Neutrino Mean Free Path in neutron matter from QMC equation of state

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Some Motivations

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- It is possible to use ab initio calculations for ground state properties, while for excited states in matter we still need to use mean field approximation.
- This work follows a previous work focused on asymmetry in the isospin channel.

Some references...

- Tamm-Dancoff response function and ν mean free path
  Cowell and Pandhariphande, *Phys. Rev. C* **70**, 035801;
  A. Lovato, O. Benhar, S. Gandolfi, C. Losa, *Phys. Rev. C 70*, 025804 (2014)
- TDLSDA for many electron systems:
  E. Lipparini and L. Serra, *Phys. rev. B* **57** R6830(R) (1998)
- This work:
  E. Lipparini and F. Pederiva, *Phys. Rev. C* **88**, *024318 (2013);* **94**, *024323 (2016)*
  E. Lipparini and F. Pederiva,

Overview

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Overview

Equation of State (EoS)
for Pure Neutron Matter (PNM) and
Spin Polarized Pure Neutron Matter (SPPNM)

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Energy-density functional

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Neutron matter

Neutron matter can be modeled as a periodic system of N neutrons interacting by a Hamiltonian of the form:

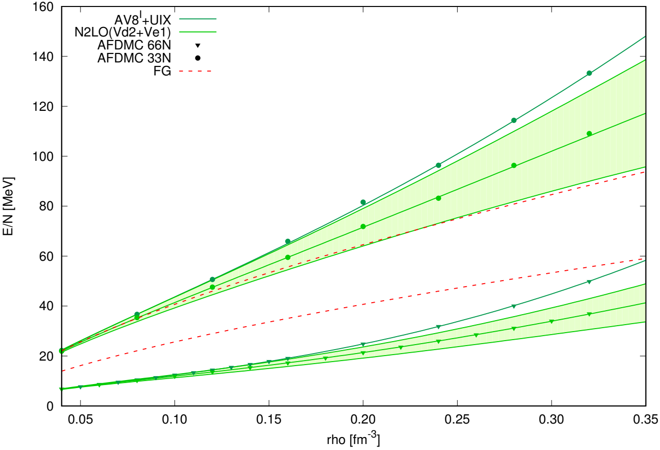
$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots \ .$$

In our calculations we used two kinds of potentials:

- phenomenological AV8'+UIX.
- chiral EFT N2LO local (D2,E1 and with R_0=R3N=1.0 fm) [Lynn et al. PRL 116, 062501 (2016)].

EoS computed by Auxiliary Field Diffusion Monte Carlo (AFDMC).

Neutron matter with AV8'+ UIX and N2LO



Uncertainties on χ-EFT (green bands) have been computed following Epelbaum et al. Eur. Phys. J. A, 51, 53 (2015).

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Energy density functional

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The interaction part of the EDF is assumed to be of the form:

$$\epsilon_V(\rho,\xi) = \epsilon_0(\rho) + \xi^2\left[\epsilon_1(\rho) - \epsilon_0(\rho)\right] \ ,$$

where:

$$\epsilon_q(\rho) = \epsilon_q^0 + a_q(\rho-\rho_0) + b_q(\rho-\rho_0)^2 + c_q(\rho-\rho_0)^3$$

where q=0,1 (spin polarization). The saturation density is *assumed* to be $\rho_0 = 0.16$ fm^{-3}.

This parametrization reproduces very well the AFDMC calculations in a wide range of density ρ (from $\rho_0/2$ to $3\rho_0$) and for both $\xi = 0, 1$.

General density excitations in nuclear matter

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We are interested in studying the density response of the system. For nucleons the response can be splitted in different channels, described by the following operators:

$$
\begin{aligned}
O_F &= \sum_i O_F(i) = \sum_i \tau^{\pm} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Fermi"} \\
\mathbf{O}_{GT} &= g_A \sum_i \mathbf{O}(i) = \sum_i \sigma_i \tau_i^{\pm} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Gamow-Teller"} \\
O_{NV} &= \sum_i O_{NV}(i) \quad \text{"Neutral-vector"} \\
&= \sum_i \left[-\sin^2\theta_W + \tfrac{1}{2}(1-2\sin^2\theta_W)\tau_i^z\right] e^{i\mathbf{q}\cdot\mathbf{r_i}} \\
\mathbf{O}_{NA} &= g_A \sum_i \mathbf{O}_{NA}(i) = g_A \sum_i \tfrac{1}{2}\tau_i^z \sigma_\mathbf{i} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Neutral-axial-vector"}
\end{aligned}
$$

Weinberg-Salam model

The relation between weak scattering processes and nuclear density response descends from the *Weinberg-Salam Lagrangian* coupling a nucleon of mass m with neutrinos through weak currents.

