Status of lattice QCD with magnetic field

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Various phenomenology predicted for QCD in a strong magnetic field

- **Effects on the QCD vacuum structure:**
  - **chiral symmetry breaking**
  - **confinement?**
- **Effects on the QCD phase diagram: $T_c(\mu)$? New phases?**
- **Equation of state: is strongly interacting matter paramagnetic or diamagnetic?**

LQCD is the ideal tool for a non-perturbative investigation of equilibrum phenomena.

QCD+QED studies of the e.m. properties of hadrons go back to the early days of LQCD

- G. Martinelli, G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. B 116, 434 (1982).

- C. Bernard, T. Draper, K. Olynyk and M. Rushton, Phys. Rev. Lett. 49, 1076 (1982).

A magnetic background does not pose any technical problem (such as a sign problem) to lattice QCD simulations.

An e.m. background field a_μ modifies the covariant derivative as follows:

$$D_\mu = \partial_\mu + i\, g A_\mu^a T^a \quad \rightarrow \quad \partial_\mu + i\, g A_\mu^a T^a + i\, q a_\mu$$

in the lattice formulation:

$$D_\mu \psi \rightarrow \frac{1}{2a} \left(U_\mu(n) u_\mu(n) \psi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu}) u_\mu^*(n-\hat{\mu}) \psi(n-\hat{\mu}) \right)$$

$U_\mu \in SU(3) \qquad \mathbf{u}_\mu \simeq \exp(\mathbf{i\, q\, a}_\mu(\mathbf{n})) \in \mathbf{U(1)}$

- $F_{ij}^{(em)} \neq 0 \implies$ non-zero magnetic field (no sign problem)
- $F_{0i}^{(em)} \neq 0 \implies$ non-zero imaginary electric field (sign problem for real e. f.)
- Uniform background field are quantized in the presence of periodic boundary conditions

Magnetic field effects on the QCD transition

There are several aspects that one would like to investigate:

- **Do deconfinement and chiral symmetry restoration stay entangled or do they split?**
- **Does the nature of the transition change?**
- **How T_c changes as a function of B?**

Various lattice studies have addressed such issues in the recent past

M. D., S. Mukherjee, F. Sanfilippo, arXiv:1005.5365

G. S. Bali et al., arXiv:1111.4956

E. -M. Ilgenfritz et al., arXiv:1203.3360

A. Tomiya, H. T. Ding, S. Mukherjee, C. Schmidt and X. D. Wang arXiv:1711.02884

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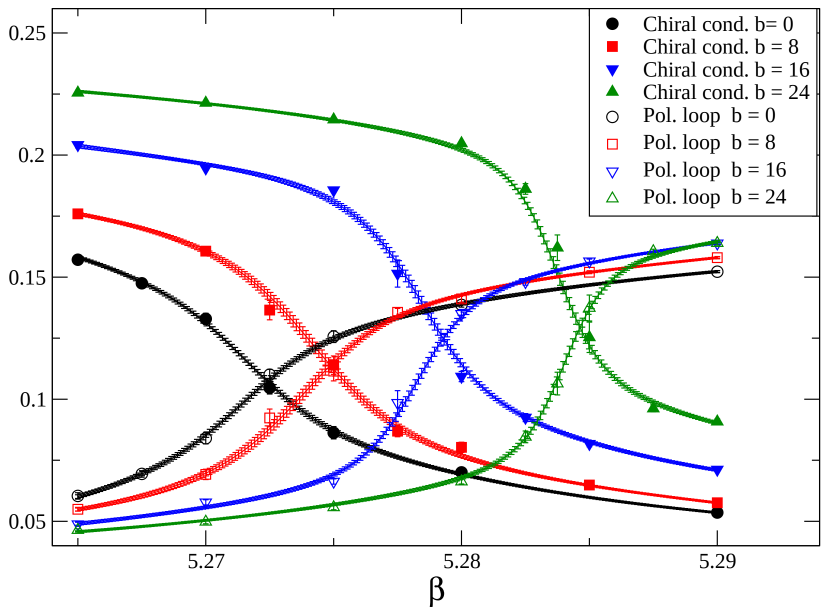
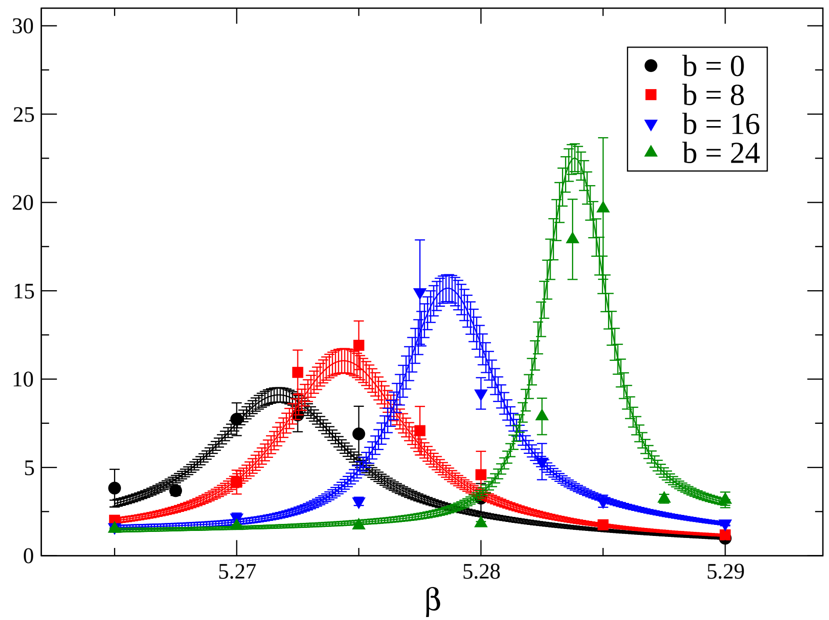


