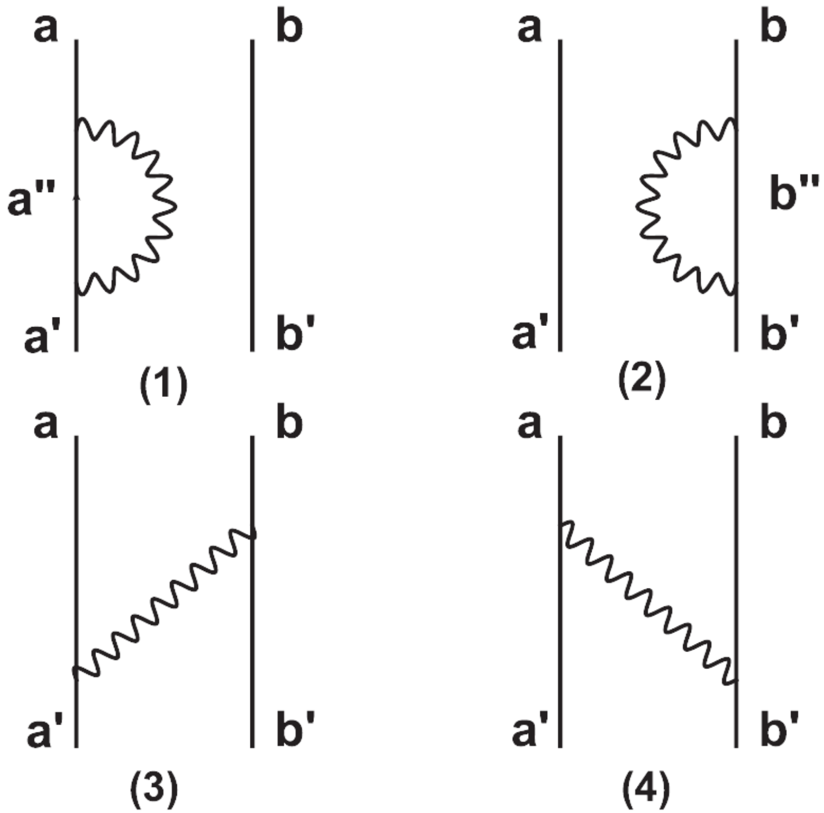
The GT strength function $S(E) = -\frac{1}{\pi}\mathrm{Im}\sum_\nu \langle 0|\hat{O}_{\mathrm{GT}\pm}|\nu\rangle^2 \frac{1}{E-\Omega_\nu + i\left(\frac{\Gamma_\nu}{2}+\Delta\right)}$

Spreading Terms

The matrix elements of the spreading term in the quasiparticle basis

$$W^{\downarrow}_{ab,a'b'} = \langle ab|V\frac{1}{E-\hat{H}}V|a'b'\rangle \; = \sum_{NN'}\langle ab|V|N\rangle\langle N|\frac{1}{E-\hat{H}}|N'\rangle\langle N'|V|a'b'\rangle,$$

where $|N\rangle = |a''b''\rangle \otimes |nL\rangle$ represents a doorway state, and a, b are quasi-particle states.

