CHIRALITY EVOLUTION IN PLASMA WITH MAGNETIC MONOPOLES

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based on work done with Yang Li arXiv:1708.08536

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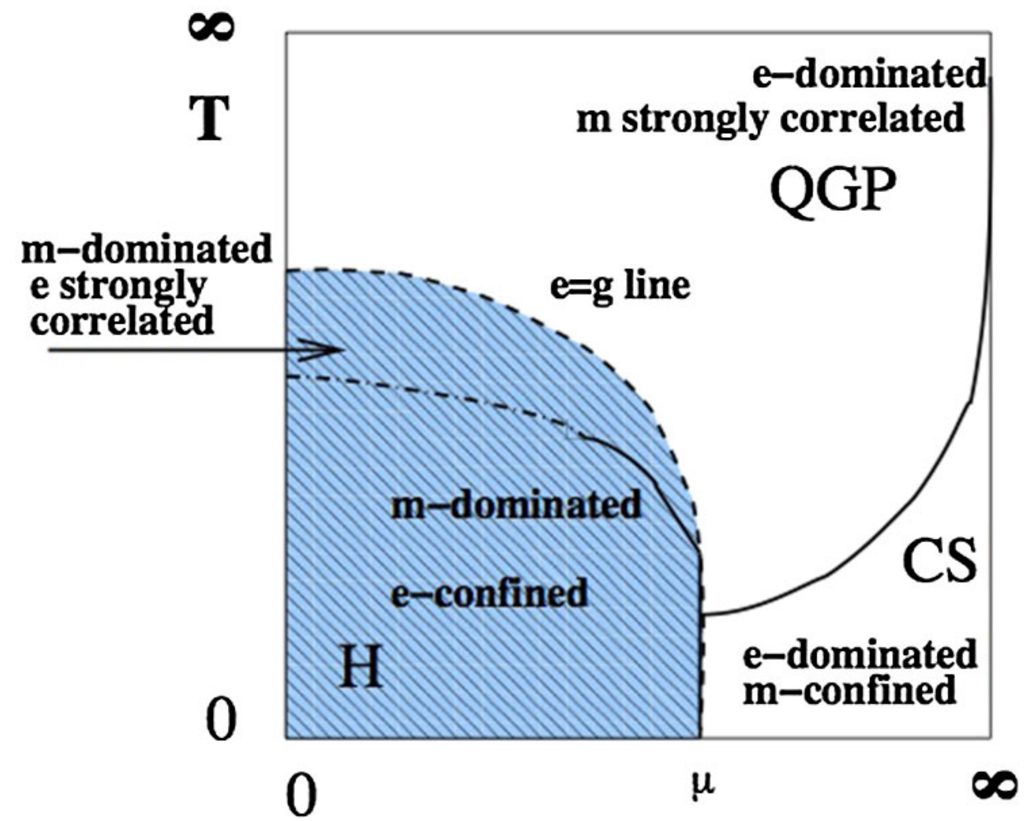
MOTIVATION & OUTLINE

Motivation:

Magnetic monopoles at T=0: dual superconductor, color confinement.

The condensate may not melt away at T_c

$\Rightarrow$ Important part of QGP dynamics



Outline

Chiral evolution without magnetic monopoles

Chiral evolution with magnetic monopoles (an Abelian model)

QED WITH CHIRAL ANOMALY WITHOUT MONOPOLES

Maxwell-Chern-Simons theory:

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}=0\,,\qquad \boldsymbol{\nabla}\times\boldsymbol{E}=-\partial_t\boldsymbol{B}\,,$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E}=\rho-c_A\,\boldsymbol{\nabla}\theta\cdot\boldsymbol{B}\,,\qquad \boldsymbol{\nabla}\times\boldsymbol{B}=\partial_t\boldsymbol{E}+\boldsymbol{j}+c_A(\partial_t\theta\,\boldsymbol{B}+\boldsymbol{\nabla}\theta\times\boldsymbol{E})$$

$$c_A=N_c\sum_f q_f^2e^2/2\pi^2$$

Pseudo-scalar field θ describes coupling of QED to the topological vacuum fluctuations of the nuclear matter (e.g. QGP).