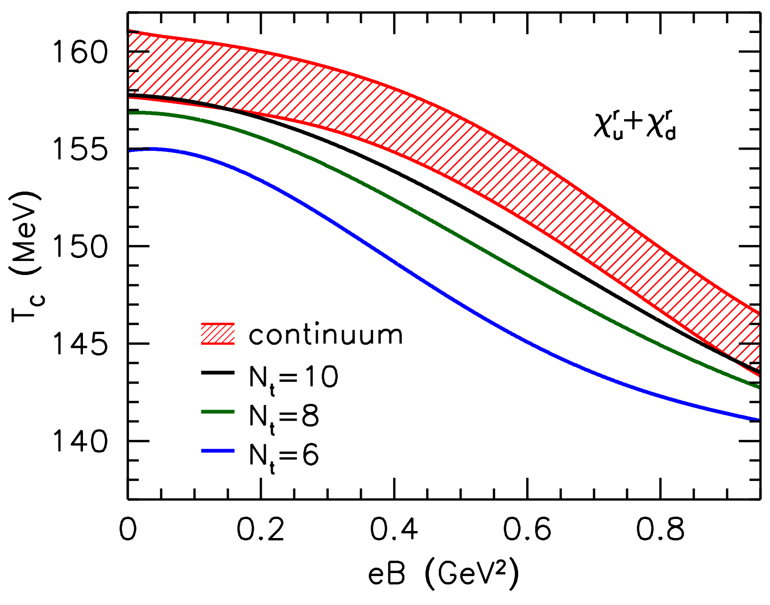


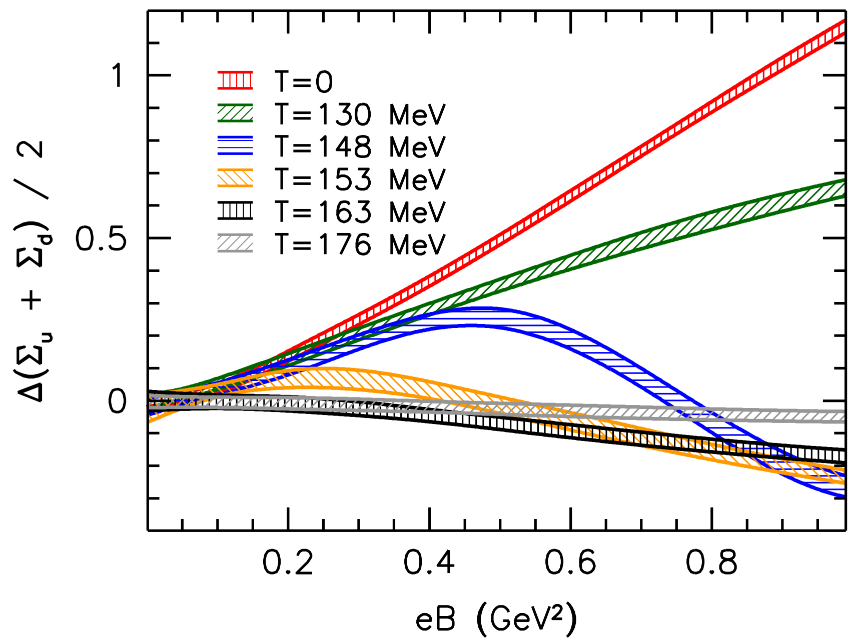
From arXiv:1005.5365, M. D., S. Mukherjee and F. Sanfilippo, unimproved rooted staggered fermions, $N_t = 4$, $L_s = 16$

Left: $\langle\bar{\psi}\psi\rangle$ and Pol. loop vs. temperature for various B quanta at $m_\pi \simeq 200$ MeV. eB up to $\sim$ 1 GeV^2.

Right: disconnected chiral susceptibility for the same parameters

- **Chiral restoration and deconfinement move together as a function of B**
- **An increase of the strength of the transition is observed**
- **There is a modest increase in T_c, of the order of 2% at $|e|B \sim$ 1 GeV^2.**
- **Magnetic catalysis is observed at all temperatures**



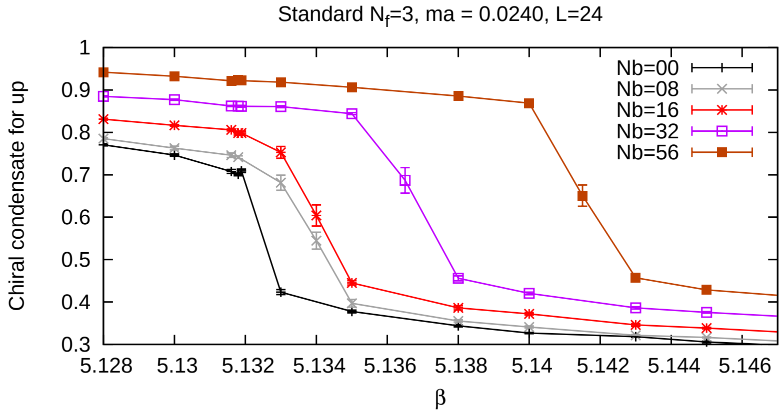


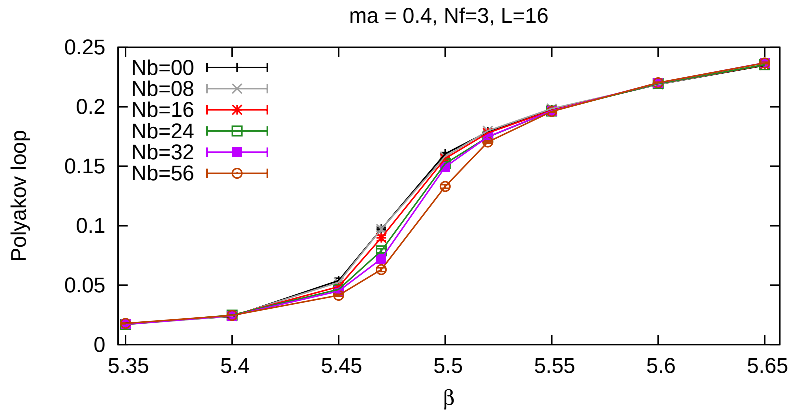
From arXiv:1111.4956 and arXiv:1206.4205, G. Bali et al.: $N_f = 2+1$, stout smeared rooted staggered fermions, physical quark masses

Left: T_c vs. eB from chiral susceptibility

Right: relative increment of $\langle\bar{\psi}\psi\rangle$ vs. eB at various temperatures

- **T_c decreases as a function of B, by about 10-20% at $|e|B \sim$ 1 GeV^2.**
- **A similar T_c change is observed from the Polyakov loop**
- **Magnetic catalysis changes sign at high T! (inverse catalysis)**
- **A slight increase in the transition strength is observed**





From arXiv:1711.02884, A. Tomiya, H. T. Ding, S. Mukherjee, C. Schmidt and X. D. Wang

