Competition between the rotational effect and the finite-size effect on relativistic fermions

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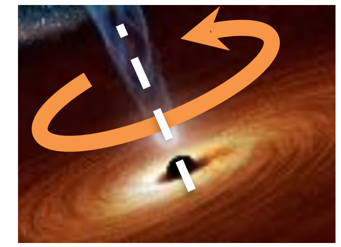
HL. Chen, K. Fukushima, XG. Huang, KM, PRD 93, 104052 (2016)
S. Ebihara, K. Fukushima, KM, PLB 764, 94 (2017)
HL. Chen, K. Fukushima, XG. Huang, KM, PRD 96.054032 (2017)

Rotating Relativistic Systems

binary star merger

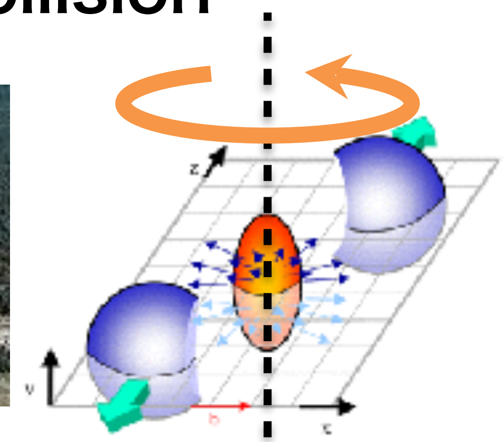
NASA/Tod Strohmayer

black holes

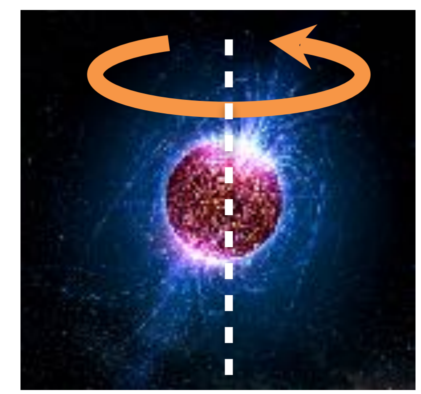


NASA/JPL-Caltech

heavy-ion collision



neutron stars



rotating (and magnetized) QCD systems

Heavy-ion collisions $eB \sim 10^{18}\,\mathrm{G}$ Skokov et al. (2009)

$\Omega \sim 10\,\mathrm{MeV}$ (local rotation) Jiang, Lin, Liao (2016) Deng, Huang (2016)

Magnetar $eB \sim 10^{15}\,\mathrm{G}$ (surface) $eB \sim 10^{18-20}\,\mathrm{G}$ (interior)

Duncan, Thompson (1992) Lai, Shapiro (1991) Ferrer et al. (2010)

$R\Omega \sim 10^{-1}$ Marshall et al. (2004)

Contents

Contents (& Goals)

Part I Finite-size system with Ω

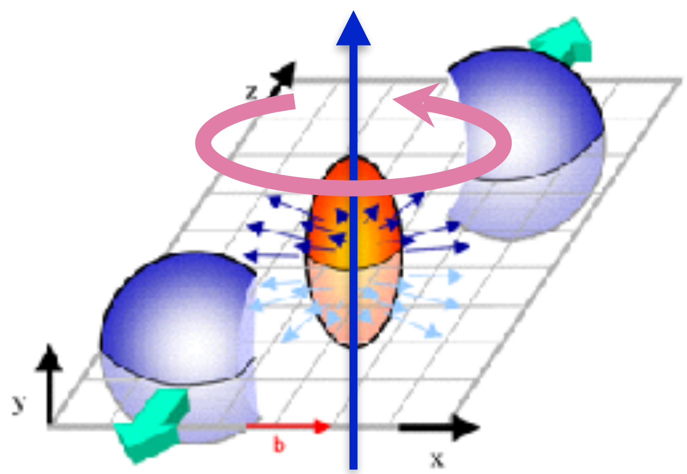
No rotational effect at zero temperature

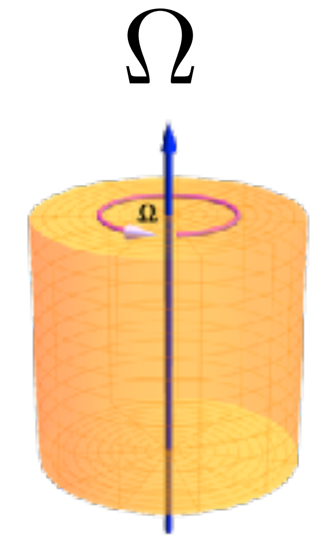
Part II Finite-size system with Ω and eB

B-field enables rotation to affect thermodynamics

Part I Finite-size system with Ω

Rigidly Rotating System





Chernodub, Gongyo (2016)

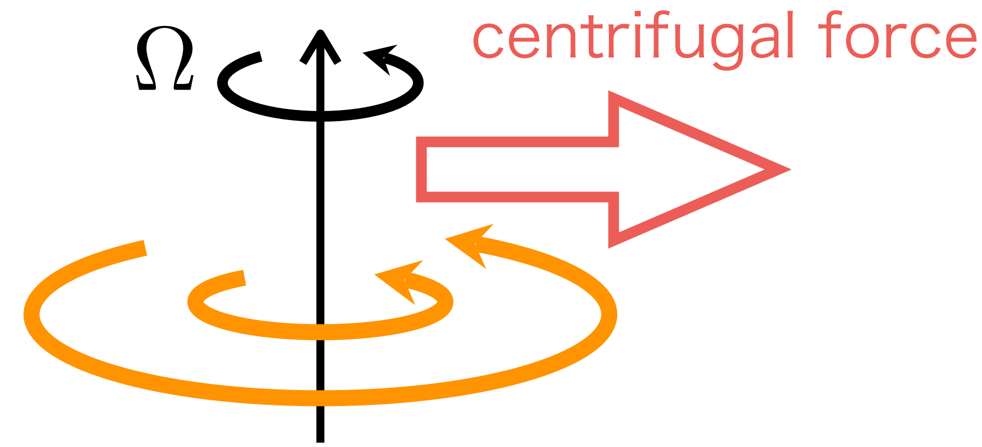
$$H \to H - \Omega \cdot \boldsymbol{L}$$

$\mu_{\rm eff}$

Ex.) rotating bosons

$$n_{\rm BE} = \frac{1}{e^{\beta(E-\Omega\ell)} - 1}$$

Landau, Lifshitz (1958)



larger orbit : favored

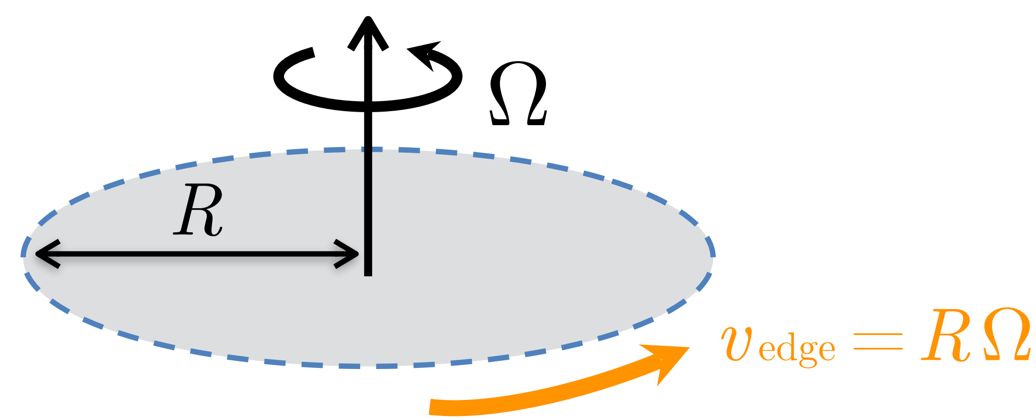
Finite-size System

$$n_{\mathrm{BE}} = \frac{1}{e^{\beta(E-\Omega\ell)} - 1}$$

$n_{\mathrm{BE}} < 0$ for large Ω ??