Weinberg-Salam model

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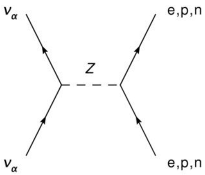
The relation between weak scattering processes and nuclear density response descends from the *Weinberg-Salam Lagrangian* coupling a nucleon of mass m with neutrinos through weak currents. E.g., for a lepton weak neutral current the coupling Lagrangian density would be:

$$\mathcal{L}_W = \frac{G_W}{\sqrt{2}} \bar{\psi}_\nu(x)\gamma_\mu(1-\gamma_5)\psi_\nu(x)\frac{1}{2}\bar{\psi}_n(x)\gamma^\mu(1-C_A\gamma_5)\psi_n$$



ν scattering rate

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The WS Lagrangian couples neutrinos to *density* and *spin density* fluctuations of neutrons.

In the **non-relativistic limit** the baryonic current can be approximated by:

$$\bar{\psi}_n(x)\gamma^\mu(1-C_A\gamma_5)\psi_n \sim \psi_n^\dagger(x)\psi_n(x)\delta_0^\mu - C_A\psi_n^\dagger(x)\sigma_i\psi_n(x)\delta_i^\mu .$$

We have two contributions: density fluctations, and spin-density fluctuations.

The scattering rate from a system of neutrons of a neutrino with 4-momentum $q^\mu \equiv (q^0, \vec{q})$ can be computed from the Fermi golden rule, averaging on the initial (neutron and/or proton) states and summing over all the final states.

ν scattering rate

The result gives the *neutrino scattering rate*. For a neutrino of incident energy E, the contribution to the scattering rate σ in a given channel can be writtten as:

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega),$$

where $S(q,\omega)$ is the *dynamical structure factor (DSF)* for the excitation operators describing the process. These in turn can be written as a combination of the DSF relative to density, and spin-density excitations.

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Time Dependent Local Density Approximation

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We use the Time Dependent Local Spin Density Approximation (TDLSDA) approach to compute the response function and the DSF.

We have worked out the response function in the transverse and longitudinal spin channels.

Following the *Kohn-Sham* method, we introduce a Local Spin Density Approximation (LSDA) for the homogeneous neutron matter defining the energy density functional as:

$$E(\rho, \xi) = T_0(\rho, \xi) + \int \epsilon_V(\rho, \xi)\ \rho\ d\mathbf{r},$$

and we applied the Hohenberg-Kohn theorem which provides a *variational principle on the energy-density functional*.

Response functions

For the longitudinal channel we can write the expression for $\chi^{n\uparrow}$ and $\chi^{n\downarrow}$, in terms χ_0 (longitudinal response function of the free Fermi gas). Summing and subtracting $\chi^{n\uparrow}$ and $\chi^{n\downarrow}$ we obtain the density-density (χ_s) and vector-density/vector-density (χ_v) response functions for arbitrary spin polarization.

A similar derivation can be done for the transverse channel. The LSDA-KS equations in the transverse channel include an effective vector potential accounting for the equilibrium spin polarization and one gets the response function χ_t in terms of the transverse response of the free Fermi gas $\chi_t^0(q,\omega)$.

Excitation strengths and sum rules

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- From the response function it is possible to determine the dynamic structure factor via the relation:

$$S^{s,v}(q,\omega) = -\frac{1}{\pi}\Im m[\chi^{s,v}]$$

- From the DSF it is also possible to compute the energy weighted sum rules:

$$m_k^{s,v} = \int_0^\infty d\omega\, \omega^k S^{s,v}(q,\omega) = \sum_n \omega_{no}^k |\langle 0|F^{s,v}|n\rangle|^2$$

  In particular the ratio m_{-1}/m_0 gives the *compressibility* of the system.

- The poles of $\chi(q,\omega)$ give the spectrum and the dispersion $\omega(q)$ of the collective excitations, for which we can also evaluate the strength.

Longitudinal response (AV8'+UIX)