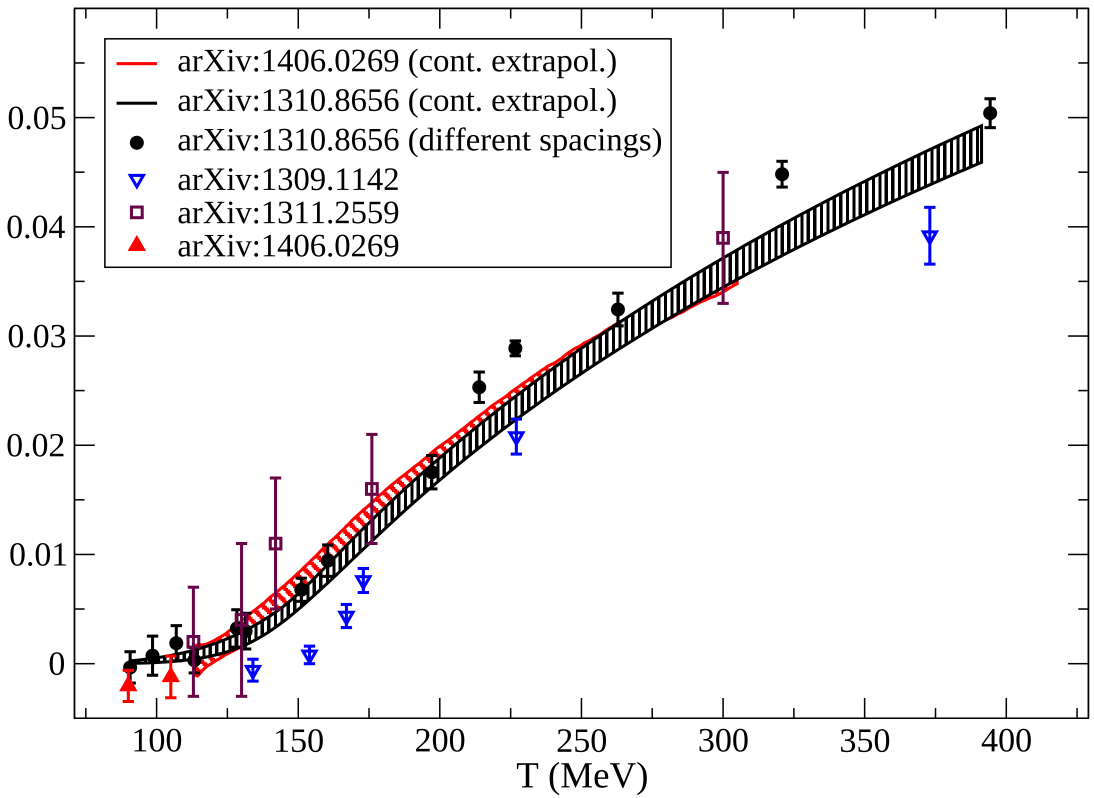
Study of $T_c(B)$ for $N_f = 3$ and various quark masses, $N_t = 4$ ($a \simeq 0.3$ fm), unimproved staggered fermions.

Left: $\langle\bar{\psi}\psi\rangle$ **for various B quanta at small mass**

Right: **Polyakov loop for various B quanta at high mass**

- **T_c increases both for low and high masses**
- **Discretization effects still unclear**

Magnetic properties of the thermal medium



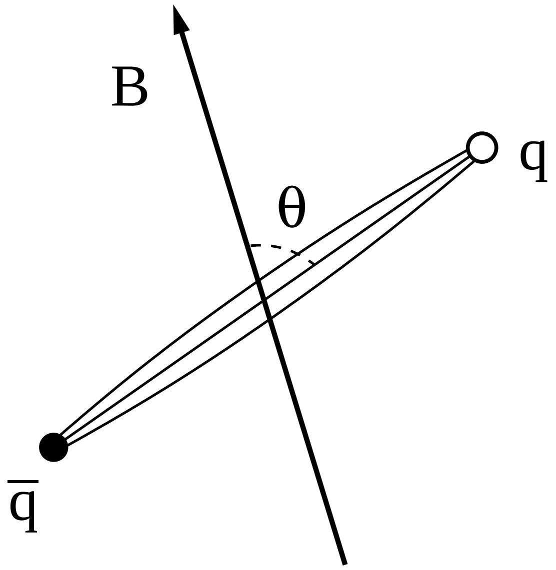
The thermal QCD medium becomes strongly paramagnetic right above T_c. In the figure: magnetic susceptibility

C. Bonati et al., arXiv:1307.8063, arXiv:1310.8656;

L. Levkova and C. DeTar, arXiv:1309.1142;

G. S. Bali et al., arXiv:1406.0269

The magnetic field has also shown to strongly influence the interaction between heavy quarks, introducing an anisotropy in the potential.

