The chiral magnetic effect in planar condensed matter systems

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Huitzil collaboration with Cristian Villavicencio, Alfredo Raya, and Saul Hernández-Ortiz.

Overview

Particle physics in table top experiments?

Electronic structure of graphene

Quantum electrodynamics in (2+1) dimensions

The pseudo-CME

Conclusions

Particle physics in table top experiments?

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LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

Quantum electrodynamics (resulting from the merger of quantum mechanics and relativity theory) has provided a clear understanding of phenomena ranging from particle physics to cosmology and from astrophysics to quantum chemistry[1–3]. The ideas underlying quantum electrodynamics also influence the theory of condensed matter[4,5], but quantum relativistic effects are usually minute in the known experimental systems that can be described accurately

behaviour shows that substantial concentrations of electrons (holes) are induced by positive (negative) gate voltages. Away from the transition region $V_g \approx 0$, Hall coefficient $R_H = 1/ne$ varies as $1/V_g$, where n is the concentration of electrons or holes and e is the electron charge. The linear dependence $1/R_H \propto V_g$ yields $n = \alpha V_g$ with $\alpha \approx 7.3 \times 10^{10}\,\mathrm{cm}^{-2}\mathrm{V}^{-1}$, in agreement with the theoretical estimate $n/V_g \approx 7.2 \times 10^{10}\,\mathrm{cm}^{-2}\mathrm{V}^{-1}$ for the surface charge density

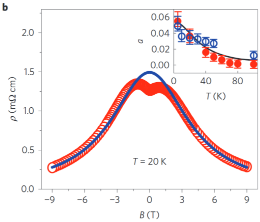
Particle physics in table top experiments?

- Graphene (2+1)D: Klein paradox
- Dirac materials (3+1)D: Chiral magnetic effect

$$J_{CME} = \frac{e^2}{2\pi^2}\mu_5 B, \qquad \mu_5 \sim E \cdot B$$

$$J_{CME} \equiv \sigma^{ik}_{CME} E^k, \sigma^{zz}_{CME} \sim B^2$$

- The magnetoresistence in $ZrTe_5$ when a magnetic field is applied parallel to an electric field is in accordance with the predictions that take into account the CME.



Li et al, Nature Phys. 12, 550 (2016).

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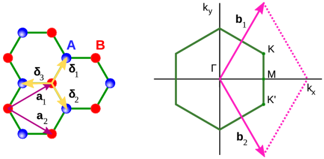
Graphene Physics



$$\mathbf{a}_1 = \frac{a}{2}\left(3, \sqrt{3}\right), \mathbf{a}_2 = \frac{a}{2}\left(3, -\sqrt{3}\right)$$

$$\mathbf{b}_1 = \frac{2\pi}{3a}\left(1, \sqrt{3}\right), \mathbf{b}_2 = \frac{2\pi}{3a}\left(1, -\sqrt{3}\right)$$

$$\delta_1 = \frac{a}{2}\left(1, \sqrt{3}\right), \delta_2 = \frac{a}{2}\left(1, -\sqrt{3}\right), \delta_3 = a(-1, 0)$$

$$\mathbf{K}_1 = \left(\frac{2\pi}{3a}, \frac{2\pi}{3\sqrt{3}a}\right), \mathbf{K}_2 = \left(\frac{2\pi}{3a}, -\frac{2\pi}{3\sqrt{3}a}\right)$$

- A single layer of carbon atoms arranged in a honeycomb lattice structure.
- Represented in terms of two triangular sublattices.
- Hexagonal reciprocal lattice.

- Tight-binding approach: nearest neighboors.
- Hopping only between sublattices.

$$H(\vec{k}) = \begin{pmatrix} 0 & tS(\vec{k}) \\ tS^*(\vec{k}) & 0 \end{pmatrix}$$

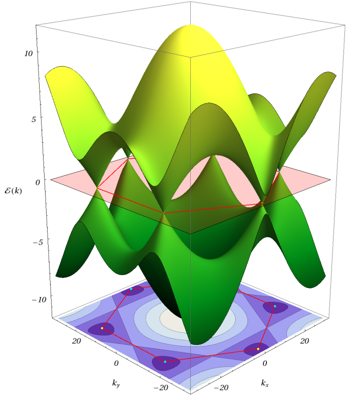
where

$$S(\vec{k}) = \sum_{\vec{\delta}} e^{i\vec{k}\cdot\vec{\delta}}$$

- The energy is:

$$E(k_x, k_y) = \pm t\sqrt{3 + 2\cos\left(\sqrt{3}k_y a\right) + 4\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\cos\left(\frac{3}{2}k_x a\right)}.$$

Zero energy around the K points.



