Theoretical and experimental constraints on the subleading $3N$ contact interaction

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work in progress

in collaboration with Alejandro Kievsky, Laura Elisa Marcucci and Michele Viviani (Pisa)

Outline

- what is the subleading $3N$ contact interaction
- why (motivation)
- when (at which order)
- how (numerics)

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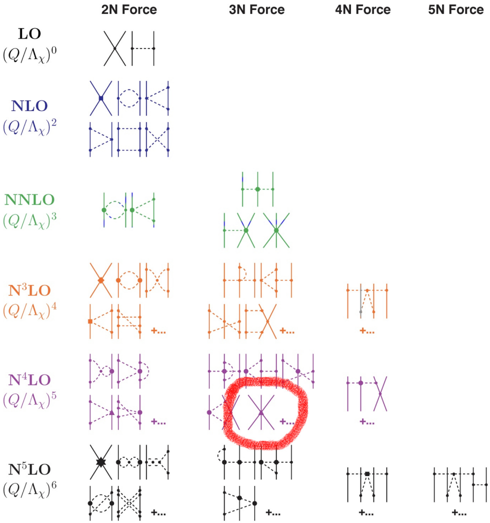
Recap of the present status

- proof of principle using the AV18

A few formal developments:

- “relativistic counting” in ChEFT
- large-N_c limit

What it is



(credit to Machleidt @CD15)

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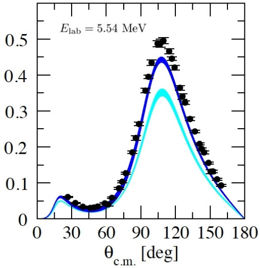

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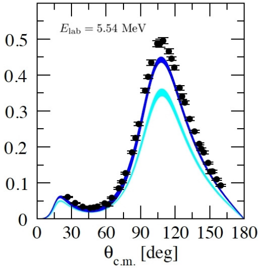

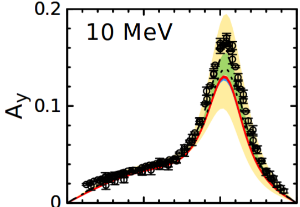

- For Nd, possibly affected by large uncertainty [LENPIC, PRC93 (2016) 044002]

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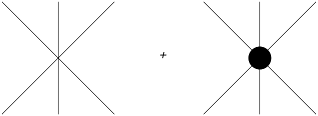

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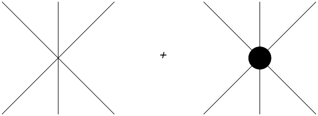

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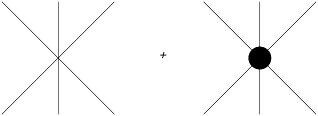

- consistency would require to consider them together with other pion-exchange 3NF at N4LO (and with a N4LO NN potential), or within $\not{\pi}$EFT
- nevertheless, contact LECs could have a prominent role, as in the case of electroweak nuclear observables
- why?

The unbearable heavyness of deuteron

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- chiral symmetry has little to say on the "unitary limit" $a \to \infty$

$$T \sim \frac{p^2}{\Lambda} \sim 20 \text{ MeV}, \quad V \sim \frac{M_\pi^3}{4\pi F_\pi^2} \sim \frac{p^3}{\Lambda F_\pi} \sim \frac{p^2}{\Lambda} \sim 20 \text{ MeV}$$

- emergence of a new light scale $\epsilon \sim 1/a$ is *unnatural*

$$\epsilon \sim T + V \sim 2 \text{ MeV} \ll 20 \text{ MeV}$$

- what are the consequences for the size of the LECs?

Naïve dimensional analysis

$$\mathcal{L} = \sum_{klm} c_{klm} A \left(\frac{\bar{N}N}{B}\right)^k \left(\frac{\partial^\mu, M_\pi}{C}\right)^l \left(\frac{\pi}{D}\right)^m , \quad c_{klm} \sim 1$$