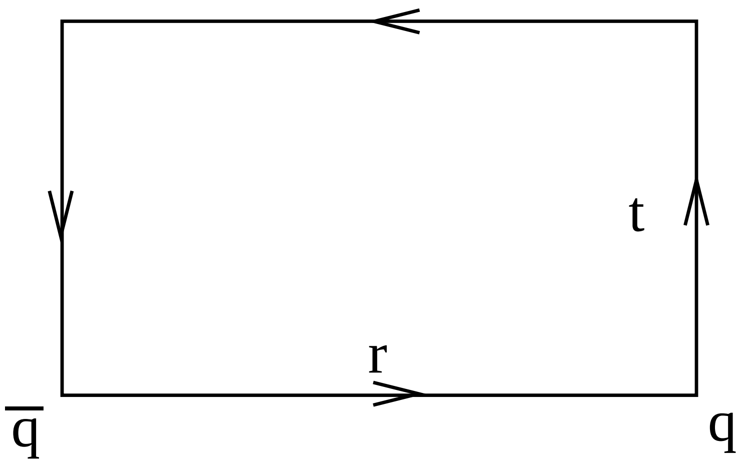


C. Bonati, MD, M. Mariti, M. Mesiti, F. Negro, A. Rucci, F. Sanfilippo, arXiv:1403.6094, arXiv:1607.08160

$N_f = 2+1$ with rooted staggered quark at the physical point

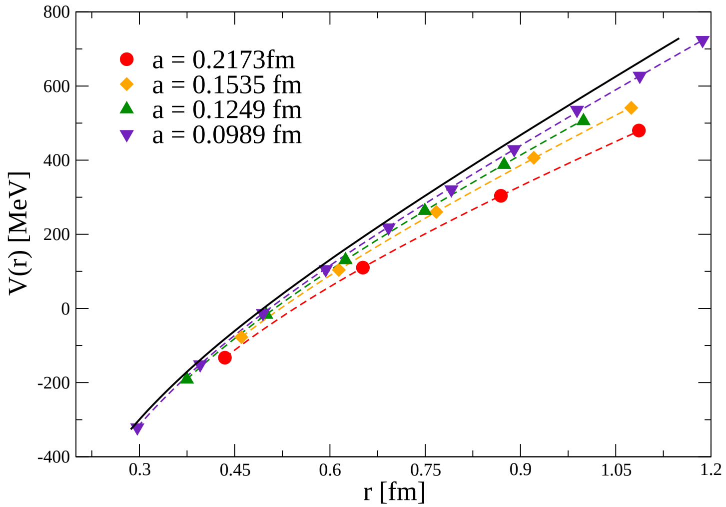
The potential is determined through Wilson loop expectation values

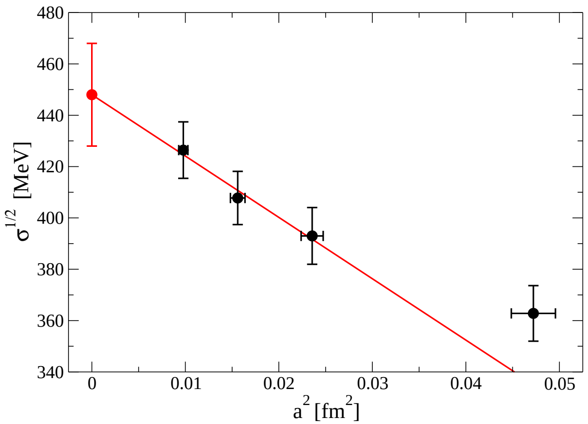
$$aV(a\vec{n}) = \lim_{n_t \to \infty} \log \left(\frac{\langle W(\vec{n}, n_t) \rangle}{\langle W(\vec{n}, n_t + 1) \rangle} \right)$$

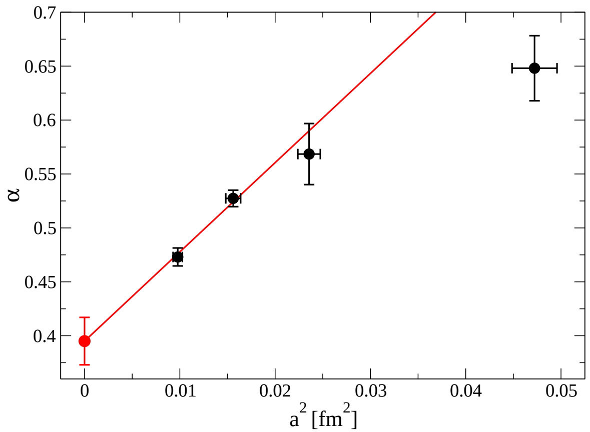


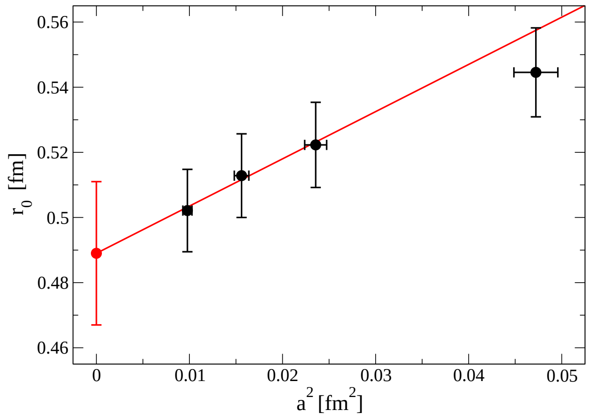
at $B = 0$ the standard Cornell potential described data for all lattice spacings

$$V(r) = -\frac{\alpha}{r} + \sigma r + V_0 \ ,$$









Continuum extrapolated results for σ, α and for the Sommer parameter r_0

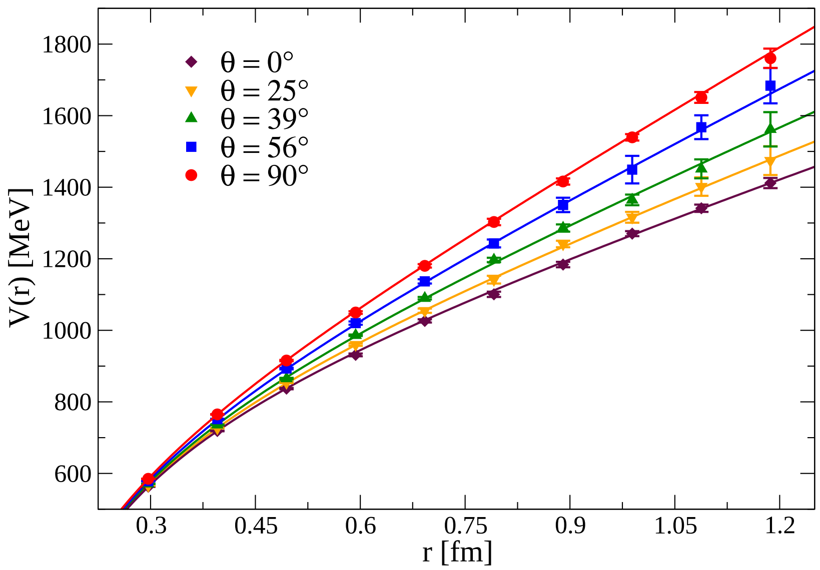
$$r_0^2 \left. \frac{dV}{dr} \right|_{r_0} = 1.65$$

α	**0.395(22)**
$\sqrt{\sigma}$	**448(20) MeV**
r_0	**0.489(20) fm**

The potential is Cornell like along each direction

$$V(r,\theta) = -\frac{\alpha(\theta,B)}{r} + \sigma(\theta,B)r + V_0(\theta,B)$$