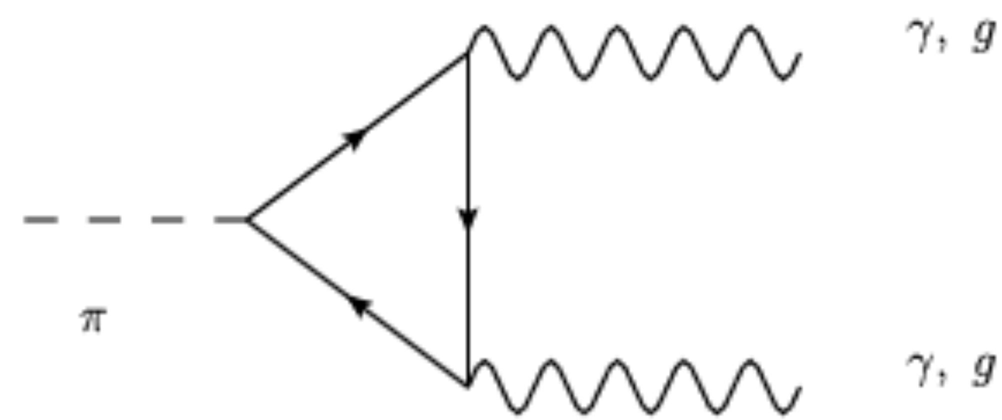
Assumption: θ is spatially homogeneous (single topological sector)

Anomalous current $\boldsymbol{j}=\sigma_\chi\boldsymbol{B}$ $\boldsymbol{j}, \boldsymbol{B}$ are T-odd ⇒ $\boldsymbol{\sigma_\chi}$ is T-even ⇒ non-dissipative

$\sigma_\chi=c_A\dot{\theta}=c_A\mu_5$ *chiral conductivity* induced by the chiral anomaly of QED

HELICITY CONSERVATION WITHOUT MONOPOLES

Chiral anomaly $\partial_\mu j^\mu_A = c_A \boldsymbol{E} \cdot \boldsymbol{B}$



Magnetic helicity: $\mathcal{H}_{\rm em} = \int \boldsymbol{A} \cdot \boldsymbol{B} \, d^3x$

Integrate over time 👉 helicity conservation

$$\frac{2V}{c_A} \langle n_A \rangle + \mathcal{H}_{\rm em} = \mathcal{H}_{\rm tot}$$

Equation of state (hot medium) $\langle n_A \rangle = \chi \, \mu_5$

$$\boxed{\frac{\sigma_\chi(t)}{\alpha} = 1 - \frac{\mathcal{H}_{\rm em}(t)}{\mathcal{H}_{\rm tot}}}$$

Helicity flows between the magnetic field and medium. Total helicity is conserved.

Fraction of the total helicity in the plasma

Fraction of the total helicity in the field

ADIABATIC SOLUTION WITHOUT MONOPOLES

Radiation gauge: $\boldsymbol{\nabla}\cdot\boldsymbol{A}=0,\ A^0=0$ $\qquad -\nabla^2\boldsymbol{A}=-\partial_t^2\boldsymbol{A}+\boldsymbol{j}+\sigma_\chi(t)\boldsymbol{\nabla}\times\boldsymbol{A}$

Expand $\quad \boldsymbol{A}=\sum_{\boldsymbol{k},\lambda}\left[a_{\boldsymbol{k}\lambda}(t)\boldsymbol{W}_{\boldsymbol{k}'\lambda'}(\boldsymbol{x})+a^*_{\boldsymbol{k}'\lambda'}(t)\boldsymbol{W}^*_{\boldsymbol{k}'\lambda'}(\boldsymbol{x})\right]$

where $\quad \boldsymbol{W}_{\boldsymbol{k}\lambda}(\boldsymbol{x})=\dfrac{\boldsymbol{\epsilon}^\lambda}{\sqrt{2kV}}e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$ $\qquad \boldsymbol{\nabla}\times\boldsymbol{W}_{\boldsymbol{k}\lambda}(\boldsymbol{x})=\lambda k\boldsymbol{W}_{\boldsymbol{k}\lambda}(\boldsymbol{x})$

Adiabatic approximation $\quad a_{\boldsymbol{k}\lambda}=a_{\boldsymbol{k}\lambda}(0)e^{-i\int_0^t\omega_{k\lambda}(t')dt'}$

$$\omega_{k\lambda}(t)=\left[-\frac{i\sigma}{2}+\lambda_1\sqrt{k^2-\sigma_\chi\lambda k-\sigma^2/4}\right] \qquad \lambda_1=\pm1$$