The scale factors are uniquely fixed by the lowest order Lagrangian

$$\mathcal{L} = \bar{N}(i\partial\!\!\!/ - m_N)N + \frac{1}{2}\partial^\mu \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} - \frac{1}{2} M_\pi^2 \boldsymbol{\pi}^2 - \frac{g_A}{2F_\pi} \bar{N}\gamma^\mu \gamma_5 \partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\tau} N + \ldots$$

to be

$$\mathcal{L} = \sum_{klm} c_{klm} \Lambda^2 F_\pi^2 \left(\frac{\bar{N}N}{F_\pi^2 \Lambda}\right)^k \left(\frac{\partial^\mu, M_\pi}{\Lambda}\right)^l \left(\frac{\pi}{F_\pi}\right)^m$$

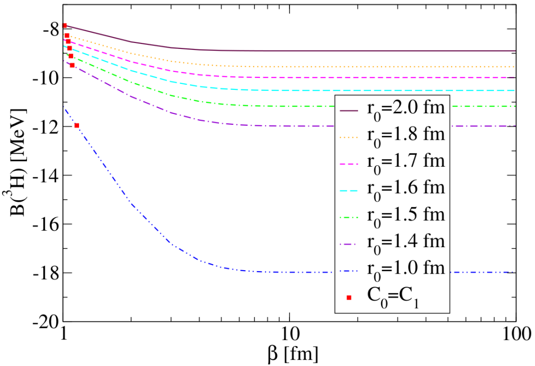
if a new scale is identified as ϵ, it must come from a further interaction

$$\Delta\mathcal{L} = -\frac{D_0}{2}(\bar{N}N)^2, \quad D_0 \sim \frac{4\pi a}{m_N} \sim \frac{4\pi}{m_N \epsilon} \sim \frac{1}{F_\pi \epsilon}$$

$$\Longrightarrow \mathcal{L} = \sum_{klm} c_{klm} \Lambda^2 F_\pi \epsilon \left(\frac{\bar{N}N}{F_\pi \Lambda \epsilon}\right)^k \left(\frac{\partial^\mu, M_\pi}{\Lambda}\right)^l \left(\frac{\pi}{F_\pi}\right)^m$$

Unitary limit in $\not\pi$EFT

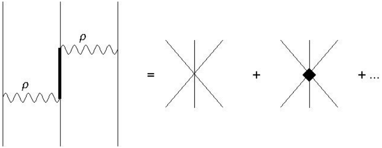
- a 3-body parameter is needed at LO to set a scale for the theory [Bedaque, Hammer, van Kolck, PRL 82 (1999) 463]
- the inclusion of OPEP doesn't change the picture



[Kievsky et al., PRC 95 (2017) 024001]

Relative promotion

- if the leading contact TNI gets promoted to LO, then also the subleading terms do the same to NLO



$\Longrightarrow$ classify all possible $3N$ contact operators involving 2 derivatives, respecting all discrete symmetries

Minimal subleading contact TNI

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\begin{aligned}
V &= \sum_{i\neq j\neq k} (E_1 + E_2 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
&\quad + (E_5 + E_6 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) S_{ij}\left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
&\quad + (E_7 + E_8 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)(\mathbf{L}\cdot\mathbf{S})_{ij}\frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
&\quad + (E_9 + E_{10} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}\boldsymbol{\sigma}_k\cdot\hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
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Spin-orbit terms suitable for the A_y puzzle [Kievsky PRC60 (1999) 034001]

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Isospin projection

- $N - d$ scattering only gives access to the $T = 1/2$ component of 3NF
- we can project each operator on isospin channels

$$o_i = P^{(1)}(o_i) + P^{(3)}(o_i) \equiv o_i P_{1/2} + o_i P_{3/2}$$

$$P_{1/2} = \frac{1}{2} - \frac{1}{6}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3), \quad P_{1/2} + P_{3/2} = 1$$

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- there is a single combination which is purely $T = 3/2$

$$o_{3/2} = 3o_1 - 2o_2 + 3o_5 + o_6 + 36o_7 + 12o_8 + 9o_9 + 3o_{10}$$

(up to cutoff effects ...)

