Quartet structure of N=Z nuclei

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XVI Conference on Theoretical Nuclear Physics in Italy
3-5 October 2017, Cortona

Outline

- Quartetting and proton-neutron pairing in N=Z nuclei
- Quartet structure of even-even N=Z nuclei:
  - the shell model perspective
  - the bosonic approach
- Conclusions

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Like-particle pairing: exact and BCS-type solutions

$$H = \sum_{i=1}^{\Omega} \epsilon_i \mathcal{N}_i - g \sum_{i,i'=1}^{\Omega} P_i^\dagger P_{i'}$$

$$\mathcal{N}_i = \sum_\sigma a_{i\sigma}^\dagger a_{i\sigma}, \quad P_i^\dagger = a_{i+}^\dagger a_{i-}^\dagger, \quad (P_i^\dagger)^\dagger = P_i$$

The exact ground state (Richardson, 1963)

$$|\Psi_{gs}\rangle = \prod_{\nu=1}^{N} B_\nu^\dagger |0\rangle, \quad B_\nu^\dagger = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P_k^\dagger, \quad E^{(\Psi)} = \sum_{\nu=1}^{N} E_\nu$$

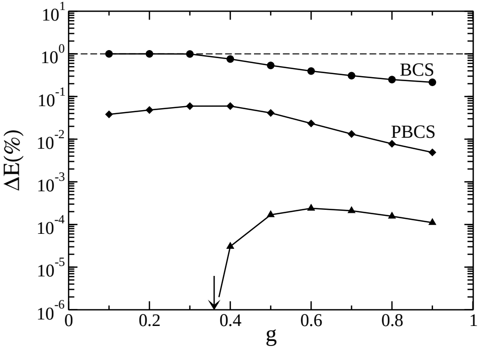
The BCS ground state

$$|BCS\rangle \propto e^{B^\dagger}|0\rangle = \sum_{n=0}^{\infty} \frac{(B^\dagger)^n}{n!}|0\rangle, \quad B^\dagger = \sum_{k=1}^{\Omega} x_k P_k^\dagger$$

The projected BCS ground state

$$|PBCS\rangle \propto (B^\dagger)^N |0\rangle$$

Ground state correlation energies for a system of
16 particles over 16 equispaced levels



M.S., PRC 85, 064326 (2012)

Isovector pairing in N=Z nuclei: BCS-type approaches

- The Hamiltonian (in a spherical mean field)

$$H^{(iv)} = \sum_i \epsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ij} V_{ij} \sum_\tau P^\dagger_{i,\tau} P_{j,\tau}$$

$$P^\dagger_{i,\tau} = [a^\dagger_i a^\dagger_i]^{T=1,J=0}_\tau \qquad (nn,\ pp,\ pn)$$

- No satisfactory description within generalized BCS-type approaches
  - BCS: particle number and isospin violations
    Two genenerate solutions (no mixing)

$$\Delta_{pn} \neq 0, \Delta_{pp} = \Delta_{nn} = 0 \qquad\qquad \Delta_{pn} = 0, \Delta_{pp} = \Delta_{nn} \neq 0$$

O. Civitarese et al., PRC 56, 1840 (1997)

  - PBCS: isospin violation
    Two approximations (none of them good)

$$|PBCS0 > \propto (B^\dagger_{pn})^{(N+Z)/2}|0\rangle \qquad\qquad |PBCS1 > \propto (B^\dagger_{pp} B^\dagger_{nn})^{(N+Z)/4}|0\rangle$$

N. Sandulescu et al., PRC 80, 044335 (2009)

The QCM and QM approaches

- The quartet condensate model (QCM) assumes that the ground state of an even-even N=Z nucleus has the form

$$|QCM\rangle = (Q^\dagger)^n|0\rangle, \qquad Q^\dagger = \sum_{i,j} c_{ij}[P_i^\dagger P_j^\dagger]^{T=0,J=0}$$

N. Sandulescu et al., PRC 85, 061303(R) (2012)

- The quartet model (QM) assumes instead that

$$|QM\rangle = \prod_{\rho=1}^{n} Q_\rho^\dagger|0\rangle, \qquad Q_\rho^\dagger = \sum_{ij} c_{ij}^{(\rho)}[P_i^\dagger P_j^\dagger]^{T=0,J=0}$$

M.S. and N. Sandulescu, PRC 88, 061303(R) (2013)

- T, J are exactly preserved.
- Quartets are constructed variationally for each nucleus.