When $k<\sigma_\chi$ $\quad Im\ \omega>0\implies$ instability 👉 inverse cascade

THE FASTEST GROWING STATE (FGS) MODEL

The fastest growing state has the largest imaginary $\omega_{k\lambda}(t)$ which happens when

$k_0 = \frac{\sigma_\chi \lambda}{2}$ for which $\omega_0(t) = -\frac{i\sigma}{2} + \frac{i}{2}\sqrt{\sigma^2 + \sigma_\chi^2(t)}$

The fastest growing state is then given by

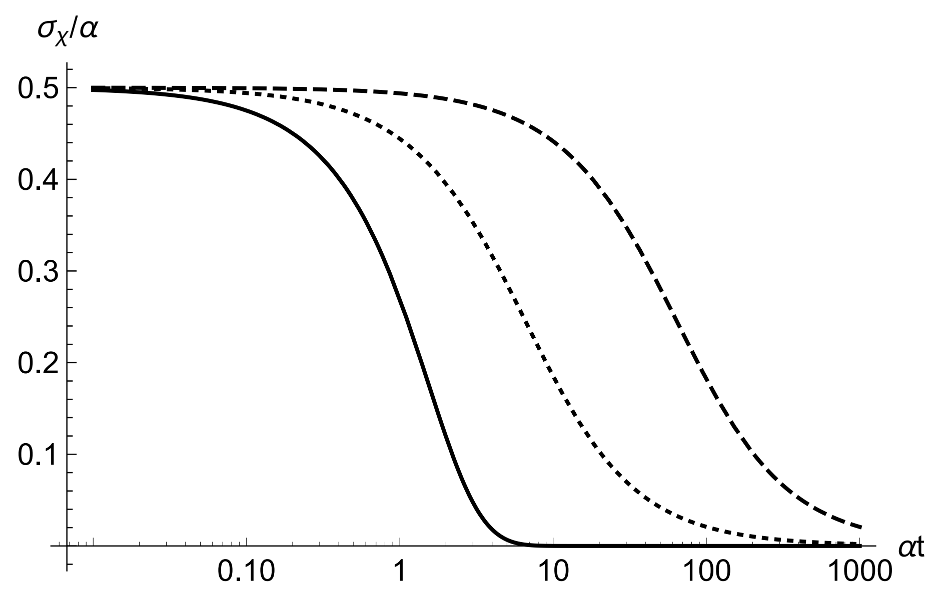
$$a_0(t) = e^{\gamma(t)/2} \qquad \gamma(t) = \int_0^t \left[\sqrt{\sigma^2 + \sigma_\chi^2(t')} - \sigma\right] dt'$$

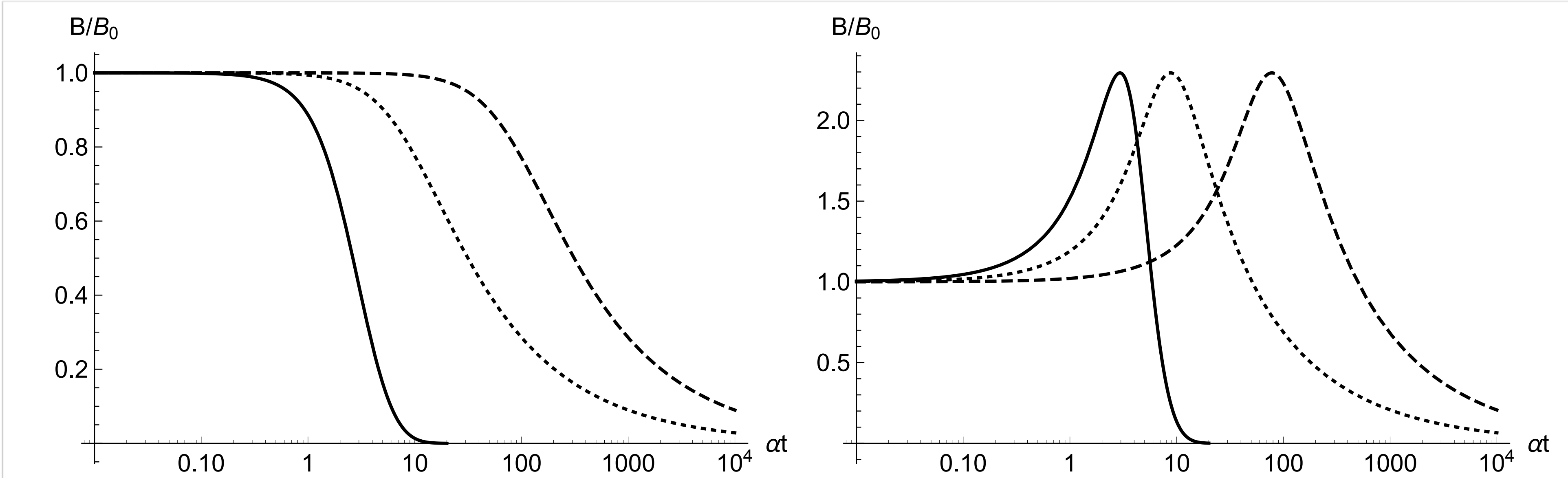
Within this model the helicity conservation equation:

$$\boxed{\dot{\sigma}_\chi = -\left(\sqrt{\sigma^2 + \sigma_\chi^2} - \sigma\right)(\alpha - \sigma_\chi)}$$

can be solved analytically ⇒ chiral evolution

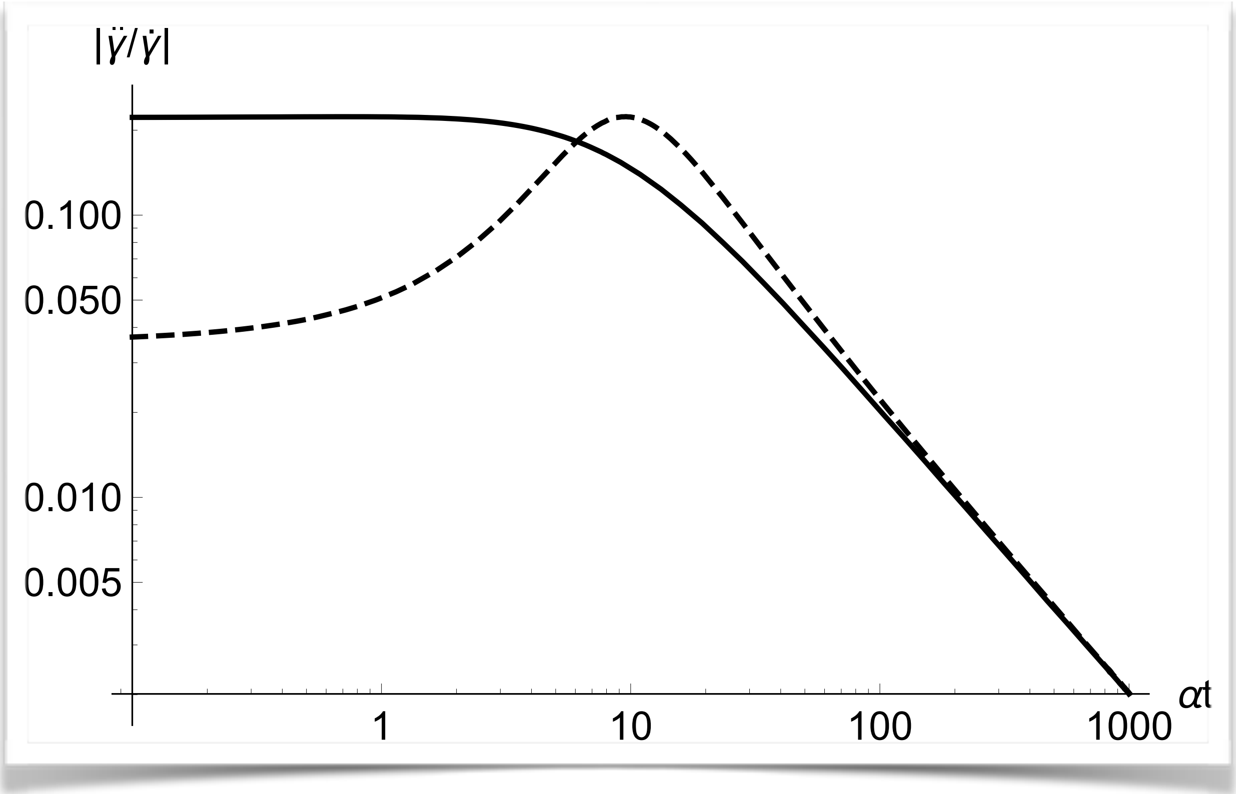
CHIRAL EVOLUTION WITHOUT MONOPOLES





Magnetic field has a peak (instability) only if $\sigma_\chi(0)/\alpha > 1/2$

VALIDITY OF ADIABATIC APPROXIMATION



Works very well!

