Chiral soliton lattice in strong magnetic fields and rotation

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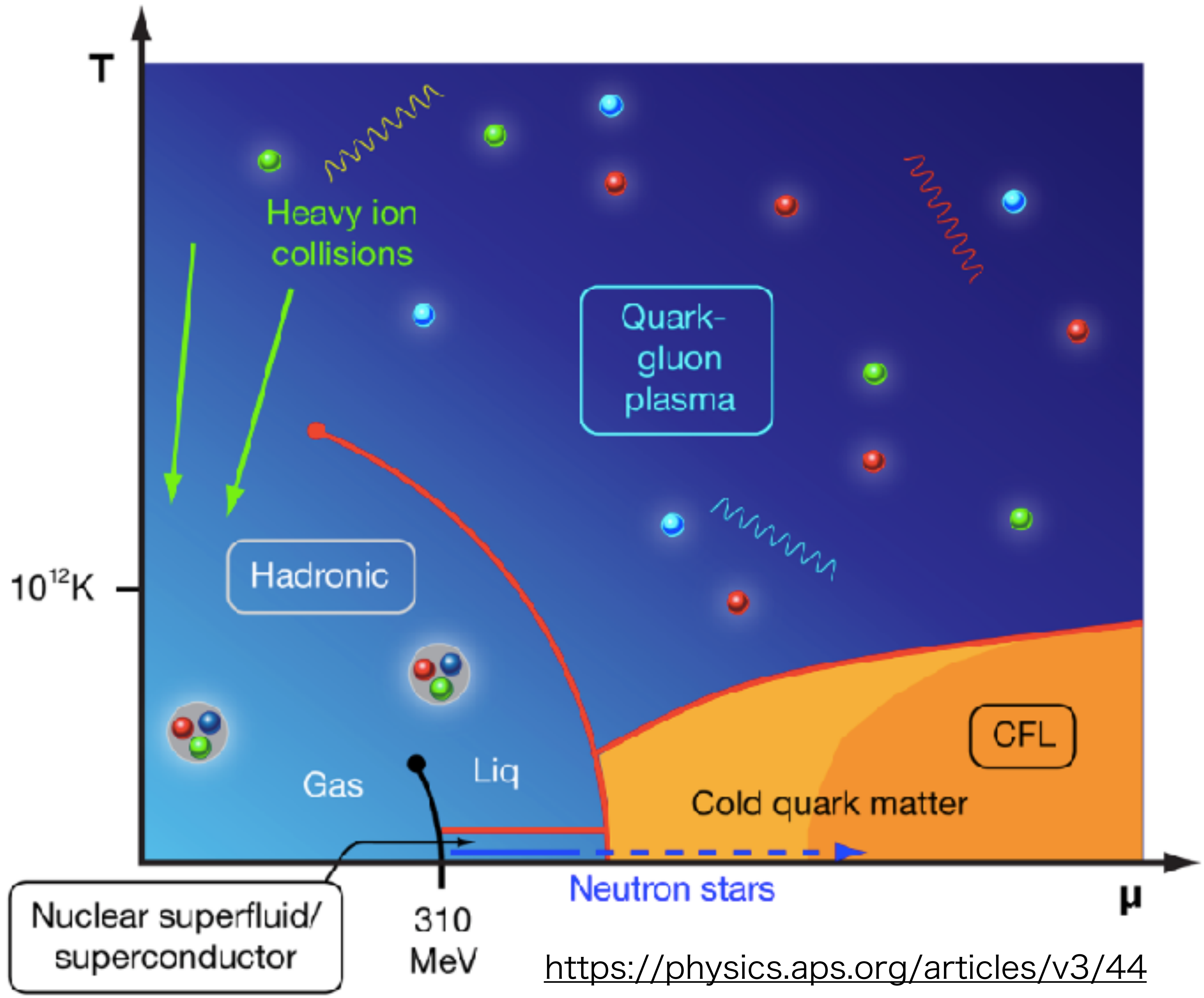
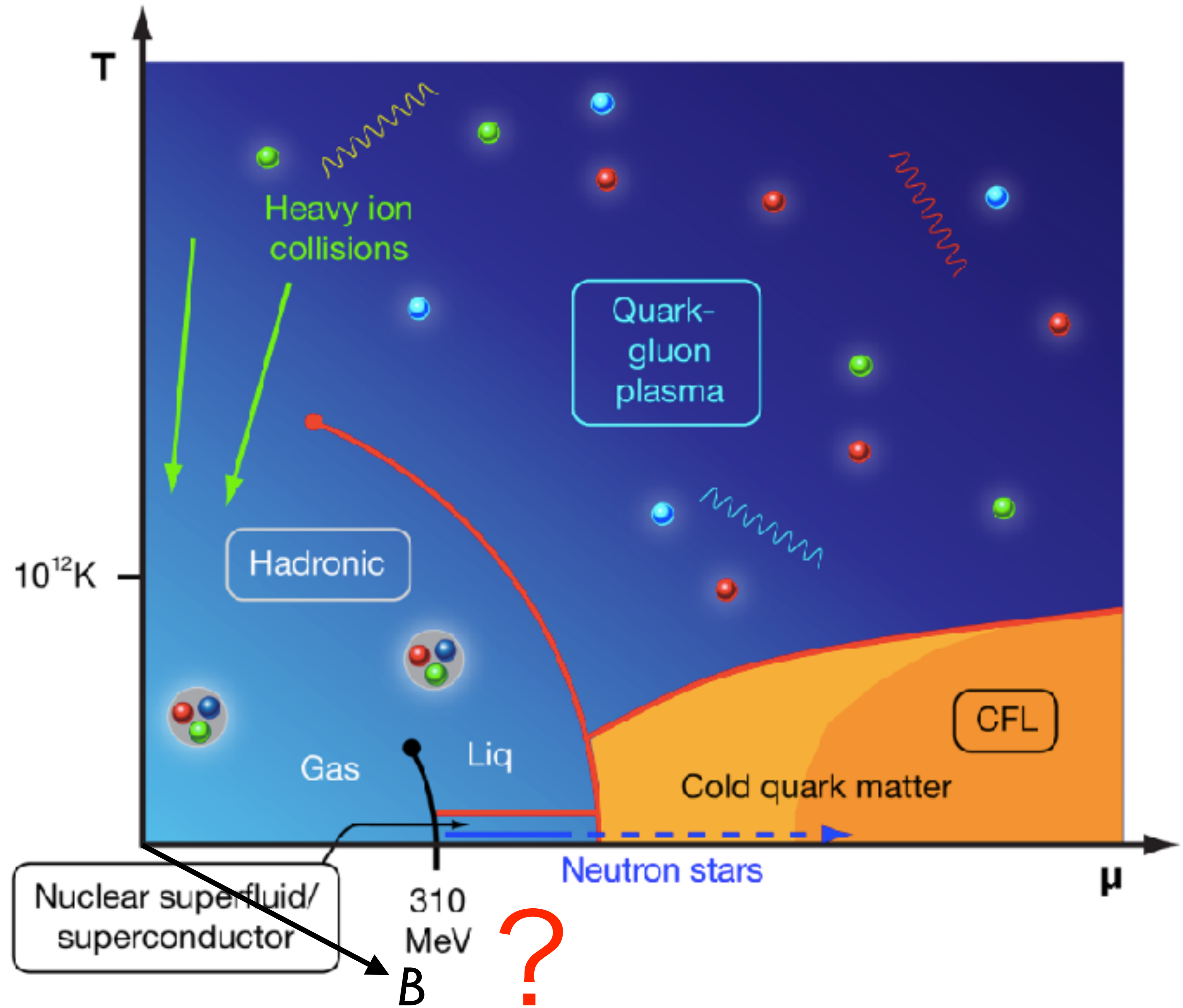
QCD phase diagram