Ground state correlation energies
(V_{ij} extracted from standard shell model interactions)

	exact	QM	QCM
^{20}Ne	-9.174	-9.174 (-)	-9.170 (0.04%)
^{24}Mg	-14.461	-14.458 (0.02%)	-14.436 (0.17%)
^{28}Si	-15.787	-15.780 (0.04%)	-15.728 (0.37%)
^{32}S	-15.844	-15.844 (-)	-15.795 (0.31%)
^{44}Ti	-5.965	-5.965 (-)	-5.964 (0.02%)
^{48}Cr	-9.579	-9.573 (0.06%)	-9.569 (0.10%)
^{52}Fe	-10.750	-10.725 (0.23%)	-10.710 (0.37%)
^{104}Te	-3.832	-3.832 (-)	-3.829 (0.08%)
^{108}Xe	-6.752	-6.752 (-)	-6.696 (0.83%)
^{112}Ba	-8.680	-8.678 (0.02%)	-8.593 (1.00%)

Isovector pairing in a deformed mean field

Ground state correlation energies

(V_{ij}=-24/A)

	exact	QM	QCM
^{20}Ne	-6.5505	-6.5505	-6.539 (0.18%)
^{24}Mg	-8.4227	-8.4227	-8.388 (0.41%)
^{28}Si	-9.6610	-9.6610	-9.634 (0.28%)
^{32}S	-10.2629	-10.2629	-10.251 (0.12%)
^{44}Ti	-3.1466	-3.1466	-3.142 (0.15%)
^{48}Cr	-4.2484	-4.2484	-4.227 (0.50%)
^{52}Fe	-5.4532	-5.4531	-5.426 (0.50%)
^{104}Te	-1.0837	-1.0837	-1.082 (0.16%)
^{108}Xe	-1.8696	-1.8696	-1.863 (0.35%)
^{112}Ba	-2.7035	-2.7034	-2.688 (0.57%)

M.S. and N. Sandulescu, PRC 88, 061303(R) (2013)

Isovector plus isoscalar pairing in N=Z nuclei

- The Hamiltonian (is a spherical mean field):

$$H = \sum_i \epsilon_i N_i \quad + \quad \sum_{i,j} V^{T=1}_{J=0}(i,j) \sum_{T_z} P^\dagger_{i,T_z} P_{j,T_z}$$
$$+ \quad \sum_{i \le j, k \le l} V^{T=0}_{J=1}(ij,kl) \sum_{J_z} D^\dagger_{ij,J_z} D_{kl,J_z}$$

$$P^\dagger_{j,T_z} = \frac{1}{\sqrt{2}} [a^\dagger_j a^\dagger_j]^{T=1,J=0}_{T_z}, \quad D^\dagger_{j_1 j_2,J_z} = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} [a^\dagger_{j_1} a^\dagger_{j_2}]^{J=1,T=0}_{J_z}$$

- The quartets:

$$Q^\dagger = Q^\dagger_{iv} + Q^\dagger_{is}$$

$$Q^\dagger_{iv} = \sum_{j_1 j_2} x_{j_1 j_2} [P^\dagger_{j_1} P^\dagger_{j_2}]^{J=0,T=0}$$

$$Q^\dagger_{is} = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4} [D^\dagger_{j_1 j_2} D^\dagger_{j_3 j_4}]^{J=0,T=0}$$

Ground state correlation energy

Spherical single-particle basis

(V_{ij} extracted from standard shell model interactions)

	Exact	QM	QCM
^{20}Ne	15.985	15.985 (-)	15.985 (-)
^{24}Mg	28.694	28.626 (0.24%)	28.595 (0.34%)
^{28}Si	35.600	35.396 (0.57%)	35.288 (0.88%)
^{44}Ti	7.019	7.019 (-)	7.019 (-)
^{48}Cr	11.649	11.624 (0.21%)	11.614 (0.30%)
^{52}Fe	13.887	13.828 (0.42%)	13.799 (0.63%)
^{104}Te	3.147	3.147 (-)	3.147 (-)
^{108}Xe	5.505	5.495 (0.20%)	5.489 (0.29%)
^{112}Ba	7.059	7.035 (0.34%)	7.017 (0.59%)

M.S. and N. Sandulescu, PRC 93, 054320 (2016)