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- we can exclude 1 LEC from the fits (e.g. E_8) and absorb its effect in the remaining LECS

Numerical implementation

The N-d scattering wave function is written as

$$\Psi_{LSJJ_z} = \Psi_C + \Psi_A$$

with Ψ_C expanded in the HH basis

$$|\Psi_C\rangle = \sum_{\mu} c_\mu |\Phi_\mu\rangle$$

and Ψ_A describing the asymptotic relative motion

$$\Psi_A \sim \Omega^R_{LS}(k,r) + \sum_{L'S'} R_{LS,L'S'}(k)\Omega^I_{L'S'}(k,r)$$

with the unknown c_μ and R-matrix elements (related to the S-matrix) to be determined so that the Kohn functional is stationary

$$[R_{LS,L'S'}] = R_{LS,L'S'} - \langle \Psi_C + \Psi_A | H - E | \Psi_C + \Psi_A \rangle$$

imposing the Kohn functional to be stationary leads to a *linear* system

$$\sum_{L''S''} R_{LS,L''S''} X_{L'S',L''S''} = Y_{LS,L'S'}$$

with the matrices

$$X_{LS,L'S'} = \langle \Omega^I_{LS} + \Psi^I_C | H - E | \Omega^I_{L'S'} \rangle \quad Y_{LS,L'S'} = -\langle \Omega^R_{LS} + \Psi^R_C | H - E | \Omega^I_{L'S'} \rangle$$

and the $\Psi_C^{R/I}$ solutions of

$$\sum_{\mu'} c_\mu \langle \Phi_\mu | H - E | \Phi_{\mu'} \rangle = -D^{R/I}_{LS}(\mu)$$

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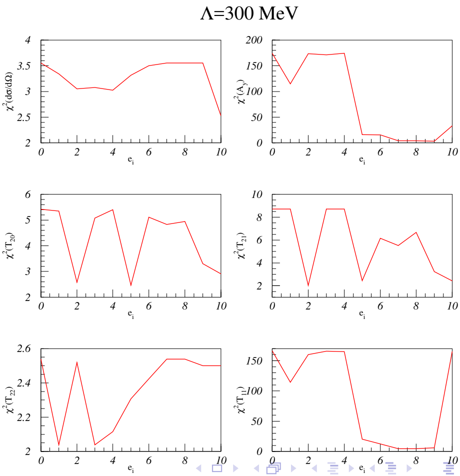
11 set of matrices are calculated once for all, and only linear systems are solved for each choice of E_i's

Results with AV18

we have 11 LECs, $E = \frac{c_E}{F_\pi^4 \Lambda}$ (LO) and $E_{i=1,\ldots,10} = \frac{e_i^{NN}}{F_\pi^4 \Lambda^3}$ (NLO) to be fitted to $B(^3H)$, $^2a_{nd}$, $^4a_{nd}$ and the p-d phaseshifts for different values of Λ

- χ^2 from 2-parameter fit with (c_E, e_i)
- strong sensitivity of A_y and iT_{11} to E_7, E_8 and E_9
- all fits are performed with POUNDerS algorithm [T. Munson et al. @ ANL]