causality constraint

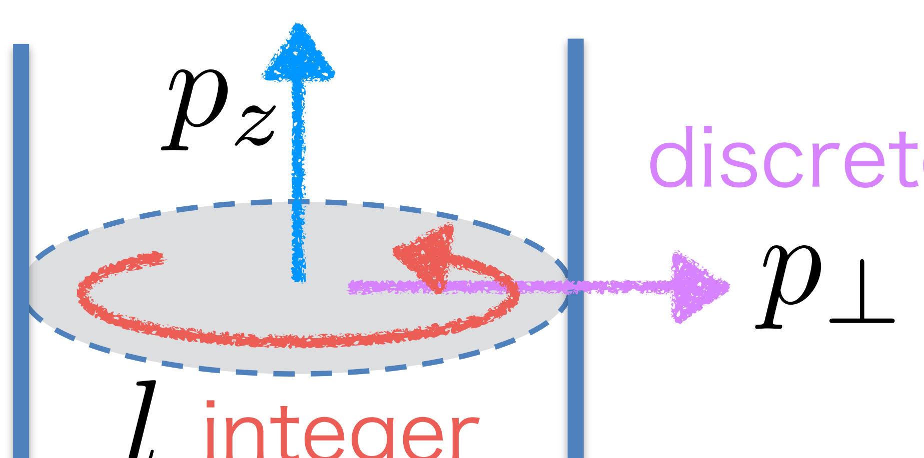
$$v_{\mathrm{edge}} = \Omega R \leq 1$$



$\longrightarrow$ $R \leq 1/\Omega < \infty$

Rotating systems must be finite-size

continuous

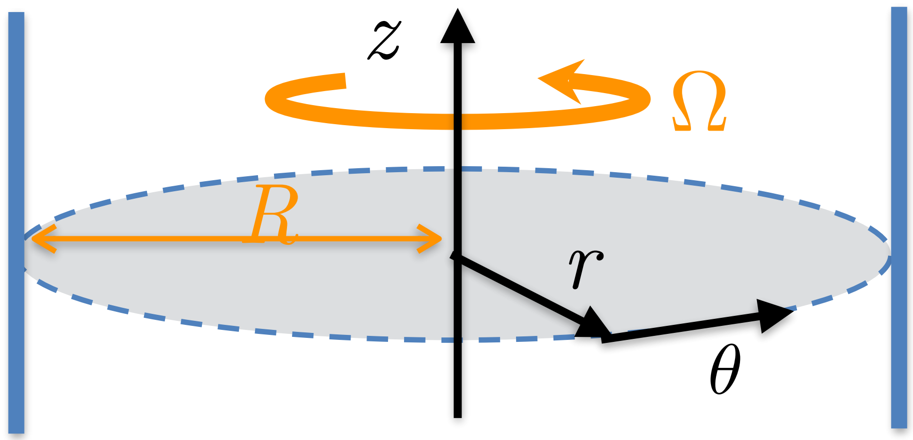


discrete ← finite-size effect

Dirac eq. in Cylinders

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2$$

$$[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m]\psi = 0$$



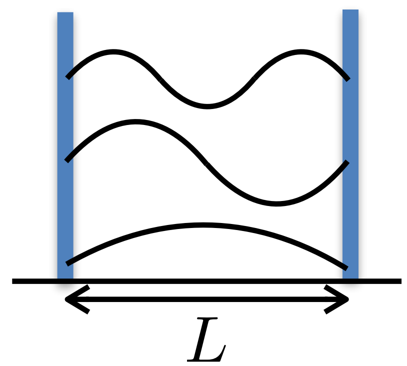
$$\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$$

$$\longrightarrow \left[-\partial_0^2 + \partial_z^2 - m^2 + \partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\theta^2\right]\psi_1 = 0$$

$$\longrightarrow \psi_1 = e^{-ip_0 t + ip_z z + il\theta} J_l(p_\perp r)$$

Momentum Discretization

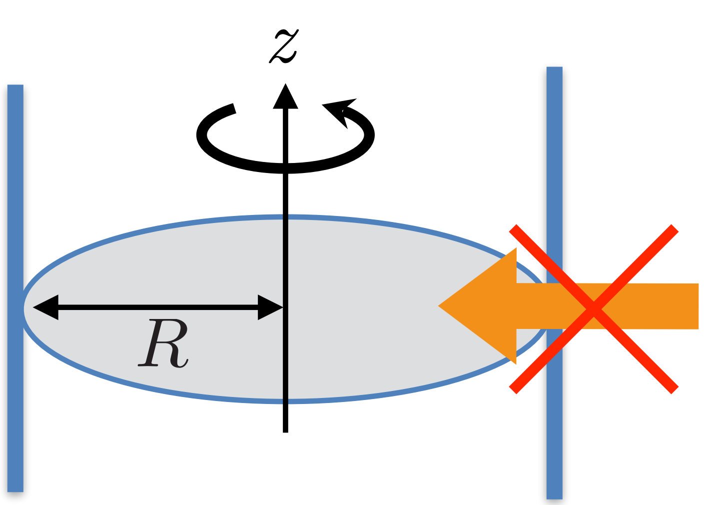
Ex.) Dirichlet b.c.