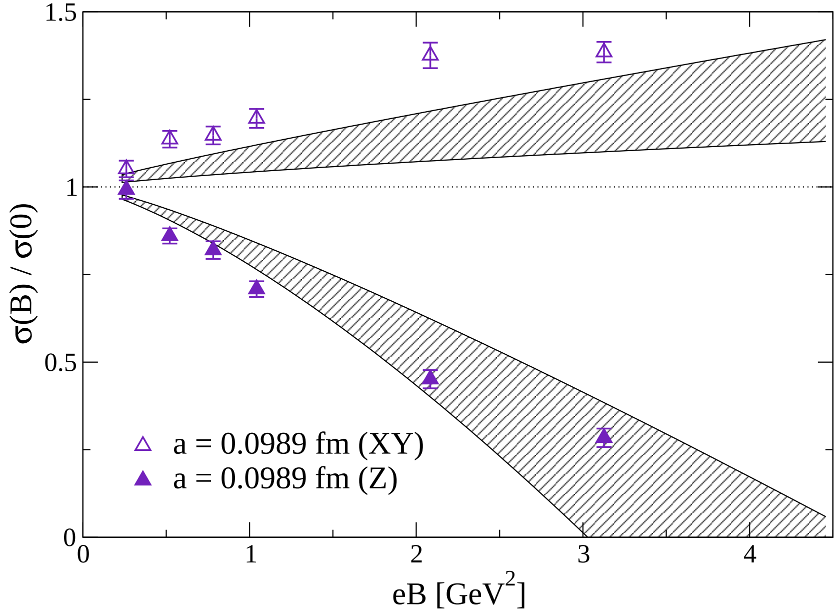
At fixed r, the potential is an increasing function of the angle and reaches a maximum for orthogonal directions



After continuum extrapolation, most of the effect seems related to an anisotropy in the string tension.

σ grows with B in the orthogonal direction

The longitudinal string tension decreases and could even vanish for $eB \sim 4$ GeV^2, but one would need $a \ll 0.1\dot{f}m$ to actually check it.



Is the deformation of the static quark-antiquark potential associated with a corresponding deformation of the color flux tube?

In principle, two different phenomena may happen:

- **The flux tube for longitudinal separation is less intense than that for transverse separation;**

- **The flux tube for transverse separation loses cilindrical symmetry and becomes anisotropic**

Lattice determinations of color flux tubes make use of correlation between Wilson loops and plaquette operators.

Connected correlators allow the determination of the field strength itself

[Di Giacomo, Maggiore, Olejnik, 1990] [Cea, Cosmai, Cuteri, Papa, 2017]

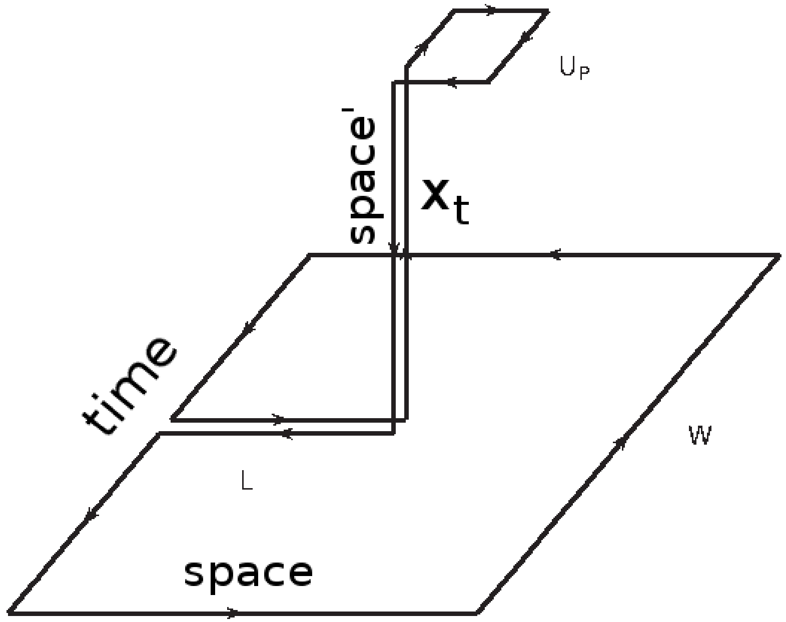
$$E_l^{chromo} = \lim_{a \to 0} \frac{1}{a^2 g} \left[\frac{\langle Tr(W L U_P L^{\dagger}) \rangle}{\langle Tr(W) \rangle} - \frac{\langle Tr(W) Tr(U_P) \rangle}{\langle Tr(W) \rangle} \right]$$