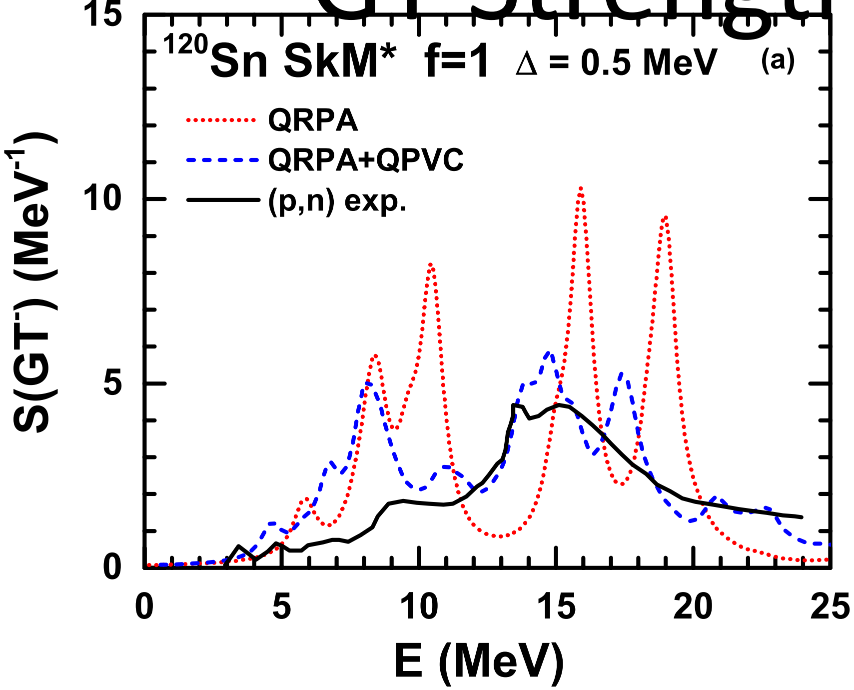


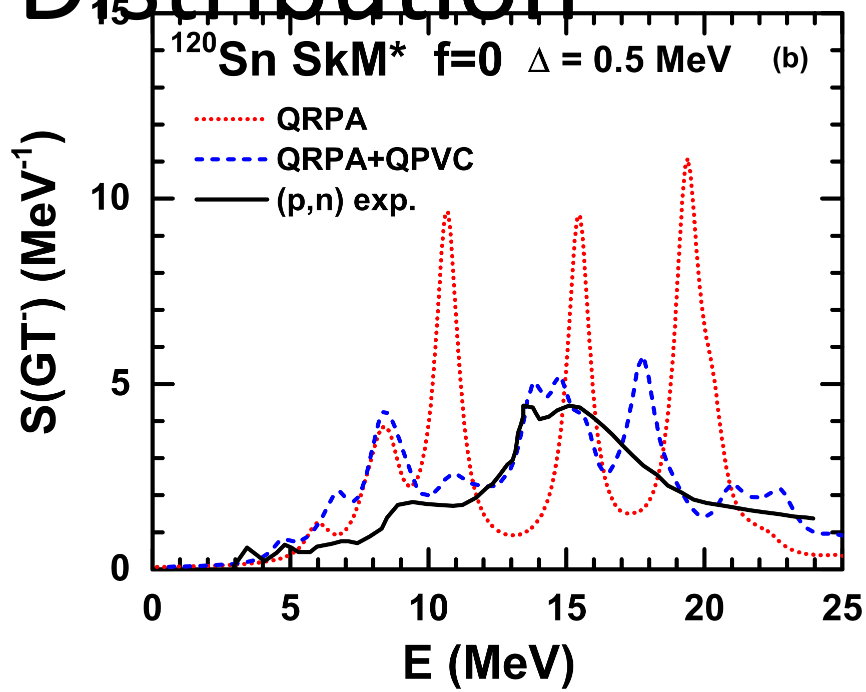
The vertex reduced matrix element:

$$\langle a||V||a'',nL\rangle = \frac{\hat{L}}{\sqrt{1+\delta_{cd}}}\sum_{cd}\left[\tilde{V}(cdLa'';a)X^{nL}_{cd} + (-1)^{j_a-j_{a''}+L}\tilde{V}(cdLa;a'')Y^{nL}_{cd}\right],$$

$$\tilde{V}(cdLa'';a) = V^{Lph}_{ada''c}(u_a u_{a''} u_c v_d - v_a v_{a''} v_c u_d)$$
$$+ V^{Lph}_{aca''d}(u_a u_{a''} v_c u_d - v_a v_{a''} u_c v_d)(-)^{j_c-j_d+L}$$
$$- V^{Lpp}_{aa''cd}(u_a v_{a''} u_c u_d - v_a u_{a''} v_c v_d)$$

