The chiral magnetic effect in planar condensed matter systems

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4th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, 22/03/2018.

Huitzil collaboration with Cristian Villavicencio, Alfredo Raya, and Saul Hernández-Ortiz.
Overview

Particle physics in table top experiments?

Electronic structure of graphene

Quantum electrodynamics in (2+1) dimensions

The pseudo-CME

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Particle physics in table top experiments?

Two-dimensional gas of massless Dirac fermions in graphene

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Quantum electrodynamics (resulting from the merger of quantum mechanics and relativity theory) has provided a clear understanding of phenomena ranging from particle physics to cosmology and from astrophysics to quantum chemistry\textsuperscript{1-3}. The ideas underlying quantum electrodynamics also influence the theory of condensed matter\textsuperscript{4,5}, but quantum relativistic effects are usually minute in the known experimental systems that can be described accurately within the framework of non-relativistic quantum mechanics. In contrast, graphene's behaviour shows that substantial concentrations of electrons (holes) are induced by positive (negative) gate voltages. Away from the transition region $V_g = 0$, Hall coefficient $R_H = 1/ne$ varies as $1/V_g$, where $n$ is the concentration of electrons or holes and $e$ is the electron charge. The linear dependence $1/R_H \propto V_g$ yields $n = \alpha V_g$ with $\alpha \approx 7.3 \times 10^{10}$ cm$^{-2}$V$^{-1}$, in agreement with the theoretical estimate $n/V_g \approx 7.2 \times 10^{10}$ cm$^{-2}$V$^{-1}$ for the surface charge density per electron.
Particle physics in table top experiments?

- Graphene (2+1)D: Klein paradox
- Dirac materials (3+1)D: Chiral magnetic effect

\[ J_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 B, \quad \mu_5 \sim E \cdot B \]

\[ J_{\text{CME}} \equiv \sigma_{\text{CME}}^{ik} E^k, \quad \sigma_{\text{CME}}^{zz} \sim B^2 \]

- The magnetoresistance in ZrTe\(_5\) when a magnetic field is applied parallel to an electric field is in accordance with the predictions that take into account the CME.

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Graphene Physics

A single layer of carbon atoms arranged in a honeycomb lattice structure.

Represented in terms of two triangular sublattices.

Hexagonal reciprocal lattice.

\[ \mathbf{a}_1 = \frac{a}{2} (3, \sqrt{3}), \mathbf{a}_2 = \frac{a}{2} (3, -\sqrt{3}) \]

\[ \mathbf{b}_1 = \frac{2\pi}{3a} (1, \sqrt{3}), \mathbf{b}_2 = \frac{2\pi}{3a} (1, -\sqrt{3}) \]

\[ \delta_1 = \frac{a}{2} (1, \sqrt{3}), \delta_2 = \frac{a}{2} (1, -\sqrt{3}), \delta_3 = a(-1, 0) \]

\[ \mathbf{K}_1 = \left( \frac{2\pi}{3a}, \frac{2\pi}{3\sqrt{3}a} \right), \mathbf{K}_2 = \left( \frac{2\pi}{3a}, \frac{-2\pi}{3\sqrt{3}a} \right) \]
Tight-binding approach: nearest neighbours.

Hopping only between sublattices.

\[ H(\vec{k}) = \begin{pmatrix} 0 & tS(\vec{k}) \\ tS^*(\vec{k}) & 0 \end{pmatrix} \]

where

\[ S(\vec{k}) = \sum_{\vec{\delta}} e^{i\vec{k} \cdot \vec{\delta}} \]

The energy is:

\[ E(k_x, k_y) = \pm t \sqrt{3 + 2 \cos \left( \sqrt{3} k_y a \right) + 4 \cos \left( \frac{\sqrt{3}}{2} k_y a \right) \cos \left( \frac{3}{2} k_x a \right)} . \]

Zero energy around the K points.
Linear dispersion relation:
\[ \mathcal{H} = \bar{\psi} \, \hbar v_F \gamma \cdot k \, \psi. \]

Dirac points: valence and conduction band touch generating no gap.

\[ \psi(k) = \begin{pmatrix}
\psi_{\uparrow}(K_+ + k) \\
\psi_{\downarrow}(K_+ + k) \\
\psi_{\uparrow}(K_- + k) \\
\psi_{\downarrow}(K_- + k)
\end{pmatrix} \]