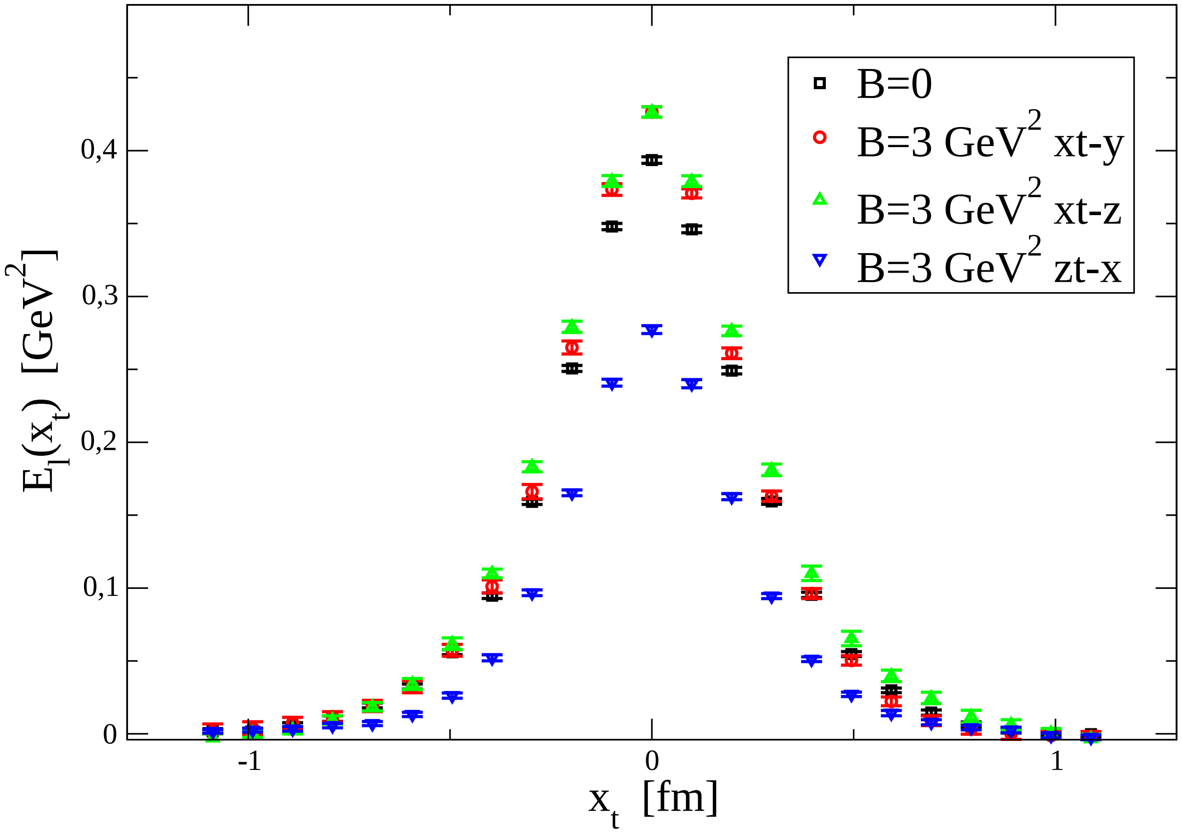
W is the open Wilson loop operator

U_P is the open plaquette operator

L is the adjoint parallel transport

A smearing procedure is adopted (1 HYP for temporal links, several APE for spatial links) as a noise reduction technique





These are the flux tube profiles for $eB \sim 3$ GeV^2 compared to $B = 0$ at a fixed number of smearing steps $N_{APE} = 80$

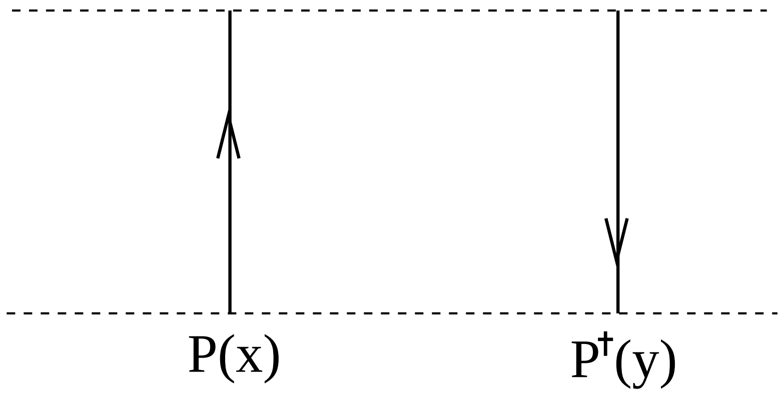
PRELIMINARY (F. Negro et al. arXiv:1710.09215)

Both kind of anisotropies emerge, even if the eccentricity of a flux tube section orthogonal to B is small

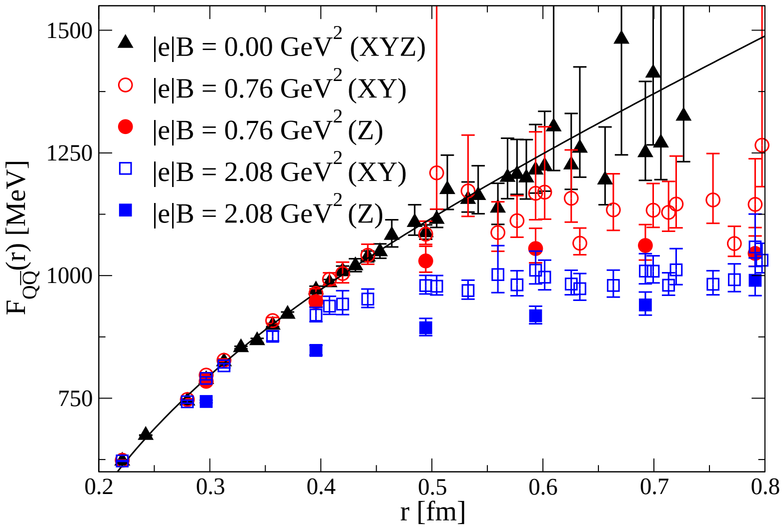
Finite T results

At finite T, the quark-antiquark potential is measured from Polyakov loop correlators

$$\langle \mathrm{Tr} P(\vec{x})\, \mathrm{Tr} P^\dagger(\vec{y}) \rangle \sim \exp\left(-\frac{F_{\bar{q}q}(r,T)}{T} \right)$$



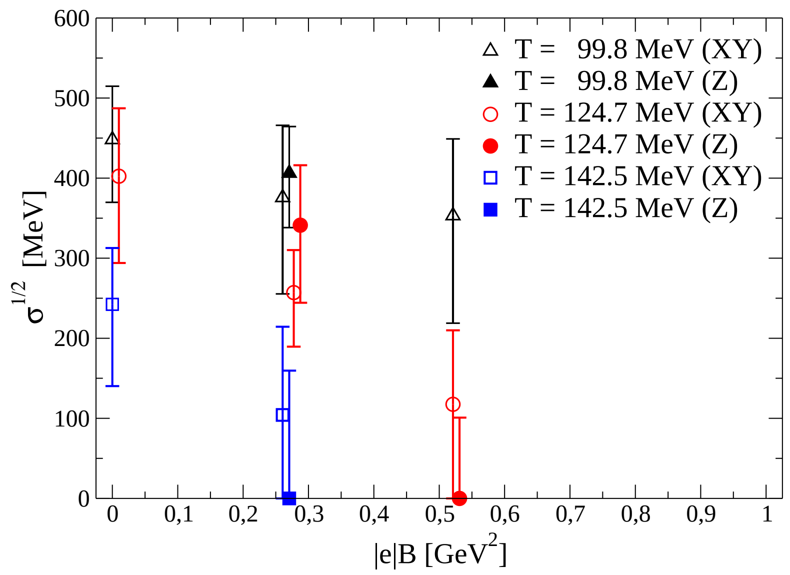
Results at $T \sim 100$ MeV on a $N_t = 20$ lattice



Although a small anisotropy is still visible, the main effect of B seems to suppress the potential in all directions