- Linear dispersion relation: $\mathcal{H} = \bar{\psi}\, \hbar v_F \boldsymbol{\gamma} \cdot \boldsymbol{k}\ \psi$.
- Dirac points: valence and conduction band touch generating no gap.

Figure extracted from http://oer.physics.manchester.ac.uk/AQM2/Notes/Notes-6.4.html

$$\psi(\boldsymbol{k}) = \begin{pmatrix} \psi_\uparrow(\boldsymbol{K}_+ + \boldsymbol{k}) \\ \psi_\downarrow(\boldsymbol{K}_+ + \boldsymbol{k}) \\ \psi_\uparrow(\boldsymbol{K}_- + \boldsymbol{k}) \\ \psi_\downarrow(\boldsymbol{K}_- + \boldsymbol{k}) \end{pmatrix}$$

Expanding around de K and K' points:

$$H_{K'}(\vec{q}) \approx \frac{3at}{2}\begin{pmatrix} 0 & \alpha(q_x+iq_y) \\ \alpha^*(q_x-iq_y) & 0 \end{pmatrix}, \qquad H_K = H^*_{K'}.$$

$$H_K = -i\hbar v_f \vec{\sigma}\nabla, \qquad H' = H_K^T.$$

Considering the 4-component spinor:

$$H = \begin{pmatrix} H_K & 0 \\ 0 & H'_K \end{pmatrix}, \qquad H_K = H^*_{K'}, \qquad H = -i\hbar v \tau_0 \otimes \vec{\sigma}\nabla.$$

The interaction with an electromagnetic field is done through the Peierls substitution:

$$a^\dagger_{n,\sigma} b_{m,\sigma} \rightarrow a^\dagger_{n,\sigma} exp\left(\frac{-ie}{\hbar c}\int_m^n \mathbf{A} d\mathbf{r}\right) b_{m,\sigma}.$$

In term of a Lagrangian:

$$\mathcal{L} = \sum_{\sigma=\pm} \bar{\psi}_\sigma(t,\mathbf{r})\left[i\gamma^0(\hbar\partial_t - i\mu_\sigma) + i\hbar v_f\gamma^1 D_x + i\hbar v_f \gamma^2 D_y\right]\psi_\sigma(t,\mathbf{r}).$$

QED in 2+1 dimensions describes graphene

- Low energy excitations in graphene are massless quasiparticles with linear dispersion relation.
- The eigenfunctions of the low energy quasiparticle excitations obey the Dirac equation. It presents a spinor structure as a consequence of the underlying lattice.
- The interactions of the quasiparticles with external electromagnetic fields is introduced using the minimal coupling prescription of quantum field theory.

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QED in (2+1)D

- QED3 fermion sector ($\hbar$=c=1):

$$\mathcal{L} = \bar{\psi}[\gamma_0(i\partial_0 + \hat{\mu}) - i(\boldsymbol{\gamma}_1 D_x + \gamma_2 D_y) - \hat{M}]\psi.$$

- $\hat{M}$ are the Dirac masses or interactions that can result in a gap in the energy bands - deformations, substrates, doping, etc.
- $\hat{\mu}$ is a generalized chemical potential including spin interaction (Zeeman term), $\mu_\sigma = \mu - \frac{\sigma g}{2\mu_B} B$.