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Neutron matter

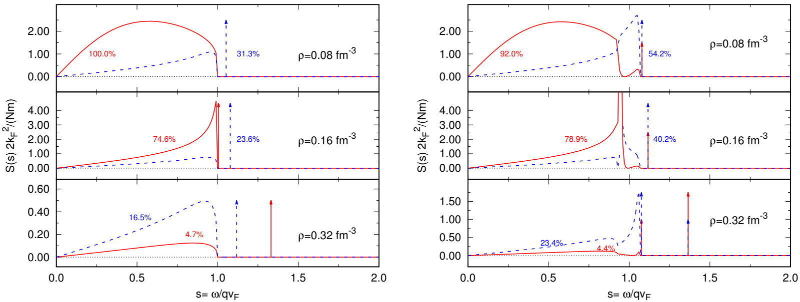
Derivation of the density functional

Response function in neutron matter and neutrino physics

Evaluation of the response function: TDLSDA

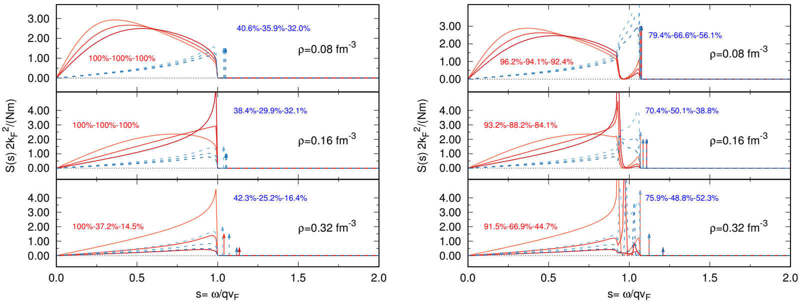
Numerical results

Conclusions



TDLSDA Dynamical structure factor for **density** (solid lines) and **spin density** (dashed lines) in the longitudinal channel. Left panel: results for PNM. Right panel: results for $\xi = 0.2$. Arrows indicate the location of the collective excitations. The percentages represent the fraction of the total strength pertinent to the particle-hole excitations.

Longitudinal response (χ-EFT)



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Transverse response

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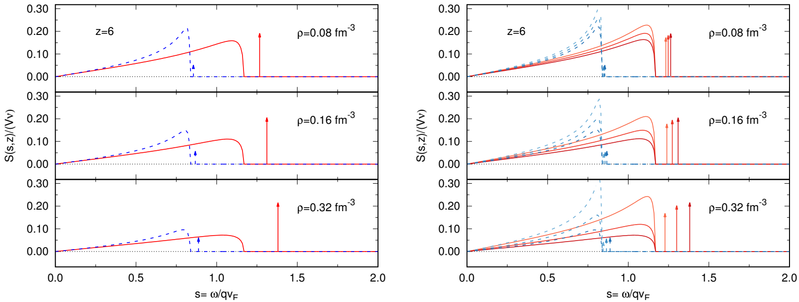
$$\nu = mk_F/(2\pi^2)$$



Excitation strengths for $z = 3q/(2k_F\xi) = 6$. The full and dashed lines indicate the particle/hole and collective strengths in the $\Delta S_z = -1$ ($s > 0$ - red) and $\Delta S_z = +1$ ($s < 0$ - blue) channels.

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Neutrino mean free path

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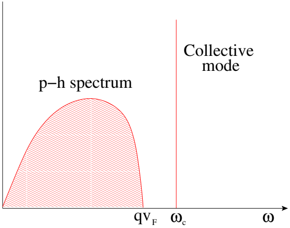
As previously discussed, the scattering rate of neutrinos can be obtained by computing the integral:

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega)$$

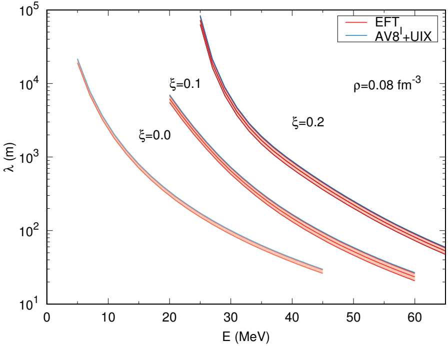
The neutrino mean free path λ is related to σ by the following relation:

$$\lambda = \frac{1}{\sigma\rho}$$



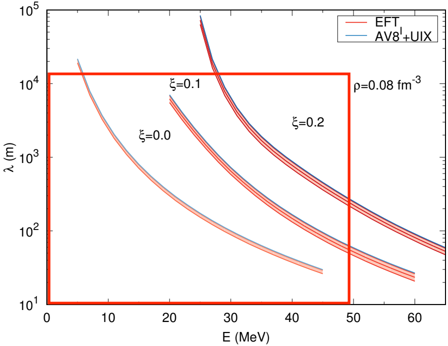
The integration has to be performed on the values of momentum kinematically accessible to neutrinos (kinematic limits).

Neutrino mean free path (transverse)



Neutrino mean free paths at different spin polarizations.

Neutrino mean free path (transverse)



Neutrino mean free paths at different spin polarizations.