https://physics.aps.org/articles/v3/44

QCD phase diagram



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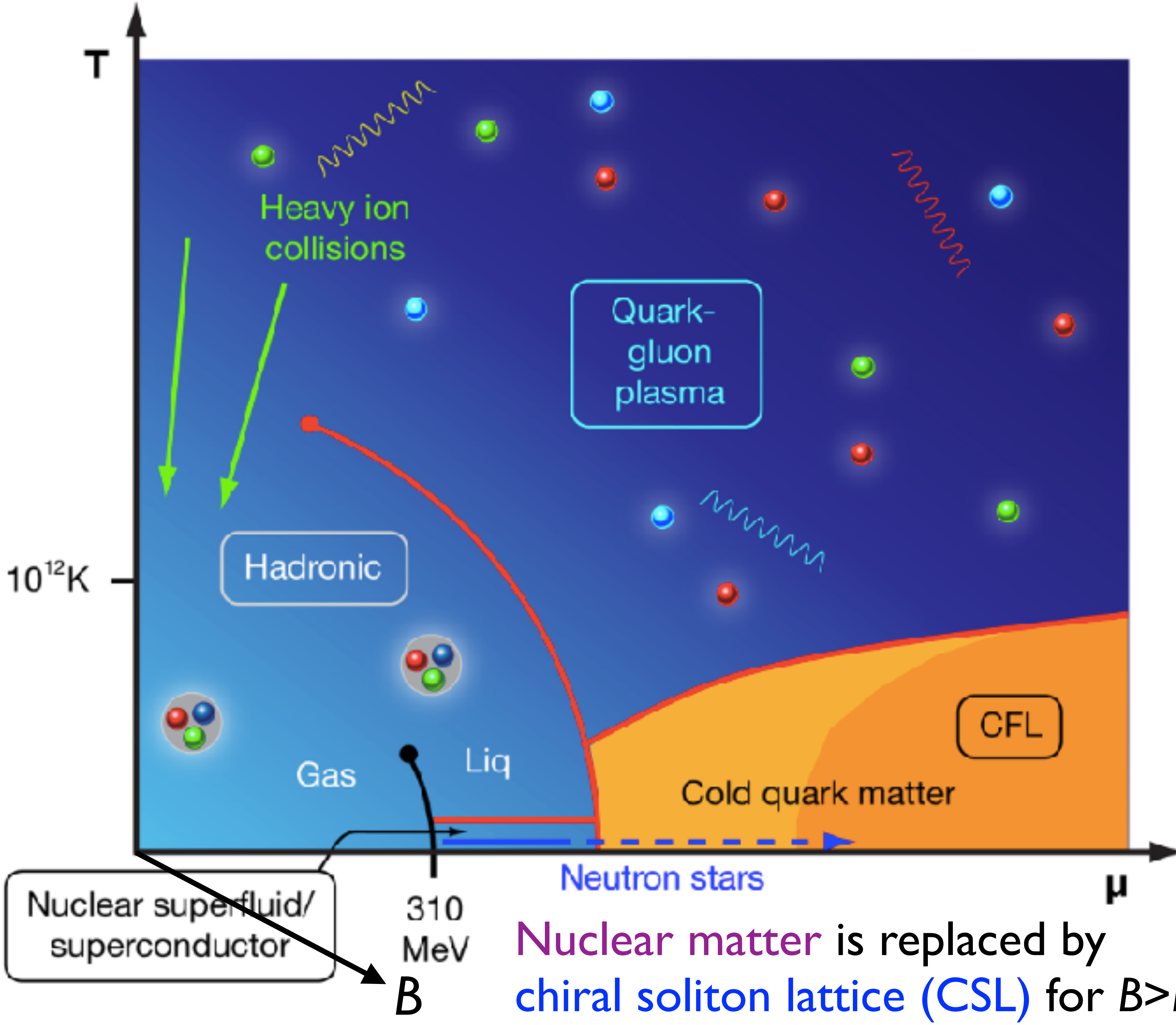

Nuclear matter is replaced by chiral soliton lattice (CSL) for $B > B_{\rm crit}$.

