Some thoughts on relativistic hydrodynamics with polarization

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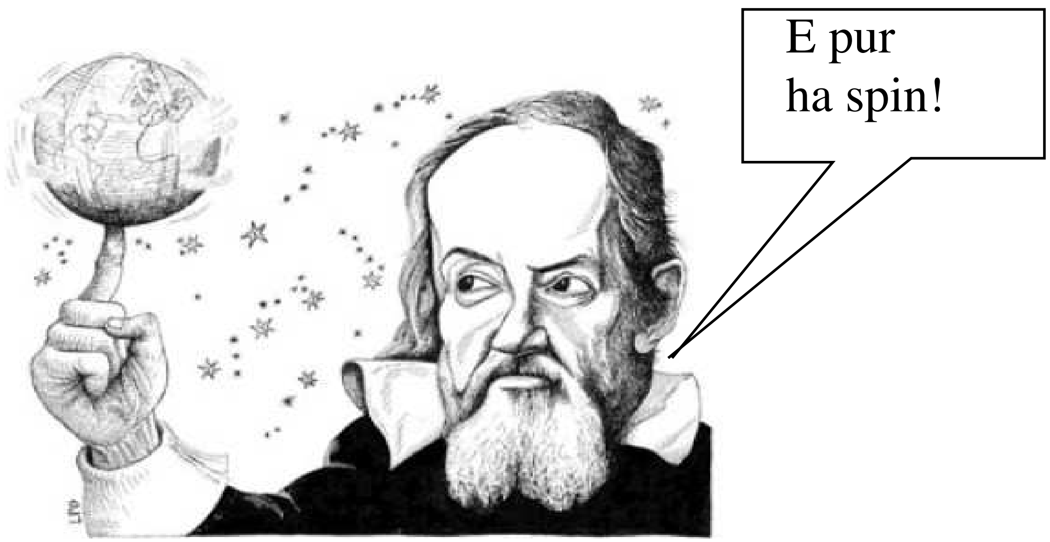
FAPESP
FUNDAÇÃO DE AMPARO À PESQUISA DO ESTADO DE SÃO PAULO



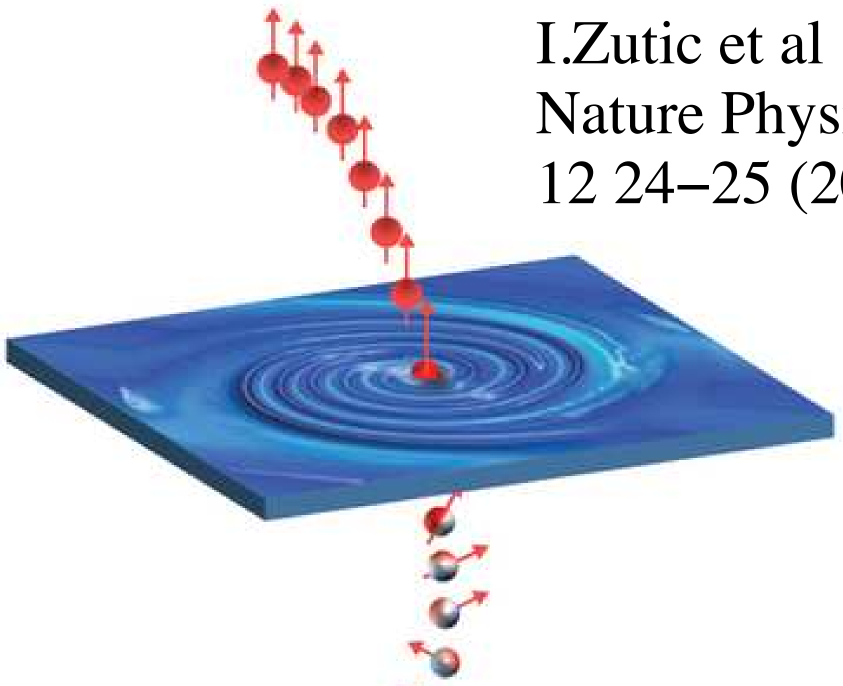
- Based on nucl-th/0708.0035,Phys. Rev. D 94, 065042 (2016) 1703.03079,1701.08263 and 1802.09011

- Many collaborators, some in the audience Leonardo Tinti, David Montenegro,Barbara Betz,Jorge Noronha,Miklos Gyulassy,...

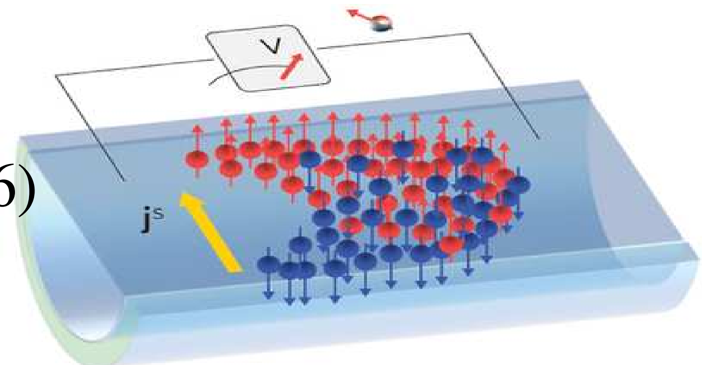
- But talk is my own, in the sense that the people above will not necessarily agree with me

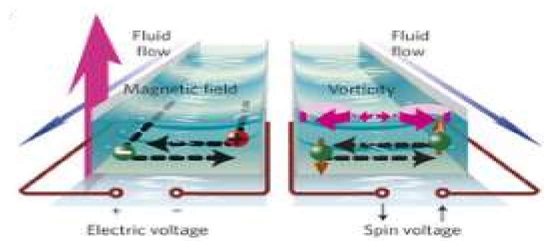
- This is a workshop, so I am not afraid to say things which are controversial, wrong or stupid to stimulate discussion.

The ideal fluid where constituents spin, so vortices make them spin around!



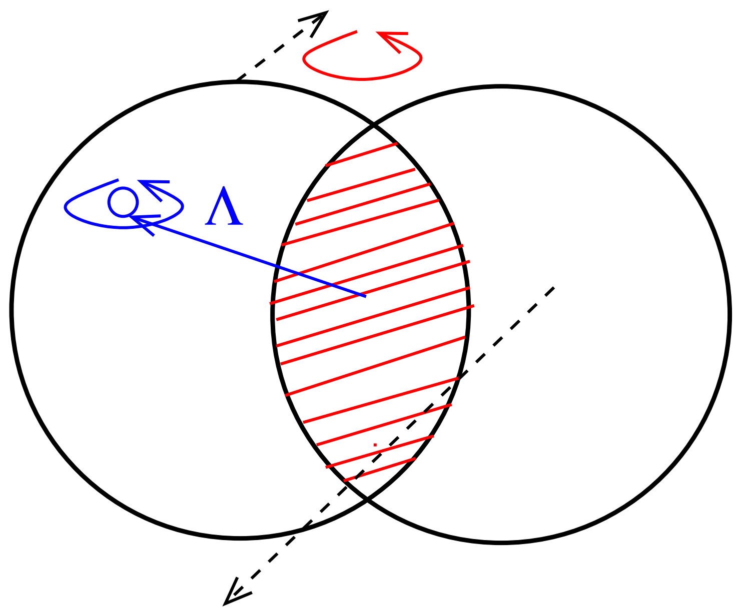
I.Zutic et al
Nature Physics
12 24–25 (2016)





Ultracold atoms: Zutic, Matos-Abiague, "Spin Hydrodynamics", Nature Physics **12** 24-25 Takahashi et al", Nature Physics **12** 52-56 (2016)

In the context of heavy ion physics...