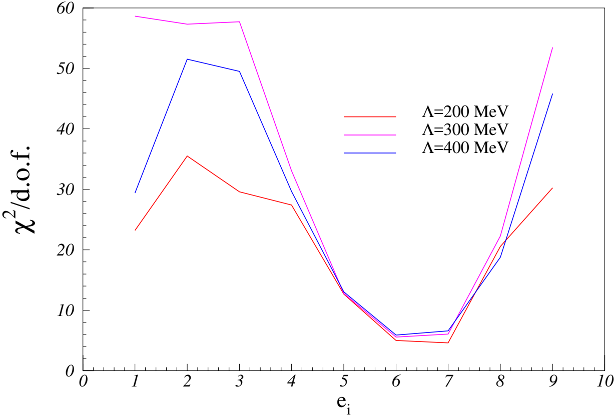


3-parameter fits

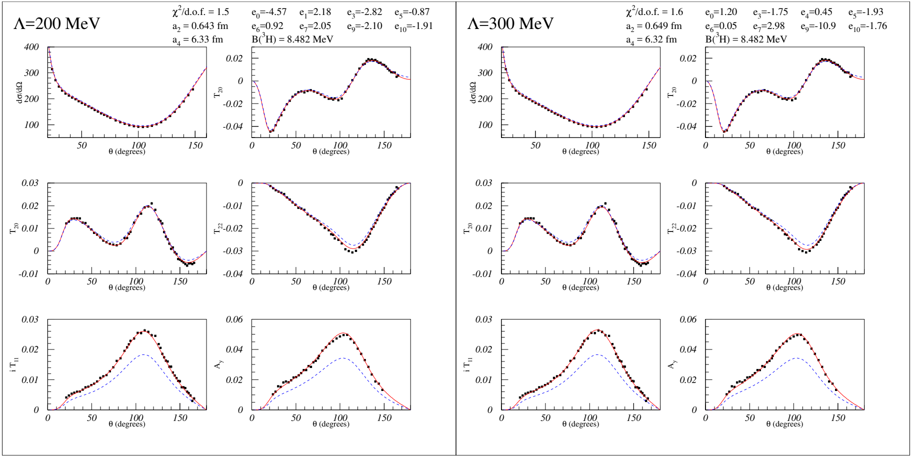
- use c_E and E_3 to account for $B(^3\mathrm{H})$ and $^2a_{nd}$

3-parameter fits

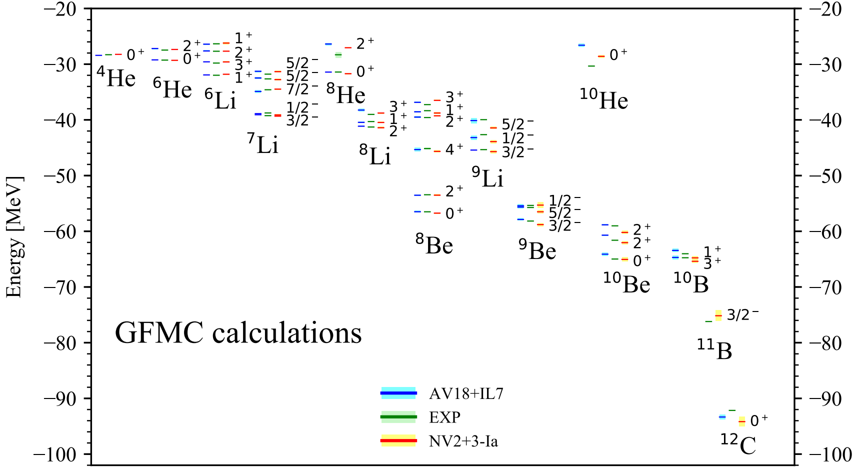
- use c_E and E_3 to account for $B(^3\text{H})$ and $^2a_{nd}$
- use another one of the E_i to fit scattering observables at 3 MeV



- the χ^2 decreases as the number of parameters increases until 7 (correlations?) The best results show $\chi^2/\mathrm{d.o.f.} = 1.5 - 1.6$



next step: use with the Norfolk potentials



M. Piarulli et al. arXiv:1707.02883

Subleading contact terms from “relativistic counting”

A new power-counting scheme for the derivation of relativistic chiral nucleon-nucleon interactions

Xiu-Lei Ren,[1] Kai-Wen Li,[2] Li-Sheng Geng,[2,3,*] Bingwei Long,[4] Peter Ring,[5,1] and Jie Meng[1,2,†]
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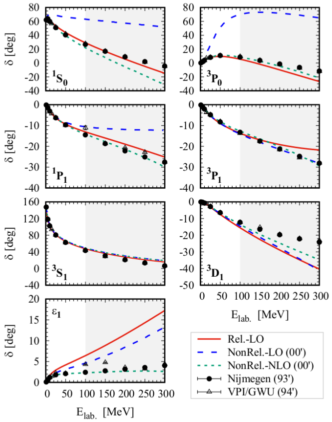
[2] *School of Physics and Nuclear Energy Engineering & International Research Center for Nuclei and Particles in the Cosmos, Beihang University, Beijing 100191, China*
[3] *Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*
[4] *Center for Theoretical Physics, Department of Physics, Sichuan University, Chengdu, Sichuan 610064, China*
[5] *Physik Department, Technische Universität München, D-85748 Garching, Germany*
(Dated: February 6, 2017)

Motivated by the successes of relativistic theories in studies of atomic/molecular and nuclear systems and the strong need for a covariant chiral force in relativistic nuclear structure studies, we develop a new covariant scheme to construct the nucleon-nucleon interaction in the framework of chiral effective field theory. The chiral interaction is formulated up to leading order with a covariant power counting and a Lorentz invariant chiral Lagrangian. We find that the covariant scheme induces all the six invariant spin operators needed to describe the nuclear force, which are also helpful to achieve cutoff independence for certain partial waves. A detailed investigation of the partial wave potentials shows a better description of the scattering phase shifts with low angular momenta than the leading order Weinberg approach. Particularly, the description of the $^1S_0$, $^3P_0$, and $^1P_1$ partial waves is similar to that of the next-to-leading order Weinberg approach. Our study shows that the relativistic framework presents a more efficient formulation of the chiral nuclear force.