Chiral Soliton Lattice

- Violates parity

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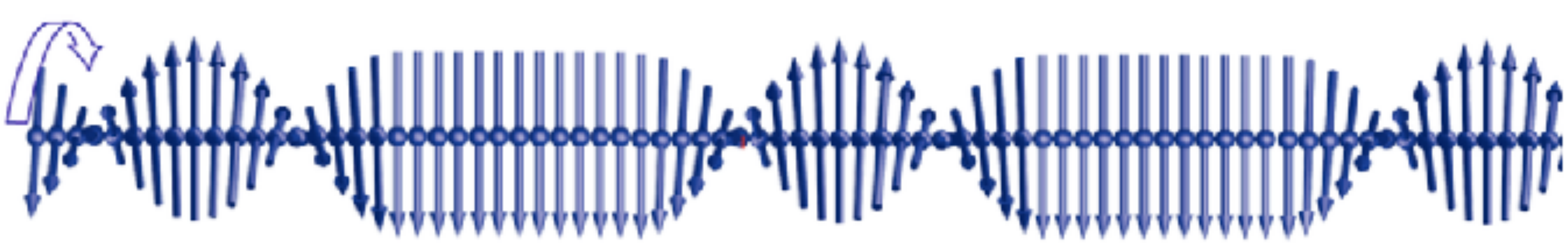
cf) CSL known to appear in chiral magnets and liquid crystals



Y. Togawa, *et al.*, PRL (2012)

Anomalous effects at finite μ

Son, Zhitnitsky; PRD (2004); Son, Stephanov, PRD (2008)

- Anomalous term in the vacuum: $\mathcal{L}_{\rm anom} = C\pi^0 \boldsymbol{E}\cdot\boldsymbol{B}, \quad C = \dfrac{1}{4\pi^2}$

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- Anomalous term in the vacuum: $\mathcal{L}_{\mathrm{anom}} = C\pi^0 \boldsymbol{E}\cdot\boldsymbol{B}, \quad C = \dfrac{1}{4\pi^2}$
- Anomalous term at finite μ and B: $\mathcal{L}_{\mathrm{anom}} = C\mu\boldsymbol{\nabla}\pi^0\cdot\boldsymbol{B}$

$$\because\ S_{\mathrm{anom}} = C\int \mathrm{d}^4x\ \pi^0(-\underbrace{\boldsymbol{\nabla}\phi}_{\text{scalar potential}})\cdot\boldsymbol{B} \sim C\int \mathrm{d}^4x\ \phi\boldsymbol{\nabla}\pi^0\cdot\boldsymbol{B}$$

Chiral perturbation theory

- Low-energy effective theory of QCD (model-independent)
- Constructed based on chiral symmetry breaking
- Systematic expansion in derivatives and quark masses

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- Systematic expansion in derivatives and quark masses
- π^0 sector to leading order: Son, Stephanov, PRD (2008)

$$\mathcal{H} = \frac{f_\pi^2}{2}(\boldsymbol{\nabla}\pi^0)^2 - \underbrace{C\mu \boldsymbol{B}\cdot\boldsymbol{\nabla}\pi^0}_{\text{anomalous term}} - \underbrace{m_\pi^2 f_\pi^2 \cos\pi^0}_{\text{mass term}} + \text{const.}$$

QCD vs. cond-mat

- QCD at finite μ and ***B*** (***B***=B***z***):

$$\mathcal{H} = \frac{f_\pi^2}{2}(\partial_z \pi^0)^2 - \underbrace{C\mu B \partial_z \pi^0}_{\text{anomalous}} - \underbrace{m_\pi^2 f_\pi^2 \cos\pi^0}_{\text{mass}} + \text{const.}$$

- Chiral magnets: Kishine, Ovchinnikov, Solid State Phys. **66** (2015)

$$\mathcal{H} = JS^2 a\left[\frac{1}{2}(\partial_z\phi)^2 - \underbrace{q_0\partial_z\phi}_{\text{Dzyaloshinskii-Moriya}} - \underbrace{m^2\cos\phi}_{\text{Zeeman}}\right] + \text{const.}$$

Two Hamiltonians are mathematically equivalent.