QED in (2+1)D

- QED_3 gauge sector:

$$\begin{aligned}\mathcal{L}_{\text{QED}_3} &= \bar{\psi}[\partial\!\!\!/ - qA\!\!\!/ + m_o\gamma_3\gamma_5]\psi \\ &\quad -\frac{1}{2\xi}(\partial\cdot A)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\theta}{4}\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho .\end{aligned}$$

- Haldane mass can be dynamically generated by the CS term.
- Parity-odd mass term in the fermion propagator radiatively induces a parity-odd contribution into the vacuum polarization tensor at one-loop level.
- Coleman-Hill theorem shows that there are no contribution from two- and higher-loops.

Reduced QED

[Gorbar, Guysinin, Miranski, PRD 64, 105028 (2001).]

- The gauge sector is not constrained to the plane.
- Coulomb rather than logarithmic interaction.
- Reduced QED: general (3+1) theory dimensionally reduced to a non-local effective (2+1) theory.

$$S = \int d^D X \left(\frac{1}{4e^2} F_{ab}^2 + A_a J^a - \frac{1}{2e^2\xi} \left(\partial_a A^a\right)^2 \right)$$

- $D = 4 \rightarrow$ Integrating over the gauge field and the third spatial dimension.
- Keeping $J^3 = 0$.
- Adding the fermion fields in (2+1)D.

$$S = \int d^3 x \left[\bar{\psi} \left(i \not{D} + m \right) \psi + \frac{1}{2} F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F^{\mu\nu} + \frac{1}{e^2\xi} \partial_\mu A^\mu \frac{1}{\sqrt{-\partial^2}} \partial_\nu A^\nu \right].$$

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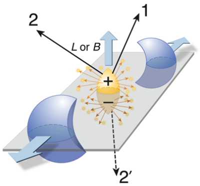


$eB \approx 10^{19} G$

Imbalance of chirality + strong magnetic field

$\Downarrow$

Electrical current in the direction of the field

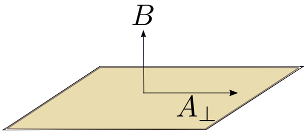
- Effective description in terms of a chiral chemical potential:

Fukushima, Kharzeev and Warringa, PRD 85, 045104 (2008).

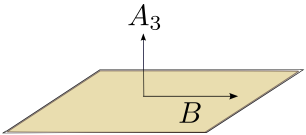
$$j_z \sim \sum_f q_f^2 B\mu_5 \quad \text{Independent of temperature and mass}$$

The pseudo-Chiral magnetic effect

Quantum Hall effect:
magnetic field perpendicular
to the plane.

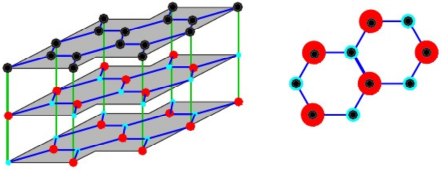


Pseudo-CME:
in plane magnetic field.



- External classical field.
- Does not contribute to quantum corrections.
- Usual Zeeman term: effective way to consider the non-relativistic approximation for the Dirac equation with a magnetic background. Equivalent to consider a non-vanishing third component of the gauge field.

- Parity breaking "mass": $M = m_3\gamma_3 + m_o\gamma_3\gamma_5$.
- Place the graphene on a Boron Nitride substrate - m_3:



- PCME Lagrangian [AJM, C. Villavicencio, A. Raya, IJMP B30, 1550257 (2015)]:

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ + \mu\gamma^0 + (eA_3^{\text{ext}} - m_3)\gamma^3 - m_o\gamma^3\gamma^5]\psi.$$

- In the chiral basis, we define: $\psi_\pm \equiv \frac{1}{2}(1 \pm \gamma^5)\psi$ and $m_\pm = m_3 \pm m_0$:

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\psi}_\chi \left[i\not\partial + \mu\gamma^0 + \left(eA_3^{ext} - m_\chi \right) \right] \psi_\chi.$$

- The Green function can be written in terms of the two chiralities:

$$G(x,x') = \frac{1}{2}(1+\gamma^5)G_+(x,x') + \frac{1}{2}(1-\gamma^5)G_-(x,x').$$

where:

$$G_\pm(r,r') = \left\langle r \mid \frac{1}{\not\Pi + (eA_3 - m_\pm)\gamma_3} \mid r' \right\rangle$$