$$\sin(px)\big|_{x=L} = 0$$

$$\longrightarrow \quad p = \frac{n\pi}{L} \geq \frac{\pi}{L}$$

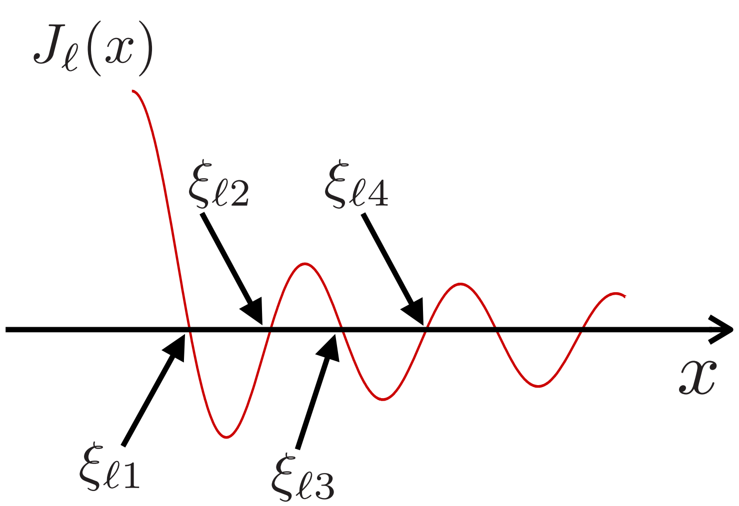
NO incoming current



$$\int dz d\theta \, \bar{\psi}\gamma^r\psi\Big|_{r=R} = 0$$

$$\longrightarrow \quad p_\perp \geq \frac{\xi_{l,1}}{R} \simeq \frac{2.4}{R}$$

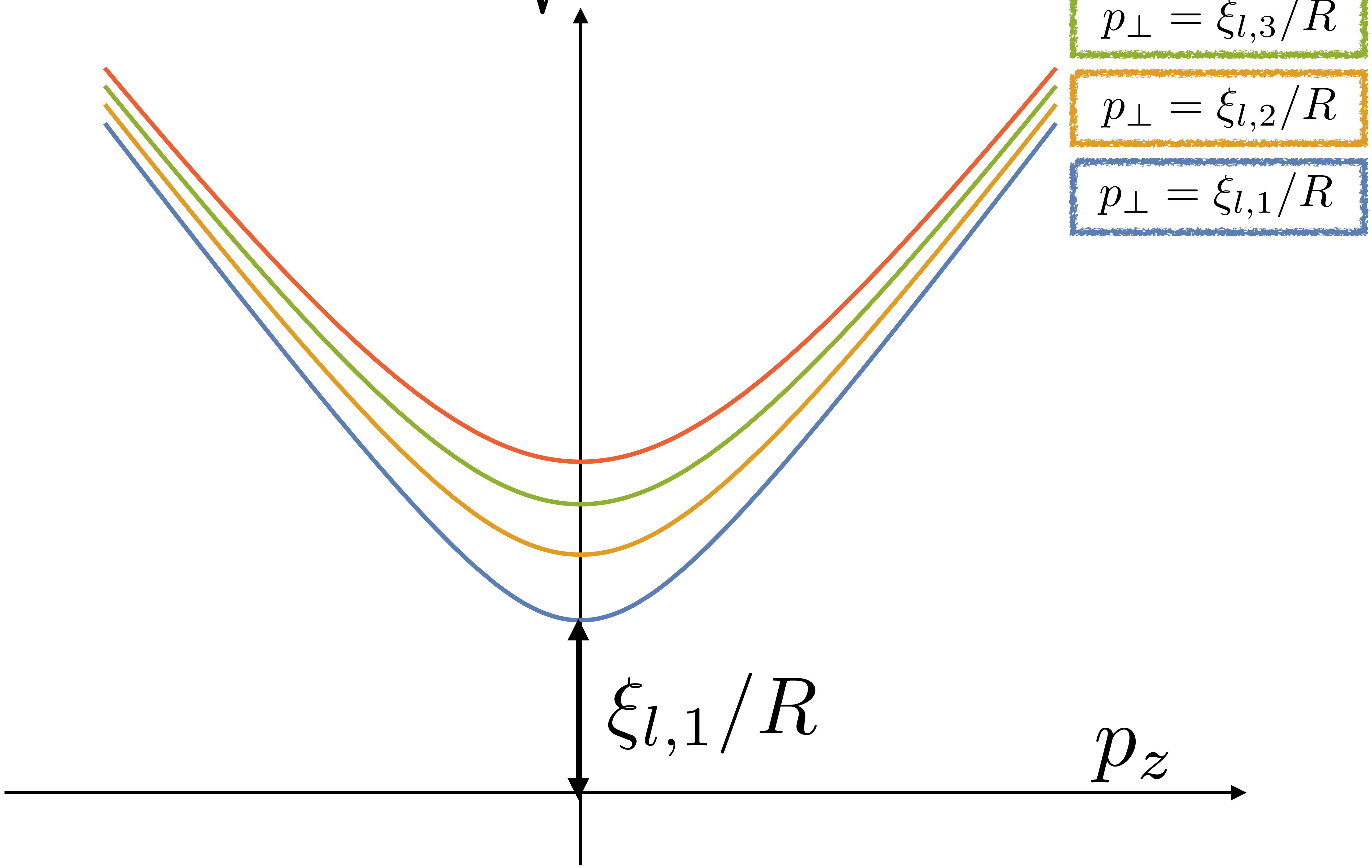
IR gapped mode



$$x = \xi_{l,k} \ : \ \text{the } k\text{th root of } J_l(x)$$

Rotational Effect at $T = 0$

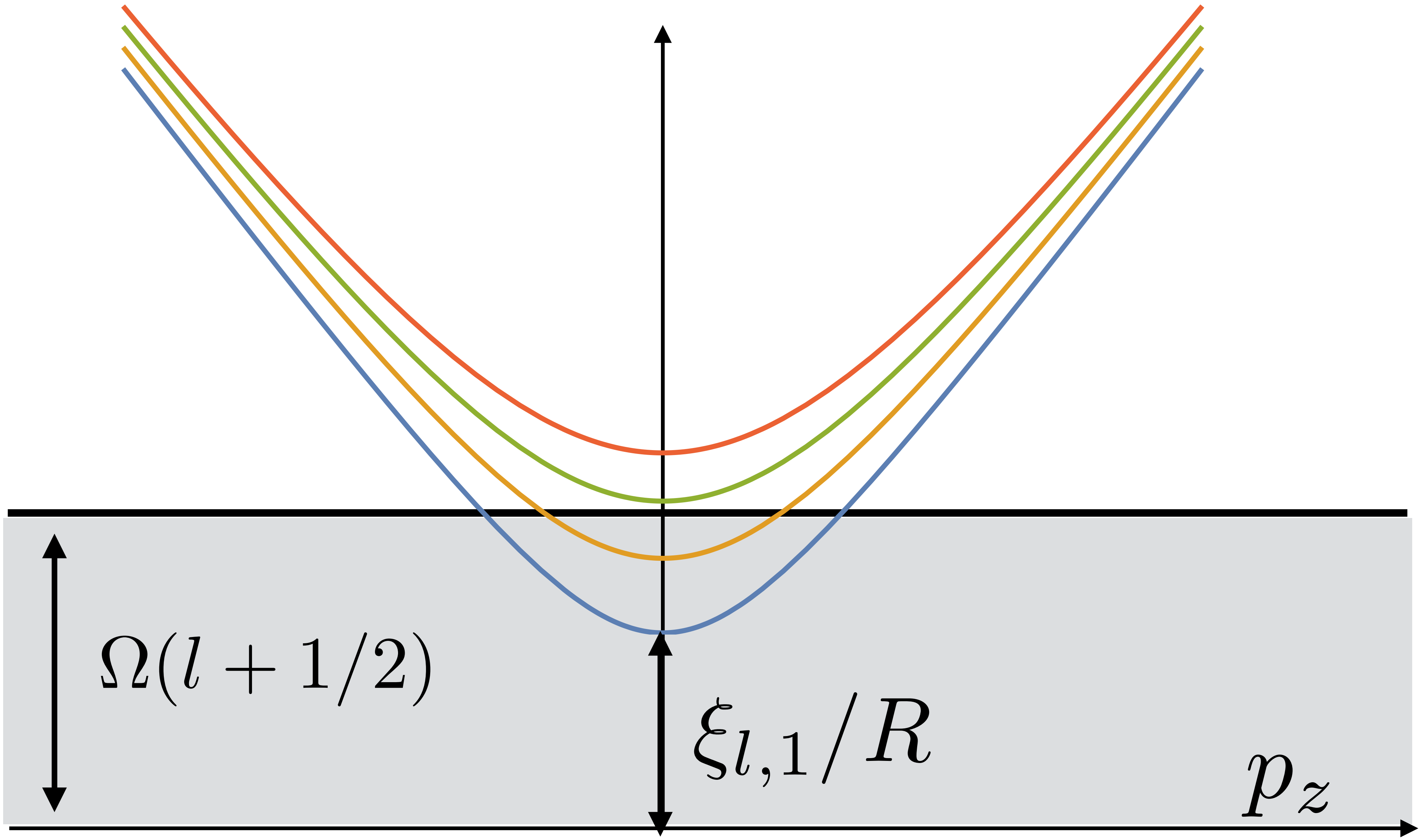
$$\varepsilon = \sqrt{p_\perp^2 + p_z^2}$$



Rotational Effect at $T = 0$

$$f(\varepsilon, j) = \frac{1}{e^{\beta(\varepsilon - \Omega j)} + 1} \longrightarrow f(\varepsilon, j) = \theta(\Omega j - \varepsilon)$$

Ωj = effective chemical potential



visible

Rotational Effect at $T = 0$

$$f(\varepsilon, j) = \frac{1}{e^{\beta(\varepsilon - \Omega j)} + 1} \longrightarrow f(\varepsilon, j) = \theta(\Omega j - \varepsilon)$$

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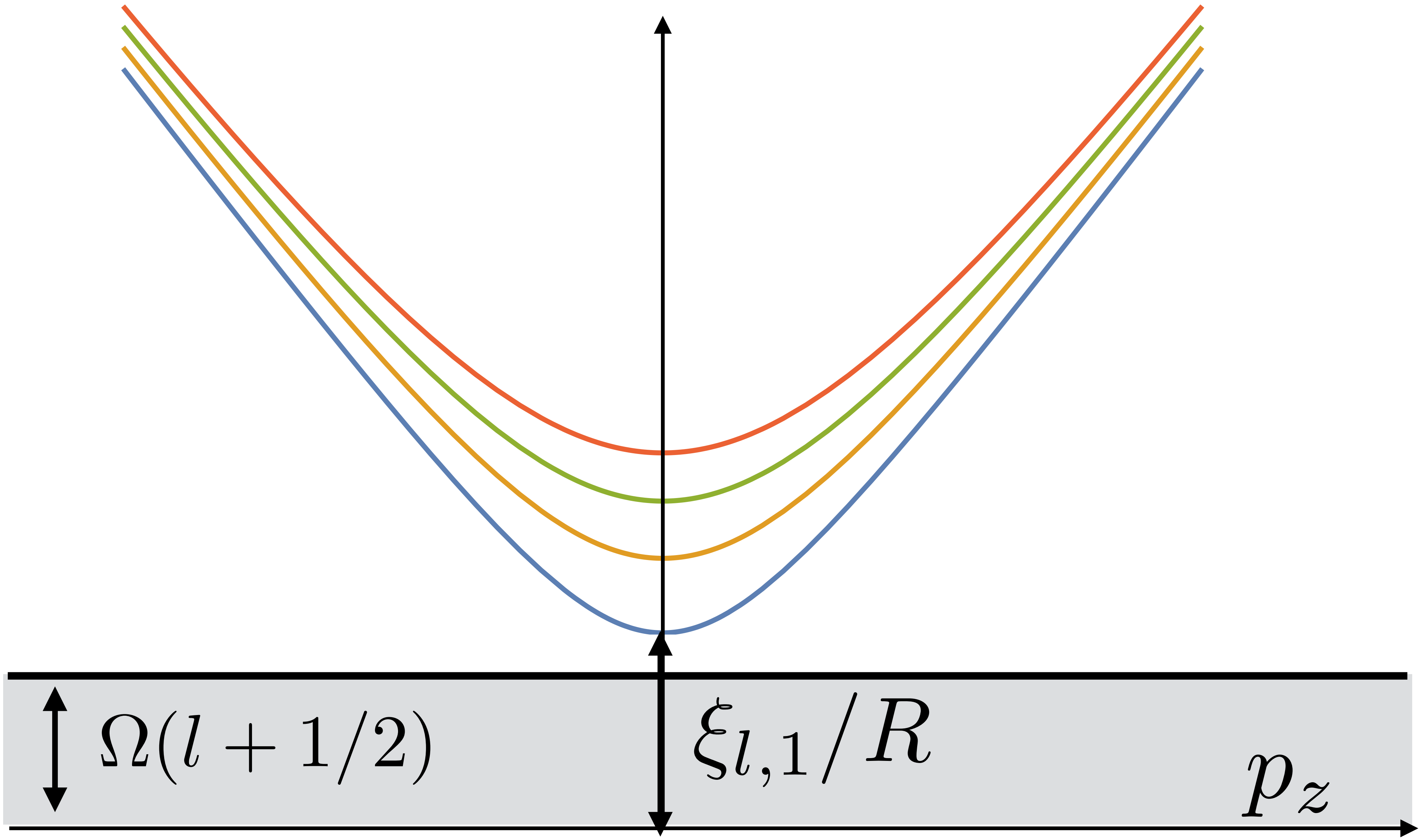


invisible

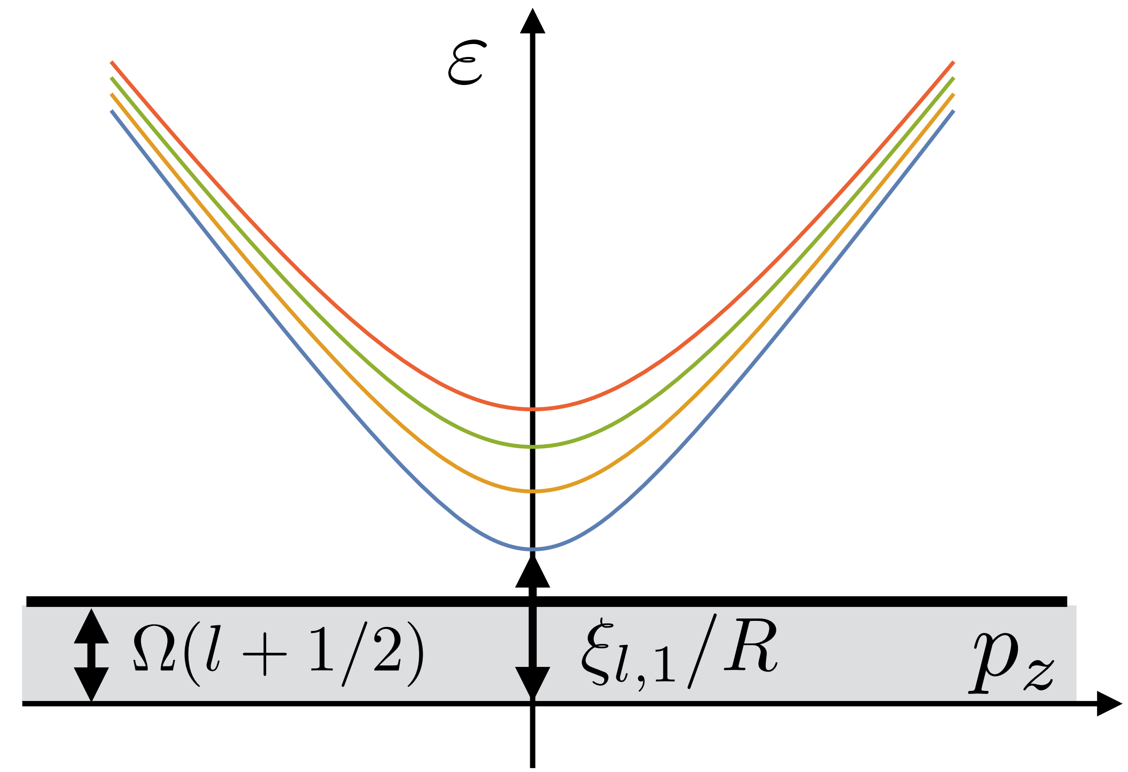
cf. Silver Blaze

Which is true?

causality $\Omega \leq 1/R$

$$\xi_{l,1}/R > \Omega(l + 1/2)$$

for arbitrary l



finite-size effect → **NO rotational effect at $T = 0$**

Ebihara, Fukushima, KM (2016)

Note : visible at high T

CVE $j_5 = \frac{T^2}{12}\Omega$

Vilenkin (1979)