Analytic solution

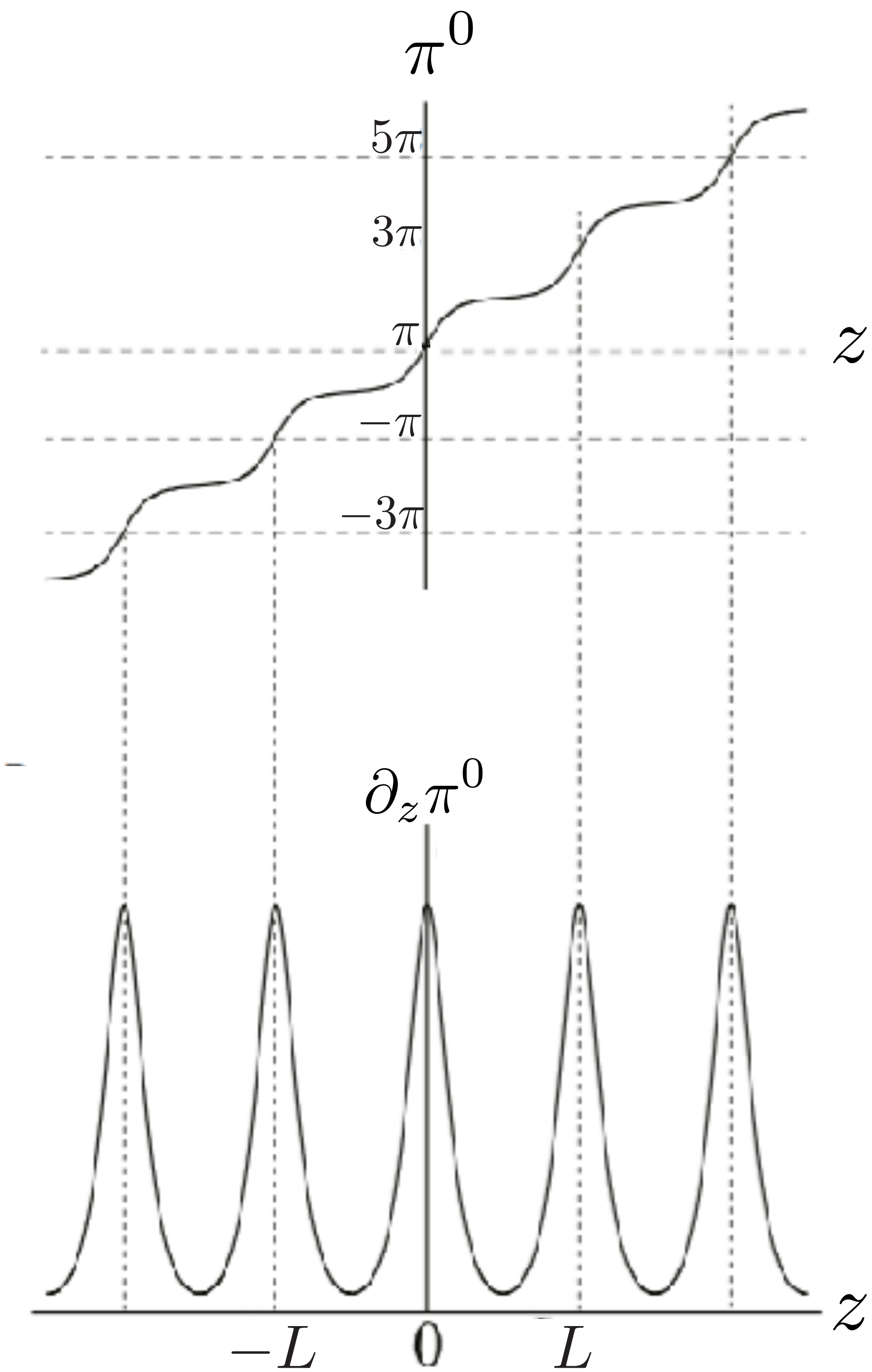
1-dim. periodic lattice of topological solitons

$$\cos\frac{\pi^0(\bar{z})}{2} = \mathrm{sn}(\bar{z}, k)\,, \quad \bar{z} = \frac{z m_\pi}{k}$$

Jacobi elliptic function

elliptic modulus

Brauner, Yamamoto, JHEP (2017)



Kishine, Ovchinnikov, Solid State Phys. **66** (2015)

Rough picture

$$\mathcal{H} = \frac{f_\pi^2}{2}(\partial_z \pi^0)^2 - C\mu B \partial_z \pi^0 - \cancel{m_\pi^2 f_\pi^2 \cos \pi^0} + \text{const.}$$

- **2nd term ≫ 3rd term:** $\langle \pi^0 \rangle = \dfrac{C\mu B}{f_\pi^2} z + \text{const.}$

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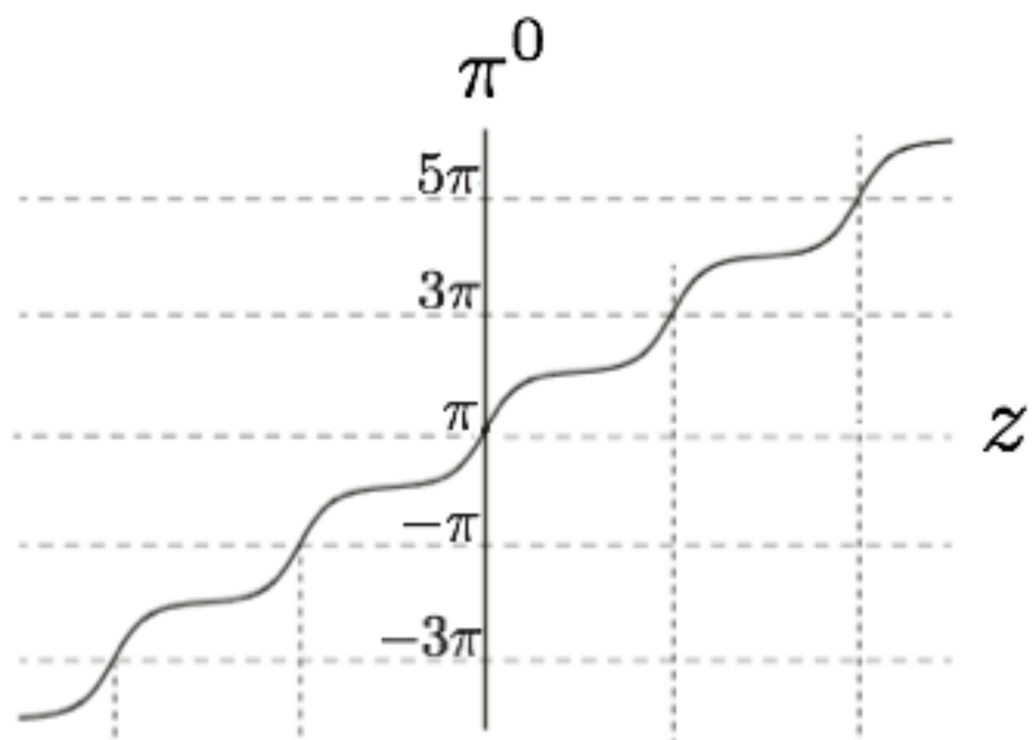