GT Strength Distribution



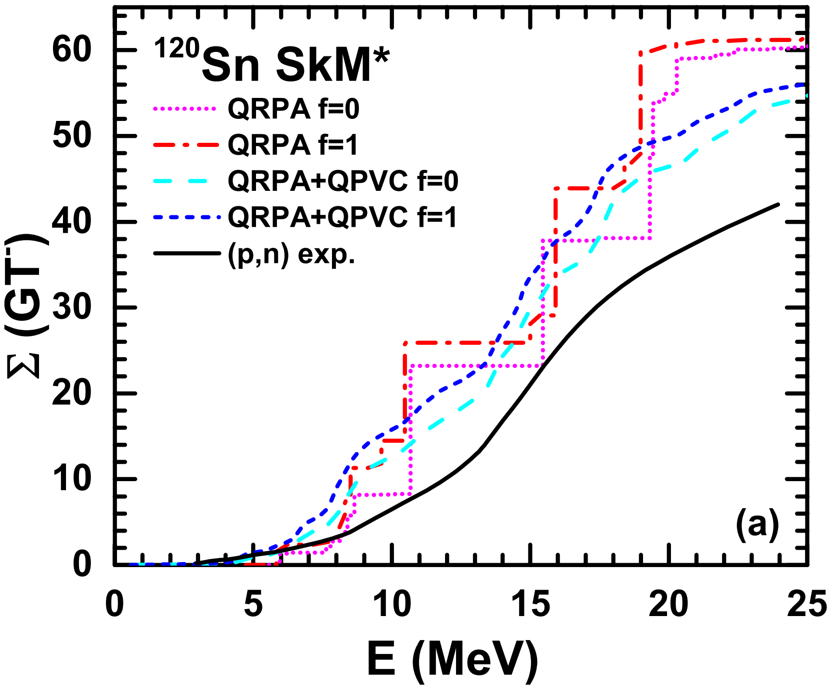


- (p,n) data: normalized by unit cross section $\sigma(0^\circ) = \hat{\sigma}\, F(q,\omega) B(\mathrm{GT})$
  Sasano, et al., PRC 79, 024602 (2009)

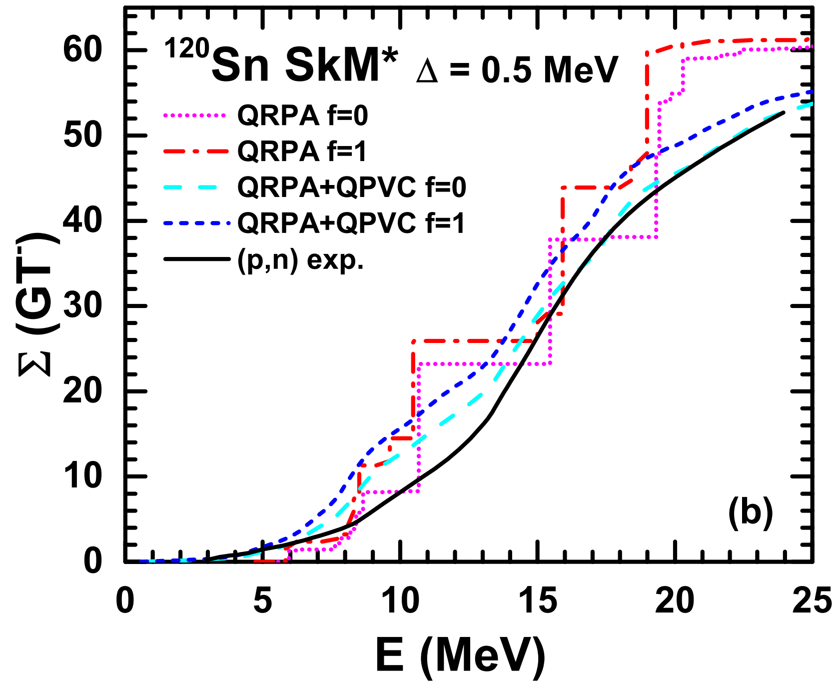
✓ QRPA + QPVC
  - Develop a width of 5.3 MeV (6.4 MeV from exp.), reproduce exp. profile in GTR
  - Overestimate the low-lying strength