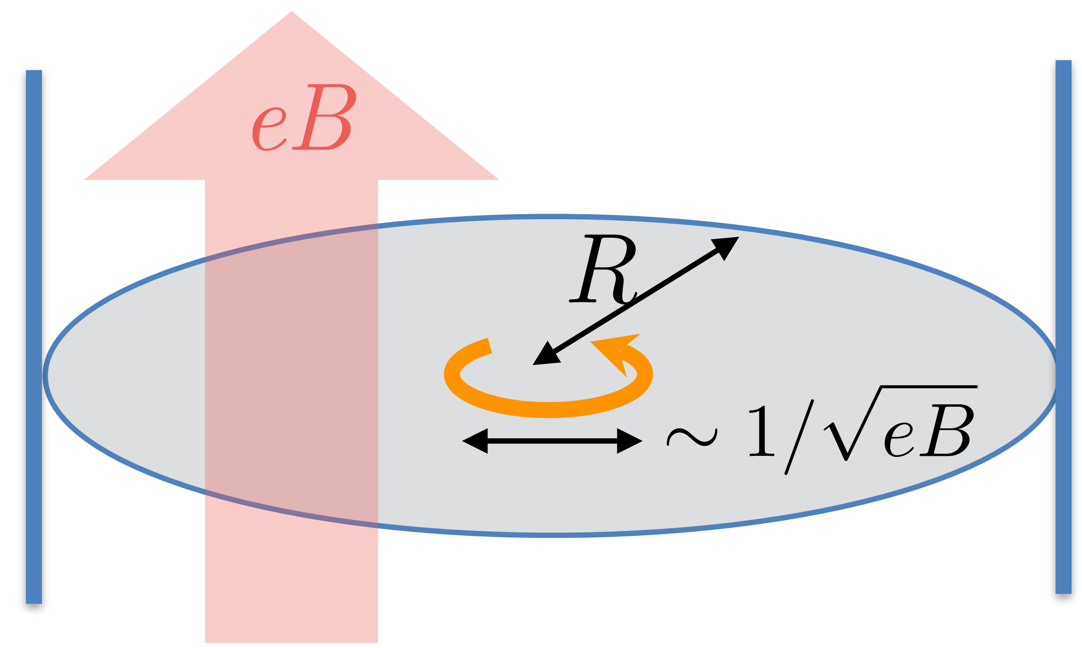
Part II Finite-size system with Ω and eB

Cyclotron Motion

(1) $1/\sqrt{eB} \ll R$

$\longrightarrow$ $p_\perp = \sqrt{2neB}$

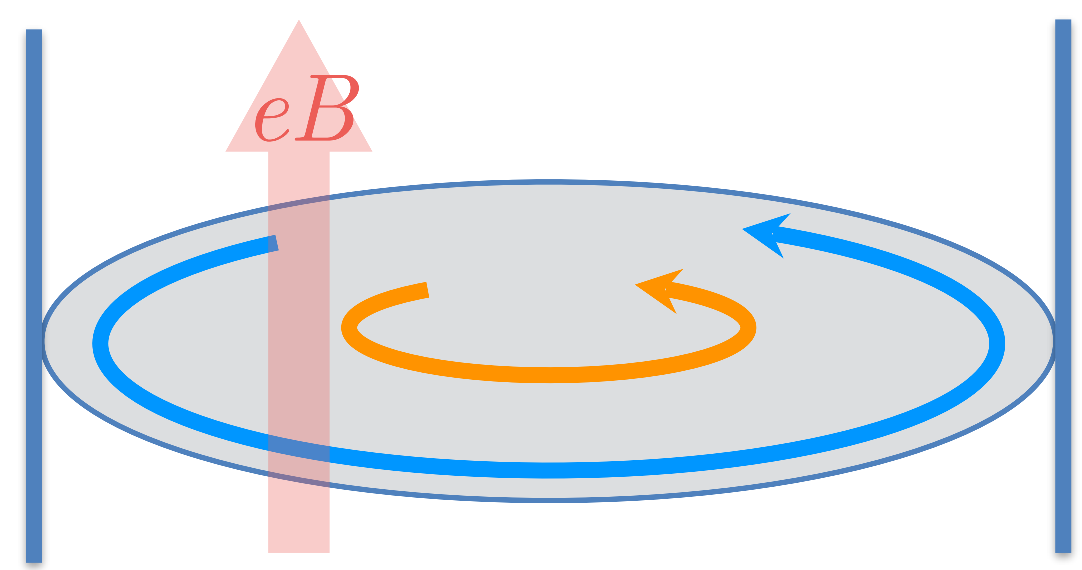
independent of l



(2) $1/\sqrt{eB} \lesssim R$

small l $\longrightarrow$ still $p_\perp \simeq \sqrt{2neB}$

large l $\longrightarrow$ modified?

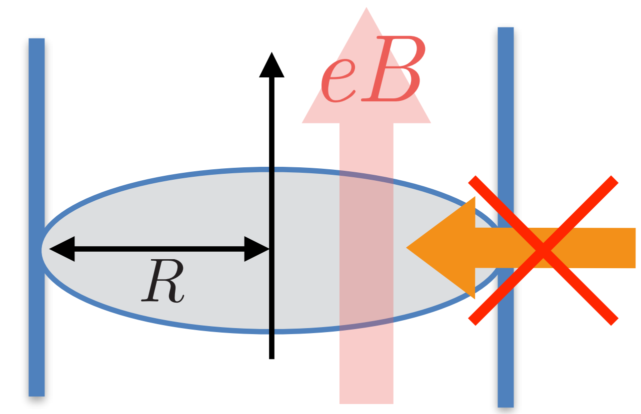


"Incomplete Landau quantization"