Neutrino mean free path (transverse)

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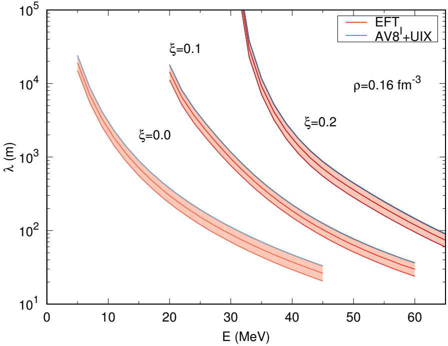

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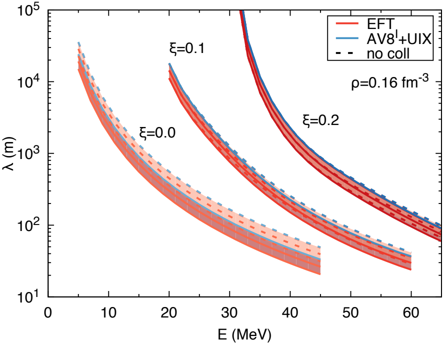
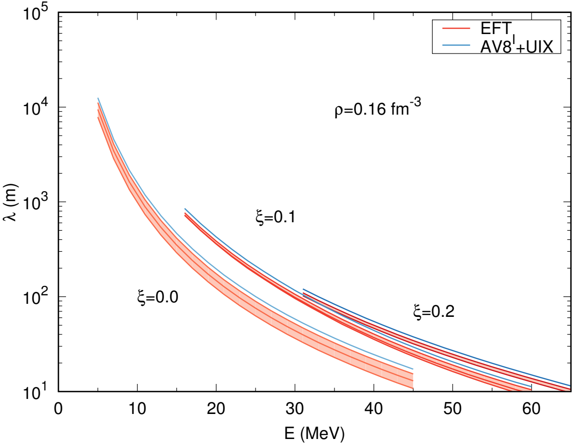
Neutrino mean free path (transverse)

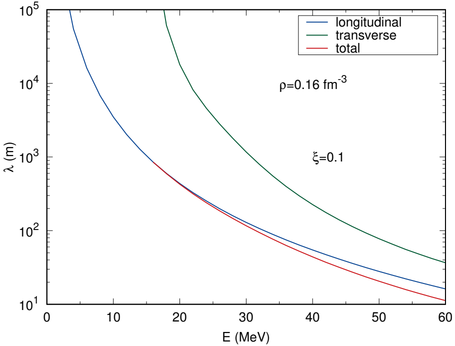


It is interesting to look at the contribution of the collective modes.

Neutrino mean free path (total)

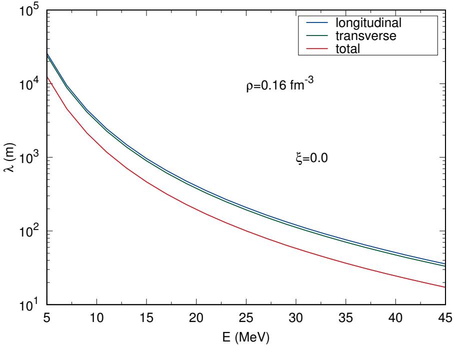


Neutrino mean free path (contribution)



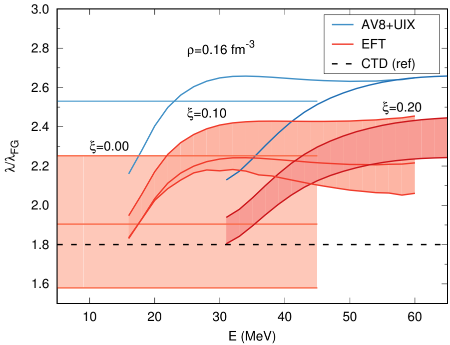
The dominant channel is the longitudinal channel, if the system is slightly spin polarized. The graph shows the results for the potential AV8′+UIX.

Neutrino mean free path (contribution)



In case of PNM, we have a similar contribution coming from both channels.

Neutrino mean free path (total)



Ratio of the NMFP in an interacting neutron matter and in a free Fermi gas at density $\rho/\rho_0 = 1$.

Conclusions

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- We computed the response function in the longitudinal and transverse channel in pure neutron matter, starting from accurate QMC calculations of (spin polarized) neutron matter.
- The time dependent local density approximation was successfully applied to estimate the response function of arbitrary spin polarized neutron matter.
- We computed the contribution of the longitudinal and transverse channels to the suppression of the neutrino mean free path in neutron matter. At the NS core conditions matter is essentially transparent, while relevant effects could be seen in the NS crust.

Magnetization estimates

- From observed B at the surface of magnetars $B \sim 10^{11}$ T.
- Maximum B allowed is $\lesssim 3 \cdot 10^{14}$ T.
  [Rhabi et al Phys. Rev. C (2015)]
- To get an estimate:

$$2m\Delta E = \mu_n B,$$

  where ΔE is separation energy between SPPNM and PNM.
- For $B \sim 10^{11}$ T we get a spin polarization $m \sim 10^{-4}$.