Ground state correlation energies

(pn pairing force extracted from standard shell model Hamiltonians)

	PBCS1	PBCS0$_{iv}$	PBCS0$_{is}$
^{20}Ne	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
^{24}Mg	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
^{28}Si	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
^{44}Ti	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
^{48}Cr	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
^{52}Fe	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
^{104}Te	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
^{108}Xe	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
^{112}Ba	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

- errors in the range $10\% \div 57\%$

M.S. and N. Sandulescu, PRC 93, 054320 (2016)

Quartetting in even-even N=Z nuclei

- We want to represent even-even N=Z nuclei in terms of the quartets

$$Q^{+}_{\alpha,JM,TT_z} = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C^{(\alpha)}_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2} \times \left[[a^{+}_{i_1} a^{+}_{j_1}]^{J_1 T_1} [a^{+}_{i_2} a^{+}_{j_2}]^{J_2 T_2} \right]^{JT}_{MT_z}$$

- Basic questions:
  - which quartets to involve?
  - how to construct them?
  - what to do with them?

The case of ^{24}Mg

$$^{24}_{12}\text{Mg}_{12} = 4 \text{ protons} + 4 \text{ neutrons outside the } ^{16}\text{O core}$$

- We want to represent its states as linear superpositions of

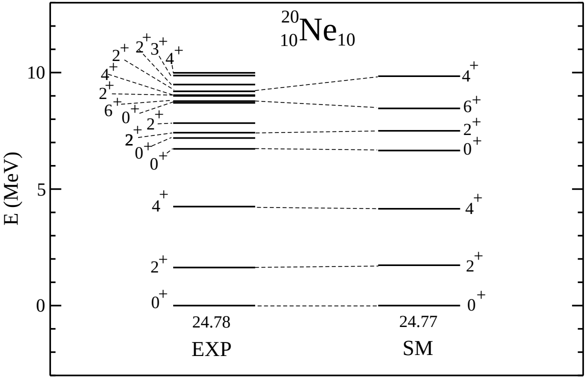
$$[Q^{+}_{\alpha J_1 T_1} Q^{+}_{\beta J_2 T_2}]^{J,T=0}_{M,T_z=0}|0\rangle$$

- We define the quartets according to the following general criterion :

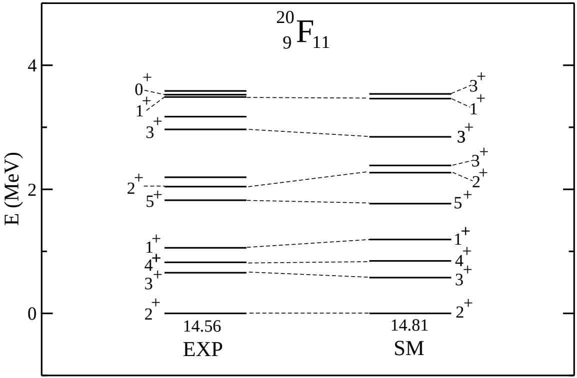
quartets describe the low-lying states of nuclei with four active particles outside the inert core of reference

- We perform a configuration interaction calculation in the quartet space.