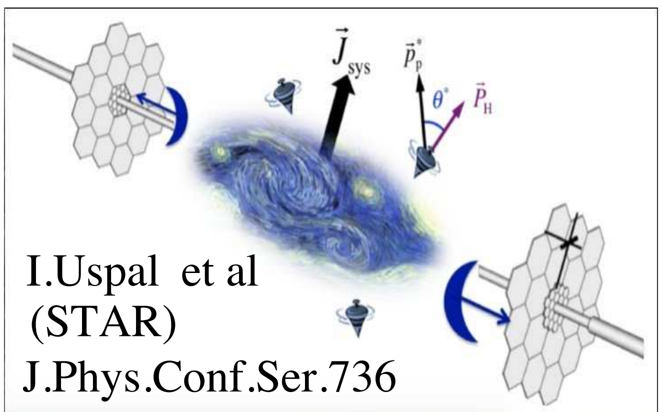


Initial rapidity gradient and initial transparency could generate initial angular momentum. NB: Different from chiral magnetic/vortaic effects. Not anomalous and all DoFs in equilibrium (no B-field, "local" microscopic spin-orbit coupling, angular momentum follows local equilibrium)

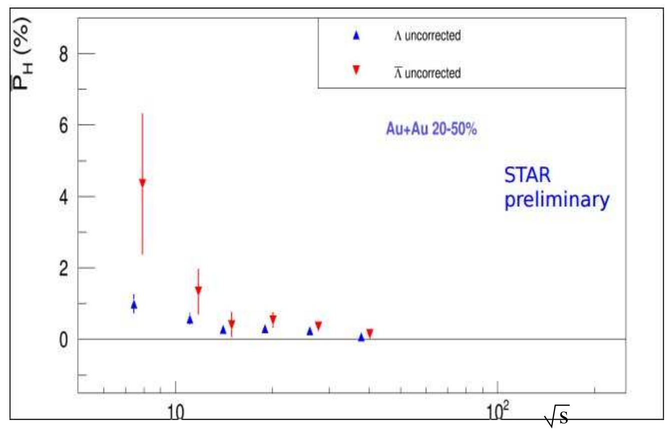
Transeferred to Λ via "spin-orbit coupling in hydro". Detectable via P-violating decays

Experimentally confirmed! published in Nature (1701.06657)!

Theory as usual lags behind! What does it mean that microscopic constituents not scalar?



I.Uspal et al (STAR) J.Phys.Conf.Ser.736 012016 (2016)



A possibly satisfactory experimental answer: Cooper-Frye the angular momentum (Becattini et al, 1303.3431)

$$\exp\left(-\frac{p_\alpha u^\alpha}{T}\right) \to \exp\left(-\frac{p_\alpha u^\alpha}{T}\right)(\bar{u},\bar{v})\exp\left[\frac{\Sigma_{\mu\nu}\omega^{\mu\nu}}{T}\right]\left(\begin{array}{c} u \\ v \end{array}\right)$$

But cannot be the whole story: Cooper-Frye is built around detailed balance over hadronization hypersurface Cooper-Frye formula based on ideal isotropic hydro.

$$d\Sigma^\mu(T_{\mu\nu}^{hydro} - T_{\mu\nu}^{particles}) = d\Sigma^\mu(s_\mu^{hydro} - s_\mu^{particles}) = 0$$

Where is polarization vs angular momentum in plasma? What is its contribution to transport? Surprisingly subtle questions

What is ideal hydro? A conceptual difficulty!

Entropy conserved always at maximum at each point in spacetime

Local isotropy in the comoving frame

Vorticity is conserved (Kelvins theorem)

Continuum limit when you break up cells, intensive results stay the same

With polarization, only the first has a chance of being realized even in the ideal limit,

Further questions

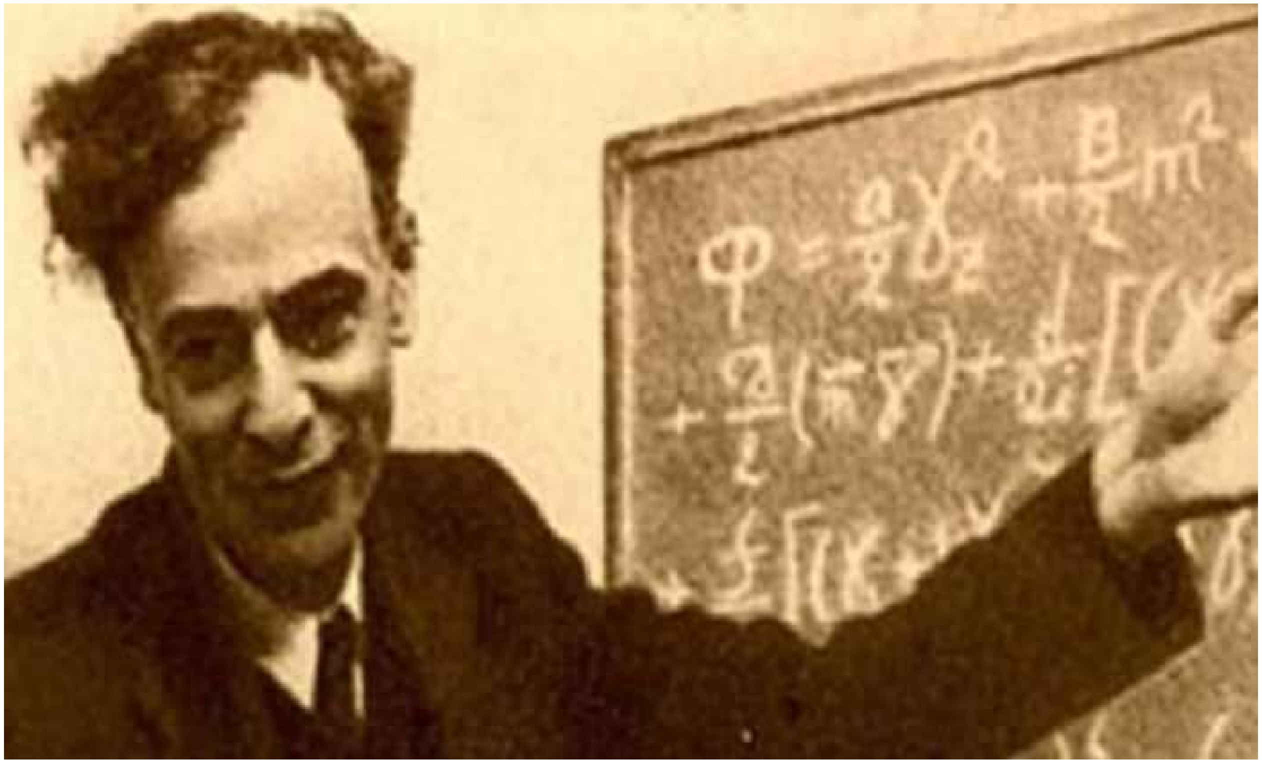
- Should you coarse-grain? Is a "small vortex" indistinguisheable from a polarization spin state

- Connection to anomalous transport, CME,CVE , magneto-hydrodynamics
  - Classical fields coupled to the fluid vs part of the fluid. (B^μ vs $\Omega^{\mu\nu}$)
  - Anomalous vs symmetry respecting ($J^\mu = ... + \Omega$ vs $T^{\mu\nu} + u_\alpha S^{\alpha\mu\nu}$)

  Very different physics but experimentally entangled . Eg, polarizability variation with baryo and isospin chemical potential can mimic the CME! all observables so far CP-respecting

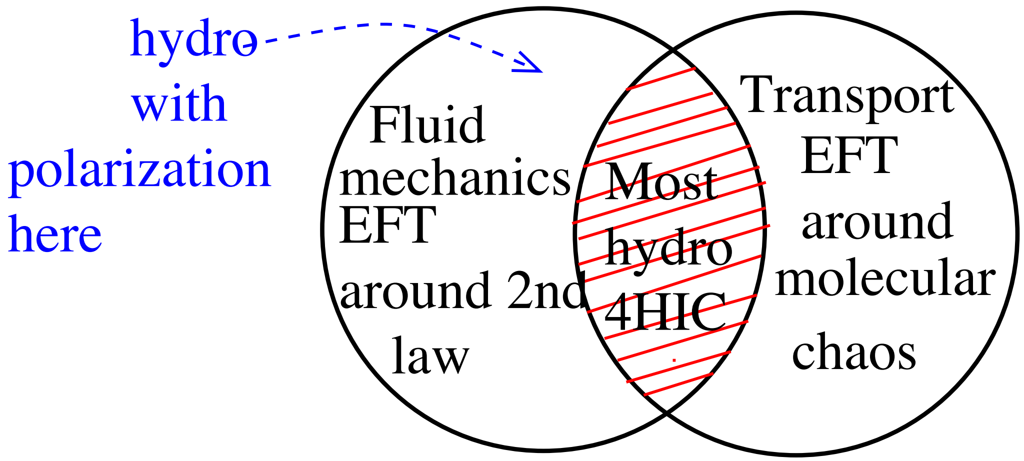
- What is the role of Gauge symmetry? In a QGP most particles with spin are gluons, but "gluon polarization" and "angular momentum" dont separate (gauge changes go between one and another).