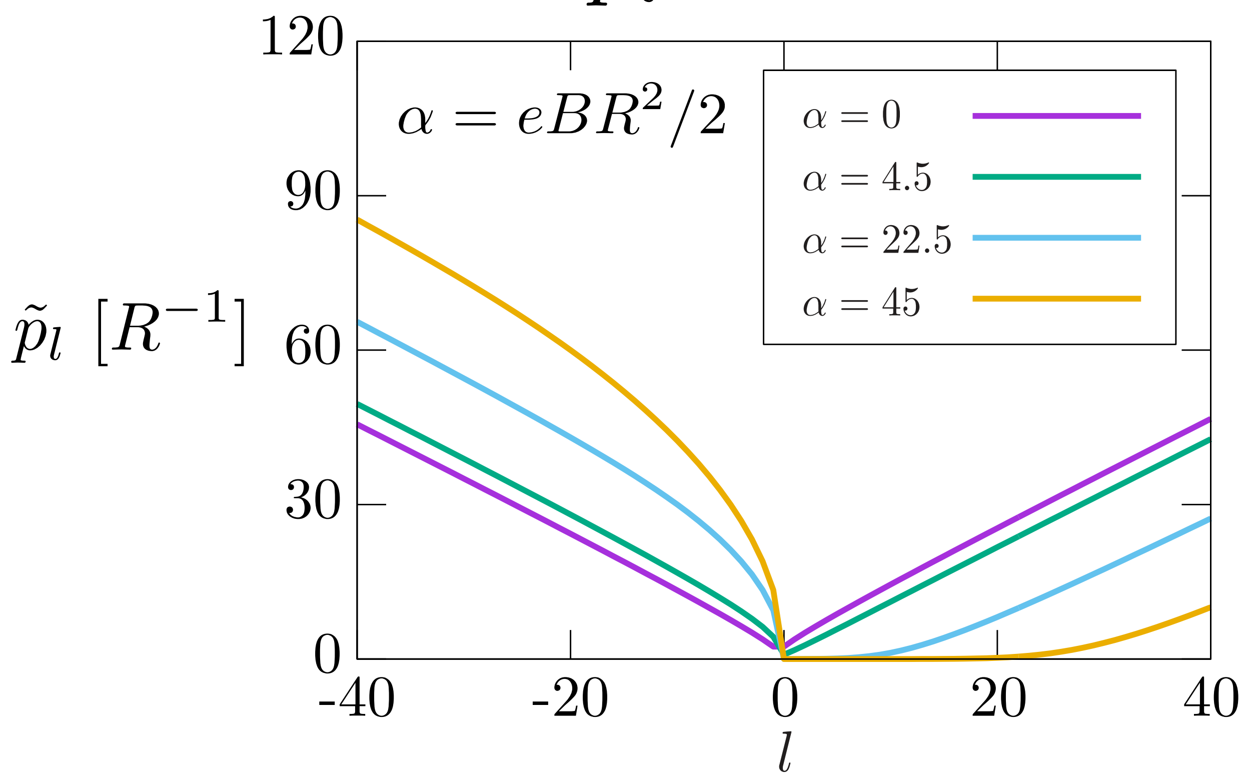
Incomplete Landau Level

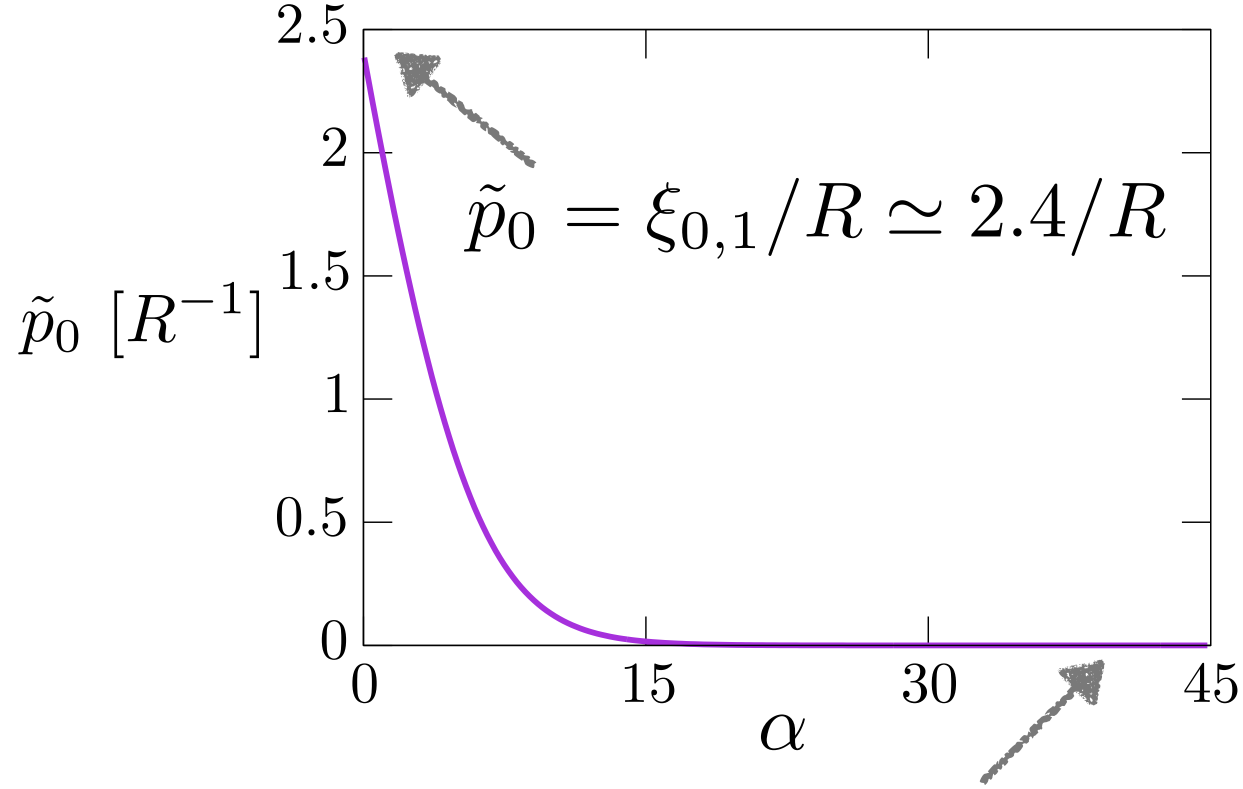
Chen, Huang, Fukushima, KM (2017)

$$[i\gamma^\mu(\partial_\mu + ieA_\mu + \Gamma_\mu)]\psi = 0$$ with



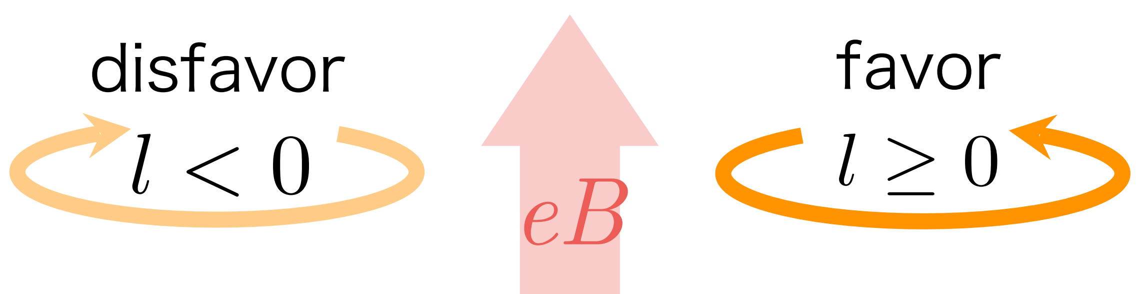
$\tilde{p}_l$ = lowest transverse momentum for l



