The influence of the magnetic fields on $v_2$ in HIC at RHIC and LHC energies

$4^{th}$ Workshop on Chirality, Vorticity and Magnetic Field in HIC

Gabriele Inghirami

(FIAS, Goethe Universität, GSI, NIC)

in collaboration with:
M. Mace, Y. Hirono, L. Del Zanna, D. Kharzeev, M. Bleicher.
Special thanks to: F. Becattini and A. Beraudo.

Firenze, 22/3/2018
About this *exploratory* work:

- **Aim 1**: explore the possible effects of $\vec{B}$ on $v_2$ at RHIC and LHC energies
- **Aim 2**: evaluate which are the most significant parameters to focus on
- **Aim 3**: find constraints on the properties of the medium
- **Method**: ideal 3D+1 MHD simulations with ECHO-QGP using Glauber i.c.
- **We focus on**: impact parameter, $\sigma \ (\sigma_{\chi})$, $T_{f.o.}$
- **Strengths**: dynamical evolution of the magnetic field coupled with the fluid
- **Weaknesses**: no resistivity, no charge distinction in produced particles, no hadronic rescattering
What is ECHO-QGP

ECHO-QGP derives from the Eulerian Conservative High-Order astrophysical code for general relativistic magnetohydrodynamics, developed by L. Del Zanna. (Del Zanna, Zanotti, Bucciantini, and Londrillo, A&A 473 (2007))

A collaboration lead by F. Becattini adapted ECHO-QGP to run second order dissipative hydrodynamical simulations of heavy ion collisions, including the computation of particle spectra following the Cooper-Frye prescription.


Floerchinger, Wiedemann, Beraudo, Del Zanna, Inghirami, Rolando, PLB 735 (2014)


Website: http://theory.fi.infn.it/echoqgp/
The basis

The fundamental equations

Energy and momentum conservation: \( d_\mu T^{\mu\nu} = 0 \)
Baryonic number conservation: \( d_\mu N^\mu = 0 \)
Second law of thermodynamics: \( d_\mu s^\mu \geq 0 \)
Maxwell equations: \( d_\mu F^{\mu\nu} = -J^\nu \) \( (d_\mu J^\mu = 0) \quad d_\mu F^{*\mu\nu} = 0 \)

The fundamental assumptions

- We neglect all dissipative effects
- We neglect polarization and magnetization effects
- We assume infinite electrical conductivity
The ideal RHMD energy-momentum tensor

Polarization and magnetization neglected

\( T^\mu_\nu = F^\mu_\lambda F^\nu_\lambda - \frac{1}{4} (F^\lambda_\kappa F^\kappa_\lambda) g^\mu_\nu \)

from Maxwell equations: \( d_\mu T^\mu_\nu = J_\mu F^\mu_\nu \)

Dissipative effects neglected:

Eckart frame = Landau frame \( \Rightarrow \) single fluid \( u^\mu \) (\( u_\mu u^\mu = -1 \))

Infinite electrical conductivity

Ohm’s law: \( J^\mu = \rho_e u^\mu + j^\mu \); \( j^\mu = \sigma^{\mu\nu} e_\nu \) \( \Rightarrow e^\mu = 0 \)

Energy-momentum tensor \( T^\mu_\nu \)

\( T^\mu_\nu = T^\mu_\nu_m + T^\mu_\nu_f \)

Matter: \( T^\mu_\nu_m = (e + p) u^\mu u^\nu + pg^{\mu\nu} \)

Electromagnetic field: \( T^\mu_\nu_f = b^2 u^\mu u^\nu + \frac{1}{2} b^2 g^{\mu\nu} - b^\mu b^\nu \)
The energy momentum tensor components

Lorentz transformations from the laboratory to the comoving frame:

\[ e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \varepsilon^{ijk} v_j B_k) \]
\[ b^\mu = (\gamma v_k B^k, \gamma B^i - \gamma \varepsilon^{ijk} v_j E_k) \]

where:

\[ \varepsilon_{ijk} \] is the Levi-Civita pseudo-tensor of the spatial three-metric
\[ \gamma = \text{Lorentz factor}, \ g_{ij} = \text{diag}(1, 1, 1) \text{ or } g_{ij} = \text{diag}(1, 1, \tau^2) \]

\[ e \text{ and } p \] are measured in the \textit{comoving fluid frame},
\[ \vec{E} \text{ and } \vec{B} \] are measured in the \textit{laboratory frame}