Cumulative Sum

Cumulative sum

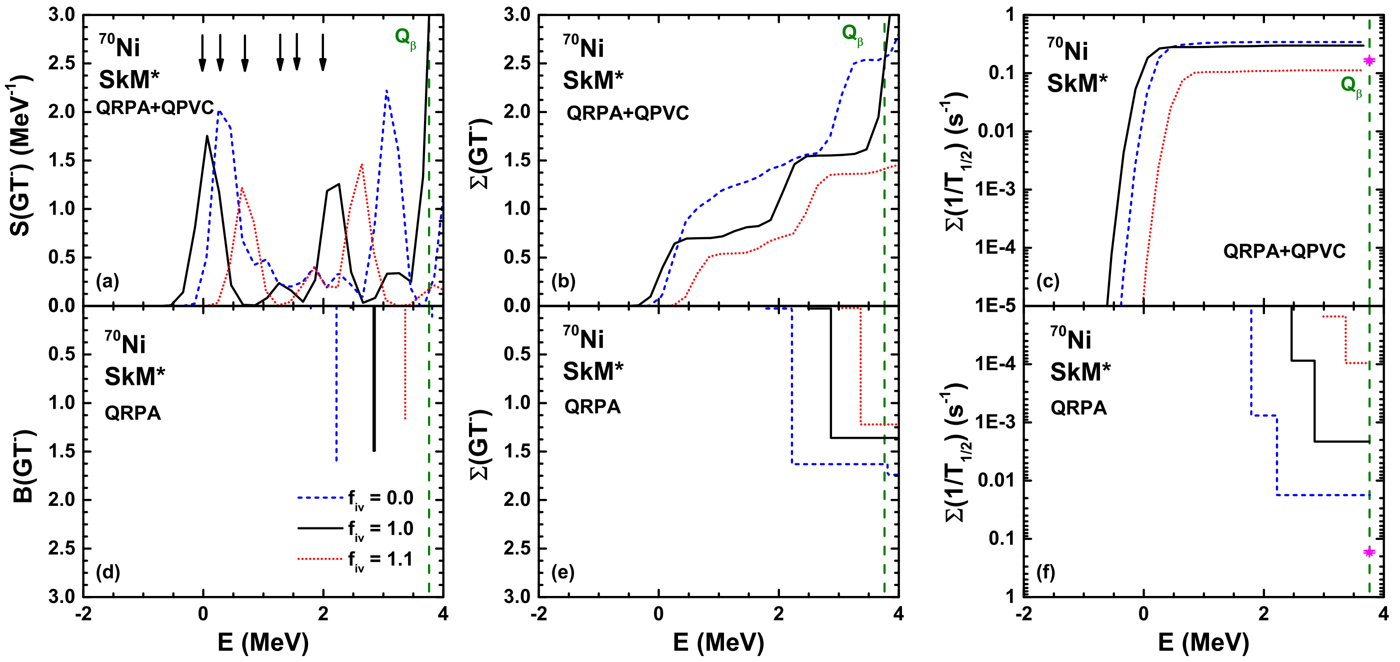


Cumulative sum with normalized data



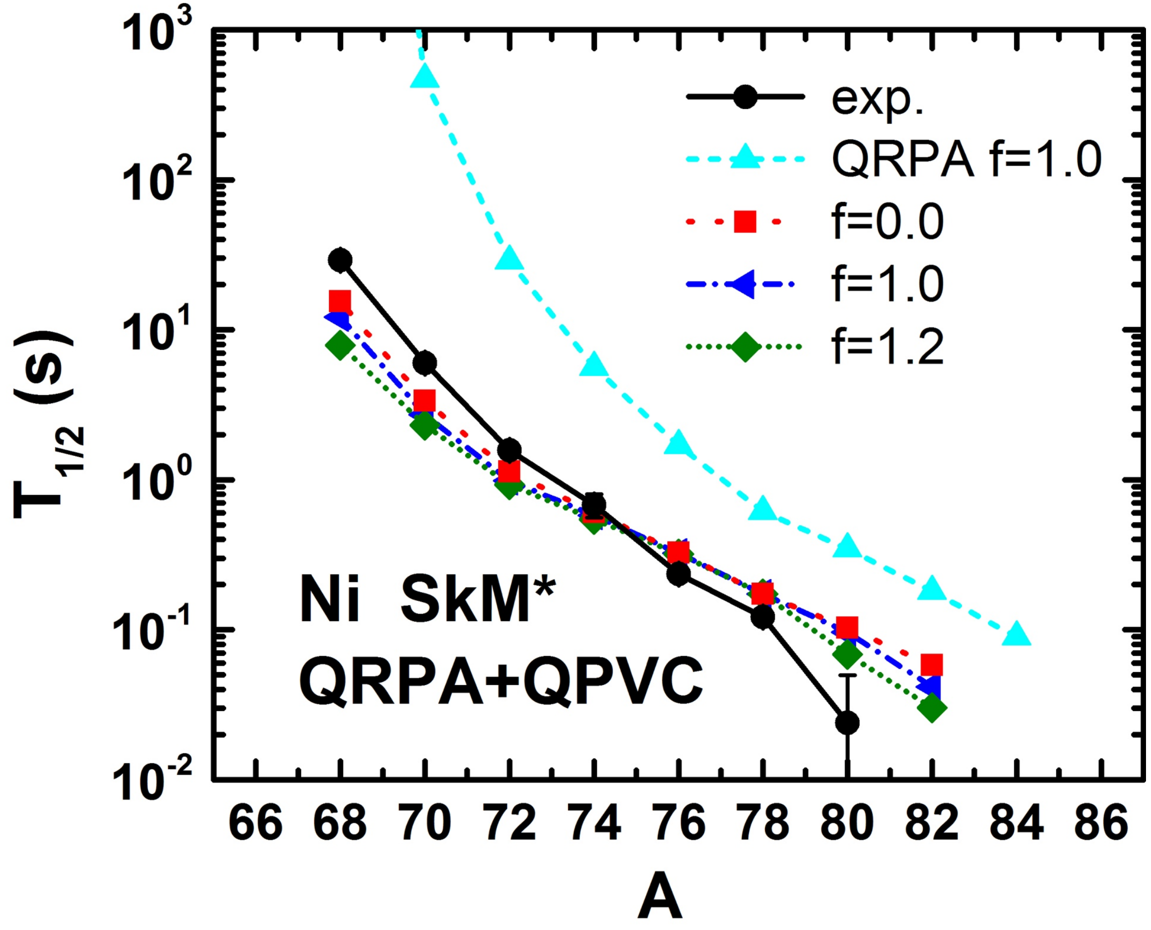
- f=0 -> f=1
  - ✓ low-lying strength is increased for both QRPA and QRPA + QPVC
- QRPA -> QRPA+QPVC:
  - ✓ better reproduces the exp. profile
  - ✓ cumulative strength is quenched by 10% at E=25 MeV
  - ✓ QRPA+QPVC strength × 0.75 = exp. strength ((p,n) data) at E=25 MeV