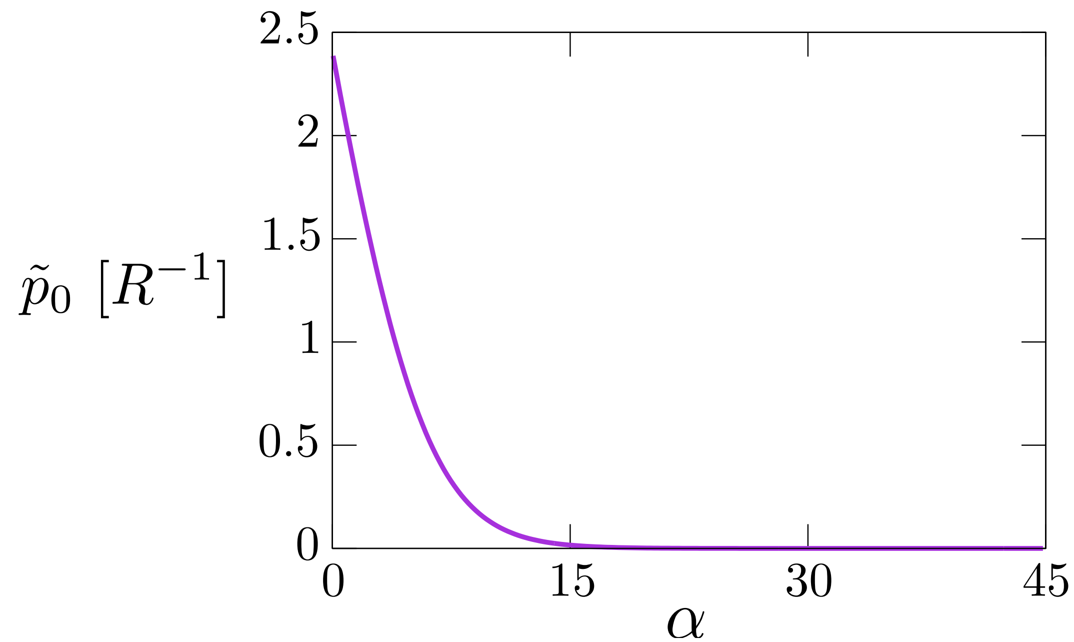


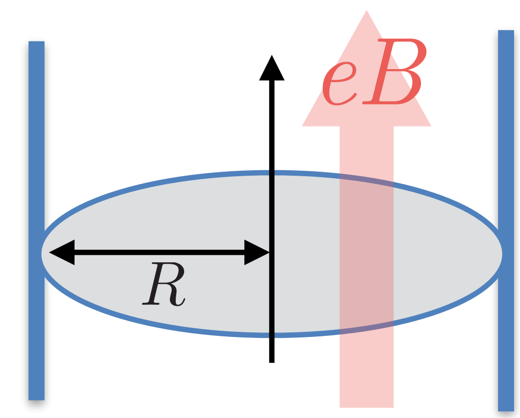
$\tilde{p}_0 = 0$

Landau zero mode

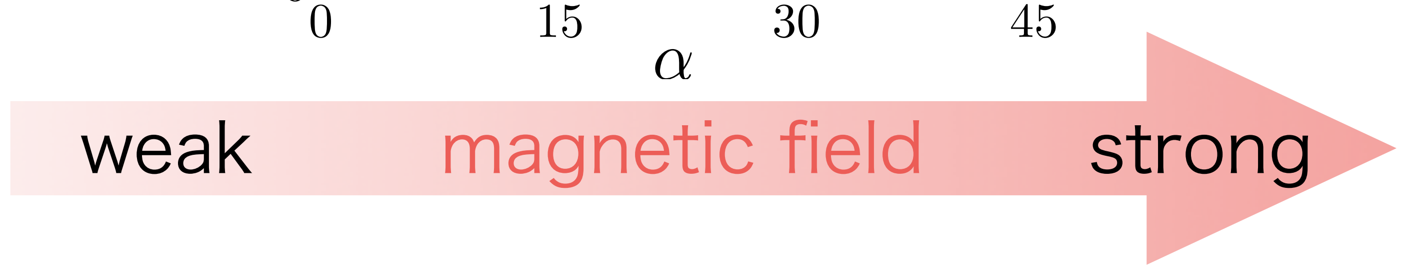


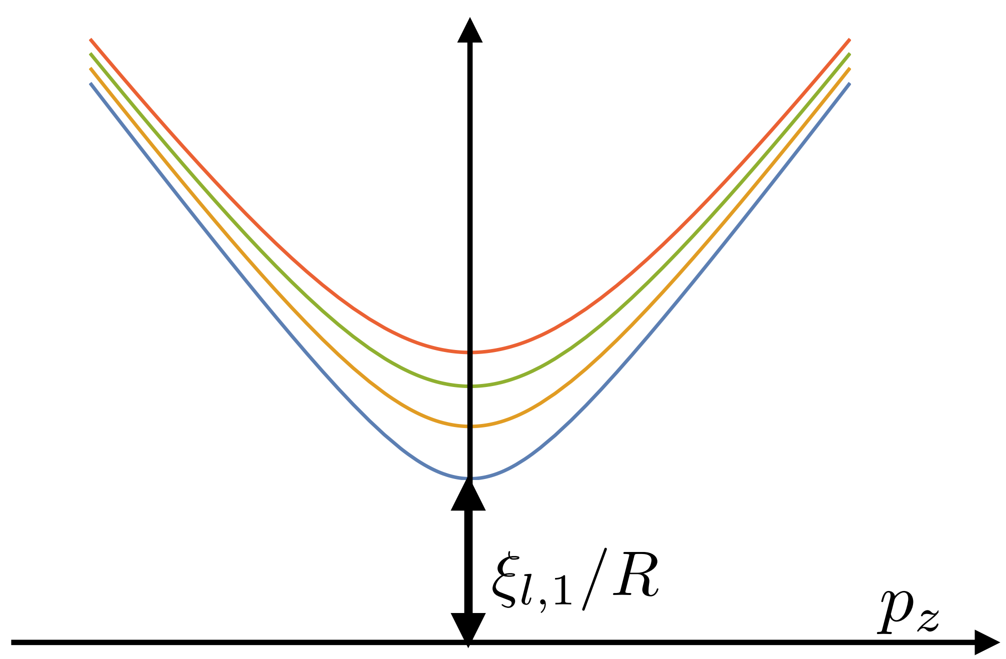
Gapped to Gapless

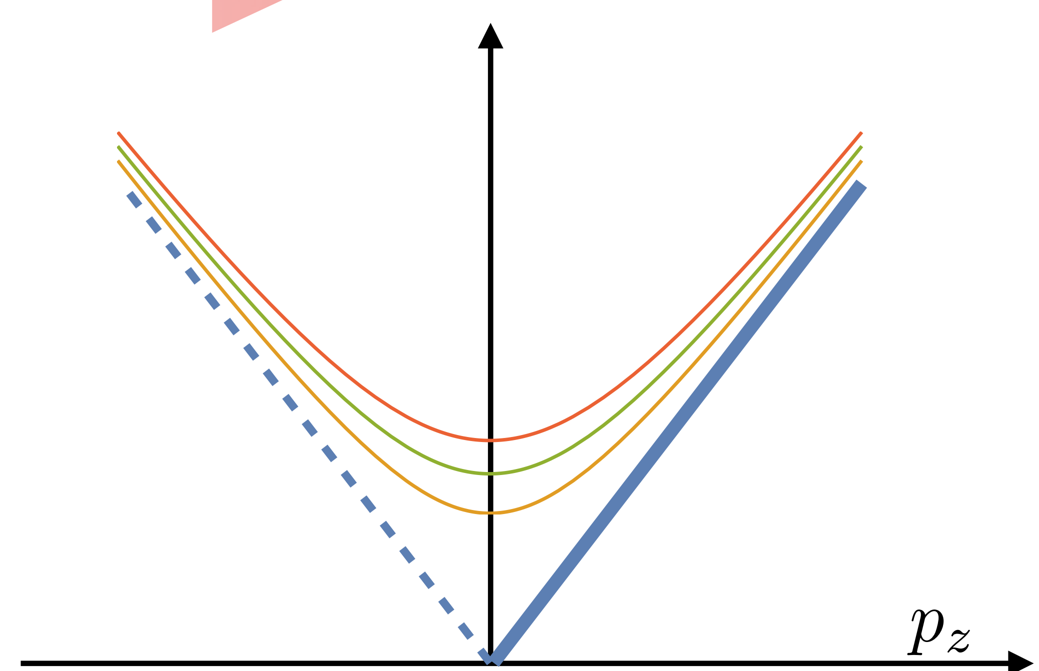




$$\alpha = eBR^2/2$$







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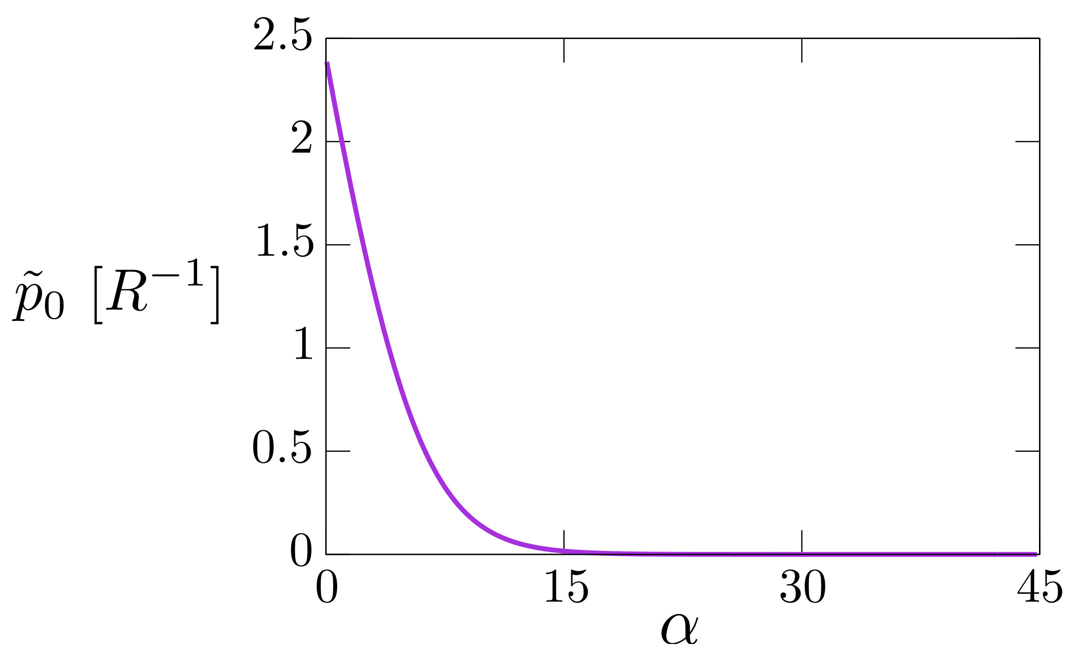
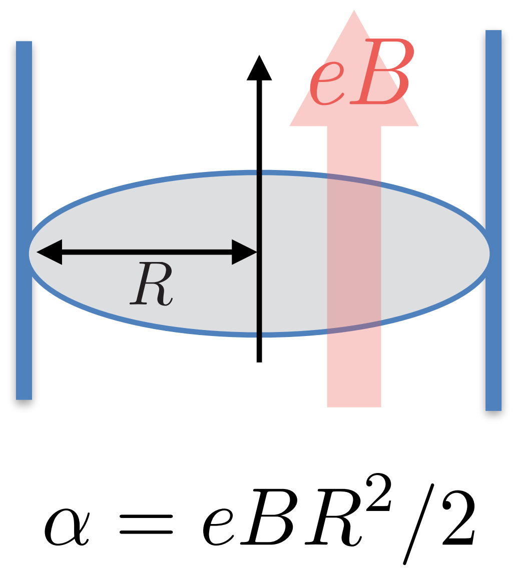
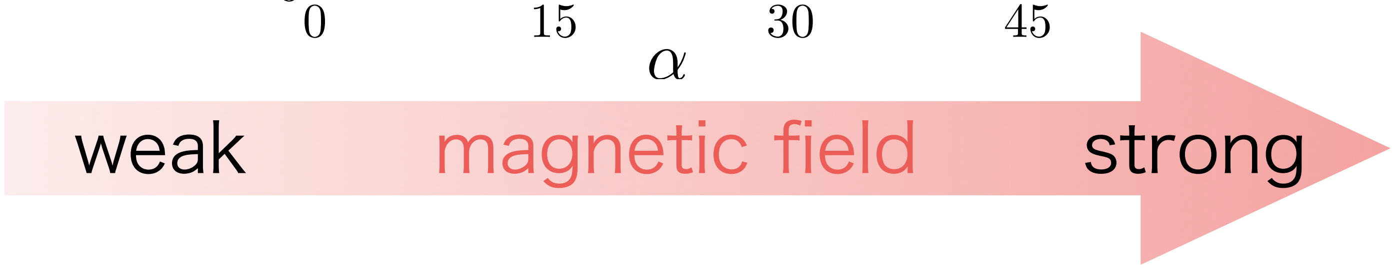
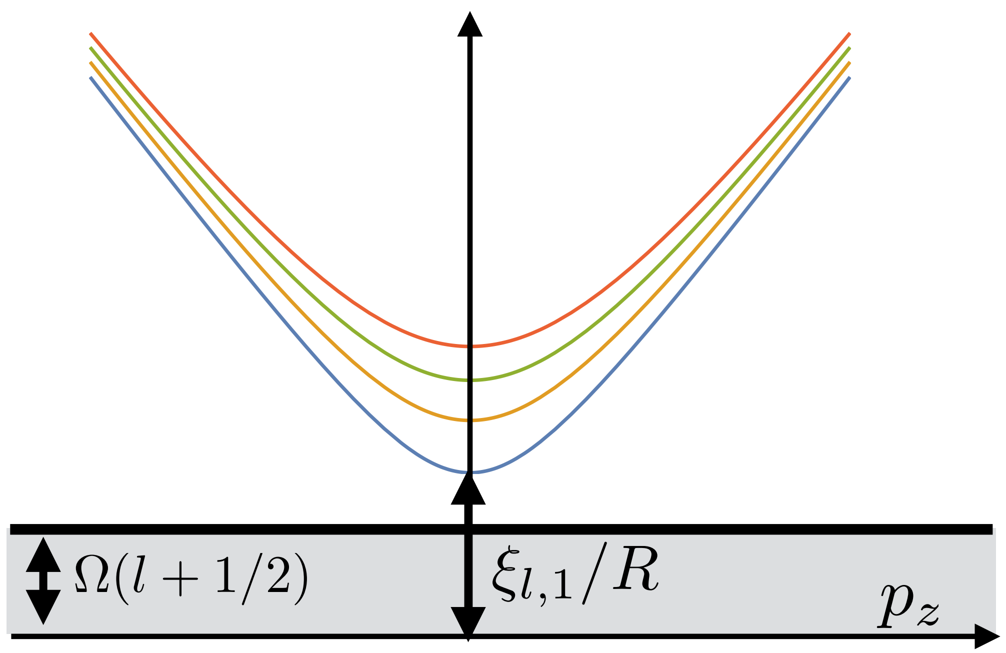
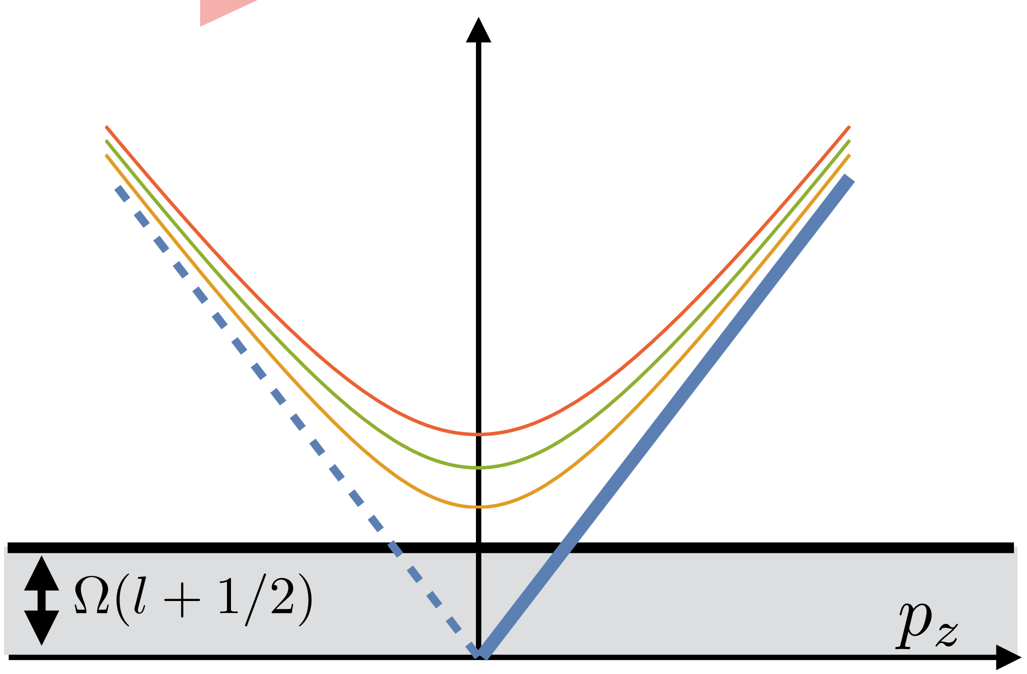






