Three-nucleon force correlations and electromagnetic response in finite nuclei with Self-Consistent Green's Functions



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## Outline

• Extension of the Algebraic Diagrammatic Construction (ADC) method with three-nucleon interactions

 Dipole Response Function and Polarisability in Oxygen and Calcium isotopes

• Effective charges in Oxygen and Nickel isotopes from realistic nuclear interactions

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# Motivations: Role of 3N forces in nuclear phenomena

See C. Barbieri's and T. Fukui's talks

Method: Self-consistent Green's function formalism

W. Dickhoff, C. Barbieri, Prog. Part. Nucl. Phys. **52**, 377 (2004) C. Barbieri, A. Carbone, Lectures notes in Phys., Vol. **936**, 571 (2017)



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Gorkov formalism: C. Barbieri, T. Duguet, V. Somà

## "What a nucleon does in the nucleus"

#### (i.e. in a strongly interacting many-fermions systems)?

Spectroscopic information (<sup>56</sup>Ni)



Binding energies and driplines (O)



- Ground state properties
- Spectroscopic informations
- One- (two-, ...) body operators matrix elements
- Optical potentials (A. Idini, C. Barbieri, Acta Phys. Pol. B 88, 273 (2017))

Microscopic nuclear Hamiltonian

Green's function (Lehmann representation)

Dyson equation

$$\hat{H} = \sum_{\alpha} \varepsilon_{\alpha}^{0} a_{\alpha}^{\dagger} a_{\alpha} - \sum_{\alpha\beta} U_{\alpha\beta} a_{\alpha}^{\dagger} a_{\beta} + \frac{1}{4} \sum_{\substack{\alpha\gamma \\ \beta\delta}} V_{\alpha\gamma,\beta\delta} a_{\alpha}^{\dagger} a_{\gamma}^{\dagger} a_{\delta} a_{\beta} + \frac{1}{65} \sum_{\substack{\alpha\gamma \\ \beta\delta}} W_{\alpha\gamma\epsilon,\beta\delta\eta} a_{\alpha}^{\dagger} a_{\gamma}^{\dagger} a_{\epsilon}^{\dagger} a_{\eta} a_{\delta} a_{\beta}$$
$$g_{\alpha\beta}(\omega) = \sum_{n} \frac{\langle \Psi_{0}^{A} | a_{\alpha} | \Psi_{n}^{A+1} \rangle \langle \Psi_{n}^{A+1} | a_{\beta}^{\dagger} | \Psi_{0}^{A} \rangle}{\omega - \varepsilon_{n}^{+} + i\eta} + \sum_{k} \frac{\langle \Psi_{0}^{A} | a_{\beta}^{\dagger} \rangle \Psi_{k}^{A-1} \rangle \langle \Psi_{k}^{A-1} | a_{\alpha} | \Psi_{0}^{A} \rangle}{\omega - \varepsilon_{k}^{-} - i\eta}$$
$$G_{\alpha\beta}(\omega) = G_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} G_{\alpha\gamma}^{(0)}(\alpha) \sum_{\gamma\delta} (\omega) G_{\delta\beta}(\omega)$$
Solf-production potential affecting

the s.p. propagation in the nuclear medium

(A. Carbone et al, Phys. Rev. C 88 (2013) 054326)

With 3N forces, # of self-energy diagrams is too cumbersome...

#### **BASIC IDEA:**

Effective interaction concept generalises the HF approximation of the twobody forces to the N-body forces and with respect to the correlated propagator



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Example of diagram with effective interaction

1p Interaction-**irreducible** second-order self-energy diagram



## Interaction-irreducible Self-Energy with 3N forces

(A. Carbone et al, Phys. Rev. C 88 (2013) 054326)

$$G_{\alpha\beta}(\omega) = G_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} G_{\alpha\gamma}^{(0)}(\omega) \Sigma_{\gamma\delta}^{\star}(\omega) G_{\delta\beta}(\omega)$$