All this means no ideal hydro limit is defined for mediums with polarization , and its connection to microsopic theory is puzzling This is conceptually difficult! I thought about it for 10 years! But its exactly what makes science fun!



I want
to understand
and calculate
things I cannot
imagine...

What won't work I: Hydro is not (just) transport!



$\langle f_{micro} f_{micro} \rangle \sim \langle f \rangle \langle f \rangle$
CLEARLY
not appropriate
for polarizeable media

Models based on microscopic distribution functions and 1-particle Wigner-functions most likely far away from ideal hydrodynamic limit because (unlike in non-polarized case) taking such a limit without spoiling convergence of the BBGKY hyerarchy impossible

the fact that transport usually cannot handle stable circulation without a mean field should be a hint! Stochastic fluctuatons with finitely many particles destroy circulation

Hydro is not (just) transport: Hydrodynamics is based on three scales

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

l_{micro} stochastic, l_{mfp} dissipative. If $l_{micro} \sim l_{mfp}$ soundwaves

Of amplitude so that momentum $P_{sound} \sim (area)\lambda\,(\delta\rho)\,c_s \gg T$

And wavenumber $k_{sound} \sim P_{sound}$

Survive (ie their amplitude does not decay to $E_{sound} \sim T$) $\tau_{sound} \gg 1/T$

fluctuating vortices, sound-waves, polarization mix. Creating an effective non-transport viscosity? (In abs/0708.0035 we confused l_{micro}, l_{mfp}).

What won't work II: AdS/CFT (at least the way people do it)

Fermion-gas coupling N_c suppressed in models with fundamental Fermions (e.g. $D3 - D7$), a straight-forward consequence of large N_c expansion.

Not surprising: At same order as microscopic fluctuations, since AdS/CFT version of

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro} \quad is \quad \frac{1}{T N_c^{2/3}} \ll \frac{\eta}{sT} \ll L_{macro}$$

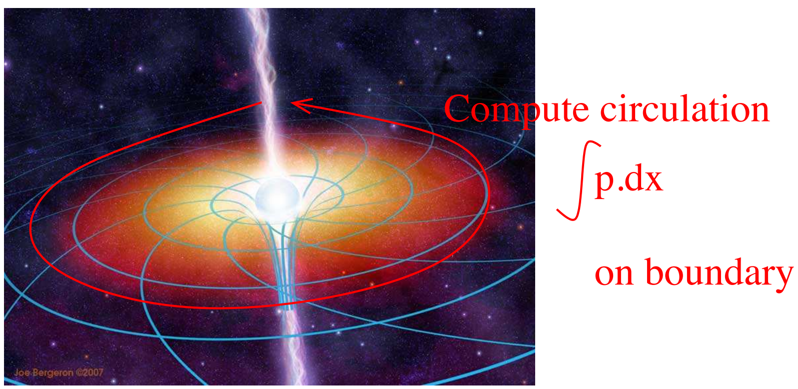
Expansion in N_c is expansion in g_s

What won't work II: AdS/CFT (at least the way people do it)

Gluon polarization not gauge invariant, not sure what happens to it at large N_c in presence of angular momentum. vorticity-polarization coupling in a medium with no flavors? not sure! What needs to be checked is

$$\left\langle \Omega^{\alpha\beta\mu}(t')\Omega_{\mu}^{\alpha\beta}(t)\right\rangle \quad and \quad \partial_{\mu}\Omega^{\alpha\beta\mu}$$

of a non-linear rotating black hole solution in AdS, calculate how circulation $\Omega_{\alpha\beta}$ around horizon decays (subtracting viscosity effect) AFAIK not yet done...



What might work: EFT techniques

- Fluid elements as fields: $\phi_I(x^\mu), I = 1...3$ position of a fluid cell
- Impose symmetries on the Lagrangian
  - isotropy, compressibility $L = F(B), B = \det_{IJ}(\partial_\mu \phi^I \partial^\mu \phi^J)$
  - chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

Can impose a well-defined u^μ by adding chemical shift symmetry
$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$

What might work: EFT techniques

$$L \to \ln \mathcal{Z} \quad \mathcal{Z} = \int \mathcal{D}\phi_i \exp\left[-T_0^4 \int F(B) d^4x \right], \langle \mathcal{O} \rangle \sim \frac{\partial \ln \mathcal{Z}}{\partial ...}$$

$$\left(eg. \quad \left\langle T_{\mu\nu}^{x} T_{\mu\nu}^{x'} \right\rangle = \frac{\partial^2 \ln \mathcal{Z}}{\partial g_{\mu\nu}(x) \partial g_{\mu\nu}(x')} \right)$$

Knudsen number an EFT scale separation , $T_0 \sim n^{-1/3}$ an effective "Planck constant" EFT expansion and lattice techniques should give all allowed terms and correlators. Coarse-graining will be handled here!

What might work: EFT techniques

Treat polarization in a manner analogous to chemical potential breaking "isotropy" direction Need local $\sim SO(3)$ charges

$$\Psi_{\mu\nu}|_{comoving} = -\Psi_{\nu\mu}|_{comoving} = \exp\left[-\sum_{i=1,2,3} \alpha_i(\phi_I)\hat{T}_i^{\mu\nu}\right]$$

For "many incoherent particles" RPA means only vector representation remains

$$\alpha_i \to \alpha_i + \Delta\alpha_i\left(\phi_I\right) \Rightarrow L(b, y_{\alpha\beta} = u_\mu \partial^\mu \Psi_{\alpha\beta})$$

but y is an auxiliary variable, since polarization not conserved

$$b \to b\left(1 - c y_{\mu\nu} y^{\mu\nu} + \mathcal{O}\left(y^4\right)\right) \quad , \quad F(b) \to F(b, y) = F\left(b\left((1 - c y^2\right)\right)$$

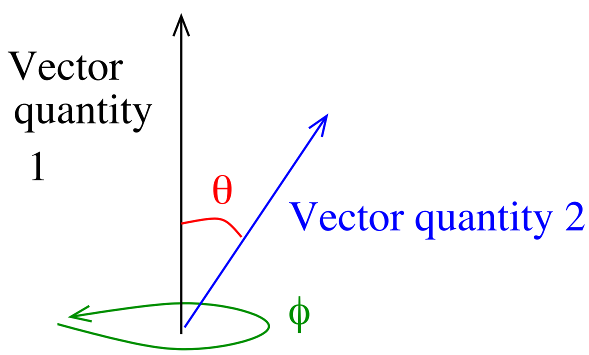
.

define ideal hydrodynamics as: Entropy always locally maximized, only transport are conserved quantities . This leads to a relationship between Lagrangian and thermodynamic relations via a free energy

$$d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial V} dV + \frac{\partial \mathcal{F}}{\partial e} de + \frac{\partial \mathcal{F}}{\partial \left[\Omega_{\mu\nu}\right]} d\left[\Omega_{\mu\nu}\right] = 0$$

Provided polarization aligned to vorticity $y^{\mu\nu} \sim \chi(T)(e+p)\left(\partial^{\mu} u^{\nu} - \partial^{\nu} u^{\mu}\right)$

If not, there is no ideal fluid limit due to non-hydrodynamic goldstone mode



Gives a complicated but uniquely defined dynamics...

$$\left\{ g_b(1-cy)\partial_\nu b + g_y 4 y_\alpha^\beta \chi(T) \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda \phi^I)} \partial^\nu (\partial_\lambda \phi^I) \right\} \times$$

$$\times [(1-cy)\frac{\partial b}{\partial(\partial_\nu \phi^I)} - (8cb) y_\alpha^\beta \chi(T) \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu \phi^I)}] + g(b,y) \times$$

$$\times (1-cy)\partial_\nu (\frac{\partial b}{\partial(\partial_\nu \phi)}) - 8c\chi(T) g(b,y) \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda \phi^I)} \left[\frac{y_\alpha^\beta}{2} \partial_\nu \partial_\lambda \phi^I \times \right.$$

$$\times \frac{\partial b}{\partial(\partial_\nu \phi^I)} + (\partial^\nu b) 4 y_\alpha^\beta \delta_\nu^\lambda + b\chi(T) (\frac{\partial(\partial^\beta u_\alpha)}{\partial(\partial_\nu \phi^I)} + \frac{\partial(\partial_\alpha u^\beta)}{\partial(\partial_\nu \phi^I)}) \times$$

$$\left. \times \partial_\nu (\partial_\lambda \phi^I) + b y_\alpha^\beta \partial_\nu ln \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu \phi^I)} \right] = 0$$

NB depends on accelleration (vorticity)! Foreseeing trouble with Ostrogradski's theorem! and indeed...