Components of the energy-momentum tensor

\[ \mathcal{E} \equiv -T^0_0 = (e + p)\gamma^2 - p + \frac{1}{2}(E_k E^k + B_k B^k) \]
\[ S_i \equiv T^0_i = (e + p)\gamma^2 v_i + \varepsilon_{ijk} E^j B^k \]
\[ T^i_j = (e + p)\gamma^2 v^i v_j + (p + \frac{1}{2}(E_k E^k + B_k B^k)) \delta^i_j - E^i E_j - B^i B_j \]
The evolution equations

**Ideal Ohm's law in the laboratory frame**

\[ e^\mu = 0 \Rightarrow E_i = -\varepsilon_{ijk} v^j B^k \]

**The evolution equations in conservative form**

\[ \partial_0 U + \partial_i F^i = S \]

where

\[
U = |g|^{1/2} \begin{pmatrix} \gamma n \\ S_j \equiv T^0_j \\ \varepsilon \equiv -T^0_0 \\ B^j \end{pmatrix}, \quad F^i = |g|^{1/2} \begin{pmatrix} \gamma n v^i \\ T^i_j \\ S^i \equiv -T^i_0 \\ v^i B^j - B^i v^j \end{pmatrix}, \quad S = |g|^{1/2} \begin{pmatrix} 0 \\ \frac{1}{2} T^{ik} \partial_j g_{ik} \\ -\frac{1}{2} T^{ik} \partial_0 g_{ik} \\ 0 \end{pmatrix}
\]
Time evolution of the magnetic field at the center of the grid


2D+1 simulation of Au+Au collision at $\sqrt{s_{NN}} = 200$ AGeV

**Magnetic field at $x=y=0$**

**Elliptic flow of positive pions**
New 3D+1 simulations: initial conditions for pressure

3D+1 simulation in Milne coordinates
Geometrical Glauber initial conditions.
Almost all parameters as in:
Pang et al. - Phys. Rev. C 93, 044919

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RHIC</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion</td>
<td>Au 197</td>
<td>Pb 208</td>
</tr>
<tr>
<td>$\sqrt{s_{NN}}$ (AGeV)</td>
<td>200</td>
<td>2760</td>
</tr>
<tr>
<td>$\tau_0$ (fm/c)</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>$T_{f.o.}$ (MeV)</td>
<td>154</td>
<td>154</td>
</tr>
<tr>
<td>$\epsilon_0$ (GeV/fm$^3$)</td>
<td>55</td>
<td>413.9</td>
</tr>
<tr>
<td>$\sigma_{in.}$ (mb)</td>
<td>42</td>
<td>64</td>
</tr>
<tr>
<td>$\alpha_{bin.coll.}$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\eta_{flat}$</td>
<td>5.9</td>
<td>7.0</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.4</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Thermal pressure at $\tau = 0.4$ fm/c,
Au+Au collision at $\sqrt{s_{NN}} = 200$ AGeV
impact parameter $b=12$ fm.
Initial conditions for $\vec{B}$: basic formula for point charge


\begin{align*}
B_\phi (t, x) &= \frac{Q}{4\pi} \cdot \frac{v \gamma x_T}{\Delta^{3/2}} \left(1 + \frac{\sigma v \gamma}{2} \sqrt{\Delta}\right) e^A, \\
B_r (t, x) &= -\sigma \chi \frac{Q}{8\pi} \cdot \frac{v \gamma^2 x_T}{\Delta^{3/2}} \cdot \left[\gamma(vt - z) + A\sqrt{\Delta}\right] e^A, \\
B_z (t, x) &= \sigma \chi \frac{Q}{8\pi} \cdot \frac{v \gamma}{\Delta^{3/2}} \cdot \\
&\quad \left[\gamma^2(vt - z)^2 \left(1 + \frac{\sigma v \gamma}{2} \sqrt{\Delta}\right) + \Delta \left(1 - \frac{\sigma v \gamma}{2} \sqrt{\Delta}\right)\right] e^A
\end{align*}

where: $\sigma$ is the electric conductivity, $\sigma \chi$ the chiral magnetic conductivity, $\Delta \equiv \gamma^2(vt - z)^2 + x_T^2$, $A \equiv (\sigma v \gamma / 2)[\gamma(vt - z) - \sqrt{\Delta}]$
Our initial conditions: formula for two colliding nuclei