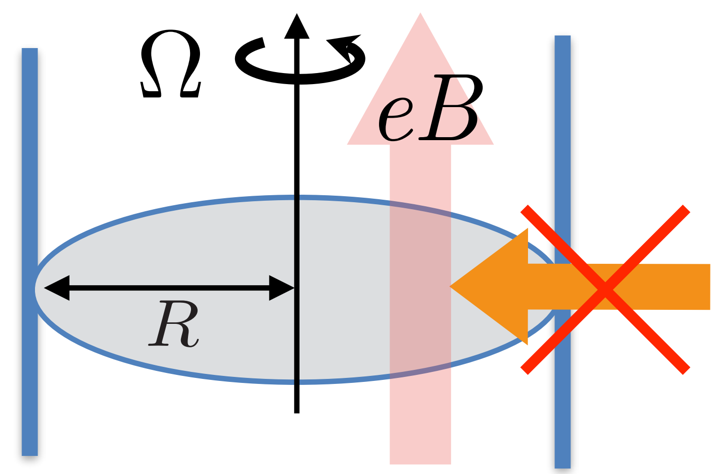




visible rotational effect due to magnetic field

Ex.1) Density Induced by Rotation

$$[i\gamma^\mu(\partial_\mu + ieA_\mu + \Gamma_\mu)]\psi = 0$$ with



Ωj = effective chemical potential $\quad f_\pm(\varepsilon) = \dfrac{1}{e^{\beta(\varepsilon \mp \Omega j)} + 1}$

$$n(r) = \langle \psi^\dagger(x)\psi(x) \rangle$$
$$= \sum_{p_z, p_\perp} \Big[f_+(\varepsilon) - f_-(\varepsilon) \Big] \times \Big(r\text{-dependence} \Big) \quad \longrightarrow \quad n(r=0) \xrightarrow[\sqrt{eB} \gg \Omega]{} \frac{eB\Omega}{4\pi^2}$$

temperature independent

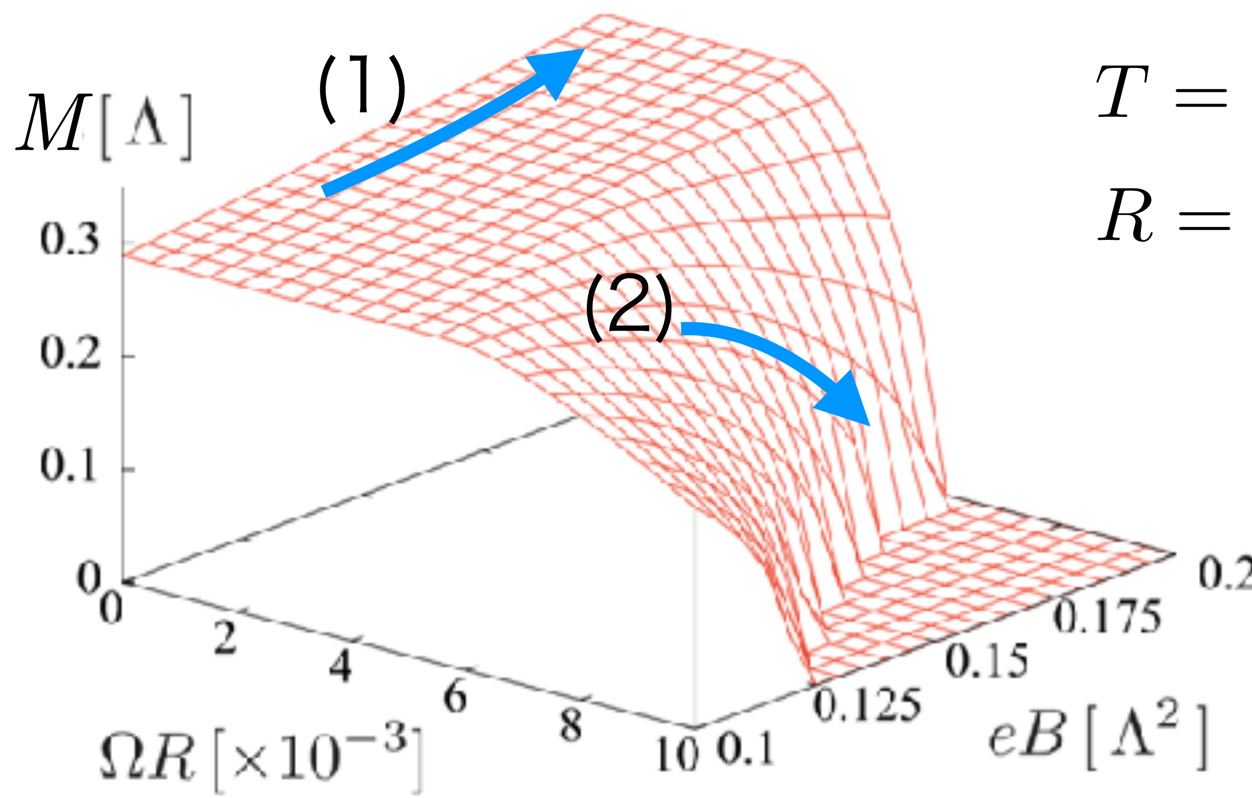
Hattori, Yin (2016)

Ebihara, Fukushima, KM (2017)

Ex.2) Chiral Symmetry Breaking

Chen, Huang, Fukushima, KM (2016)

NJL model (mean field approx.) + homogeneity



$T = 0$

$R = 10^3\ [\Lambda^{-1}]$

(1) eB increases → M increases — Magnetic Catalysis

(2) eB increases → M decreases — Inverse MC

Cf.) Ebert, Klimenko (1999) Preis, Rebhan, Schmitt (2012)

“rotational magnetic inhibition”

Summary & Outlook

Summary

- No rotational effect at $T = 0$
- B-field leads to a visible rotational effect (even at $T = 0$)
- Rotational magnetic inhibition : inverse phenomenon for the MC

Outlook

- Rotation yields abundant phase structures

Chen, Huang, Fukushima, KM (2016)
Jiang, Liao (2016) Chernodub, Gongyo (2016)
Liu, Zahed (2017) Huang, Nishimura, Yamamoto (2017)

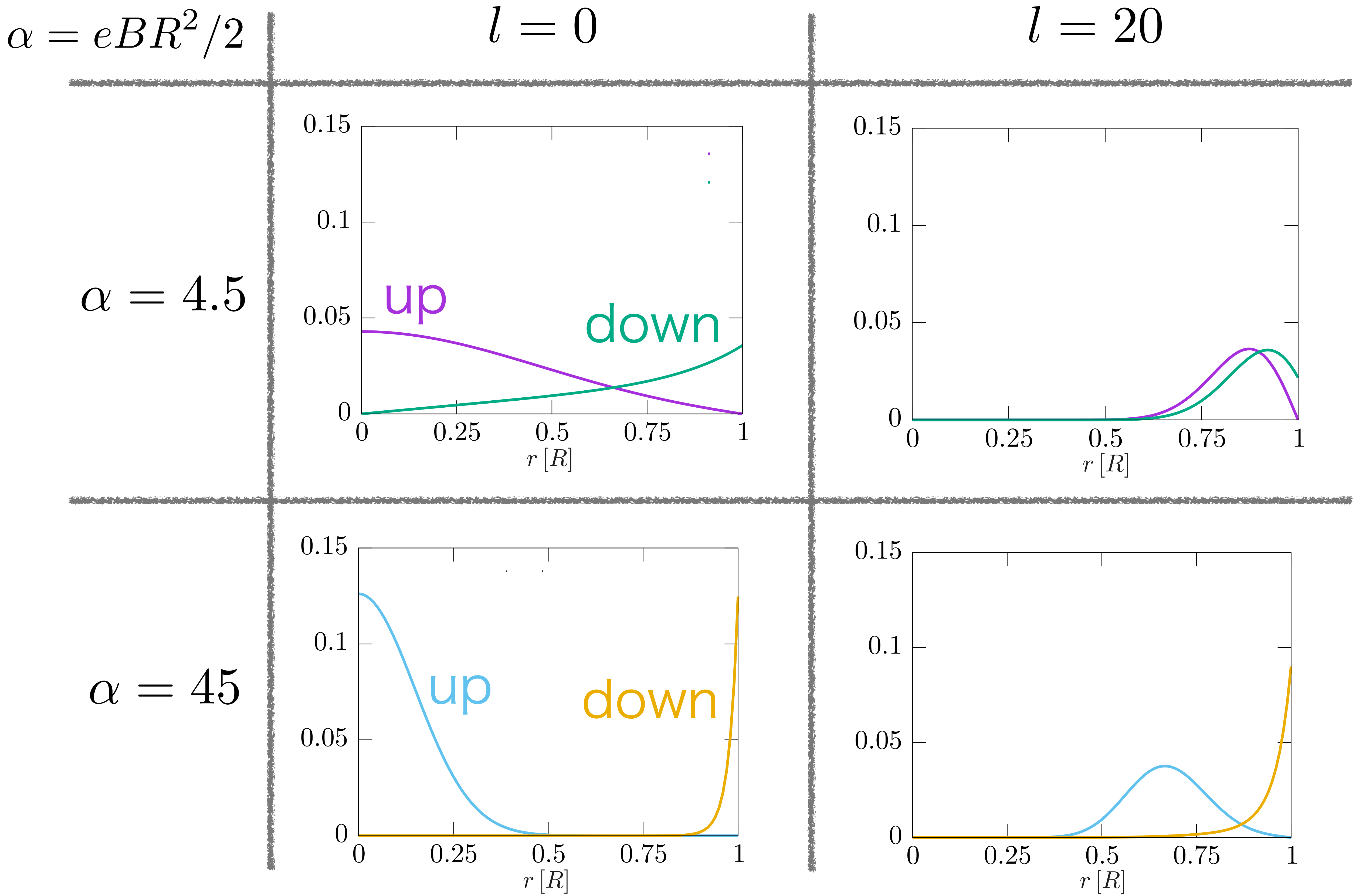
- Novel anomalous (magneto-vorticical) correction $T^{ij} = \# \, eB^i\Omega^j$