Second-order diagrams with 3N forces



#### Third-order diagrams with 3N forces



## Interaction-irreducible Self-Energy with 3N forces

(A. Carbone et al, Phys. Rev. C 88 (2013) 054326)

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**Diagrams with effective 2N forces** (A. Cipollone et al, Phys. Rev. C 92 (2015) 014306)

#### **Diagram with irreducible 3N forces**

(F.R., C. Barbieri, Proceeding of NTSE (2016)) (F.R., C. Barbieri, arXiv:1709.04330 (2017))

Third-order diagrams with 3N forces



# Algebraic Diagrammatic Construction method at order 3

J. Schirmer and collaborators:

Phys. Rev. A26, 2395 (1982) Phys. Rev. A28, 1237 (1983)

#### Self-energy expansion is treated NON-perturbatively:

Entire classes of self-energy diagrams (ladder and ring) are summed at infinite order by means of a geometric series



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Entire classes of self-energy diagrams (ladder and ring) are summed at infinite order by means of a geometric series









The set of ladder diagrams is a geometric series



## How does ADC(n) work practically

General form of the irreducible self-energy

$$\Sigma_{\alpha\beta}(\omega) = \mathcal{M}^{\dagger} \frac{1}{\hbar\omega - E_{ph} - \mathcal{C}} \mathcal{M}$$
  
E2p1h, E3p2h, ...

Formal expansion of  $\mathcal{M}$  in powers of interactions  $\mathcal{M} = \mathcal{M}^{(I)} + \mathcal{M}^{(II)} + \mathcal{M}^{(III)} + \dots$ 

## How does ADC(n) work practically

General form of the irreducible self-energy

$$\Sigma_{\alpha\beta}(\omega) = \mathcal{M}^{\dagger} \frac{1}{\hbar\omega - E_{ph} - \mathcal{C}} \mathcal{M}$$
  

$$\sum_{\substack{\epsilon \geq 2p1h, \epsilon \leq 3p2h, \dots}} \mathcal{K}$$
First order in the interaction

Formal expansion of  $\mathcal{M}$  in powers of interactions  $\mathcal{M} = \mathcal{M}^{(I)} + \mathcal{M}^{(II)} + \mathcal{M}^{(III)} + \dots$ 

Explicit expressions for  $\mathcal{M}$  and  $\mathcal{C}$  are found by comparing with derived expressions of self-energy Goldstone diagrams up to the same order

$$\mathcal{M}^{\dagger} \frac{1}{\hbar\omega - E_{ph} - \mathcal{C}} \mathcal{M} = \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(II)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(II)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{C} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \text{fourth order} + \dots$$

## How does ADC(n) work practically

General form of the irreducible self-energy

$$\Sigma_{lphaeta}(\omega) = \mathcal{M}^{\dagger} \frac{1}{\hbar\omega - E_{ph} - \mathcal{C}} \mathcal{M}$$
  
Explanation is a single set of the First order in the

interaction

Formal expansion of  $\mathcal{M}$  in powers of interactions  $\mathcal{M} = \mathcal{M}^{(I)} + \mathcal{M}^{(II)} + \mathcal{M}^{(III)} + \dots$ 

Explicit expressions for  $\mathcal{M}$  and C are found by comparing with derived expressions of self-energy Goldstone diagrams up to the same order

$$\mathcal{M}^{\dagger} \frac{1}{\hbar\omega - E_{ph} - \mathcal{C}} \mathcal{M} = \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(II)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{C} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{C} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{C} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{C} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{C} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)} + \mathcal{M}^{(I)\dagger} \frac{1}{\hbar\omega - E_{ph}} \mathcal{M}^{(I)} +$$

## Features of Self-Energy in ADC(n)

Compatible with the Lehmann representation Principle of Causality

Hermitian

 $\Sigma_{\alpha\beta}(\omega)$ 

Non perturbative resummation

Dyson equation is solved as eigenvalue problem poles and residues of the propagator are found as eigenvalues and eigenvectors of the Self-Energy Hermitian matrix