Linearizing and calculating dispersion relation shows violation of causality
We decompose perturbation into sound and vortex $\phi_I = \nabla\phi + \nabla \times \vec{\Omega}$

$$\left(\begin{array}{c} \varphi \\ \vec{\Omega} \end{array} \right) = \int dw d^3k \left(\begin{array}{c} \varphi_0 \\ \vec{\Omega}_0 \end{array} \right) \exp\left[i \left(\vec{k_{\phi,\Omega}}.\vec{x} - w_{\phi,\Omega} t \right) \right]$$

The part parallel to k ("sound-wave") will have a dispersion relation

$$w_\phi^2 - c_s^2 k_\phi^2 + 2\beta k_\phi w_\phi^3 = 0$$

The vector part will be

$$(3k_\Omega^2 - 2k_\Omega w_\Omega)_j (\vec{k}_\Omega \times \vec{\Omega}_0)_i w_\Omega^2 + w^4 \Omega = 0$$

Vorticity causes sound and vortices to mix, alters the dispersion relation to a generally non-causal one...

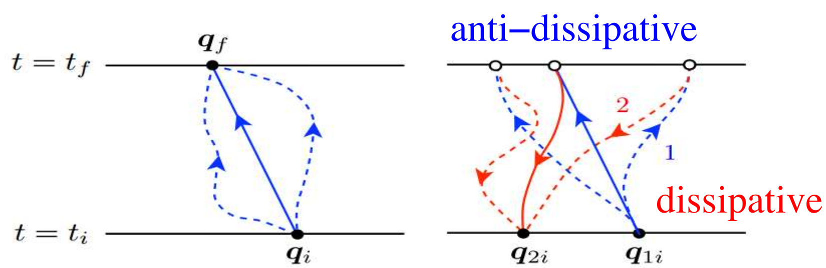
A possible interpretation: Minimal viscosity from polarization!

Need Israel-Stewart like terms at first order to restore causality. e.g. (wrong in paper!)

$$\tau_\Omega u_\alpha \partial^\alpha \Omega_{\mu\nu} + \Omega_{\mu\nu} = \chi(T,y)^{-1} y_{\mu\nu} \quad , \quad \Omega_{\mu\nu} \to u^\beta \epsilon_{\mu\nu\alpha\gamma} \partial^\alpha T^\gamma_\beta$$

Dissipative techniques can be added to Lagrangian using field doubling

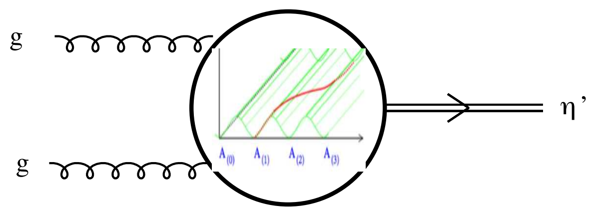
C.Galley
1210.2745

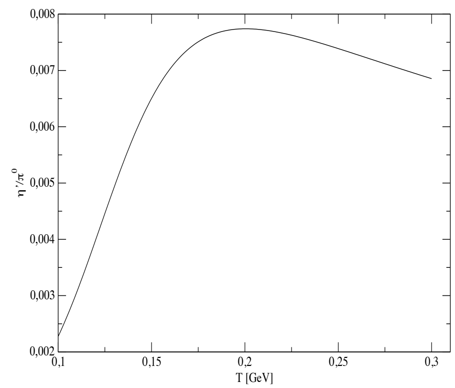


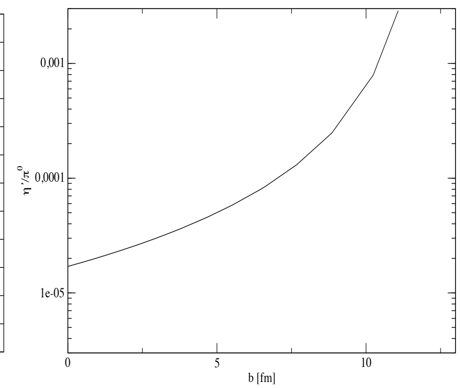
$$2L = \underbrace{m\dot{x}^2 - wx^2}_{SHO} \rightarrow \underbrace{\left(m\dot{x}_+^2 - wx_+^2\right)}_{\mathcal{L}_1} - \underbrace{\left(m\dot{x}_-^2 - wx_-^2\right)}_{\mathcal{L}_2} + 2\underbrace{\dot{x}_+ x_- - \dot{x}_- x_+}_{\mathcal{K}}$$

Standard techniques give you two sets of equations, one with a damped harmonic oscillator, the other "anti-damped"

PS an experimental aside (1802.09011)





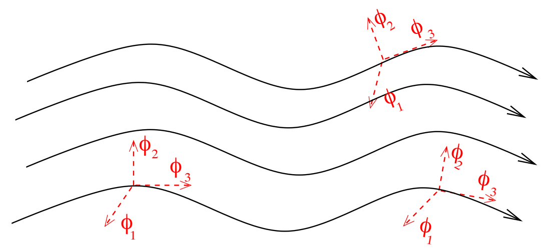


Gluons in a QGP producing η' mesons at coalescence should be susceptible to polarization. $U_A(1)$ violation forces matrix element to be

$$T_{ab}^{\alpha\beta}(p,q,P) = H_f(p^2,q^2,P^2)P(p,q) \quad , \quad P(p,q,\alpha,\beta) = \epsilon_{\mu\nu\lambda\gamma}\, p^\mu\, q^\nu\, \epsilon^\lambda(p)\, \epsilon^\gamma(q)$$

Second term, together with thermal gluons, contains vorticity! η' was known to be a probe of CP-odd phases for years but Success of statistical model suggests using the centrality dependence of η'/π^0 , both decaying into $\gamma\gamma$ as a probe of gluon polarization. A strong centrality dependence (absent from most ratios) would suggest it.

Some non-conclusions



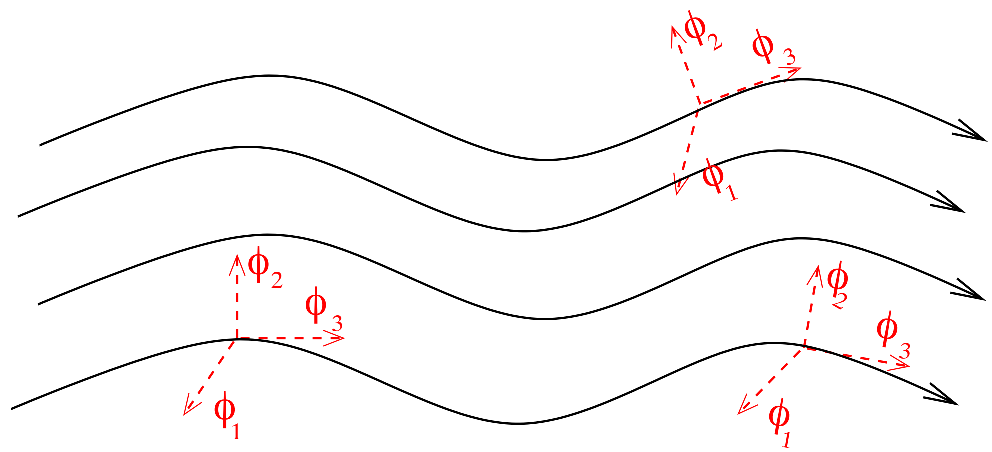
- Thermalization of "spin-angular momentum" interactions an unsolved problem. Its solution conceptually very non-trivial, phenomenologically essential to understand a whole range of effects

- only method which I think is capable to tackle it is to treat hydro as a lagrangian field theory. Work in progress

- Mostly theory work, but some experimental results I am waiting for: η'/π^0 ratio as a function of centrality, Λ vs $\overline{\Lambda}$ polarization as a function of centrality/system size, chemical potential

Hydro as EFT fields: (Nicolis et al,1011.6396 (JHEP))

Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t=0, x^i), I = 1...3$. (Lagrangian hydro . NB: no conserved charges)



The system is a Fluid if it's Lagrangian obeys some symmetries (Ideal hydrodynamics $\leftrightarrow$ Isotropy in comoving frame) Solutions generally break these, Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons".