Niu, Colo, Vigezzi, Bai, Sagawa, **PRC** 94, 064328 (2016)



^{70}Ni
SkM*
QRPA+QPVC
Q_β
S(GT$^-$) (MeV^{-1})
(a)
^{70}Ni
SkM*
QRPA
B(GT$^-$)
f_{iv} = 0.0
f_{iv} = 1.0
f_{iv} = 1.1
(d)
E (MeV)
Σ(GT$^-$)
(b)
(e)
$\Sigma(1/T_{1/2})$ (s^{-1})
(c)
(f)

β-Decay Half-Lives in Ni isotopes

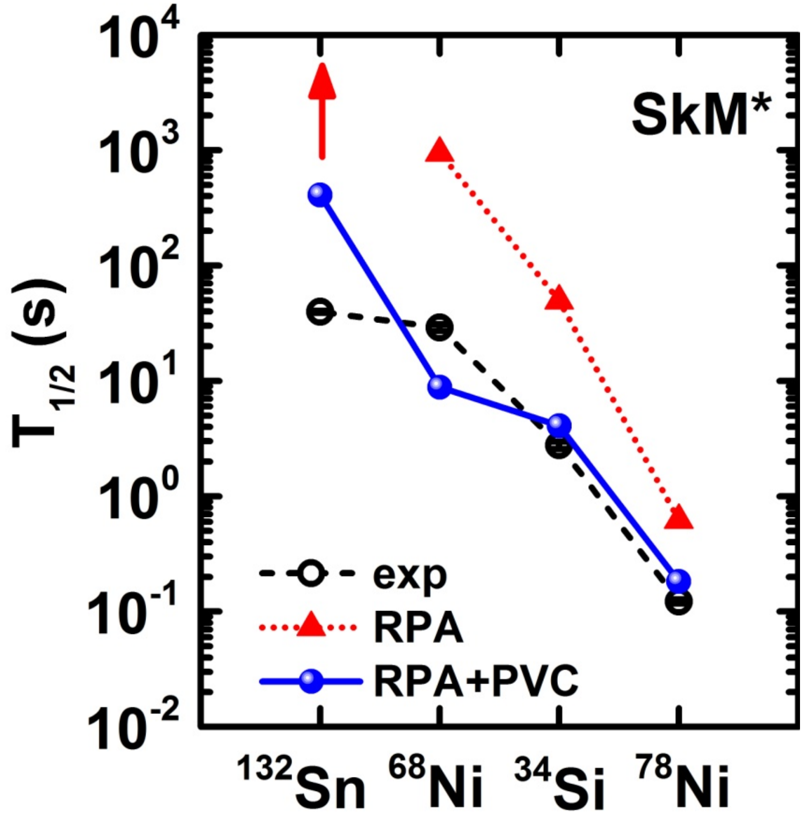


- Isoscalar Pairing:
  not so effective for Ni isotopes
  (nuclei before N=50 closed shell)

- QPVC:
  reduce the half-lives

RPA+PVC: β-Decay Half-Lives

- **Improved description of β-decay half-lives**



✓ **Reduce half-lives systematically** ✓ **Reproduce β-decay half-lives**

Y.F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, **PRL** 114, 142501 (2015).