Figure extracted from http://oer.physics.manchester.ac.uk/AQM2/Notes/Notes-6.4.html
Expanding around de $K$ and $K'$ points:

\[
H_{K'}(\bar{q}) \approx \frac{3at}{2} \begin{pmatrix}
0 & \alpha(q_x + iq_y) \\
\alpha^*(q_x - iq_y) & 0
\end{pmatrix}, \quad H_K = H_{K'}^*.
\]

\[
H_K = -i\hbar v_f \vec{\sigma} \nabla, \quad H' = H_K^T.
\]

Considering the 4-component spinor:

\[
H = \begin{pmatrix}
H_K & 0 \\
0 & H_K'
\end{pmatrix}, \quad H_K = H_{K'}^*, \quad H = -i\hbar v_0 \tau_0 \otimes \vec{\sigma} \nabla.
\]

The interaction with an electromagnetic field is done through the Peierls substitution:

\[
a_{n,\sigma}^\dagger b_{m,\sigma} \rightarrow a_{n,\sigma}^\dagger \exp \left( \frac{-ie}{\hbar c} \int_m^n A dr \right) b_{m,\sigma}.
\]

In term of a Lagrangian:

\[
\mathcal{L} = \sum_{\sigma = \pm} \bar{\psi}_\sigma(t, r) \left[ i\gamma^0 (\hbar \partial_t - i\mu_\sigma) + i\hbar v_f \gamma^1 D_x + i\hbar v_f \gamma^2 D_y \right] \psi_\sigma(t, r).
\]
QED in 2+1 dimensions describes graphene

- Low energy excitations in graphene are massless quasiparticles with linear dispersion relation.
- The eigenfunctions of the low energy quasiparticle excitations obey the Dirac equation. It presents a spinor structure as a consequence of the underlying lattice.
- The interactions of the quasiparticles with external electromagnetic fields is introduced using the minimal coupling prescription of quantum field theory.
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QED in (2+1)D

- QED3 fermion sector ($\hbar = c = 1$):

$$\mathcal{L} = \bar{\psi} \left[ \gamma_0 (i \partial_0 + \hat{\mu}) - i (\gamma_1 D_x + \gamma_2 D_y) - \hat{M} \right] \psi.$$

- $\hat{M}$ are the Dirac masses or interactions that can result in a gap in the energy bands - deformations, substrates, doping, etc.

- $\hat{\mu}$ is a generalized chemical potential including spin interaction (Zeeman term), $\mu_\sigma = \mu - \frac{\sigma g}{2\mu_B} B$. 
QED in (2+1)D

- QED$_3$ gauge sector:

$$\mathcal{L}_{\text{QED}_3} = \bar{\psi} \left[ \partial - qA + m_o \gamma_3 \gamma_5 \right] \psi - \frac{1}{2\xi} (\partial \cdot A)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\theta}{4} \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho.$$  

- Haldane mass can be dynamically generated by the CS term.

- Parity-odd mass term in the fermion propagator radiatively induces a parity-odd contribution into the vacuum polarization tensor at one-loop level.

- Coleman-Hill theorem shows that there are no contribution from two- and higher-loops.
Reduced QED

[Gorbar, Guysinin, Miranski, PRD 64, 105028 (2001).]

- The gauge sector is not constrained to the plane.
- Coulomb rather than logarithmic interaction.
- Reduced QED: general (3+1) theory dimensionally reduced to a non-local effective (2+1) theory.

\[ S = \int d^D X \left( \frac{1}{4e^2} F_{ab}^2 + A_a J^a - \frac{1}{2e^2 \xi} (\partial_a A^a)^2 \right) \]

- \( D = 4 \rightarrow \) Integrating over the gauge field and the third spatial dimension.
- Keeping \( J^3 = 0 \).
- Adding the fermion fields in (2+1)D.

\[ S = \int d^3 x \left[ \bar{\psi} \left( i \slashed{D} + m \right) \psi + \frac{1}{2} F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F^{\mu\nu} + \frac{1}{e^2 \xi} \partial_\mu A^\mu \frac{1}{\sqrt{-\partial^2}} \partial_\nu A^\nu \right] \]
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Imbalance of chirality + strong magnetic field

Electrical current in the direction of the field

$eB \approx 10^{19} G$

- Effective description in terms of a chiral chemical potential:
  

  $j_z \sim \sum_f q_f^2 B \mu_5$ Independent of temperature and mass
The pseudo-Chiral magnetic effect

Quantum Hall effect: magnetic field perpendicular to the plane.