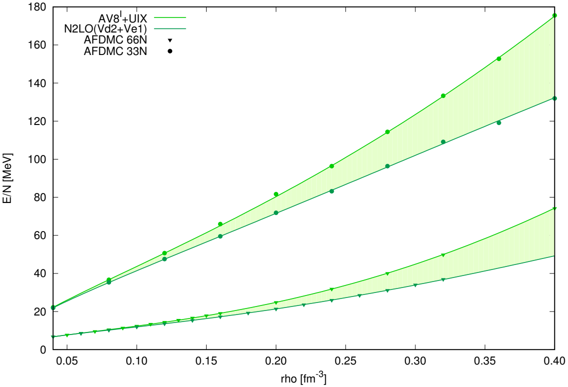
Procedure

- EoS computed by Auxiliary Field Diffusion Monte Carlo (AFDMC).
- Computations carried out with 33 and 66 neutrons for PNM and SPPNM respectively.
- Projection from a wavefunction of the form:

$$\psi_T(\mathbf{R}, S) = \phi_{\mathcal{S}}(\mathbf{R})\phi_{\mathcal{A}}(\mathbf{R}, S) ,$$

where the first factor is a (linearized) Jastrow factor including operators, and the second factor is a Slater determinant of plane waves (normal Fermi liquid).

Neutron matter with AV8'+ UIX and N2LO



AV8'+UIX PNM results from Gandolfi et al. Eur. Phys. J. A, 50(2) (2014), χ-EFT PNM results from Lynn et. al.

Argonne NN potential

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To describe the two body interaction we choose Argonne AV8' potential, which is not a simple truncation of AV18, but re-projection The potential can be written as a sum of operators in coordinate space:

$$O_{i,j}^{p=1,8} = (1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \mathbf{L}_{ij} \cdot \mathbf{S}_{ij}) \times (1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \ ,$$

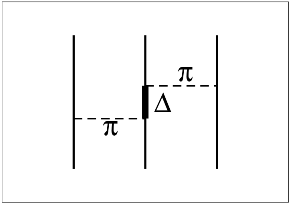
multiplied by radial functions $v_p(r_{ij})$, which depends the distance between the nucleons i and j.

Urbana IX Three-body force

The three nucleon potential is introduced to correct the limitations of only two nuclear forces and it plays a crucial role at densities higher that the saturation densities ($\rho_0 = 0.16$ fm^{-3}). The Urbana IX potential contains two terms:

$$V_{ijk} = V_{ijk}^{2\pi,P} + V_{ijk}^{R} \ ,$$

where the first term describes two pion exchange.



Variational Monte Carlo (VMC) - configurations

The EoS has been computed by means of Monte Carlo calculations.
The variational energy relative to some wavefunction $|\psi_T\rangle$ can be written as:

$$E_V = \frac{\langle\psi_T|\, H \,|\psi_T\rangle}{\langle\psi_T|\psi_T\rangle} \geq E_0 \ ,$$

where the integrals are computed by means of Metropolis Monte Carlo techniques.

The variatonal energy E_V gives an upper bound to the ground state energy E_0.

Wavefunctions

Trial wavefunction used in the Monte Carlo algorithm has the form:

$$\psi_T(\mathbf{R}, S) = \phi_{\mathcal{S}}(\mathbf{R})\phi_{\mathcal{A}}(\mathbf{R}, S) \ ,$$

where the first term is a Jastrow operatorial correlation function and can be written in terms of two and three body correlation, which depend on operators and some parameters (quenching factors and strength parameters).

The second term is usually written as a Slater determinant and we chose the ground state single-particle orbitals of a Fermi Gas in a periodic box.

Auxiliary Field Diffusion Monte Carlo (AFDMC)

This is an extension of Diffusion Monte Carlo, which projects out the ground state of a given Hamiltonian from a trial wavefunction.
The propagation is in imaginary time (in the limits $\Delta\tau \to 0$ and $\tau \to \infty$ the walkers have the GS distribution of the Hamiltonian).
It is engeneered to include quadratic spin and isospin operators (via HS transformation).
For the sign problem we used Fixed Phase Approximation (walkers have the same phase of the importance function ψ_I).

Time Dependent Local Density Approximation

We use the Time Dependent Local Spin Density Approximation (TDLSDA) approach to compute the response function and the DSF.

We have worked out the response function in the transverse and longitudinal spin channels.