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \mathrm{det} B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B (actually $b = \sqrt{B}$)
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}} \epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\,\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K$$

(A useful formula is $\frac{db}{d\partial_\mu\phi_I}\partial_\nu\phi_I = u^\mu u^\nu - g^{\mu\nu}$)

Equation of state chosen by specifying $F(b)$. “Ideal”: $\Leftrightarrow F(B) \propto b^{2/3}$

b is identified with the entropy and $b\frac{dF(B)}{dB}$ with the microscopic temperature.

u^μ fixed by $u^\mu\partial_\mu\phi^{\forall I} = 0$

You can also show that

$$\partial_\mu \left(\underbrace{b}_{=s} u^\mu \right) = 0 \quad , \quad s = -\frac{dP}{dT} = \frac{p+\rho}{T}$$

Ie, b is the conserved quantity corresponding to our earlier group. Up to dimensional factor corresponds to microscopic entropy. Can also write everything in terms of $K^\mu = bu^\mu$

Chemical potentials (neglected here) would be implemented by complexifying ϕ_I and promoting them to internal space vectors

An infinite number of global conserved charges for every closed path, vorticity is conserved. Corresponding to infinite-D diffeomorphism invariance

Ideal hydrodynamics and the microscopic scale

The most general Lagrangian is

$$L = T_0^4 F\left(\frac{B}{T_0^4}\right) \quad , \quad B = T_0^4 \det B^{IJ} \quad , \quad B^{IJ} = \left|\partial_\mu \phi^I \partial^\mu \phi^J\right|$$

Where $\phi^{I=1,2,3}$ is the comoving coordinate of a volume element of fluid.

NB: $T_0 \sim \Lambda g$ microscopic scale, includes thermal wavelength and $g \sim N_c^2$ (or μ/Λ for dense systems). $T_0 \to \infty \Rightarrow$ classical limit
It is therefore natural to identify T_0 with the microscopic scale!

Kn behaves as a gradient, T_0 as a Planck constant!!!

At $T_0 < \infty$ quantum and thermal fluctuations can produce sound waves and vortices, “weighted” by the usual path integral prescription!

$$\mathcal{Z} = \int \mathcal{D}\phi_i \exp\left[-T_0^4 \int F(B) d^4x\right], \langle \mathcal{O} \rangle \sim \frac{\partial \ln \mathcal{Z}}{\partial \ldots} \left(eg. \quad \left\langle T^{x}_{\mu\nu} T^{x'}_{\mu\nu} \right\rangle = \frac{\partial^2 \ln \mathcal{Z}}{\partial g_{\mu\nu}(x) \partial g_{\mu}} \right.$$

$T_0 \sim n^{-1/3}$, unlike Knudsen number, behaves as a ”Planck constant”

For analytical calculations fluid can be perturbed around a hydrostatic ($\phi_I = \vec{x}$) background

$$\phi_I = \vec{x} + \underbrace{(\vec{\pi}_L)}_{sound} + \underbrace{(\vec{\pi}_T)}_{vortex}$$

Polarization likely to dramatically change things here

And we discover a fundamental problem: Vortices carry arbitray small energies but stay put! No S-matrix in hydrostatic solution!

$$L_{linear} = \underbrace{\dot{\vec{\pi}}_L^{\,2} - c_s^2(\nabla.\vec{\pi}_L)^2}_{sound\quad wave} + \underbrace{\dot{\pi_T}^2}_{vortex} + Interactions(\mathcal{O}\left(\pi^3, \partial\pi^3, ...\right))$$

Unlike sound waves , Vortices can not give you a theory of free particles, since they do not propagate: They carry energy and momentum but stay in the same place! Can not expand such a quantum theory in terms of free particles.

Physically: "quantum vortices" can live for an arbitrary long time, and dominate any vacuum solution with their interactions. This does not mean the theory is ill-defined, just that its strongly non-perturbative!

Polarization might help here!

The big problem with Lagrangians... usually only non-dissipative terms

A first order term in the Lagrangian can always be reabsorbed as a field redefinition, i.e. is topological

But there are a few ways to fix it. We focus on coordinate doubling (Galley,but before Morse+Feschbach)

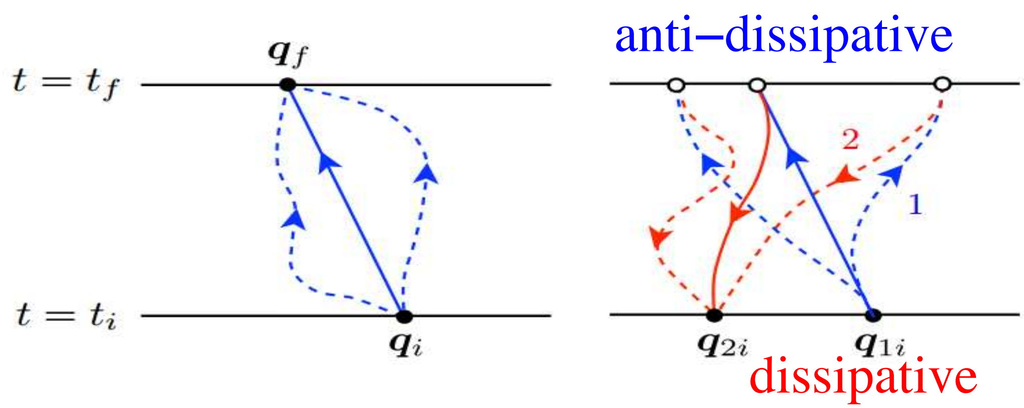
$$\phi_I \to \hat{\phi}_I = (\phi_I^+, \phi_I^-)$$

Action given by two copies plus an interaction term

$$S_{CTP} = \int_{t_f}^{t_i} d^4x \left\{ \mathcal{L}_s[\phi^+] - \mathcal{L}_s^*[\phi^-] + \mathcal{K}[\hat{\phi_\pm}] \right\}$$

The first two terms are non-dissipative, action doubled. Third term can be used to model dissipation

Dissipative extension of Hamiltons principle



$$L = \frac{1}{2}\Big(\underbrace{m\dot{x}^2 - wx^2}_{SHO}\Big) \to \Bigg[\underbrace{\frac{1}{2}\left(m\dot{x}_+^{\,2} - wx_+^2\right)}_{\mathcal{L}_1}\Bigg] - \Bigg[\underbrace{\frac{1}{2}\left(m\dot{x}_-^{\,2} - wx_-^2\right)}_{\mathcal{L}_2}\Bigg] + \Big[\underbrace{\dot{x}_+ x_- - x}_{\mathcal{K}}$$

Standard techniques give you <u>two</u> sets of equations, one with a damped harmonic oscillator, the other "anti-damped"

Navier-Stokes (GT,D.Montenegro, PRD, in press)

In terms of $K^\mu = bu^\mu$ the bulk term is

$$\mathcal{L}_{CTP}^{(1)} = T_o^4 \sum_{i,j,k} z_{ijk}(K^{l\gamma}K_\gamma^m) B \partial^\mu \phi^{iI} \partial^\nu \phi^{jJ} \partial_\mu K_\nu^k.$$

and the shear term is

$$\mathcal{L}_{CTP}^{(1)} = T_o^4 \sum_{i,j,k} z_{ijk}(K^{l\gamma}K_\gamma^m) B B_{IJ}^{-1} \partial^\mu \phi^{iI} \partial^\nu \phi^{jJ} \partial_\mu K_\nu^k.$$

These are the simplest terms compatible with most symmetries. But shear term also breaks volume-preserving diffeomorphism invariance. Effect of fundamental length?