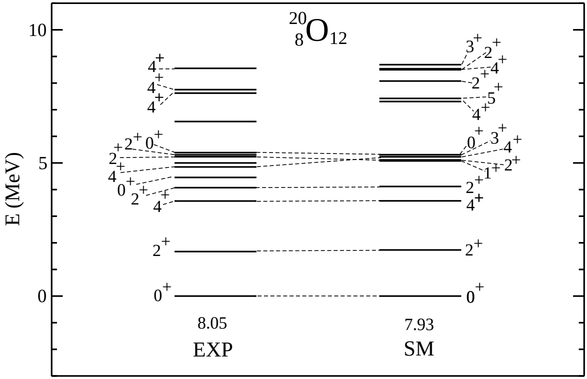
^{20}Ne: T=0 quartets



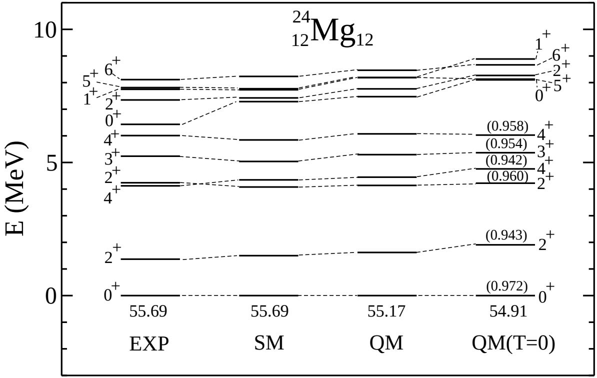
^{20}F: T=1 quartets



^{20}O: T=2 quartets

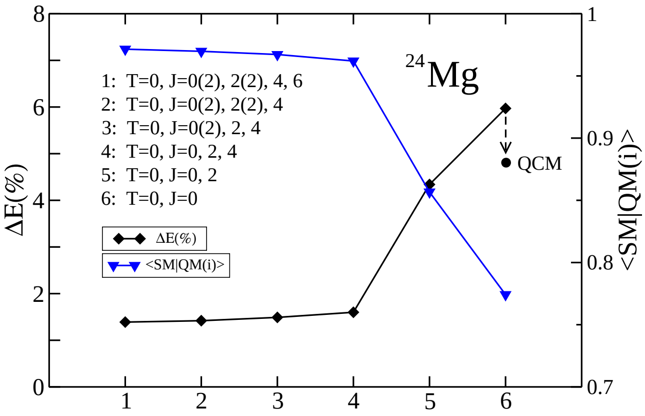


^{24}Mg: the spectrum

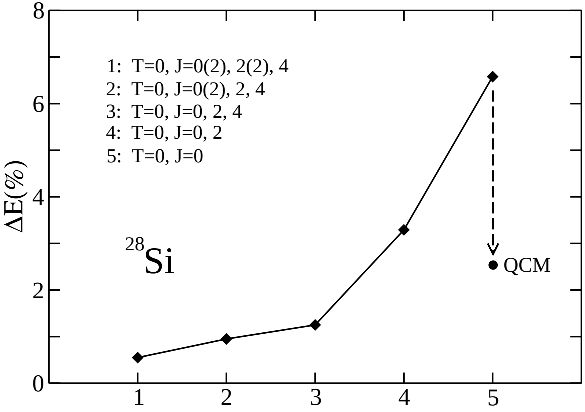


M.S. and N. Sandulescu, PRL 115, 112501 (2015)

^{24}Mg: ground state correlation energy



^{28}Si: ground state correlation energy



Further applications

- "Quartetting and spin-aligned proton-neutron pairs in heavy N=Z nuclei"
  M. S. and N. Sandulescu, PRC 91(2015)064318

- "Quartetting in odd-odd self-conjugate nuclei"
  M. S. and N. Sandulescu, PLB 763(2016)151

Bosonic approach to quartetting in even-even N=Z nuclei

- We assume two basic building-blocks:

$$Q^+_{J=0,T=0} \equiv S^+ \qquad Q^+_{J=2,T=0} \equiv D^+$$

- We replace them with two elementary bosons:

$$S^+ \Longrightarrow s^+$$
$$D^+ \Longrightarrow d^+$$

- We construct the most general one- plus two-body Hamiltonian

$$H_B = \epsilon_s \widehat{n}_s + \epsilon_d \widehat{n}_d + \sum_{\lambda_1\lambda_2\lambda_1'\lambda_2'\Lambda} V^{(\Lambda)}_{\lambda_1\lambda_2\lambda_1'\lambda_2'} [[b^+_{\lambda_1} b^+_{\lambda_2}]^{(\Lambda)} [\tilde{b}_{\lambda_1'} \tilde{b}_{\lambda_2'}]^{(\Lambda)}]^{(0)}$$

- It is: $H_B \equiv H_B^{(IBM)}$ but $N_B = \frac{N_B^{(IBM)}}{2}$
- Previous use of this formalism:
  J. Dukelsky et al, Phys. Lett. 115B, 359 (1982)

Application of the boson formalism to ^{28}Si

$$^{28}_{14}\mathrm{Si}_{14} = 6 \text{ protons} + 6 \text{ neutrons outside the } ^{16}\mathrm{O} \text{ core}$$

$$\Longrightarrow 3 \text{ alpha-like quartets} \Longrightarrow N_B = 3$$

- Single-boson energies ϵ_s and ϵ_d:

$$\begin{aligned} \epsilon_s &\equiv \mathrm{E}(0_1^+) \text{ of } ^{20}\mathrm{Ne} \\ \epsilon_d &\equiv \mathrm{E}(2_1^+) \text{ of } ^{20}\mathrm{Ne} \end{aligned}$$