TDLSDA (long)

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S. Gandolfi

Following the *Kohn-Sham* method, we introduce a Local Spin Density Approximation (LSDA) for the homogeneous neutron matter defining the energy functional as:

$$E(\rho, m) = T_0(\rho, m) + \int \epsilon_V(\rho, m)\ \rho\ d\mathbf{r}$$

where $T_0(\rho, m)$ is the kinetic energy of the *non interacting* system with density $\rho = \rho_{n_\uparrow} + \rho_{n_\downarrow}$, and spin polarization $m = \rho_1/\rho$ with $\rho_1 = \rho_{n_\uparrow} - \rho_{n_\downarrow}$.
Energy minimization with respect to the neutron up and neutron down densities gives the set of equations:

$$\left[-\frac{1}{2m}\nabla^2_{\mathbf{r}} + v(\mathbf{r}) + w(\mathbf{r})\eta_\sigma \right] \varphi_i^\sigma(\mathbf{r}) = \varepsilon_{i,\sigma}\, \varphi_i^\sigma(\mathbf{r}) \ ,$$

where $\eta^\sigma = \pm 1$ depending on the spin of the neutron.

TDLSDA (long)

The effective interactions $v(\mathbf{r})$ and $w(\mathbf{r})$ are related to the energy-density functional by:

$$v(\mathbf{r}) = \frac{\partial \left\{\rho\epsilon_V[\rho(\mathbf{r}), m(\mathbf{r})]\right\}}{\partial\rho(\mathbf{r})} \qquad w(\mathbf{r}) = \frac{\partial \left\{\rho\epsilon_V[\rho(\mathbf{r}), m(\mathbf{r})]\right\}}{\partial m(\mathbf{r})}$$

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The Hohenberg-Kohn theorem provides a *variational principle on the energy-density functional*.

TDLSDA (short)

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where $T_0(\rho, m)$ is the kinetic energy of the *non interacting* system with density $\rho = \rho_{n_\uparrow} + \rho_{n_\downarrow}$, and spin polarization $m = \rho_1/\rho$ with $\rho_1 = \rho_{n_\uparrow} - \rho_{n_\downarrow}$.

The Hohenberg-Kohn theorem provides a *variational principle on the energy-density functional*.

Longitudinal channel (short)

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Since we are interested in a homogeneous system, the linearized solutions of the KS-equations have the form (for both spin up and spin down) are :

$$\rho_n(\mathbf{r},t) = \rho_n + \delta\rho_n(\mathbf{r},t)\,,$$

where:

$$\delta\rho_n(\mathbf{r},t) = \delta\rho_n\left[e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} + e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\right].$$

The quantities $\delta\rho_n$ have to be determined from the KS equations. In order to determine $\delta\rho_{n_\uparrow}$ and $\delta\rho_{n_\downarrow}$, we insert $\rho_{n_\uparrow}(\mathbf{r},t)$,$\rho_{n_\downarrow}(\mathbf{r},t)$ in the KS equations, and linearize. After this procedure one obtains:

$$\lambda\chi^{n\uparrow}(q,\omega) = \lambda'_{n_\uparrow}\,\chi_0^{n_\uparrow}(q,\omega)$$
$$\lambda\chi^{n\downarrow}(q,\omega) = \lambda'_{n_\downarrow}\,\chi_0^{n_\downarrow}(q,\omega)$$

Longitudinal channel (long)

We seek for solutions for a homogeneous matter in presence of a (spin-)density excitation. For example, in the longitudinal channels:

$$\sum_{k=1}^{A} \lambda_\sigma^k \left(e^{i(\mathbf{q}\cdot\mathbf{r}_k - \omega t)} + e^{-i(\mathbf{q}\cdot\mathbf{r}_k - \omega t)} \right)$$

where λ is the excitation strength and $\lambda_\sigma^k = \lambda$ in the scalar channel and $\lambda_\sigma^k = \lambda\eta_\sigma$ in the vector channel. We can then write time dependent Kohn-Sham equations become:

$$\begin{aligned} i\frac{\partial}{\partial t}\varphi_i^\sigma(\mathbf{r},t) &= \Big\{-\frac{1}{2m}\nabla_\mathbf{r}^2 + v[\rho_{n_\uparrow}(\mathbf{r},t), \rho_{n_\downarrow}(\mathbf{r},t)] \\ &\quad + w[\rho_{n_\uparrow}(\mathbf{r},t), \rho_{n_\downarrow}(\mathbf{r},t)]\eta_\sigma + \lambda_\sigma \big[e^{i(\mathbf{q}\cdot\mathbf{r} - \omega t)} \\ &\quad + e^{-i(\mathbf{q}\cdot\mathbf{r} - \omega t)}\big]\Big\}\varphi_i^\sigma(\mathbf{r},t) \end{aligned}$$

Longitudinal channel (long)