- Using the Schwinger proper time method:

$$\tilde{G}(k;\xi) = \int_{-\infty}^{\infty} ds\, r_s(k^0, K_\parallel^0)\, e^{isK_\parallel{}^2 - i\left[k_2{}^2+\xi^2\right]\tan(eBs)/eB} \left\{ \not{K}_\parallel \left[1+\gamma^2\gamma^3\tan(eBs)\right] + \left[k_2\gamma^2+\xi\gamma^3\right]\sec^2(eBs)\right\},$$

$$K_\parallel = (k_0+\mu, k_1, 0), \qquad \xi = eB(x_2+x_2')/2 + m_\pm .$$

- We calculate the currents at finite temperature:

$$J_\mu(x) = -e\langle\bar{\Psi}\gamma_\mu\Psi\rangle, \qquad J_{\mu5}(x) = -e\langle\bar{\Psi}\gamma_\mu\gamma_5\Psi\rangle$$

- Electric current density $j_i = \langle\bar{\Psi}\gamma^i\Psi\rangle$:

$$j_1(y) = j(y-y_+) - j(y-y_-)$$

$$j_{1,5}(y) = j(y-y_+) + j(y-y_-)$$

$$j(\eta) = 2\frac{e^2BT}{2\pi}\sum_n \int_{-\infty}^{\infty} ds\, r_s(\omega_n,\mu)\,(\omega_n - i\mu)\left[\frac{\tan(eBs)}{eBs}\right]^{1/2} e^{-s(\omega-i\mu)^2 - eB\eta^2}.$$

- Only $i = 1$ component is non-vanishing.

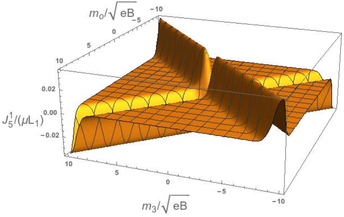


Figure: $L_2\sqrt{eB} = 0.2$

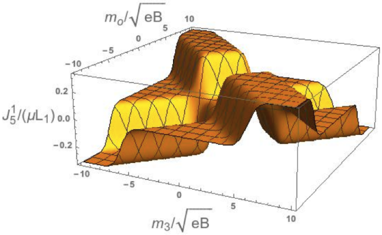
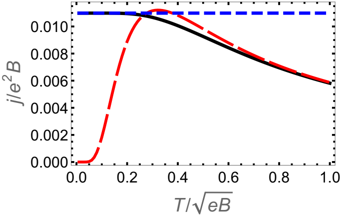


Figure: $L_2\sqrt{eB} = 8$

Electric current

AJM, A. Raya and C. Villavicencio, hep/ph:1803.05794.



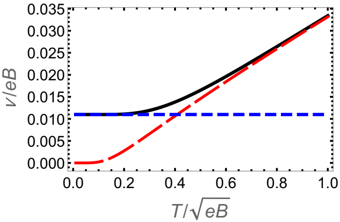
For $|eB| \ll (\pi T)^2 - \mu^2$:

$$j(\eta) = \frac{e^2 B}{2\pi}[n_F(eB\eta - \mu) - n_F(eB\eta + \mu)], \qquad \text{where } n_F = (1 + e^{x/T})^{-1}.$$

For $|eB| \gg (\pi T)^2 - \mu^2$:

$$j(\eta) = \frac{e^2 B}{\sqrt{|eB|}} \frac{\mu}{\pi^{3/2}} e^{-|eB|\eta^2}.$$

Density number



For $|eB| \ll (\pi T)^2 - \mu^2$:

$$\nu(\eta) = \frac{eB\eta}{\pi} T^2 \left[\frac{|eB\eta|}{T} ln\left(\frac{1+e^{(|eB\eta|-\mu)/T}}{1+e^{(|eB\eta|+\mu)/T}} \right) + Li_2\left(-e^{(|eB\eta|-\mu)/T}\right) + Li_2\left(-e^{(|eB\eta|+\mu)/T}\right) \right].$$