- Two-body matrix elements $V^{(\Lambda)}_{\lambda_1\lambda_2\lambda_1'\lambda_2'}$:

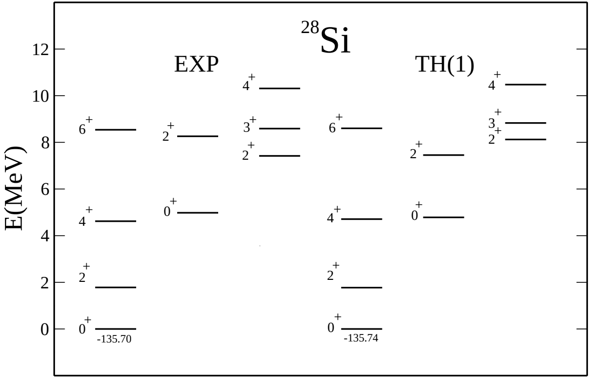
  fit to the experimental data

- E2 transitions:

$$T^{(E2)} \equiv T^{(E2)}_{IBM} = e_b([d^+ s + s^+ \tilde{d}]^{(2)}) + \chi[d^+\tilde{d}]^{(2)})$$

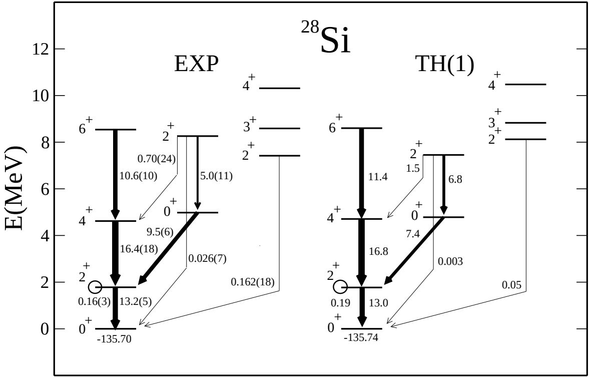
e_b, χ: fit to the experimental data

^{28}Si: the low-lying spectrum



EXP: Sheline et al., PLB 119(1982)263

^{28}Si: low-lying spectrum and E2 transitions



M.S. and N. Sandulescu, in preparation

Fermion-boson correspondence

FERMIONIC SPACE		BOSONIC SPACE
Q_i^+	$\longleftrightarrow$	b_i^+
$[Q_i, Q_j^+] = \widehat{\Delta}_{ij}$	$\longleftrightarrow$	$[b_i, b_j^+] = \delta_{ij}$
$F^{(N)} = \{Q_{k_1}^+ Q_{k_2}^+ \cdots Q_{k_N}^+ \lvert 0\rangle\}_{k_i<k_j}$	$\longleftrightarrow$	$B^{(N)} = \{b_{k_1}^+ b_{k_2}^+ \cdots b_{k_N}^+ \lvert 0)\}_{k_i<k_j}$
$\lvert N, k\rangle$	$\longleftrightarrow$	$\lvert N, k)$
$\langle N, k \lvert N, k'\rangle \neq \delta_{k,k'}$	$\longleftrightarrow$	$(N, k \lvert N, k') = \delta_{k,k'}$

Boson mapping of the fermion Hamiltonian

- The boson image H_B of the fermion Hamiltonian H_F is defined such that

$$(N,l|H_B|N,m) = \sum_{ij} R^{(N)}_{li} \langle N,i|H_F|N,j\rangle R^{(N)}_{jm}$$

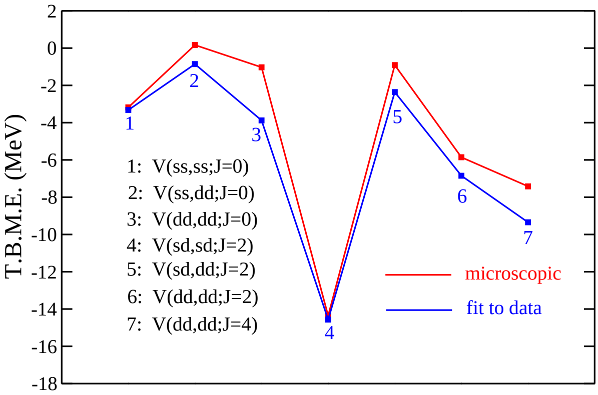
being

$$R^{(N)}_{li} = {\sum_k}^* f^{(N)}_{lk} \frac{1}{\sqrt{\mathcal{N}^{(N)}_k}} f^{(N)}_{ik}, \qquad \sum_l \langle N,i|N,l\rangle f^{(N)}_{lj} = \mathcal{N}^{(N)}_j f^{(N)}_{ij}$$