Topological charges

$$\mathcal{H} = \frac{f_\pi^2}{2}(\partial_z \pi^0)^2 - C\mu B \partial_z \pi^0 - m_\pi^2 f_\pi^2 \cos \pi^0 + \text{const.}$$

- CSL carries two topological charge densities:
  - Baryon number: $n_{\mathrm{B}}(z) = -\frac{\partial \mathcal{H}}{\partial \mu} = CB\partial_z \pi^0(z)$
  - Magnetization: $m(z) = -\frac{\partial \mathcal{H}}{\partial B} = C\mu \partial_z \pi^0(z)$

Son, Stephanov, PRD (2008)

Topological charges

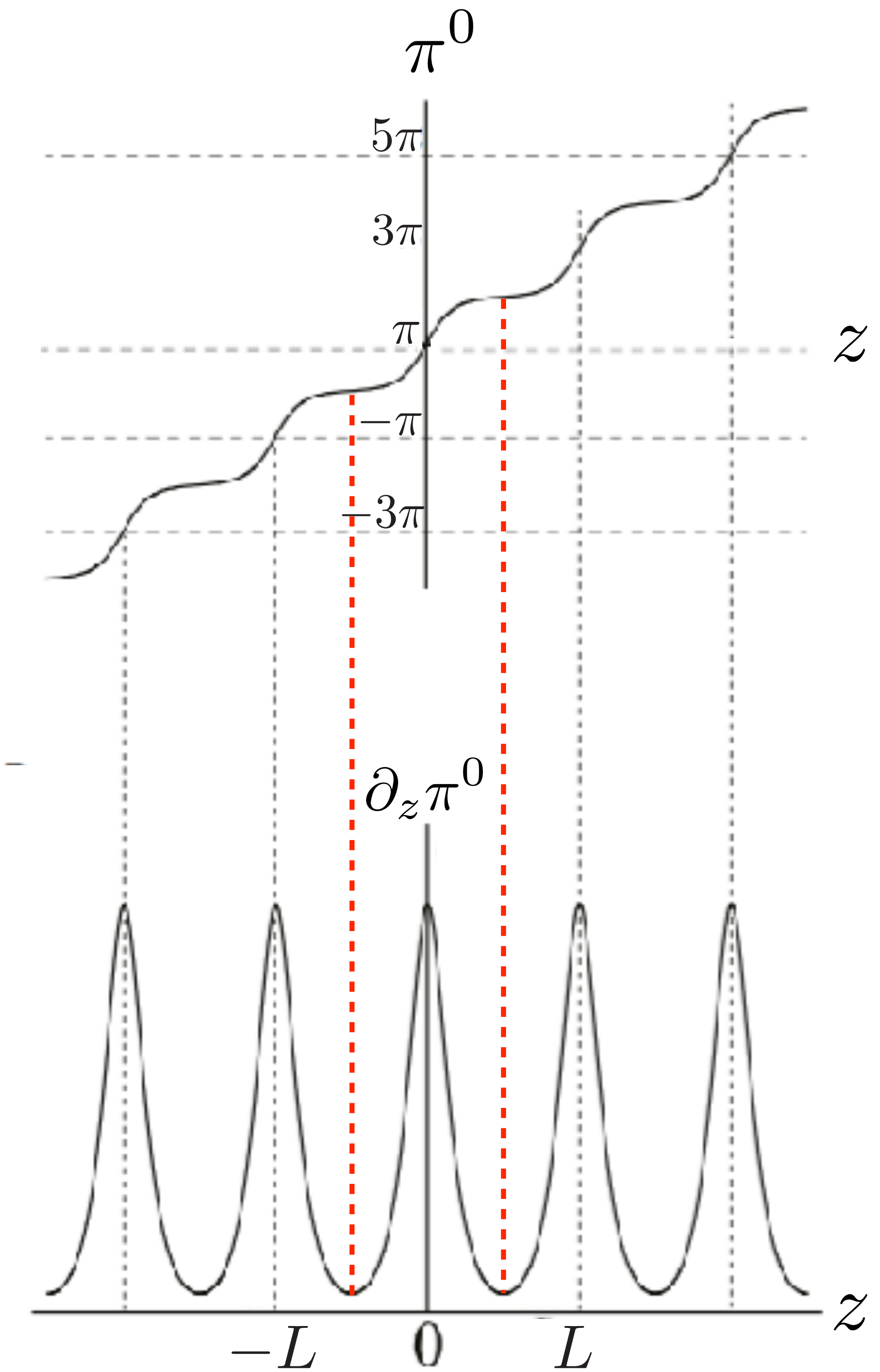
- Each domain wall has baryon charge:

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} n_{\rm B}(z) = CB\left[\underbrace{\pi^0\left(\frac{L}{2}\right)}_{2\pi} - \underbrace{\pi^0\left(-\frac{L}{2}\right)}_{0}\right] = \frac{B}{2\pi}$$

- Baryon number and magnetization:

$$\frac{N_{\rm B}}{S} = \frac{B}{2\pi}\,, \quad \frac{M}{S} = \frac{\mu}{2\pi}$$

independent of the detailed form of π^0



Ground state and excitations

- CSL is favored than vacuum and nuclear matter for $B>B_{\rm CSL}$:

$$\frac{E(k)}{k} = \frac{\mu B}{16\pi m_\pi f_\pi^2} \qquad \therefore\ B_{\rm CSL} = \frac{16\pi m_\pi f_\pi^2}{\mu}$$

K(*k*), *E*(*k*): complete elliptic integral of the 1st/2nd kind

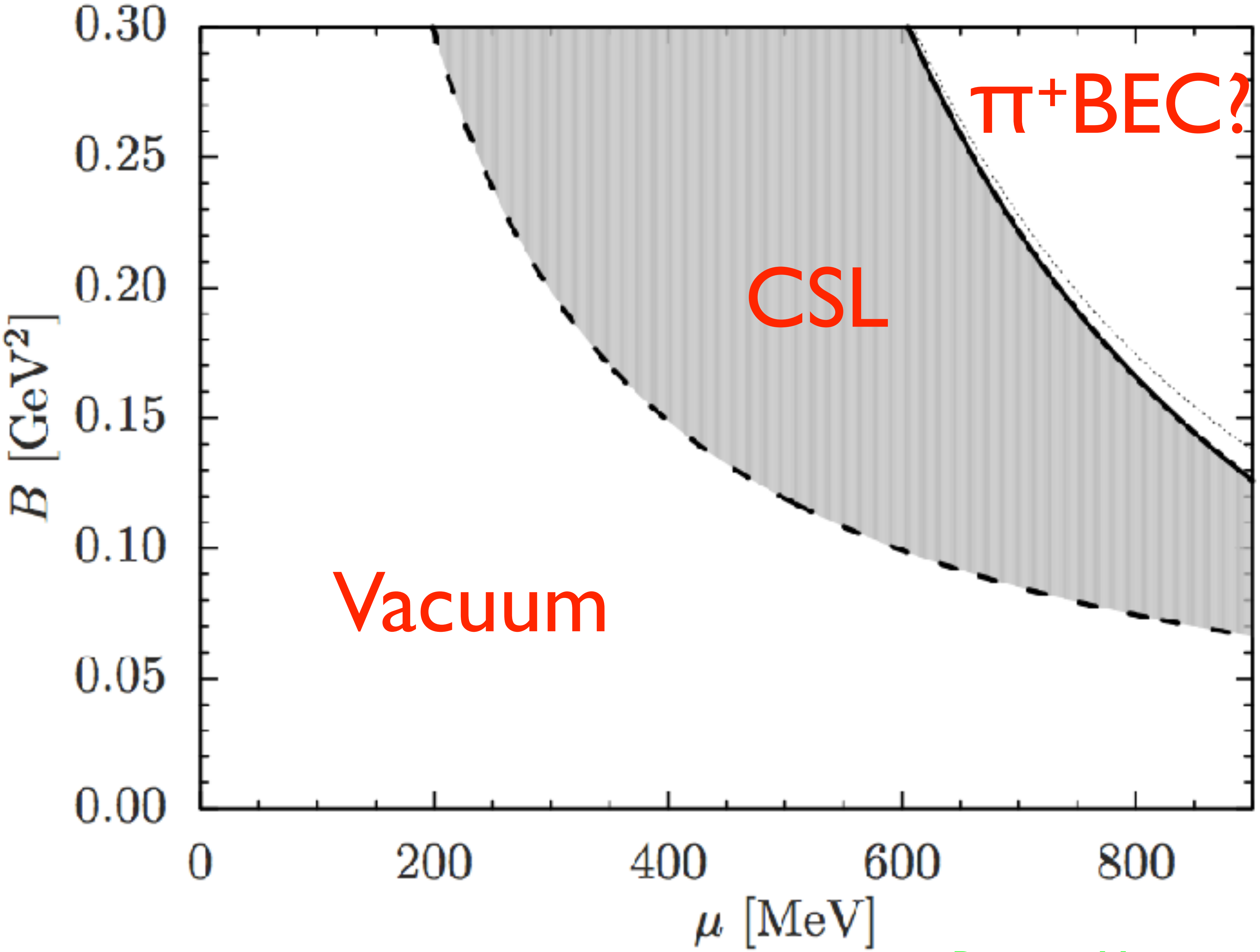
When m_q=0, QCD vacuum is unstable in an infinitesimally small *B*

- Phonon dispersion:

$$\omega^2 = p_x^2 + p_y^2 + (1-k^2)\left[\frac{K(k)}{E(k)}\right]^2 p_z^2 + \mathcal{O}(p_z^4)$$

Brauner, Yamamoto, JHEP (2017)

Phase diagram



Brauner, Yamamoto, JHEP (2017)

CSL under rotation

Huang, Nishimura, Yamamoto, JHEP (2018)

- (Non-renormalized) chiral vortical effect: $\boldsymbol{j}_5^{a=3} = \frac{\mu_{\mathrm{B}}\mu_{\mathrm{I}}}{\pi^2}\boldsymbol{\Omega}$
- "Anomaly matching" of CVE → anomalous term for pions:

$$\mathcal{L}_{\mathrm{anom}} = \frac{\mu_{\mathrm{B}}\mu_{\mathrm{I}}}{2\pi^2 f_\pi}\boldsymbol{\nabla}\pi_0 \cdot \boldsymbol{\Omega}$$