Pseudo-CME: in plane magnetic field.

▶ External classical field.
▶ Does not contribute to quantum corrections.
▶ Usual Zeeman term: effective way to consider the non-relativistic approximation for the Dirac equation with a magnetic background. Equivalent to consider a non-vanishing third component of the gauge field.
Parity breaking “mass”: \( M = m_3 \gamma_3 + m_0 \gamma_3 \gamma_5 \).

Place the graphene on a Boron Nitride substrate - \( m_3 \):

PCME Lagrangian [AJM, C. Villavicencio, A. Raya, IJMP B30, 1550257 (2015)]:

\[
\mathcal{L} = \bar{\psi} [i \partial^\mu + \mu \gamma^0 + (e A_3^{\text{ext}} - m_3) \gamma^3 - m_0 \gamma^3 \gamma^5] \psi.
\]
In the chiral basis, we define: \( \psi_\pm \equiv \frac{1}{2} (1 \pm \gamma^5) \psi \) and \( m_\pm = m_3 \pm m_0 \):

\[
\mathcal{L} = \sum_{\chi = \pm} \bar{\psi}_\chi \left[ i \partial^\mu + \mu \gamma^0 + (eA_3^{\text{ext}} - m_\chi) \right] \psi_\chi.
\]

The Green function can be written in terms of the two chiralities:

\[
G(x, x') = \frac{1}{2} (1 + \gamma^5) G_+(x, x') + \frac{1}{2} (1 - \gamma^5) G_-(x, x').
\]

where:

\[
G_\pm(r, r') = \left\langle r \left| \frac{1}{i \gamma^0 + (eA_3 - m_\pm) \gamma_3} \right| r' \right\rangle.
\]
Using the Schwinger proper time method:

\[
\tilde{G}(k; \xi) = \int_{-\infty}^{\infty} ds \, r_s(k^0, K^\|) \, e^{isK^\|} \left[ k_2^2 + \xi^2 \right] \frac{\tan(eBs)}{eB} \left\{ K^\| \left[ 1 + \gamma^2 \gamma^3 \tan(eBs) \right] + \left[ k_2^2 + \xi \gamma^3 \right] \sec^2(eBs) \right\},
\]

\[ K^\| = (k_0 + \mu, k_1, 0), \quad \xi = eB(x_2 + x_2')/2 + m_\pm. \]

We calculate the currents at finite temperature:

\[ J_\mu(x) = -e\langle \bar{\psi} \gamma_\mu \psi \rangle, \quad J_\mu 5(x) = -e\langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle \]

Electric current density \( j_i = \langle \bar{\psi} \gamma^i \psi \rangle \):

\[ j_1(y) = j(y - y_+) - j(y - y_-) \]

\[ j_{1,5}(y) = j(y - y_+) + j(y - y_-) \]

\[ j(\eta) = 2\frac{e^2 BT}{2\pi} \sum_n \int_{-\infty}^{\infty} ds \, r_s(\omega_n, \mu) (\omega_n - i\mu) \left[ \frac{\tan(eBs)}{eB} \right]^{1/2} e^{-s(\omega - i\mu)^2 - eB\eta^2}. \]
Only $i = 1$ component is non-vanishing.