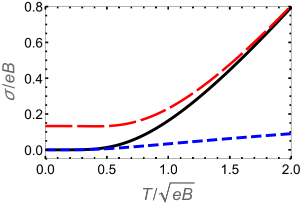
For $|eB| \gg (\pi T)^2 - \mu^2$:

$$\nu(\eta) = \sqrt{|eB| \frac{\mu}{\pi^3/2} e^{-|eB|\eta^2}}.$$

$$J_1 = |e| sign(B) N_5$$

Condensates

- Condensate $\sigma_3 = \langle \bar{\psi}\gamma_3\psi \rangle$



For $|eB| \ll (\pi T)^2 - \mu^2$:

$$\sigma_{T,B} = -\frac{eB\eta}{\pi} T \left[ln\left(1 + e^{-\beta(|eB\eta| - \mu)}\right) + ln\left(1 + e^{-\beta(|eB\eta| + \mu)}\right) \right]$$

For $|eB| \gg (\pi T)^2 - \mu^2$:

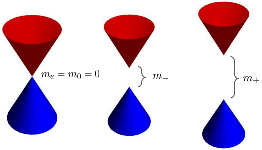
$$\sigma_{T,B}(\eta) = \frac{2eB\eta}{\pi} |2eB|^{1/4} T^{1/2} e^{-|eB|\eta^2}$$

$$\times \left[Li_{1/2}\left(-e^{-(\sqrt{2|eB|} - \mu)/T}\right) + Li_{1/2}\left(-e^{-(\sqrt{2|eB|} + \mu)/T}\right) \right].$$

Chiral chemical potential

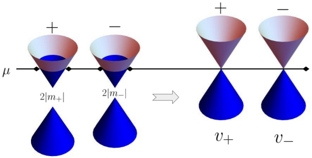
[AJM, A. Raya and C. Villavicencio, hep/ph:1803.05794.]

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ + \mu\gamma^0 + (eA_3^{\rm ext} - m_3)\gamma^3 - m_o\gamma^3\gamma^5]\psi.$$



Expanding around $p_\chi = \sqrt{\mu^2 - m_\chi^2}$,

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\psi}'_\chi \left[i\gamma^0\partial_0 - v_\chi \gamma \cdot \nabla \right] \psi'_\chi.$$

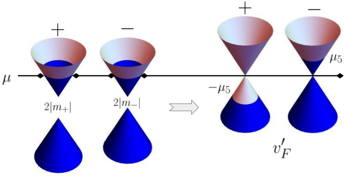


Expanding around $p_\chi = \frac{m_\chi v'_F}{\sqrt{1-v'^2_F}}$,

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\Psi}'[i\gamma^0\partial_0 - v'_F \boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + \mu_5\gamma_0\gamma_5]\Psi'_\chi$$

$$\mu' = \mu - \frac{|m_-|+|m_+|}{2\sqrt{1-v\prime^2_F}}$$

$$\mu_5 = \frac{|m_-|-|m_+|}{2\sqrt{1-v\prime^2_F}},$$



Description in terms of the chiral chemical potential μ_5.

Work in progress: RQED

- What kind of interaction gives rise to $M = m_3\gamma_3 + m_o\gamma_3\gamma_5$?
- In QED_3 the Haldane mass is directly related to the Chern-Simons term.
- Known contributions from 1-loop radiactive corrections.
- Coleman-Hill theorem states that there are no higher order contributions.
- First step: does Coleman-Hill theorem work for RQED?
- The answer is YES! Proof based on the Ward identities of the theory (in collaboration with D. Dudal and P. Pais, hep/ph:1801.08853).
- Dyson-Schwinger calculation to relate the CS term to m_o in RQED (in collaboration with D. Dudal, A. Raya and C. Villavicencio, in preparation).

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Conclusions and final remarks

- Relativistic-like fermions in condensed matter systems provide a potential link between high energy physics and condensed matter physics.
- In particular, it is possible to construct an analogy for the chiral magnetic effect: macroscopical quantum effects.
- CME detected in $ZrTe_5$, but detecting it in graphene would be very interesting for fundamental physics and from a technological point of view.
- Goal: how to generate the kind of interactions we need to reproduce the CME? Do the numbers allow it to be measured?