Going further, second order term?

Problem Causality problem for first order terms (Lagrangian unbounded), second order terms with no local equilibrium (Ostrogradski's theorem)

Solution: introduce a new degree of freedom. Keep transversality condition but drop gradient dependence

$$\Pi_{\mu\nu} = X_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J$$

X_{IJ} are 6 new degrees of freedom to be fixed by initial conditions... Equivalent of Israel-Stewart off-diagonal terms

Israel-Stewart/Anisotropic hydrodynamics emerge naturally in Lagrangian approach

I-S in a lagrangian approach

$\Pi_{\mu\nu} = X_{IJ}\bar{A}^{IJ}_{\mu\nu}$ As these are not conserved quantities, the equation of motion has to be obtained from Lagrange's equations

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial X)} = \frac{\partial \mathcal{L}}{\partial X}$$

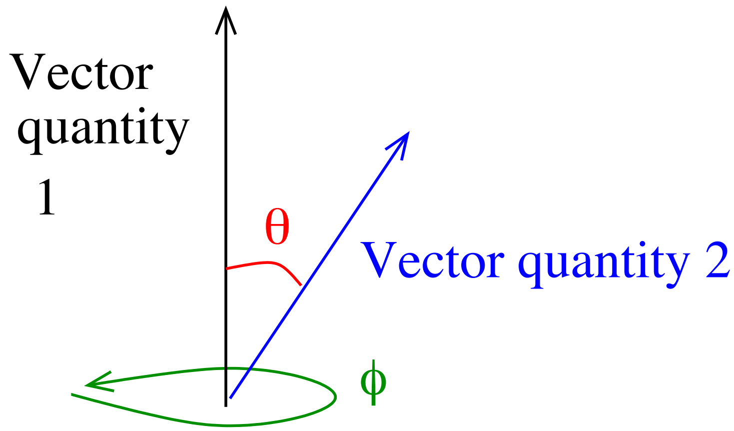
The Israel-Stewart equations of motion Follows easily from the Lagrangian

$$\mathcal{L} = T_0^4 F(B) + \frac{1}{2}\tau_\pi^\eta (\Pi_-^{\mu\nu} u_+^\alpha \partial_\alpha \Pi_{\mu\nu+} - \Pi_+^{\mu\nu} u_-^\alpha \partial_\alpha \Pi_{\mu\nu-})$$

$$+\frac{1}{2}\Pi_\pm^{\mu\nu}\Pi_{\mu\nu\pm} + \frac{X_{IJ}\pm}{6} \underbrace{\left[(A^\circ)^{IJ}_{\mu\nu}\partial^\mu K^\nu\right]_\pm}_{\sim\sigma_{\mu\nu}} + \mathcal{O}\left((\partial u)^2\right)$$

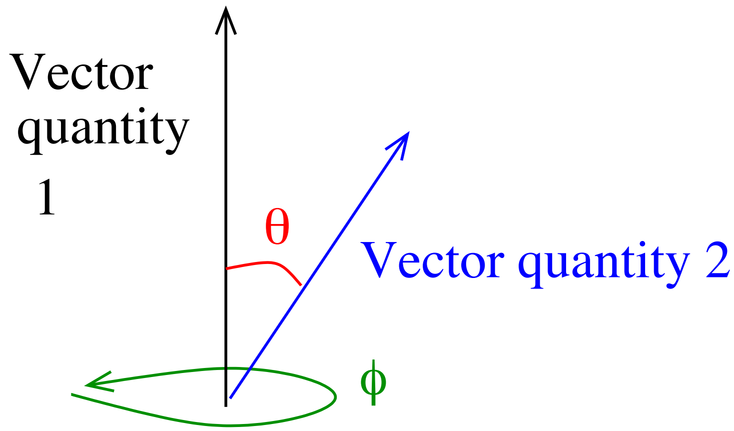
Last term non-dissipative, worked out in J. Bhattacharya, S. Bhattacharyya and M. Rangamani,1211.1020

An important implication for relaxation dynamics



The ideal limit is not just $\eta \to 0$ but also a well-defined $\tau_\pi \to 0$, "instant" relaxation. This implies a well-defined relaxed state.

An important implication for relaxation dynamics



A system with two local vector observables (eg polarization and vorticity!) which are not aligned generally does not have it. The ϕ degeneracy generates a Goldstone mode and topological constraints which make a $\tau_\pi \to 0$ limit unstable! this will be important later

We are now ready to combine polarization with the ideal hydrodynamic limit, defined as

(i) The dynamics within each cell is faster than macroscopic dynamics, and it is expressible only in term of local variables and with no explicit reference to four-velocity u^{μ} (gradients of flow are however permissible, in fact required to describe local vorticity).

(ii) Dynamics is dictated by local entropy maximization, within each cell, subject to constraints of that cell alone. Macroscopic quantities are assumed to be in local equilibrium inside each macroscopic cell

(iii) Only excitations around a hydrostatic medium are sound waves,vortices

(i-iii) ,with symmetries and EFT define the theory

Conserved charges (Dubovsky et al, 1107.0731(PRD))

Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

obviously can generalize to more complicated groups

So how do we implement polarization?

In comoving frame, polarization described by a representation of a "little group" of the volume element.

Need local $\sim SO(3)$ charges and unambiguus definition of u^μ $(s^\mu \propto J^\mu)$

$$\Psi_{\mu\nu}|_{comoving} = -\Psi_{\nu\mu}|_{comoving} = \exp\left[-\sum_{i=1,2,3} \alpha_i(\phi_I)\hat{T}_i^{\mu\nu}\right]$$

For particle spinor, vector, tensor... repreentations possible.

For "many incoherent particles" RPA means only vector representation remains

Chemical shift symmetry, $SO(3)_{\alpha_{1,2,3}} \to SO(3)_{\alpha_{1,2,3}(\phi^I)}$

$$\alpha_i \to \alpha_i + \Delta\alpha_i\,(\phi_I) \Rightarrow L(b, y_{\alpha\beta} = u_\mu \partial^\mu \Psi_{\alpha\beta})$$

$y_{\mu\nu} \equiv \mu_i$ for polarization vector components in comoving frame

This way we ensured spin current flows with u^μ.

Note that it is not a proper chemical potential (it it would be there would be 3 phases attached to each ϕ_I) as $y_{\mu\nu}$ not invariant under symmetries of ϕ_I. $y_{\mu\nu}$ "auxiliary" polarization field

How to combine polarization with local equilibrium?

Since polarization decreases the entropy by an amount proportinal to the DoFs and independent of polarization direction

$$b \to b\left(1 - c y_{\mu\nu} y^{\mu\nu} + \mathcal{O}\left(y^4\right)\right) \quad , \quad F(b) \to F(b,y) = F\left(b\left((1 - c y^2\right)\right)$$

.

Other terms break requirement (i)

First law of thermodynamics,

$$dE = TdS - pdV - Jd\Omega \to dF(b) = db\frac{dF}{db} + dy\frac{dF}{d(yb)}$$

Energy-momentum tensor

Not uniquely defined

Canonical defined as the Noether charge for translations, could be negative because of $\sim \frac{\partial L}{\partial(\partial\psi_i)}\partial\psi_j$

Belinfante-Rosenfeld $\sim \frac{\delta S}{\delta g_{\mu\nu}}$ symmetric independent of spin, no non-relativistic limit

Which is the source for $\partial_\mu T^{\mu\nu} = 0$? Not clear as...

The problem: Too many degrees of freedom

8 degrees of freedom,5 equations $(e, p, u_{x,y,z}, y^{\mu\nu})$. One can include the antisymmetric part of $T_{\mu\nu}$ and match equations but...