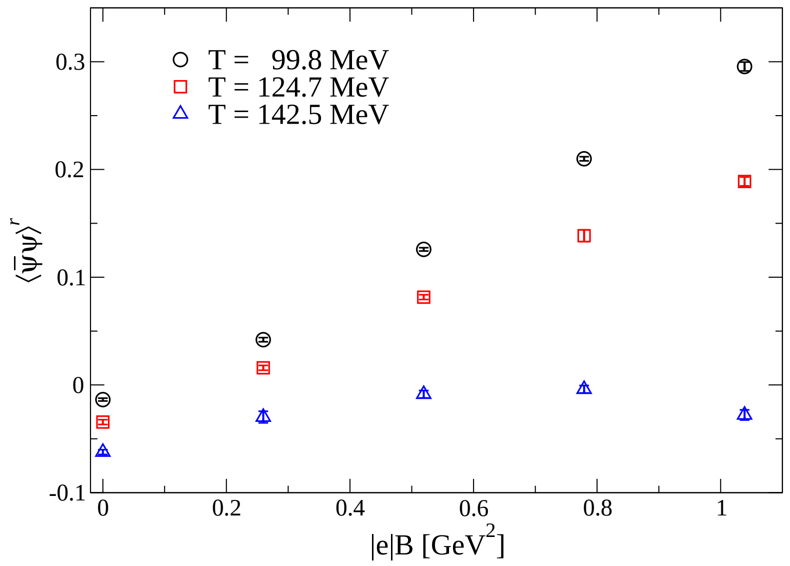
The string tension tends to disappear

A fit to the Cornell potential works in a limited range of distances and permits to obtain a determination of σ, which shows a steady decrease in all directions.



We can call this effect deconfinement catalysis

It is interesting to notice that this happens before (in temperature) inverse magnetic catalysis is visible in the chiral condensate



Is the decrease of T_c as a function of B related to a change in the confining properties?

Above T_c

Deep in the deconfined phase, heavy quark interactions are related to the screening properties of the Quark-Gluon plasma.

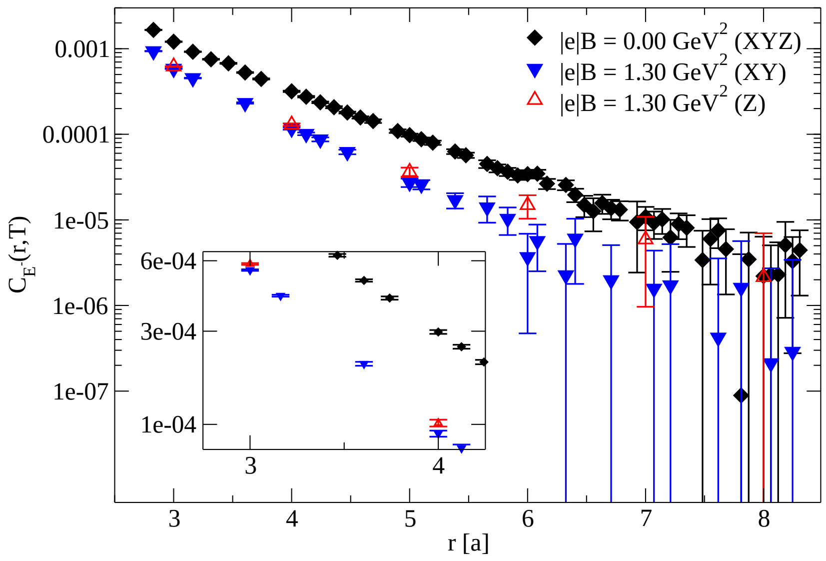
It is known that, contrary to electro-magnetic plasmas, interactions mediated by magnetostatic gluon are dominant at large distances.

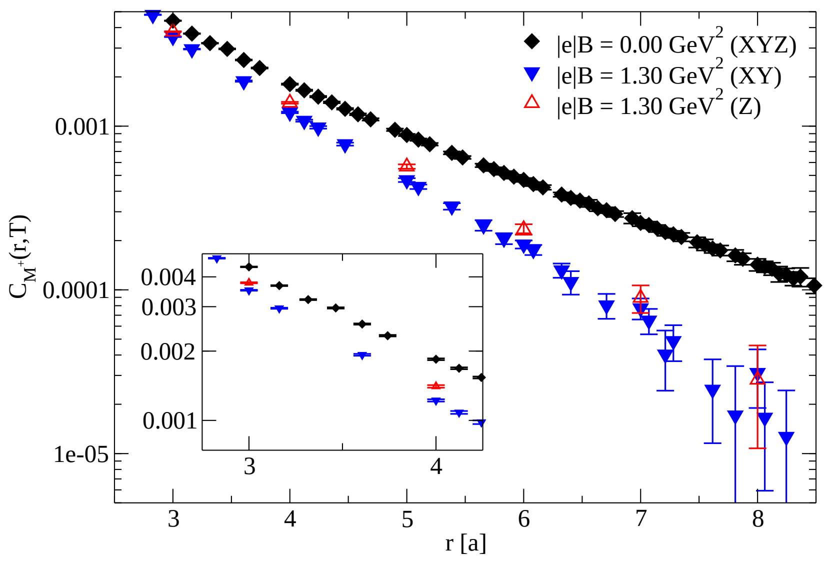
Nevertheless, it is possible to separate the electric and magnetic channels and define two different gauge invariant screening masses:

(Braaten-Nieto, 1994; Arnold-Yaffe, 1995)

$$C_{M^+} = + \frac{1}{2}\mathrm{Re}\big[C_{LL} + C_{LL^\dagger}\big] - |\langle \mathrm{Tr} L\rangle|^2 = \langle \mathrm{Tr}\mathrm{Re} L(\mathbf{0}) \mathrm{Tr}\mathrm{Re} L(\mathbf{r})\rangle$$

$$C_{E^-} = - \frac{1}{2}\mathrm{Re}\big[C_{LL} - C_{LL^\dagger}\big] = \langle \mathrm{Tr}\mathrm{Im} L(\mathbf{0}) \mathrm{Tr}\mathrm{Im} L(\mathbf{r})\rangle \ .$$