- The spectrum of H_B in $B^{(N)}$ is identical to that of H_F in $F^{(N)}$ (plus a number of zero's equal to the number of $\mathcal{N}^{(N)}_j = 0$)

- H_B is, in general, N-body

Two-boby matrix elements of H_B



H_B and its geometric structure

- Any IBM-type Hamiltonian has associated with it an intrinsic geometric structure.
- This geometric stucture results from the minimization of

$$E(N,\beta,\gamma) = \langle N;\beta,\gamma|H_B|N;\beta,\gamma\rangle$$

where

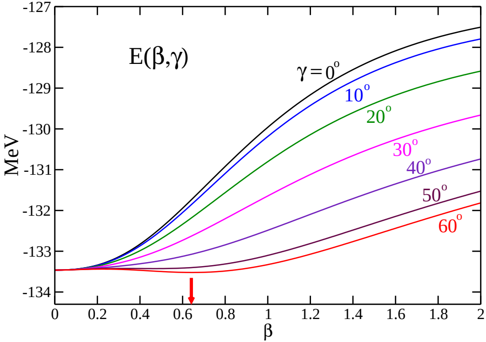
$$|N;\beta,\gamma\rangle = \frac{1}{\sqrt{N!(1+\beta^2)^N}}(B^+)^N|0\rangle$$

with

$$B^+ = s^+ + \beta[cos\gamma d_0^+ + \frac{1}{\sqrt{2}}sin\gamma(d_{+2}^+ + d_{-2}^+)]$$

- β_0 and γ_0 at the minimum define the equilibrium "shape"
- It is: $\beta^{(IBM)} \propto \beta^{(GM)}$ $\gamma^{(IBM)} = \gamma^{(GM)}$
- For a standard (two-body) IBM-type Hamiltonian, it is either $\gamma_0 = 0^o$ (prolate shape) or $\gamma_0 = 60^o$ (oblate shape)

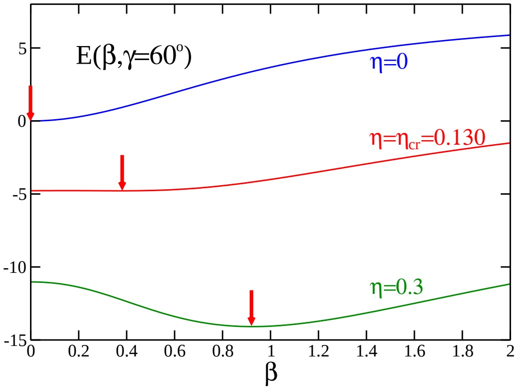
The intrinsic shape of ^{28}Si



^{28}Si emerges as an oblate "β-unstable" nucleus
(in agreement with Das Gupta et al., NPA 94(1967)602)

The potential energy surface of $H_{IBM}^{(T)}$ $(\gamma = 60^o)$

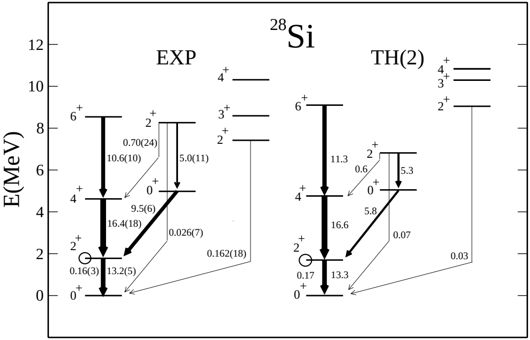
$$H_{IBM}^{(T)} = (1-\eta)\hat{n}_d - \eta(Q^+ \cdot Q^+), \qquad Q^+ = [d^+ s + s^+ \tilde{d}]^{(2)} + \frac{\sqrt{7}}{2}[d^+ \tilde{d}]^{(2)}$$



The spectrum of $H_{IBM}^{(T)}$ at $\eta = \eta_{cr}$

$$H_{IBM}^{(T)} = (1-\eta_{cr})\hat{n}_d - \eta_{cr}(Q^+ \cdot Q^+), \qquad Q^+ = [d^+ s + s^+ \tilde{d}]^{(2)} + \frac{\sqrt{7}}{2}[d^+ \tilde{d}]^{(2)}$$