- CSL is favored than vacuum and nuclear matter for $\Omega > \Omega_{\mathrm{CSL}}$:

$$\Omega_{\mathrm{CSL}} = \frac{8\pi m_\pi f_\pi^2}{\mu_{\mathrm{B}}|\mu_I|}$$

Axion electrodynamics in CSL

Electro-magnetism

- Effective theory for electromagnetic fields:

$$\mathcal{L} = \frac{\varepsilon}{2}\boldsymbol{E}^2 - \frac{1}{2\mu}\boldsymbol{B}^2 + C\langle\pi^0\rangle\boldsymbol{E}\cdot\boldsymbol{B} - j^\mu A_\mu$$

CSL solution

Axion electrodynamics

- Modified Maxwell's equations:

$$\epsilon \boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - C \langle \boldsymbol{\nabla} \pi^0 \rangle \cdot \boldsymbol{B},$$

$$\frac{1}{\mu} \boldsymbol{\nabla} \times \boldsymbol{B} = \epsilon \partial_t \boldsymbol{E} + \boldsymbol{j} + \underbrace{C \langle \boldsymbol{\nabla} \pi^0 \rangle \times \boldsymbol{E}}_{\text{Anomalous Hall effect}}$$

Properties of electromagnetic waves (photons) are modified

Non-relativistic photons

- For helicity +1, $\omega \sim \frac{f_\pi^2}{\mu B_{\rm ex}} k^2$: non-relativistic gapless photon
- For helicity -1, $\omega \sim \frac{\mu B_{\rm ex}}{f_\pi^2}$: gapped photon

Yamamoto, PRD (2016); Ozaki, Yamamoto, JHEP (2017);
Qiu, Cao, Huang, PRD (2017); Brauner, Kadam, JHEP (2017)

CSL distinguishes between R & L-handed photons = polarizer

Summary

- New ground state of QCD: chiral soliton lattice (CSL)
- Axion electrodynamics and non-relativistic photons in CSL
- QCD phase diagram in (T, μ, B or Ω)? Physical observables?

Backup slides

Chiral magnets

Kishine, Ovchinnikov, Solid State Phys. **66** (2015)

- Heisenberg Hamiltonian:

$$H = -J\sum_{j=1}^{N} \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1} - h\sum_{j=1}^{N} S_j^z \simeq JS^2a^2\left[\frac{1}{2}(\partial_z\phi)^2 - \frac{h}{JSa^2}\cos\phi\right] + \text{const.}$$

$$z = ja$$

- Dzyaloshinskii-Moriya (DM) interaction:

$$H_{\text{DM}} = \sum_{j=1}^{N} \boldsymbol{D}\cdot(\boldsymbol{S}_j \times \boldsymbol{S}_{j+1}) \simeq -DS^2a\partial_z\phi$$

$$\longrightarrow \mathcal{H} = JS^2a\left[\frac{1}{2}(\partial_z\phi)^2 - \underbrace{q_0\partial_z\phi}_{\text{Dzyaloshinskii-Moriya}} - \underbrace{m^2\cos\phi}_{\text{Zeeman}}\right] + \text{const.}$$

Elliptic functions

- Jacobi elliptic functions:

$$u = \int_0^x \frac{\mathrm{d}x}{\sqrt{(1-x^2)(1-k^2x^2)}} \equiv \mathrm{sn}^{-1}(x,k)$$

$$\mathrm{cn}^2(u,k) \equiv 1 - \mathrm{sn}^2(u,k), \quad \mathrm{dn}^2(u,k) \equiv 1 - k^2\mathrm{sn}^2(u,k)$$

- Complete elliptic integral of the 1st/2nd kind:

$$K(k) = \int_0^{\pi/2} \frac{\mathrm{d}\phi}{\sqrt{1-k^2\sin^2\phi}}\,, \quad E(k) = \int_0^{\pi/2} \mathrm{d}\phi\sqrt{1-k^2\sin^2\phi}$$