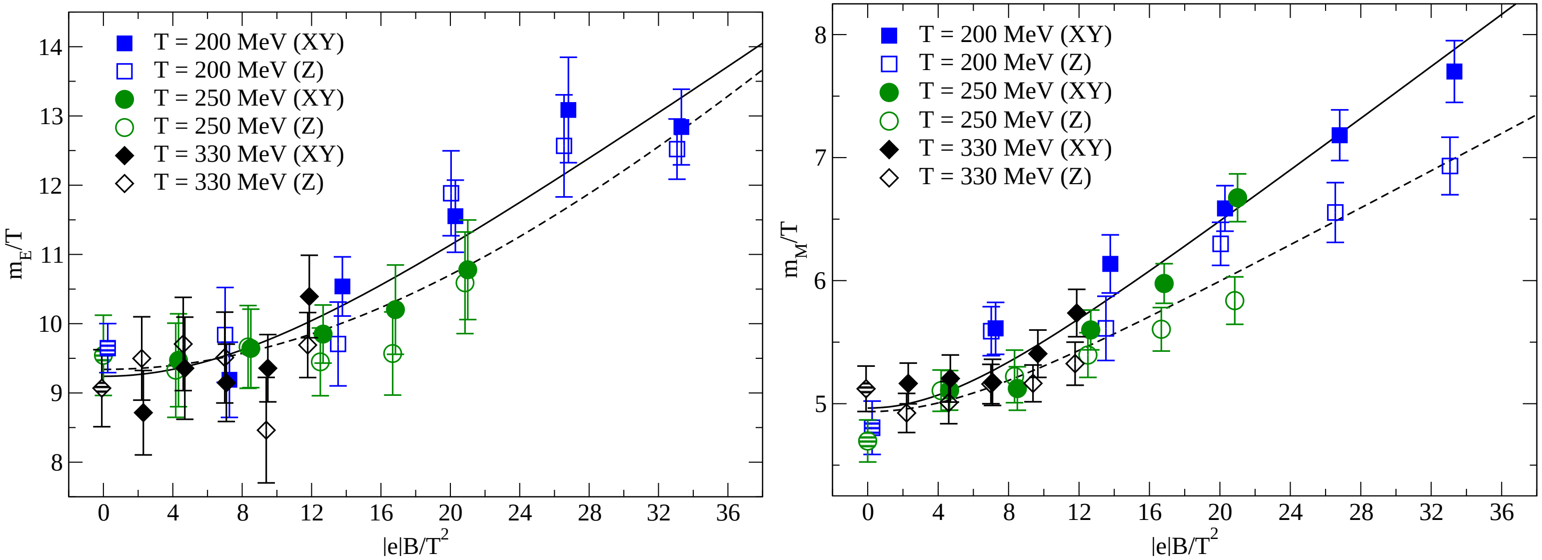




$$C_{E^-}(\mathbf{r},T)\Big|_{r\to\infty} \simeq \frac{e^{-m_E(T)r}}{r}$$

$$C_{M^+}(\mathbf{r},T)\Big|_{r\to\infty} \simeq \frac{e^{-m_M(T)r}}{r}$$

Electric and magnetic screening masses show a sizable dependence on the magnetic background



Such masses show a clear (increasing) dependence on B: the magnetic background field enhances the color screening properties of the QGP

$$\frac{m^d_{E/M}}{T} = a^d_{E/M}\left[1 + c^d_{1;E/M}\frac{|e|B}{T^2}\text{atan}\left(\frac{c^d_{2;E/M}}{c^d_{1;E/M}}\frac{|e|B}{T^2}\right)\right] ,$$

from C. Bonati, MD, M. Mariti, M. Mesiti, F. Negro, A. Rucci, F. Sanfilippo, 1703.00842

Discussion and Conclusions

Location and order of the transition:

- T_c moves downwards for increasing B. Studies with unimproved fermions do not agree but this is likely a discretization effect

- Effect on chiral or confining properties?

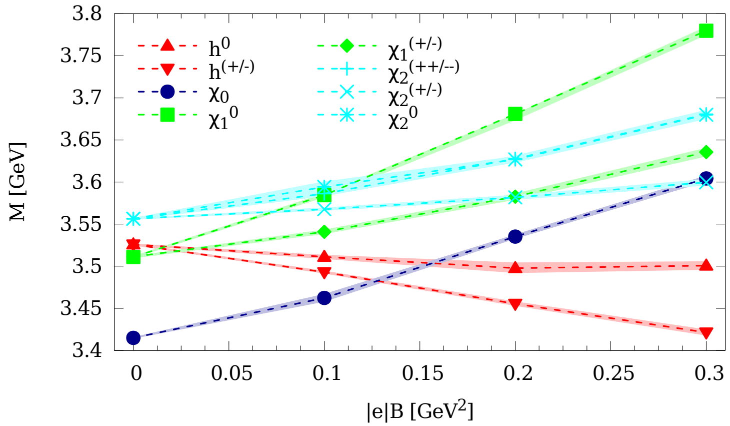
- Dependence on the quark mass still entangled with discretization effects. Need for better studies in the future.

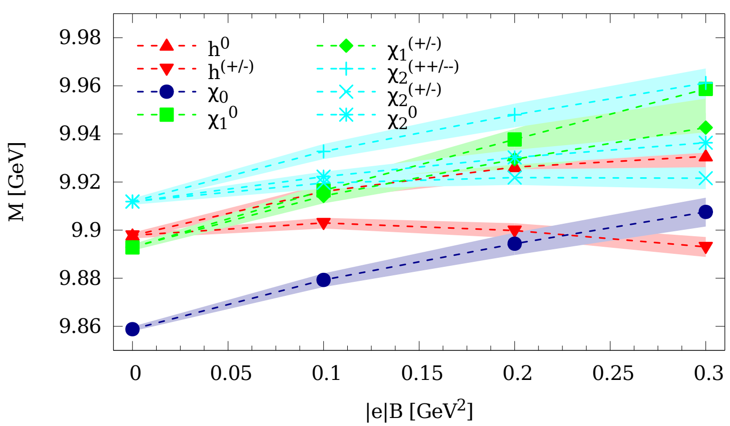
- First order transition at very high magnetic field?
Evidence from some results and models (G. Endrodi, arXiv:1504.08280)

- Breaking of string tension even at $T = 0$ at very high magnetic field?

Effects on heavy quark interactions:

Modifications of the static quark potential at $T = 0$ have consequences on quarkonia spectra which might be relevant to the early stages of heavy ion collisions





Charmonia **Bottomonia**

From C. Bonati, MD, A. Rucci, 1506.07890

Screening lengths decrease as a function of B:

does B have any influence on heavy quarkonia suppression in the QGP? Not clear, provided B survives thermalization, one should also know how the quarkonia wave function is modified by B

A direct determination of quarkonia spectral functions in the presence of B would be the most direct way to check