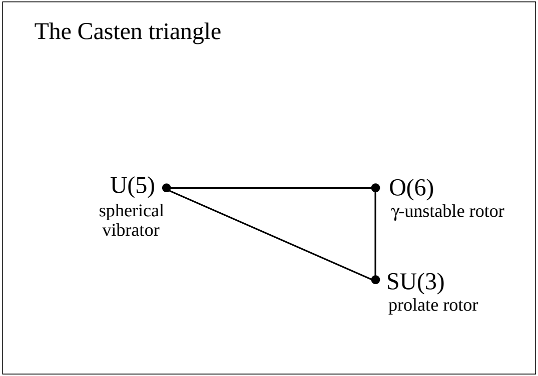
$$H_B \equiv \rho H_{IBM}^{(T)}, \qquad T^{(E2)} \equiv \tau Q^+$$



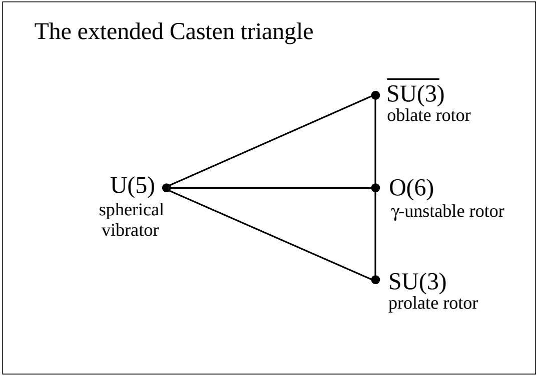
Conclusions

- Quartetting plays a key role in the structure of both even-even (and odd-odd) self-conjugate nuclei.
- T=0 quartets emerge as the basic building blocks of even-even N=Z nuclei.
- T=0, J=0,2,... quartets in N=Z nuclei appear to play a role analogous to that of S,D,... pairs in ordinary nuclei.
- The bosonic implementation of quartetting can provide a powerful tool for the study of N=Z nuclei.

The IBM structural diagram

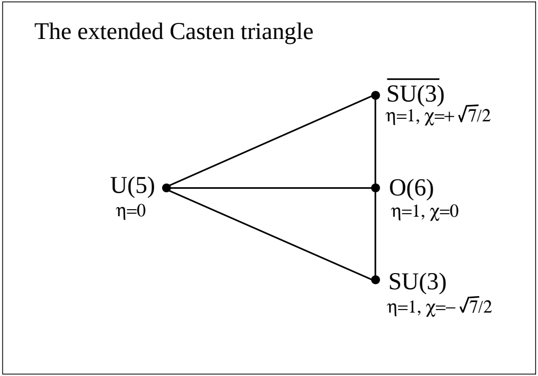


The IBM structural diagram



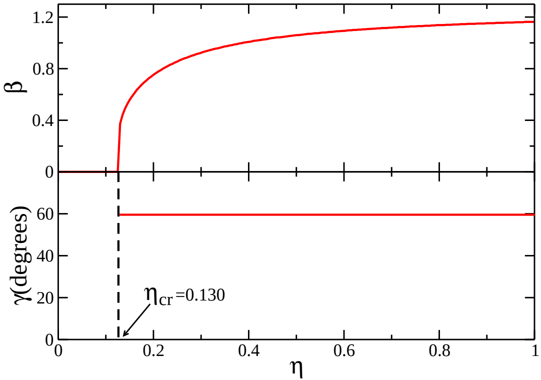
A toy IBM Hamiltonian

$$H_{IBM}^{(T)} = (1-\eta)\widehat{n}_d - \eta(Q^+ \cdot Q^+), \qquad Q^+ = [d^+ s + s^+ \tilde{d}]^{(2)} + \chi[d^+ \tilde{d}]^{(2)}$$