Figure: $L_2 \sqrt{eB} = 0.2$

Figure: $L_2 \sqrt{eB} = 8$
Electric current


For $|eB| \ll (\pi T)^2 - \mu^2$:

$$j(\eta) = \frac{e^2 B}{2\pi} [n_F(eB\eta - \mu) - n_F(eB\eta + \mu)], \quad \text{where } n_F = (1 + e^{x/T})^{-1}.$$

For $|eB| \gg (\pi T)^2 - \mu^2$:

$$j(\eta) = \frac{e^2 B}{\sqrt{|eB|}} \frac{\mu}{\pi^{3/2}} e^{-|eB|\eta^2}.$$
Density number

\[
\nu(\eta) = \frac{eB\eta}{\pi} T^2 \left[ \frac{|eB\eta|}{T} \ln \left( \frac{1 + e^{(|eB\eta| - \mu)/T}}{1 + e^{(|eB\eta| + \mu)/T}} \right) + Li_2 \left( -e^{(|eB\eta| - \mu)/T} \right) + Li_2 \left( -e^{(|eB\eta| + \mu)/T} \right) \right].
\]

For \(|eB| \ll (\pi T)^2 - \mu^2|

\[
\nu(\eta) = \frac{eB\eta}{\pi} \sqrt{|eB|} \frac{\mu}{\pi^{3/2}} e^{-|eB|\eta^2}.
\]

\[
J_1 = |e| \text{sign}(B) N_5
\]
Condensates

Condensate $\sigma_3 = \langle \bar{\psi} \gamma_3 \psi \rangle$

For $|eB| \ll (\pi T)^2 - \mu^2$:

$$\sigma_{T,B} = -\frac{eB\eta}{\pi} T \left[ \ln \left( 1 + e^{-\beta(\sqrt{eB}|\eta| - \mu)} \right) + \ln \left( 1 + e^{-\beta(\sqrt{eB}|\eta| + \mu)} \right) \right]$$

For $|eB| \gg (\pi T)^2 - \mu^2$:

$$\sigma_{T,B}(\eta) = \frac{2eB\eta}{\pi} |2eB|^{1/4} T^{1/2} e^{-|eB|\eta^2}$$

$$\times \left[ Li_{1/2} \left( -e^{-\left( \sqrt{2|eB|}\mu \right)/T} \right) + Li_{1/2} \left( -e^{-\left( \sqrt{2|eB|}\mu \right)/T} \right) \right].$$
Chiral chemical potential

\[ \mathcal{L} = \bar{\psi} [i \partial + \mu \gamma^0 + (e A_3^{ext} - m_3) \gamma^3 - m_0 \gamma^3 \gamma^5] \psi. \]

Expanding around \( p_\chi = \sqrt{\mu^2 - m_\chi^2} \),

\[ \mathcal{L} = \sum_{\chi = \pm} \bar{\psi}_{\chi}' \left[ i \gamma^0 \partial_0 - v_\chi \gamma \cdot \nabla \right] \psi_{\chi}'. \]
Expanding around $p_\chi = \frac{m_\chi v'_F}{\sqrt{1-v'^2_F}}$,

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\psi}' [i\gamma^0 \partial_0 - v'_F \gamma \cdot \nabla + \mu_5 \gamma_0 \gamma_5] \psi'_\chi$$

$$\mu' = \mu - \frac{|m_-| + |m_+|}{2 \sqrt{1-v'^2_F}}$$

$$\mu_5 = \frac{|m_-| - |m_+|}{2 \sqrt{1-v'^2_F}} ,$$

Description in terms of the chiral chemical potential $\mu_5$. 
Work in progress: RQED

- What kind of interaction gives rise to $M = m_3 \gamma_3 + m_0 \gamma_3 \gamma_5$?
- In QED$_3$ the Haldane mass is directly related to the Chern-Simons term.
- Known contributions from 1-loop radiative corrections.
- Coleman-Hill theorem states that there are no higher order contributions.
- First step: does Coleman-Hill theorem work for RQED?
- The answer is YES! Proof based on the Ward identities of the theory (in collaboration with D. Dudal and P. Pais, hep/ph:1801.08853).
- Dyson-Schwinger calculation to relate the CS term to $m_0$ in RQED (in collaboration with D. Dudal, A. Raya and C. Villavicencio, in preparation).
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Conclusions and final remarks

- Relativistic-like fermions in condensed matter systems provide a potential link between high energy physics and condensed matter physics.
- In particular, it is possible to construct an analogy for the chiral magnetic effect: macroscopical quantum effects.
- CME detected in ZrTe$_5$, but detecting it in graphene would be very interesting for fundamental physics and from a technological point of view.
- Goal: how to generate the kind of interactions we need to reproduce the CME? Do the numbers allow it to be measured?