The nuclei freely propagate into a medium with finite conductivity, before and after the collision. For each of them (uniformly charged disk approximation):

\[ B_Z(x_-, b_1) = \int 2\sqrt{R_A^2 - b'^2}\rho B(x_-, |b_1 - b'|)(-\sin \psi_1 \hat{x} + \cos \psi_1 \hat{y})d^2b \]  

A similar expression holds for the other nucleus and the total magnetic field is the sum of the two.

Orientation of the magnetic field

Magnetic field of a lead ion at LHC energy.

Magnetic field of *classic* origin, at
\( \tau = 0.2 \text{ fm}/c, \eta = 0, \ b=8 \text{ fm}, \ \sigma = 5.8 \text{ MeV} \)

Magnetic field of *chiral* origin, at
\( \tau = 0.2 \text{ fm}/c, \eta = 0, \ b=8 \text{ fm}, \ \sigma_\chi = 1.5 \text{ MeV} \)
\[\nabla \cdot \vec{B} = 0: \ B_z \neq 0 \text{ not plotted !} \]
Ratio between initial magnetic and thermal pressure

3D+1 simulation of Au+Au collision at $\sqrt{s_{NN}} = 200$ AGeV
Conductivities of the medium ($\tau \leq \tau_0$): $\sigma = 5.8$ MeV, $\sigma_\chi = 1.5$ MeV

$B_y$ at $\tau = 0.4$ fm/c and $\eta = 0$

Magnetic/thermal pressure ratio
Magnetic field at later times

RHIC energy, $b=12$ fm,

$\tau = 2$ fm/$c$

RHIC energy, $b=12$ fm,

$\tau = 4$ fm/$c$
Importance of the initial B magnitude

RHIC energy, $b=12$ fm

LHC energy, $b=12$ fm
$v_2$ at different impact parameters

RHIC energy, $b=2$ fm

RHIC energy, $b=8$ fm
Impact of the initial conductivity

RHIC energy, $b=12$ fm

RHIC energy, $b=12$ fm, $B_{in.} = 3B_0$

Preliminary results, to be checked!
Separate effects of classic/chiral contribution

The magnetic field of chiral origin seems to play only a minor role (we are neglecting the dynamical evolution of axial currents).

RHIC energy, $b=12$ fm, $B_{in.} = 4B_0$

LHC energy, $b=12$ fm, $B_{in.} = 4B_0$
The impact of $\vec{B}$ on $v_2$ seems to be limited both at RHIC and LHC. However, the many uncertainties in the pre-equilibrium phase might have led to an underestimate of the initial $\vec{B}$ field. In this case, the magnetic field might produce an enhancement of $v_2$ in peripheral collisions. But probably resistivity and viscosity strongly suppress this effect. And hadronic rescattering might make measurements more difficult. Moreover, assuming an initial null velocity field is a rough approximation. Anyway, the basic MHD assumption $e^\mu = 0$ is not realistic: arXiv:1803.06695. Moreover, we have seen in the talk by N. Tanji that $\mathbf{E} \cdot \mathbf{B} \neq 0$. Eventually, taking into account EM fields in the Cooper-Frye (arXiv:1705.07842) prescription requires to go beyond ideal MHD.
Future perspectives

- Ongoing collaboration (also with M. Kaminski) to include resistivity and dynamical evolution of axial charges (formalism almost completed by Y. Hirono, however numerical implementation might be challenging)
- Event by event simulations coupled with UrQMD (hadronic rescattering, resonances decay)
- Use of the $\gamma$ correlator or the brand new $R_{\Psi_m}(\Delta S)$ (see R. Lacey’s talk)
- Comparisons with $\vec{B}$ estimates based on $\Lambda - \bar{\Lambda}$ polarization splitting (see M. Lisa’s talk)
- Connections with lattice QCD results (see M. D’Elia’s talk)