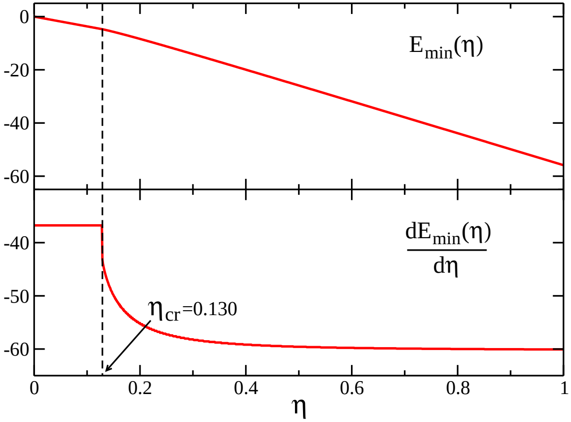


The $\mathrm{U}(5) - \overline{\mathrm{SU}(3)}$ shape-phase transition

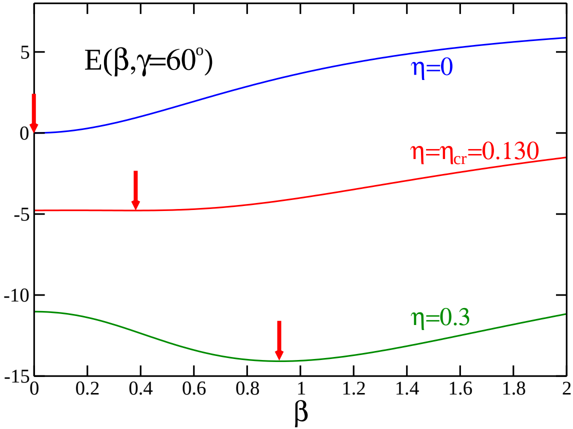
$$H_{IBM}^{(T)} = (1-\eta)\widehat{n}_d - \eta(Q^+ \cdot Q^+), \qquad Q^+ = [d^+ s + s^+ \tilde{d}]^{(2)} + \frac{\sqrt{7}}{2}[d^+ \tilde{d}]^{(2)}$$



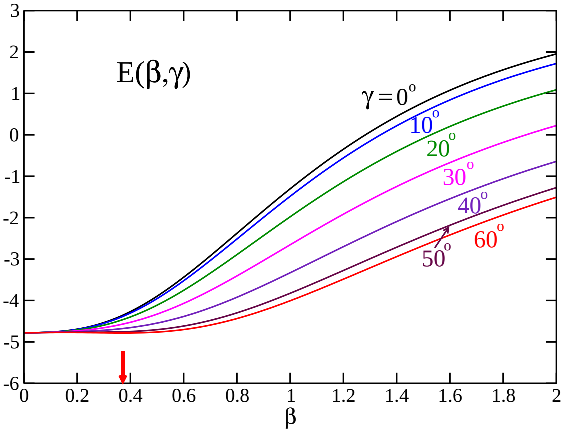
The $\mathrm{U}(5) - \overline{\mathrm{SU}(3)}$ shape-phase transition: first-order



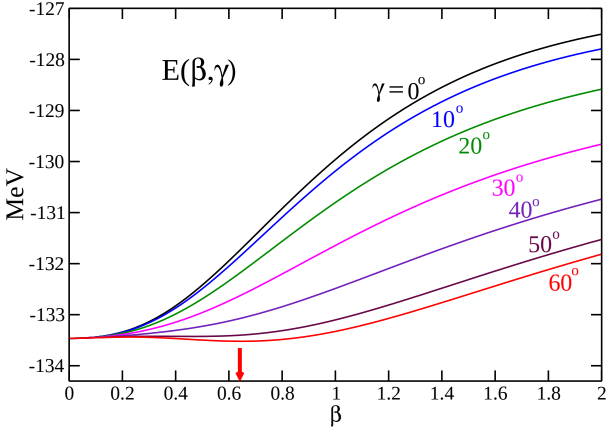
The potential energy surface of $H_{IBM}^{(T)}$ ($\gamma = 60^o$)



The potential energy surface of $H^{(T)}_{IBM}$ at $\eta = \eta_{cr}$



The potential energy surface of ^{28}Si



The spectrum of $H_{IBM}^{(T)}$ at $\eta = \eta_{cr}$

$$H_{IBM}^{(T)} = (1-\eta_{cr})\hat{n}_d - \eta_{cr}(Q^+ \cdot Q^+), \qquad Q^+ = [d^+ s + s^+ \tilde{d}]^{(2)} + \frac{\sqrt{7}}{2}[d^+ \tilde{d}]^{(2)}$$

$$H_B \equiv \rho H_{IBM}^{(T)}, \qquad T^{(E2)} \equiv \tau Q^+$$