Allowed GT approximation Engel PRC 60, Niu plB 723, De Shalit

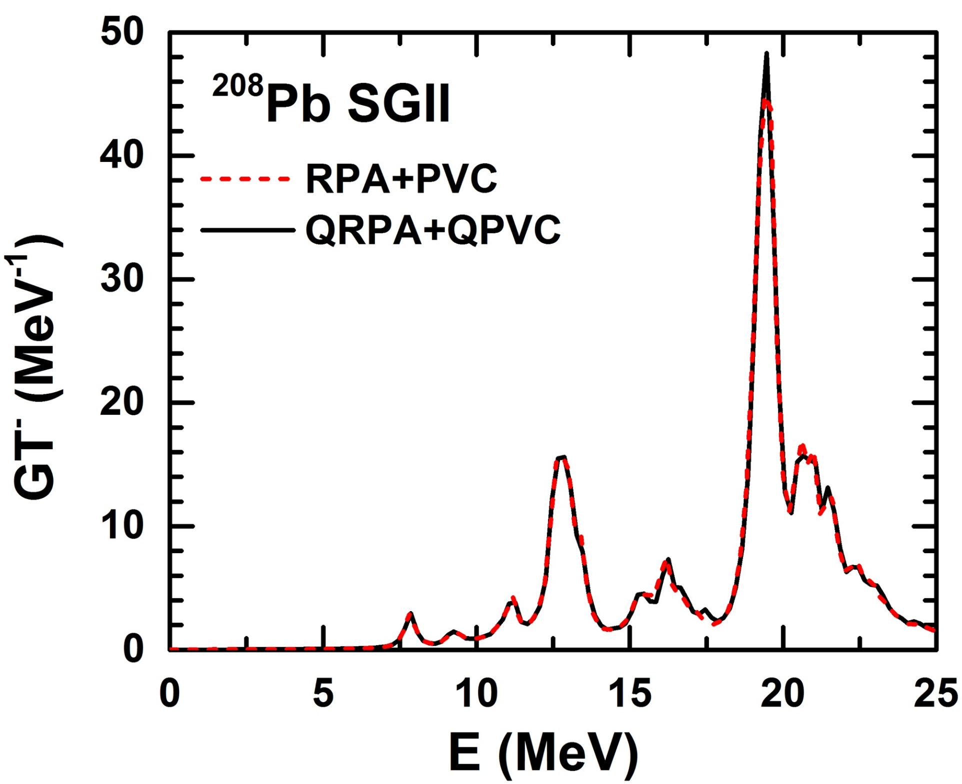
Why isoscalar pairing is sensitive to spin orbit

Why isoscalar pairing has a strong impact on strength
Distribution as son as a level becomes occupied
In Niksic et al prc 71 014308, they discuss the effect of isoscalar pairing on p. 12-13, an
say that ‘Because the neutron vu 1h_9/2 has low occupancy, the T=0 pairing has
a strong effect and reduces the calculated half-lives to the experimental values.
Actually, the lifetime of 134 Sn is reduced by two orders of magnitude when they add
isoscalar pairing.
What I don’t understand very well, is how we we go from a situation where v^2 =0 and
lifetime is 10^2 s, to the case where v^2 is only 0.024 (their Table V) and the lifetime is
1 s.