Complete set of ADC(3) working equations can be found in:

(F.R., C. Barbieri, Proceeding of NTSE (2016))

(F.R., C. Barbieri, ArXiv:1709.04330 (2017))

Work in progress: Implementation in BcDor Code



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# Electric Dipole Polarizability $\alpha_D$

#### In general:

 $\alpha_D \propto E1$  electromagnetic response (quality of the nuclear wave function correlations)

#### **Recent studies:**

- Reinhard et al, PRC 81 051303(R) 2010
- Piekarewicz *et al*, PRC 85 041302(R) 2012



 $\alpha_{\rm D}$  as input quantity for constraining the isovector part of the nuclear interaction

· Hagen et alii, Nature Physics 12, 186 (2015)



Theory input for determining the Radius of Neutron stars

## Electromagnetic response in SCGF

OBSERVABLES   

$$\sigma_{\gamma}(E) = 4\pi^{2} \alpha E R(E)$$
PHOTOABSORPTION CROSS SECTION
$$\alpha_{D} = 2\alpha \int dE \frac{R(E)}{E}$$
ELECTRIC DIPOLE POLARIZABILITY

Response R(E) depends on excited states of the nuclear system, when "probed" with dipole operator  $\hat{D}$ 

$$R(E) = \sum_{\nu} |\langle \psi_{\nu}^{A} | \hat{D} | \psi_{0}^{A} \rangle |^{2} \, \delta_{E_{\nu},E}$$

## Electromagnetic response in SCGF

 $\sigma_{\gamma}(E) = 4\pi^{2} \alpha E R(E) \text{ photoabsorption cross section}$   $\alpha_{D} = 2\alpha \int dE \, \frac{R(E)}{E} \quad \text{electric dipole polarizability}$ 

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$$R(E) = \sum_{\nu} |\langle \psi_{\nu}^{A} | \hat{D} | \psi_{0}^{A} \rangle|^{2} \delta_{E_{\nu},E}$$
$$\sum_{ab} \langle a | \hat{D} | b \rangle \langle \psi_{\nu}^{A} | c_{a}^{\dagger} c_{b} | \psi_{0}^{A} \rangle$$

S.p. matrix element of the dipole one-body operator

Nuclear structure correlations: g<sup>II</sup> RPA level (first order) g<sup>I</sup> "dressed" ADC(3) Results: cross section and dipole polarisability

<sup>16</sup>O <sup>22</sup>O <sup>40</sup>Ca <sup>48</sup>Ca

# Results for Oxygen isotopes

 $\sigma$  from RPA response (discretized spectrum) vs  $\sigma$  from photoabsorption and Coulomb excitation



#### **NNLO**sat

- GDR position of <sup>16</sup>O reproduced
- · Hint of a soft dipole mode on the neutron-rich isotope

Dipole polarizability $\alpha_D$ (fm <sup>3</sup> )				
Nucleus	SCGF	$\rm CC/LIT$	$\operatorname{Exp}$	
<sup>16</sup> O	0.50	0.57(1)	0.585(9)	
$^{22}O$	0.72	0.86(4)	0.43(4)	

# **Results for Calcium isotopes**

#### $\sigma$ from RPA response (discretized spectrum) vs $\sigma$ from photoabsorption and Coulomb excitation



#### **NNLO**sat

GDR positions reproduced

Total sum rule reproduced but poor strength distribution (Lack of correlations)

Dipole polarizability $\alpha_D$ (fm <sup>3</sup> )					
Nucleus	SCGF	CC/LIT	$\operatorname{Exp}$		
<sup>40</sup> Ca	1.79	$1.47 \ (1.87)_{thresh}$	1.87(3)		
<sup>48</sup> Ca	2.08	2.45	2.07(22)		

## Comparison with CC-LIT (Coupled Cluster- Lorentz Integral Transform method)

In collaboration with M. Miorelli and S. Bacca (TRIUMF, University of Mainz)