This procedure determines the self-consistent KS mean-field potential entering the KS equations.
The KS equations in this case can be regarded as those of a system of non interacting particles in an oscillatory field with effective strengths:

$$\lambda'_{n_\uparrow} = \delta\rho_{n_\uparrow} V_{n_\uparrow,n_\uparrow} + \delta\rho_{n_\downarrow} V_{n_\uparrow,n_\downarrow} + \lambda$$
$$\lambda'_{n_\downarrow} = \delta\rho_{n_\uparrow} V_{n_\downarrow,n_\uparrow} + \delta\rho_{n_\downarrow} V_{n_\downarrow,pn_\downarrow} \pm \lambda$$

For such a system, the density response functions are the single-particle free responses $\chi_0^{n_\uparrow}(q,\omega)$, $\chi_0^{n_\downarrow}(q,\omega)$:

$$\chi_0^n(q,\omega) = \frac{V\delta\rho_n}{\lambda'_n}$$
$$\chi_0^p(q,\omega) = \frac{V\delta\rho_p}{\lambda'_p}$$

from which we obtain (Dyson-like equations):

$$\lambda\chi^n(q,\omega) = \lambda'_n\,\chi_0^n(q,\omega)$$
$$\lambda\chi^p(q,\omega) = \lambda'_p\,\chi_0^p(q,\omega)$$

Longitudinal channel

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The previous two equations, together with the definition of the linear response function, allow for solving for $\chi^{n_\uparrow}$ and $\chi^{n_\downarrow}$, given χ_0 (longitudinal response function of the free Fermi gas). Summing and subtracting $\chi^{n_\uparrow}$ and $\chi^{n_\downarrow}$ we obtain the density-density and vector-density/vector-density response functions for arbitrary spin polarization.

TDLSDA longitudinal response functions (scalar and vector)

$$\frac{\chi^s(q,\omega)}{V} = \frac{\frac{\chi_0^{n_\uparrow}}{V}[1-(V_{n_\downarrow n_\downarrow}-V_{n_\uparrow n_\downarrow})\frac{\chi_0^{n_\downarrow}}{V}]+\frac{\chi_0^{n_\downarrow}}{V}[1-(V_{n_\uparrow n_\uparrow}-V_{n_\downarrow n_\uparrow})\frac{\chi_0^{n_\uparrow}}{V}]}{(1-V_{n_\downarrow n_\downarrow}\frac{\chi_0^{n_\downarrow}}{V})(1-V_{n_\uparrow n_\uparrow}\frac{\chi_0^{n_\uparrow}}{V})-V_{n_\uparrow n_\downarrow}\frac{\chi_0^{n_\uparrow}}{V}V_{n_\downarrow n_\uparrow}\frac{\chi_0^{n_\downarrow}}{V}},$$

$$\frac{\chi^v(q,\omega)}{V} = \frac{\frac{\chi_0^{n_\uparrow}}{V}[1-(V_{n_\downarrow n_\downarrow}+V_{n_\uparrow n_\downarrow})\frac{\chi_0^{n_\downarrow}}{V}]+\frac{\chi_0^{n_\downarrow}}{V}[1-(V_{n_\uparrow n_\uparrow}+V_{n_\downarrow n_\uparrow})\frac{\chi_0^{n_\uparrow}}{V}]}{(1-V_{n_\downarrow n_\downarrow}\frac{\chi_0^{n_\downarrow}}{V})(1-V_{n_\uparrow n_\uparrow}\frac{\chi_0^{n_\uparrow}}{V})-V_{n_\uparrow n_\downarrow}\frac{\chi_0^{n_\uparrow}}{V}V_{n_\downarrow n_\uparrow}\frac{\chi_0^{n_\downarrow}}{V}}.$$

Excitation strengths: longitudinal channel (long)

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It is convenient to express the response function in term of the adimensional variable $s = \omega/qv_F$, where v_F is the Fermi velocity at a given density ρ. The longitudinal Fermi gas response function as a function of s in the low q limit, and for an arbitrary spin polarization reads:

$$\frac{\chi_0^{n_\uparrow,n_\downarrow}(\mathbf{q},\omega)}{V} = -\nu^{n,p}\left[1 + \frac{s}{2(1\pm m)^{1/3}}\ln\frac{s-(1\pm m)^{1/3})}{s+(1\pm m)^{1/3})}\right]$$

where $\nu^{n_\uparrow,n_\downarrow} = \mathrm{m}k_F^{n_\uparrow,n_\downarrow}/\pi^2 = \mathrm{m}k_F(1\pm m)^{1/3}/\pi^2$, $k_F = (3\pi^2\rho)^{1/3}$. The plus sign holds for $\chi_0^{n_\uparrow}$ and the minus sign for $\chi_0^{n_\downarrow}$.