Numerical Check

Numerical Details

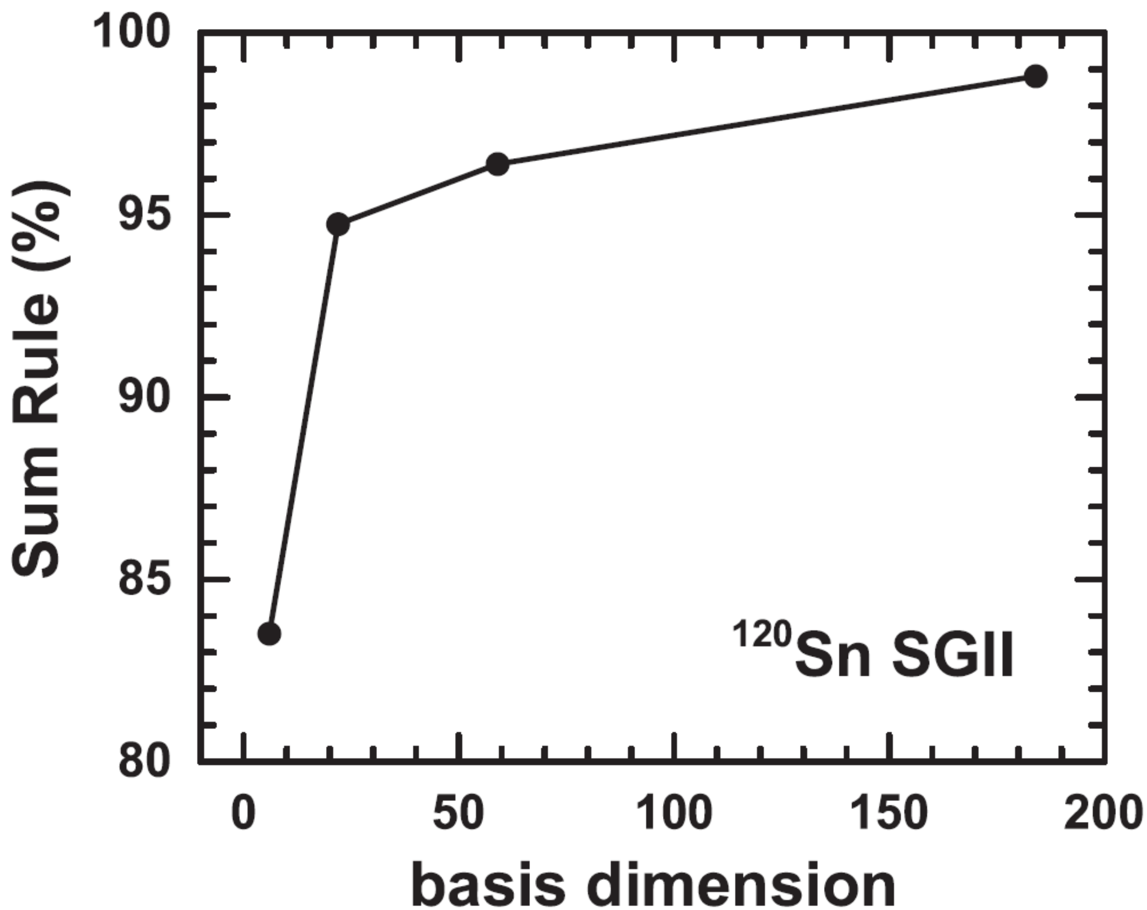
- HFB Bennaceur, et al., CPC, 168 96 (2005)
  - solved in coordinate space R = 20 fm rstep = 0.1 fm
  - Surface pairing, which reproduces the exp. pairing gap 1.34 MeV
- QRPA
  - Ecut = 100 MeV, uv > 0.001 for quasiparticle configuration space



Sum Rule

- Fulfillment of Ikeda Sum Rule for Gamow-Teller response at QRPA+QPVC level

Ikeda Sum Rule $$S_{GT^-} - S_{GT^+} = 3(N - Z)$$



Phonon Properties

TABLE II. The energy and reduced transition probability of the lowest phonons of different multipolarities included in the QRPA+QPVC calculation for ^{120}Sn. The experimental data are taken from NNDC [52]. The theoretical results are obtained by the QRPA approach with the interactions SAMi, SGII, and SkM*.

	E (MeV)				$B(EL, 0 \to L)$ (e^2 fm^{2L})			
Phonons	Expt.	SAMi	SGII	SkM*	Expt.	SAMi	SGII	SkM*
2^+	1.171	2.708	1.941	1.420	2.016×10^3	1.463×10^3	1.766×10^3	2.632×10^3
3^-		3.595	3.313	3.297		1.880×10^5	1.396×10^5	1.089×10^5
4^+		4.029	3.757	3.230		2.496×10^6	1.568×10^6	1.453×10^6
5^-		4.603	3.669	3.536		4.454×10^7	2.555×10^7	3.103×10^7