- · CC-Singles-Doubles (analogous to 2<sup>nd</sup> RPA)
- · LIT reduces a continuum state problem to a bound-state-like problem

Different treatment of the correlations:

#### SCGF

Reference state correlated RPA (first-order two-body correlator) **CC-SD-LIT** 

HF Reference state Singles-Doubles

### Role of the correlations included in the reference state



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Methods: Particle-Vibration coupling in the Self-consistent Green function formalism

#### Theoretical effective charges (as opposed to the ones extracted from experiment)

Our purpose is to calculate effective charges without resorting to any measurement of electromagnetic observables

Basic idea: calculate the core-polarization effect felt by the single-particle orbital of interest because of the energy-dependent effective potential, calculated at ADC(3) level



Effective charge as the ratio between the transition strengths (with and without the core-polarization) of a given multipole field:

$$\frac{\langle \tilde{\alpha} | \hat{\phi}^{(\lambda \mu_{\lambda})} | \tilde{\beta} \rangle}{\langle \alpha | \hat{\phi}^{(\lambda \mu_{\lambda})} | \beta \rangle} = 1 + \frac{\tilde{\Sigma}_{\alpha \beta}^{(\lambda \mu)}}{\langle \alpha | \hat{\phi}^{(\lambda \mu_{\lambda})} | \beta \rangle}$$

 $|\tilde{lpha}
angle\equiv$  s.p. state with correlations induced by the nuclear interaction and electromagnetic operator

Results: Theoretical effective charges of Oxygen and Nickel isotopes for E2 operator

# Features of the calculation

- Medium-mass isotopes:
  - Oxygen isotopes in *sd* and *psd* valence space: <sup>14</sup>O, <sup>16</sup>O, <sup>22</sup>O and <sup>24</sup>O
  - Nickel isotopes in  $0f1p0g_{9/2}$ : <sup>48</sup>Ni, <sup>56</sup>Ni, <sup>68</sup>Ni and <sup>78</sup>Ni
- NN and 3N nuclear interaction NNLOsat (Phys. Rev. C 91, 051301(R))
- Electric quadrupole operator E2  $\hat{\phi}^{(2\mu)} = \sum_{i} r_i^2 Y_{2\mu}(\hat{r}_i)$
- Dyson equation solved with self-energy truncated at ADC(3) level:



 Nuclear many-body wave function expanded in HO wave functions with N<sub>max</sub>=13 and hΩ=20 MeV

# Results for Oxygen isotopes



Standard values of e x p e r i m e n t a l effective charges in *psd* nuclei are  $e_p=1.3$ and  $e_n=0.5$ 

Neutron-rich nuclei have weaker core polarisation (quench of neutron effective charge)
Significant isotopic dependence especially for neutrons (compared with Bohr-Mottelson Eq. 6-386b with Sagawa parametrisation of PRC 70, 054316, 200 e\_{\pi}^{eff} = e + a\_{\overline{A}}^{Z} + b\_{\overline{A}}^{N-Z} - \left(c + d\_{\overline{A}}^{Z} \frac{N-Z}{A}\right))  $e_{\nu}^{eff} = a_{\overline{A}}^{Z} + b_{\overline{A}}^{N-Z} + \left(c + d_{\overline{A}}^{Z} \frac{N-Z}{A}\right)$ 

Single-particle state dependence also significant (yet to be studied and understood...)

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# **Conclusions and Perspectives**

- ADC(n) as a non-perturbative method for many-body physics
- Set of effective charges for Oxygen and Nickel isotopes
   calculated from realistic potential (ready to be used as input in Shell
   Model calculations)
- Expected isospin-dependence of neutron effective charges is found
- Dipole response and polarisability calculated from first principles
- Continuum to be included, but dipole polarisability seems quite insensitive to it
- Correlations: comparison with CC-LIT and extension of ADC to polarization propagator