CSL Solution

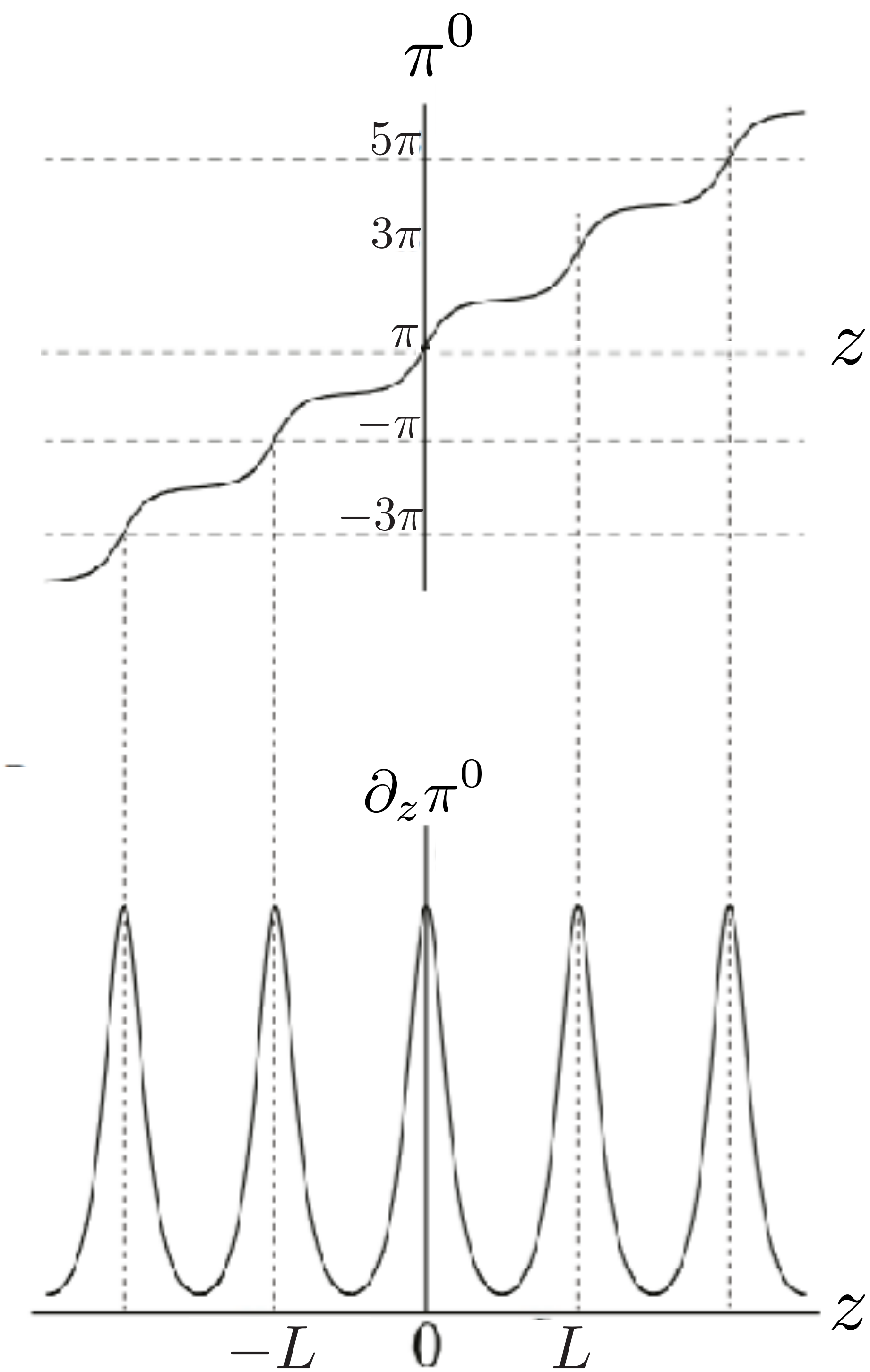
1-dim. periodic lattice of topological solitons

$$\cos\frac{\pi^0(\bar{z})}{2} = \mathrm{sn}(\bar{z}, k)\,, \quad \bar{z} = \frac{z m_\pi}{k}$$

Jacobi elliptic function　　elliptic modulus

- Minimization of H: $\dfrac{E(k)}{k} = \dfrac{\mu B}{16\pi m_\pi f_\pi^2}$
- Period of solution: $L = \dfrac{2kK(k)}{m_\pi}$

K, *E*: complete elliptic integral of the 1st/2nd kind



Kishine, Ovchinnikov, Solid State Phys. **66** (2015)

Chiral perturbation theory

- Kinetic and mass terms:

$$\mathscr{L} = \frac{f_\pi^2}{4}\left[\mathrm{Tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) + 2m_\pi^2 \,\mathrm{Re}\,\mathrm{Tr}\,\Sigma\right] \qquad \langle\Sigma\rangle \equiv \Sigma_0 = e^{\mathrm{i}\tau_3\phi}$$

$$\mathscr{L}_{\mathrm{bilin}} = \frac{1}{2}(\partial_\mu\vec{\pi})^2 + (A^\mu - \partial^\mu\phi)(\pi_1\partial_\mu\pi_2 - \pi_2\partial_\mu\pi_1) + \frac{1}{2}A^\mu A_\mu(\pi_1^2+\pi_2^2) - \frac{1}{2}m_\pi^2\vec{\pi}^2\cos\phi.$$

- Wess-Zumino-Witten (WZW) term:

$$\mathscr{L}_{\mathrm{WZW}}^{\mathrm{bilin}} = \frac{1}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}A_\mu^{\mathrm{B}}\partial_\nu(\phi F_{\alpha\beta}) - \frac{1}{16\pi^2 f_\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}\partial_\alpha\phi(\pi_1\partial_\beta\pi_2 - \pi_2\partial_\beta\pi_1)$$

Anomaly matching of CVE

Huang, Nishimura, Yamamoto, JHEP (2018)

- (Non-renormalized) chiral vortical effect: $\boldsymbol{j}_5^{a=3} = \frac{\mu_\mathrm{B}\mu_\mathrm{I}}{\pi^2}\boldsymbol{\Omega}$
- Anomaly matching:

$$\delta S_\mathrm{QCD} = \int \mathrm{d}^4x \boldsymbol{\nabla}\theta_3 \cdot \boldsymbol{j}_5^3 \longleftrightarrow \delta S_\mathrm{EFT} = \int \mathrm{d}^4x \boldsymbol{\nabla}\left(\frac{\pi_0}{2f_\pi}\right) \cdot \boldsymbol{j}_5^3$$

- Anomalous term in Ω: $\mathcal{L}_\mathrm{anom} = \frac{\mu_\mathrm{B}\mu_\mathrm{I}}{2\pi^2 f_\pi}\boldsymbol{\nabla}\pi_0 \cdot \boldsymbol{\Omega}$
- CSL is favored for $\Omega > \Omega_\mathrm{CSL}$: $\Omega_\mathrm{CSL} = \frac{8\pi m_\pi f_\pi^2}{\mu_\mathrm{B}|\mu_I|}$