Hattori, Huang, KM (in preparation)

- Finite-size system under B-field EdH effect for chiral fermions

Fukushima, Hirono, Huang, Kharzeev, KM (in preparation)

Wave Functions of Lowest Modes



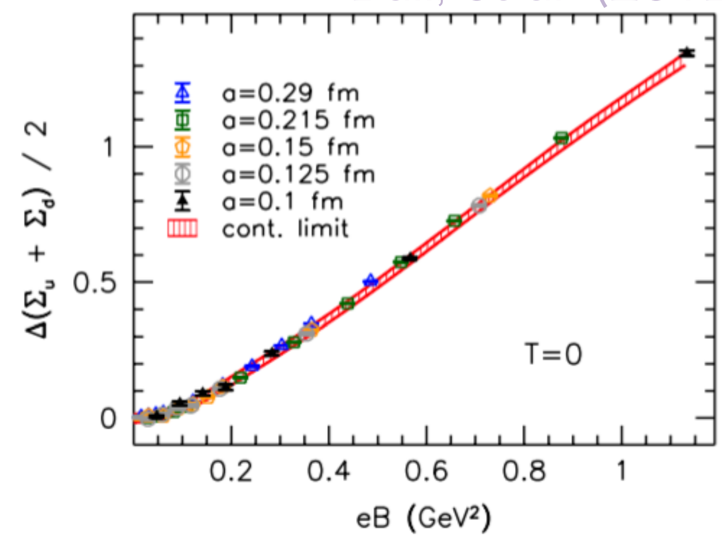
Magnetic Catalysis

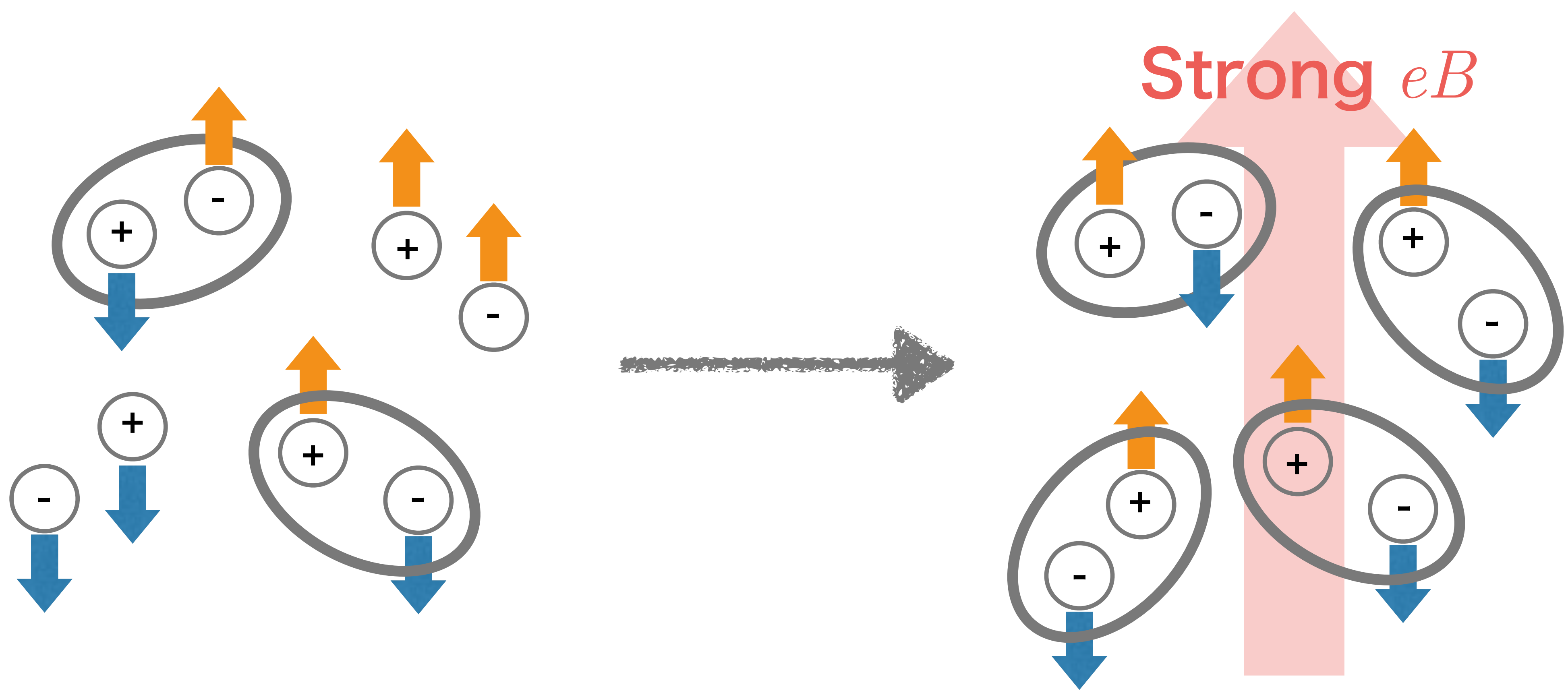
eB increases → M increases

$$M \sim \Lambda e^{-\frac{2\pi^2}{GeB}}$$

Klimenko (1992)
Gyusynin, Miransky, Shovkovy (1994)

Bail, et al. (2012)





Inverse Magnetic Catalysis

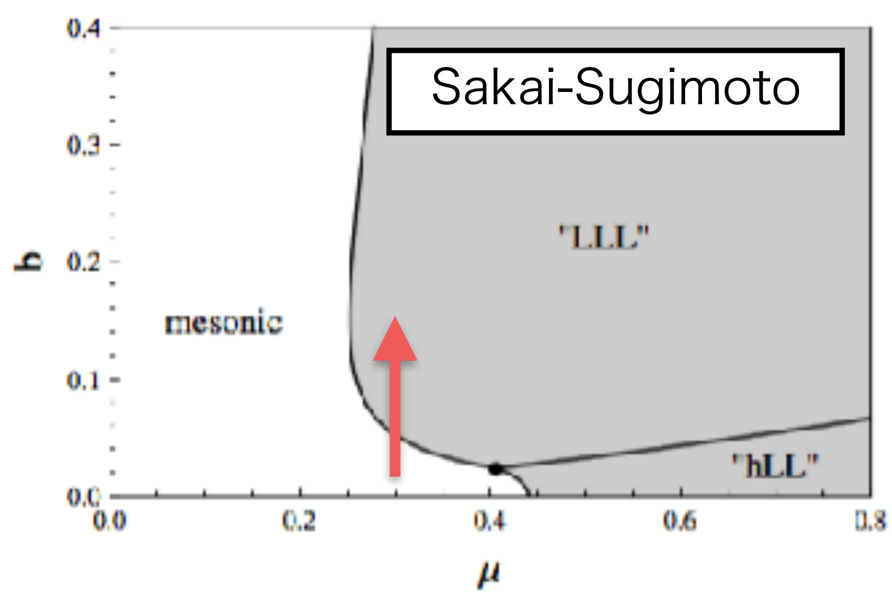
- magnetic field

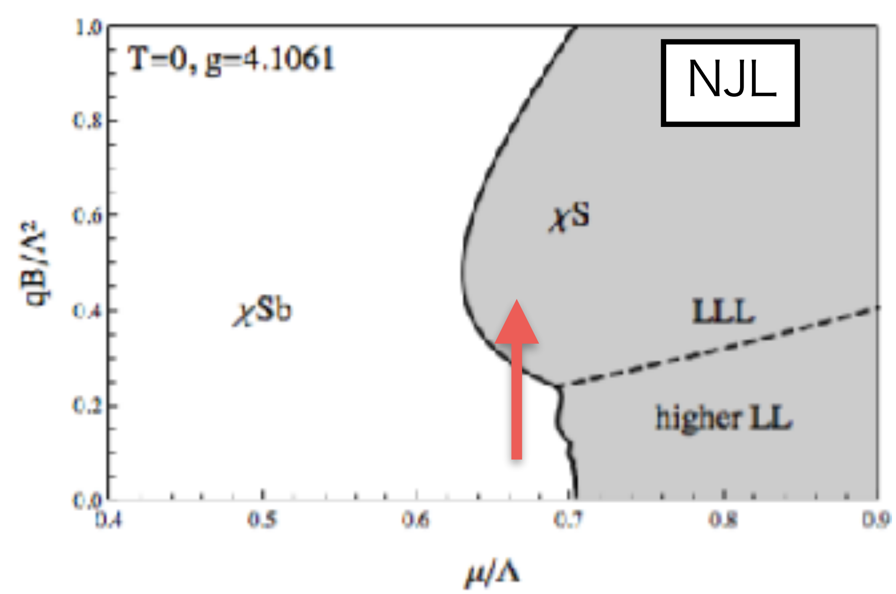
eB increases → m increases

- magnetic field + density ($\mu \neq 0$)

Ebert, Klimenko (1999)
Preis, Rebhan, Schmitt (2012)

eB increases → m decreases





Preis, Rebhan, Schmitt (2012)