No entropy maximization If spin waves and sound waves separated, in comoving volume their ratio is arbitrary... but it should be decided by <u>entropy maximization</u>!

Solution clear: make polarization always proportional to vorticity,

$$y^{\mu\nu} \sim \chi(T)(e+p)\left(\partial^\mu u^\nu - \partial^\nu u^\mu\right)$$

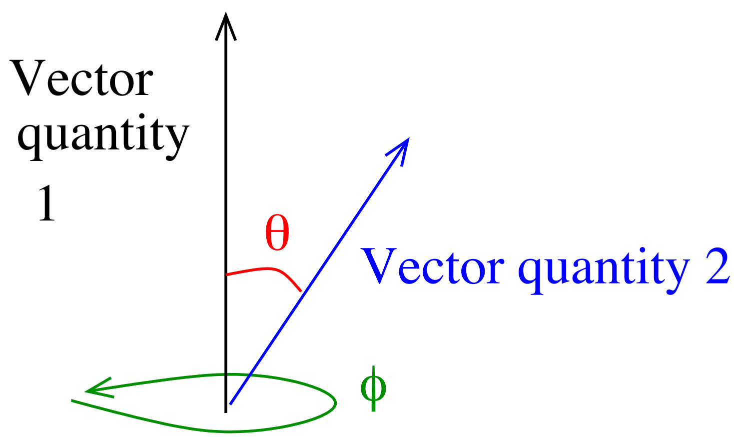
extension of Gibbs-Duhem to angular momentum uniquely fixes χ via entropy maximization. For a free energy $\mathcal{F}$ to be minimized

$$d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial V} dV + \frac{\partial \mathcal{F}}{\partial e} de + \frac{\partial \mathcal{F}}{\partial \left[\Omega_{\mu\nu}\right]} d\left[\Omega_{\mu\nu}\right] = 0$$

where $\left[\Omega_{\mu\nu}\right]$ is the vorticity in the comoving frame.
THis fixes χ . It also constrains the Lagrangian to be a Legendre transform of the free energy just as in the chemical potential case, in a straightforward generalization of Nicolis,Dubovsky et al. Free energy always at (local) minimum! (requirement (ii))

A qualitative explanation

Instant thermalization means vorticity instantly adjusts to angular momentum, and is parallel to angular momentum. Corrections to this will be of the relaxation type a-la Israel-Stewart



Note that microscopic physics could allow an arbitrary angle between vorticity and polarization. but such systems would have no hydrodynamic limit due to requirement (iii) and the necessity for stability of relaxation dynamics

These techniques lead to a well-defined Euler-Lagrange equation of motion

$$\left\{ g_b(1-cy)\partial_\nu b + g_y 4 y^\beta_\alpha \chi(T) \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda \phi^I)} \partial^\nu (\partial_\lambda \phi^I) \right\} \times$$

$$\times [(1-cy)\frac{\partial b}{\partial(\partial_\nu \phi^I)} - (8cb) y^\beta_\alpha \chi(T) \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu \phi^I)}] + g(b,y) \times$$

$$\times (1-cy)\partial_\nu (\frac{\partial b}{\partial(\partial_\nu \phi)}) - 8c\chi(T) g(b,y) \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda \phi^I)} \left[\frac{y^\beta_\alpha}{2} \partial_\nu \partial_\lambda \phi^I \times \right.$$

$$\times \frac{\partial b}{\partial(\partial_\nu \phi^I)} + (\partial^\nu b) 4 y^\beta_\alpha \delta^\lambda_\nu + b\chi(T) (\frac{\partial(\partial^\beta u_\alpha)}{\partial(\partial_\nu \phi^I)} + \frac{\partial(\partial_\alpha u^\beta)}{\partial(\partial_\nu \phi^I)}) \times$$

$$\left. \times \partial_\nu (\partial_\lambda \phi^I) + b y^\beta_\alpha \partial_\nu ln \frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu \phi^I)} \right] = 0$$

NB depends on accelleration, so $\Delta S = 0 \Rightarrow \partial_\mu \partial_\nu \frac{\partial F}{\partial(\partial_\mu \partial_\nu \phi^I)} = \partial_\mu \frac{\partial F}{\partial(\partial_\mu \phi^I)}$

Which can be linearized, $\phi_I = X_I + \pi_I$

The "free" (sound wave and vortex kinetic terms) part of the equation will be

$$\mathcal{L} = (-F'(1))\left\{\frac{1}{2}(\dot{\pi})^2 - c_s^2[\partial\pi]^2\right\}+$$
$$+f\zeta\left\{\ddot{\pi}^i\partial_i\dot{\pi}_j + \ddot{\pi}_i\ddot{\pi}_j + \partial_j\dot{\pi}^i\partial_i\dot{\pi}_j + \partial_j\dot{\pi}_i\ddot{\pi}_j+\right.$$
$$\left.+(2\ddot{\pi}^i\partial_j\dot{\pi}_i - 2\ddot{\pi}_j\partial^i\dot{\pi}_j) + (\ddot{\pi}_i^2 - \ddot{\pi}_j^2) + (\partial_j\dot{\pi}_i^2 - \partial_i\dot{\pi}_j^2)\right\}$$

- Accelleration terms survive linearization
- Vortices and sound wave modes mix at "leading" order. Change in temperature due to sound wave changes polarizability, and that changes vorticity

We decompose perturbation into sound and vortex $\phi_I = \nabla\phi + \nabla \times \vec{\Omega}$

$$\begin{pmatrix} \varphi \\ \vec{\Omega} \end{pmatrix} = \int dw d^3k \begin{pmatrix} \varphi_0 \\ \vec{\Omega}_0 \end{pmatrix} \exp\left[i \left(\vec{k_{\phi,\Omega}}.\vec{x} - w_{\phi,\Omega} t \right) \right]$$

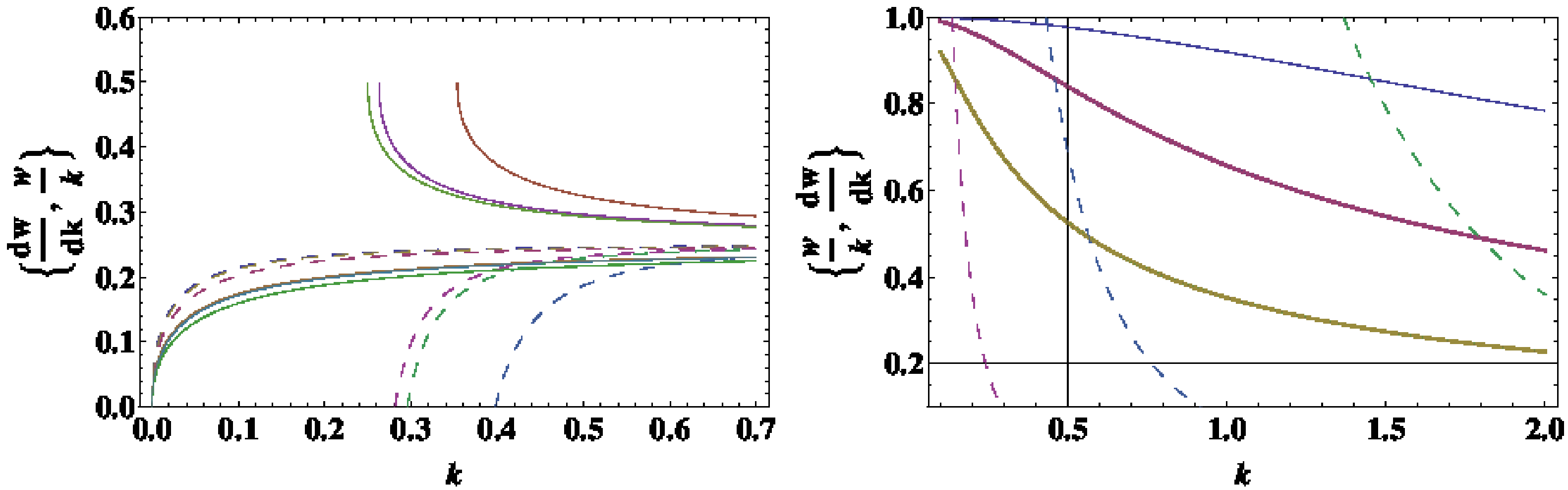
The part parallel to k ("sound-wave") will have a dispersion relation

$$w_\phi^2 - c_s^2 k_\phi^2 + 2\beta k_\phi w_\phi^3 = 0$$

The vector part will be

$$(3k_\Omega^2 - 2k_\Omega w_\Omega)_j (\vec{k}_\Omega \times \vec{\Omega}_0)_i w_\Omega^2 + w^4 \Omega = 0$$

Dispersion relations show violation of causality!