PACS numbers: 13.75.Cs,21.30.-x

$$
\begin{aligned}
\mathcal{L}_{NN}^{(0)} &= \frac{1}{2}\Big[C_S(\bar\Psi\Psi)(\bar\Psi\Psi) + C_A(\bar\Psi\gamma_5\Psi)(\bar\Psi\gamma_5\Psi) \\
&+ C_V(\bar\Psi\gamma_\mu\Psi)(\bar\Psi\gamma^\mu\Psi) + C_{AV}(\bar\Psi\gamma_\mu\gamma_5\Psi)(\bar\Psi\gamma^\mu\gamma_5\Psi) \\
&+ C_T(\bar\Psi\sigma_{\mu\nu}\Psi)(\bar\Psi\sigma^{\mu\nu}\Psi)\Big],
\end{aligned}
$$

“relativistic corrections are in the data”



Relativistic counting applied to contact TNI

There are 25 $C-$, $P-$ and $T-$ relativistic invariant operators

$$
\begin{aligned}
o_{1,2} &= (\bar\psi\psi)_1(\bar\psi\psi)_2(\bar\psi\psi)_3[\mathbf{1}, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2] \\
o_{3,4,5} &= (\bar\psi\psi)_1(\bar\psi\gamma_5\psi)_2(\bar\psi\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{6,7,8} &= (\bar\psi\psi)_1(\bar\psi\gamma^\mu\psi)_2(\bar\psi\gamma_\mu\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{9,10,11} &= (\bar\psi\psi)_1(\bar\psi\gamma^\mu\gamma_5\psi)_2(\bar\psi\gamma_\mu\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{12,13,14} &= (\bar\psi\psi)_1(\bar\psi\sigma^{\mu\nu}\psi)_2(\bar\psi\sigma_{\mu\nu}\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{15} &= (\bar\psi\gamma_5\psi)_1(\bar\psi\gamma^\mu\psi)_2(\bar\psi\gamma_\mu\gamma_5\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{16} &= (\bar\psi\sigma^{\mu\nu}\psi)_1(\bar\psi\gamma_\mu\psi)_2(\bar\psi\gamma_\nu\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{17} &= (\bar\psi\sigma^{\mu\nu}\psi)_1(\bar\psi\gamma_\mu\gamma_5\psi)_2(\bar\psi\gamma_\nu\gamma_5\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{18} &= (\bar\psi\sigma^{\mu\nu}\psi)_1(\bar\psi\sigma_{\mu\alpha}\psi)_2(\bar\psi\sigma^\alpha_\nu\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{19,20,21} &= (\bar\psi\gamma_5\psi)_1(\bar\psi\sigma^{\mu\nu}\psi)_2(\bar\psi\sigma_{\mu\nu}\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{22,23,24,25} &= (\bar\psi\gamma_\mu\psi)_1(\bar\psi\gamma_\nu\gamma_5\psi)_2(\bar\psi\sigma^{\mu\nu}\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3), \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2-\boldsymbol{\tau}_3)]
\end{aligned}
$$

After deriving all sort of Fierz identities like

$$(\sigma^{\mu\alpha})[\sigma_\alpha{}^\nu] - \mu \leftrightarrow \nu = i(\sigma^{\mu\nu}][) - i(][\sigma^{\mu\nu}) + i(\sigma^{\mu\nu}\gamma_5][\gamma_5) - i(\gamma_5][\sigma^{\mu\nu}\gamma_5)$$

using the 3×25 linear relations we are left with 5 operators

$$o_1, \quad o_3, \quad o_6, \quad o_9, \quad o_{12}$$

$\Longrightarrow$ test the relativistic counting by including only 5 combinations of the 10 LECs