Since $\chi^{s,v}$ depends on q and ω only via the dependence of $\chi_0^{n_\uparrow}$ and $\chi_0^{n_\downarrow}$ on these variables, also the response functions of the interacting system turn out to be functions of s only.

Transverse channel (short)

A similar derivation can be done for the transverse channel.
In this case the LSDA-KS equations are:

$$\left[-\frac{1}{2}\nabla^2_{\mathbf{r}} + \frac{1}{2}\omega_L \sigma_z + v(\mathbf{r}) + w(\mathbf{r})\sigma_z \right] \varphi_i^{\sigma}(\mathbf{r}) = \varepsilon_{i,\tau}\, \varphi_i^{\sigma}(\mathbf{r})$$

The second term in the l.h.s is an effective vector potential accounting for the equilibrium spin polarization (due to the presence of strong magnetic fields). The parameter ω_L can be related to spin imbalance by imposing that the variation of the LSDA energy with respect to m be zero.

Transverse channel (short)

The resulting interaction (single particle) Hamiltonian, when a density fluctuation is considered, becomes:

$$H_{\text{int}} \sim \sigma^- e^{i\mathbf{q}\cdot\mathbf{r}-\imath\omega t} + \sigma^+ e^{-i\mathbf{q}\cdot\mathbf{r}+\imath\omega t}.$$

The response function is defined as the difference of the respose relative σ^- and σ^+ and, eventually the final result for the response function is:

$$\frac{\chi_t(q,\omega)}{V} = \frac{\chi_t^0(q,\omega)}{1-\frac{2}{V}\mathcal{W}(\rho,m)\chi_t^0(q,\omega)},$$

where the $\chi_t^0(q,\omega)$ is the transverse response of the free Fermi gas.

Transverse channel (long)

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In the $\Delta S_z = \pm 1$ channel, for a spin polarization different from zero in the ground state the LSDA equations can be rewritten as:

$$\left[-\frac{1}{2}\nabla^2_{\mathbf{r}} + \frac{1}{2}\omega_C \sigma_z + v(\mathbf{r}) + \mathcal{W}(\rho, |\mathbf{m}|)\mathbf{m} \cdot \sigma \right] \varphi_i^\sigma(\mathbf{r}) = \varepsilon_{i,\tau}\, \varphi_i^\sigma(\mathbf{r}) \, .$$

Transverse response (long)

Introducing the adimensional quantities $s = \frac{\omega}{q v_F}$, $z = \frac{3q}{2k_F m}$, the free Fermi gas transverse response function reads:

$$\frac{\chi_t^0(q,\omega)}{V\nu} \equiv \frac{\chi_t^0(s,z)}{V\nu} = \Omega_\pm(s,z)\,, \qquad (1)$$

with

$$\nu = \mathrm{m} k_F/\pi^2,$$

$$\Omega_\pm(s,z) = -\left(1 + s/2 \ln \frac{s-1-1/z}{s+1-1/z}\right),$$

The interacting transverse response function becomes then:

$$\frac{\chi_t(q,\omega)}{V\nu} \equiv \frac{\chi_t(s,z)}{V\nu} = \frac{\Omega_\pm(s,z)}{1 - 2\nu\mathcal{W}(\rho,m)\Omega_\pm(s,z)}\,.$$

Neutrino mean free path

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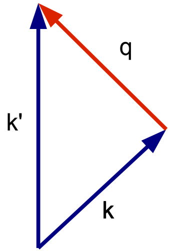
The integration must be performed on the values of momentum kinematically accessible to neutrinos.
Notation:

$$k^\mu = (k^0, \vec{k}) \quad k'^\mu = (k'^0, \vec{k}')$$

are the incoming and outgoing 4-momenta of the neutrino.

$$q^\mu = (\omega, \vec{q})$$

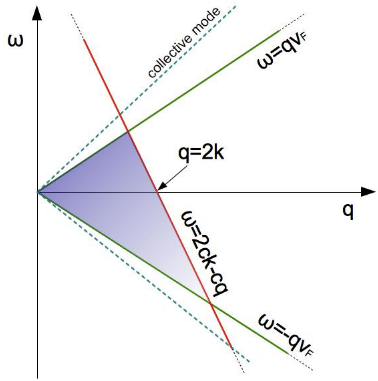
is the transferred 4-momentum.



Neutrinos are assumed to be *ultra-relativistic*.

Neutrino mean free path

Kinematic limits



The transferred momentum must satisfy the following inequality:

$$|\omega| < cq < |\omega - 2ck|$$

This implies that:

$$\omega < c(2k - q)$$

This represents the integration bound, that has to be intersected with the limits coming from $S(q, \omega)$.

This holds for non-degenerate neutrinos.