Both phase and group velocity will generally go above unity

What I think is going on I: A lower limit of viscosity for polarized hydro

the Free energy $\mathcal{F}$, and hence the local dynamics, is sensitive to an accelleration. As is well-known (Ostrogradski's theorem, Dirac runaway solutions) such Lagrangians are unstable and lead to causality violation. Note that one needs Lagrangians to see this!

To fix this issue, one would need to update the proportionality of y on Ω to an Israel-Stewart type equation

$$\tau_\Omega u_\alpha \partial^\alpha y_{\mu\nu} + y_{\mu\nu} = \chi(T, y)\Omega_{\mu\nu}$$

with an appropriate relaxation time τ_Ω would resolve this issue. Just like with Israel-Stewart, this requires the introduction of new DoFs with relaxation-type dynamics, but, unlike non-polarized hydro,such terms are required from the idea limit Polarization and vorticity conservation

When polarization is not dynamical ($y_{\mu\mu}$ constant), vorticity conservation arises as a non-local Noether current of the diffeomorphism invariance of the theory, specifically

$$\oint_{\Omega} dx_i u^i \frac{dF(b)}{db} = -\int_0^1 d\tau \int d^3x \frac{\partial L}{\partial(\partial_0 \phi^I)} \frac{d\Omega^I}{d\tau} \delta^3 \left(\phi^J - \Omega^J(\tau)\right)$$

LHS Vorticity defined along closed loop Ω

RHS Noether current of the diffeomorphism moving ϕ^I along closed path Ω in terms of parameter τ

$$\zeta^I_{\Omega}(\phi^J) = -\int_0^1 d\tau \frac{d\Omega^I}{d\tau} \delta^3 \left(\phi^J - \Omega^J(\tau)\right)$$

Polarization and vorticity conservation

If polarization is not zero, the fact that the equation above only moves around ϕ_I and not $y^{\mu\nu}$ breaks the symmetry, by an amount

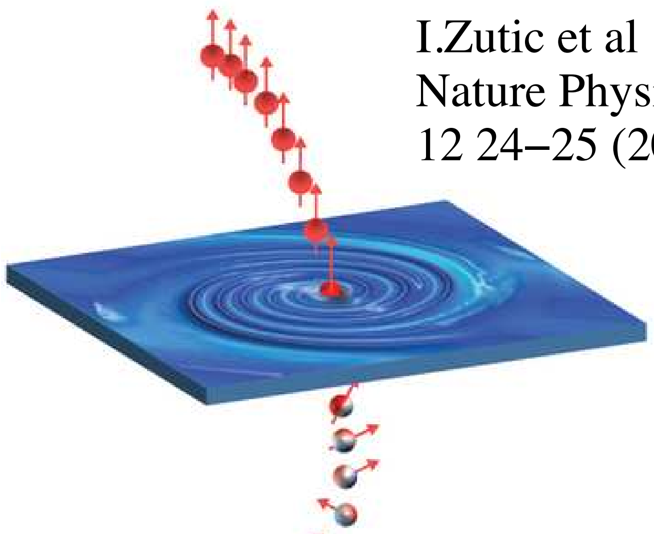
$$\frac{dy^{\mu\nu}}{d\tau} = \int d^3x \partial^\alpha y^{\mu\nu} \partial_\alpha \phi^I \delta^3 \left(\phi_J - \Omega_J(\tau)\right)$$

Hence, over a closed path we expect vorticity conservation to break down by an amount

$$\frac{d}{dt} \oint_\Omega dx_i u^i \frac{dF(b)}{db} = \dot{y}_{\alpha\beta} \frac{dL}{\partial(\partial_\mu y^{\alpha\beta})} \partial^\mu \zeta_\Omega(\phi^J) \equiv \frac{1}{2} g(b,y) \dot{y}^2 \int_0^1 \frac{d\Delta^{\alpha\beta}}{d\tau} \frac{\partial \Omega}{\partial \Delta^{\alpha\beta}} d\tau$$

$$\frac{d}{dt}\oint_{\Omega} dx_i u^i \frac{dF(b)}{db} = \frac{1}{2} g(b,y)\dot{y}^2 \int_0^1 \frac{d\Delta^{\alpha\beta}}{d\tau}\frac{\partial\Omega}{\partial\Delta^{\alpha\beta}} d\tau$$

The LHS is in principle a calculable but non-local quantity representing the transfer between local polarization and non-local vorticity degrees of freedom, the relativistic ideal hydrodynamic equivalent of



I.Zutic et al
Nature Physics
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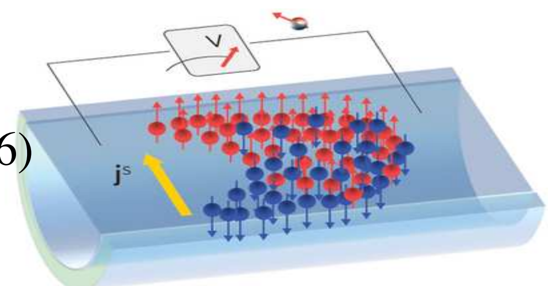
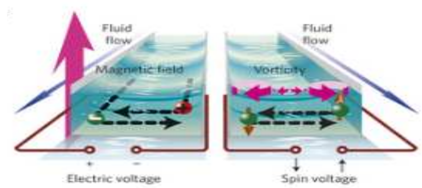
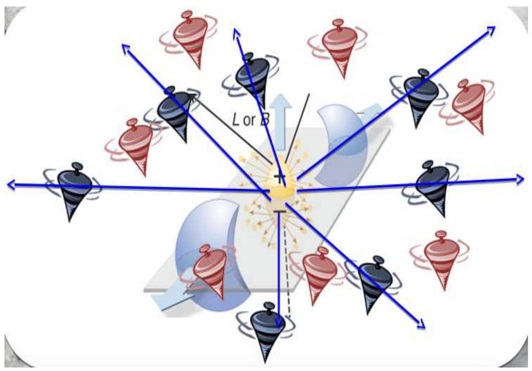
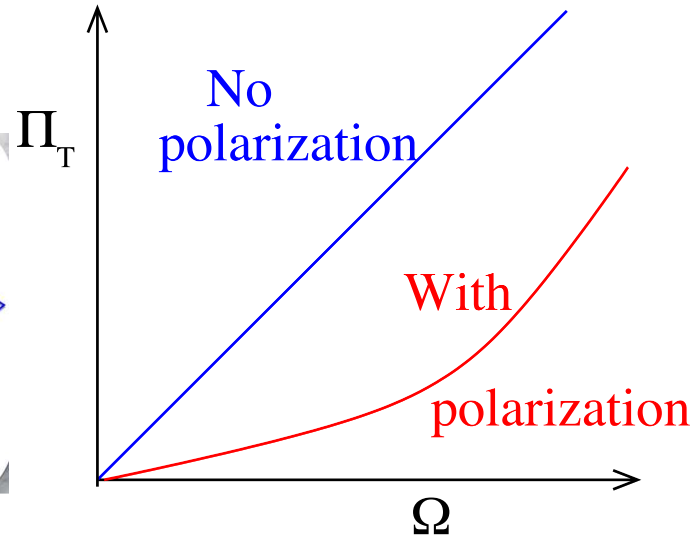




What I <u>think</u> is going on II





Polarization makes vorticity aquire a "soft gap" wrt angular momentum. At small amplitudes, creating polarization is more advantageus than creating vorticity. This means small amplitude vortices get quenched. Stabilizes theory against perturbations, might act as effective viscosity! Working on an expansion to prove it