Insight from the large-N_c limit

- initially proposed by 't Hooft in 1974, to define a *weak coupling* limit of QCD, $g^2 N_c$ =const giving rise to substantial simplifications over QCD, but with similar physical properties
- a topological expansion emerges in which only *planar diagrams* survive, and no dynamical quark loops
- extended to baryons by Witten in 1979
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- as a result, one finds e.g.

$$\mathbf{1} \sim \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim O(N_c)$$

while

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \sim \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim O(1/N_c)$$

Large-N_c and Pauli principle

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$$\mathcal{L} = c_1 N^\dagger N N^\dagger N + c_2 N^\dagger \sigma_i N N^\dagger \sigma_i N + c_3 N^\dagger \tau^a N N^\dagger \tau^a N + c_4 N^\dagger \sigma_i \tau^a N N^\dagger \sigma_i \tau^a N \equiv \sum_i c_i o_i$$

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reobtaining the well-established fact that $C_S >> C_T$

3NF and large-N_c

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[D.R.Phillips and C.Schat, PRC88 (2013) 034002]

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$$\begin{aligned}\mathcal{L} \equiv & -\sum_i^6 E_i O_i = -E_1 N^\dagger N N^\dagger N N^\dagger N - E_2 N^\dagger \sigma^i N N^\dagger \sigma^i N N^\dagger N \\ & -E_3 N^\dagger \tau^a N N^\dagger \tau^a N N^\dagger N - E_4 N^\dagger \sigma^i \tau^a N N^\dagger \sigma^i \tau^a N N^\dagger N \\ & -E_5 N^\dagger \sigma^i N N^\dagger \sigma^i \tau^a N N^\dagger \tau^a N - E_6 \epsilon^{ijk} \epsilon^{abc} N^\dagger \sigma^i \tau^a N N^\dagger \sigma^j \tau^b N N^\dagger \sigma^k \tau^c N\end{aligned}$$

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- only E_1, E_4 and E_6 are $O(N_c)$
- but since the 6 operators are all proportional, the LEC associated to any choice will be $\sim O(N_c)$
- operators with different scaling properties in $1/N_c$ get mixed

Large-N_c constraints on subleading 3N contact interaction

- applying Phillips and Schat counting to our redundant operators we get 13 leading structures
- using Fierz identities we find 4 vanishing LECs in the large-N_c limit

$$E_2 = E_3 = E_5 = E_9 = 0$$

thus reducing the number of subleading LECs to 6 but...

Is large-N_c at work in NN scattering?

- at leading order $C_S \gg C_T$
- at subleading order $C_1, C_4, C_6 \gg$ others
- at N2LO $D_1, D_4, D_6 \gg$ others

Maria's fit to observables up to $E = 10$ MeV, $\Lambda = 200$ MeV, $\chi^2 \sim 1.8$

LO (fm^2)	N2LO (fm^6)
$C_S = -4.525$	$D_1 = -2.136$
$C_T = 0.166$	$D_2 = -0.276$
NLO (fm^4)	$D_3 = 0.011$
$C_1 = -3.824$	$D_4 = 0.326$
$C_2 = -0.483$	$D_5 = 0.430$
$C_3 = -0.099$	$D_6 = 0.101$
$C_4 = -1.189$	$D_7 = -0.696$
$C_5 = 0.009$	$D_8 = 0.041$
$C_6 = -1.098$	$D_9 = 1.675$
$C_7 = -1.054$	$D_{10} = -2.494$
	$D_{11} = -0.076$
	$D_{12} = 0.381$
	$D_{13} = -0.425$
	$D_{14} = 0.110$
	$D_{15} = -0.134$

prediction satisfied *but for spin-orbit operators*

Evidence for large-N_c violation in the vacuum channel?

- as is well known, spin-orbit couplings are generated by scalar-isoscalar exchange (e.g. σ)
- large-N_c violation *is observed* in the 0^+ channel in the meson sector, in the form of OZI rule violation

$$R_{32} = \frac{\langle \bar{u}u \rangle_{(m_u=m_d=m_s=0)}}{\langle \bar{u}u \rangle_{(m_u=m_d=0;m_s\neq 0)}} = 1 - 0.54 \pm 0.27$$

[Moussallam, EPJC 14 (2000) 111]

- this is possibly related to a proximity of a chiral phase transition, as a function of the number of light quark flavours N_f
- it would be wonderful if nuclear physics would reveal such subtle properties of the QCD vacuuum!