QED WITH CHIRAL ANOMALY AND WITH MONOPOLES

$$\nabla \cdot \boldsymbol{B} = 0\,, \qquad -\nabla \times \boldsymbol{E} = \partial_t \boldsymbol{B} + \boldsymbol{j}_m$$

$$\nabla \cdot \boldsymbol{E} = 0\,, \qquad \nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j}_e + \sigma_\chi \boldsymbol{B}\,,$$

Assume linear response: $\boldsymbol{j}_e = \sigma_e \boldsymbol{E},\ \boldsymbol{j}_m = \sigma_m \boldsymbol{B},$

$$-\nabla^2 \boldsymbol{B} + \partial_t^2 \boldsymbol{B} = -(\sigma_e + \sigma_m)\partial_t \boldsymbol{B} - \sigma_e \sigma_m \boldsymbol{B} + \sigma_\chi(t) \nabla \times \boldsymbol{B}$$

Define “vector potential” $\boldsymbol{E} = -\partial_t \boldsymbol{A} - \sigma_m \boldsymbol{A} \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}. \qquad \nabla \cdot \boldsymbol{A} = 0$

Chiral anomaly equation $\partial_t \sigma_\chi = c_A^2/(\chi V) \int \boldsymbol{E} \cdot \boldsymbol{B}\, d^3x\,,$

👉 $$\boxed{\beta^{-1}\partial_t \sigma_\chi = -\partial_t \mathcal{H}_{\rm em} - 2\sigma_m \mathcal{H}_{\rm em}}$$

$\Rightarrow$ Magnetic current dissipates magnetic helicity,

but conserves energy because is σ_m is T-even

SUPERCONDUCTING PHASE

$\boldsymbol{E} = -\partial_t \boldsymbol{A} - \sigma_m \boldsymbol{A}$ reminiscent of the London theory of superconductivity

Note, however, is that A does not transform under the gauge transform as a vector potential has to.

Define the “normal” and “super” fields $\boldsymbol{E}_n = -\partial_t \boldsymbol{A}\,, \qquad \boldsymbol{E}_s = -\sigma_m \boldsymbol{A}\,.$

and currents $\boldsymbol{j}_n = \sigma_e \boldsymbol{E}_n\,, \qquad \boldsymbol{j}_s = \sigma_e \boldsymbol{E}_s = -\sigma_e \sigma_m \boldsymbol{A}\,.$

charge conservation $\boldsymbol{\nabla} \cdot \boldsymbol{j}_n = \boldsymbol{\nabla} \cdot \boldsymbol{j}_s = 0.$

London equations: $\boldsymbol{\nabla} \times \boldsymbol{j}_s = -\,\sigma_e \sigma_m \boldsymbol{B}, \qquad \partial_t \boldsymbol{j}_s = +\,\sigma_e \sigma_m \boldsymbol{E}_n,$

⇒ when $\boldsymbol{E}_{\mathrm{n}}=0$, $\boldsymbol{j}_{\mathrm{s}}$ is finite

MEISSNER EFFECT

Stationary limit: $\nabla^2 \boldsymbol{B} = \sigma_e \sigma_m \boldsymbol{B} - \sigma_\chi \boldsymbol{\nabla} \times \boldsymbol{B}\,,$

$$\nabla^2 \boldsymbol{E}_s = \sigma_e \sigma_m \boldsymbol{E}_s - \sigma_\chi \boldsymbol{\nabla} \times \boldsymbol{E}_s$$

Spectrum:

$$k = \frac{\lambda \sigma_\chi}{2} \pm \sqrt{\frac{1}{4}\sigma_\chi^2 - \sigma_e \sigma_m}.$$

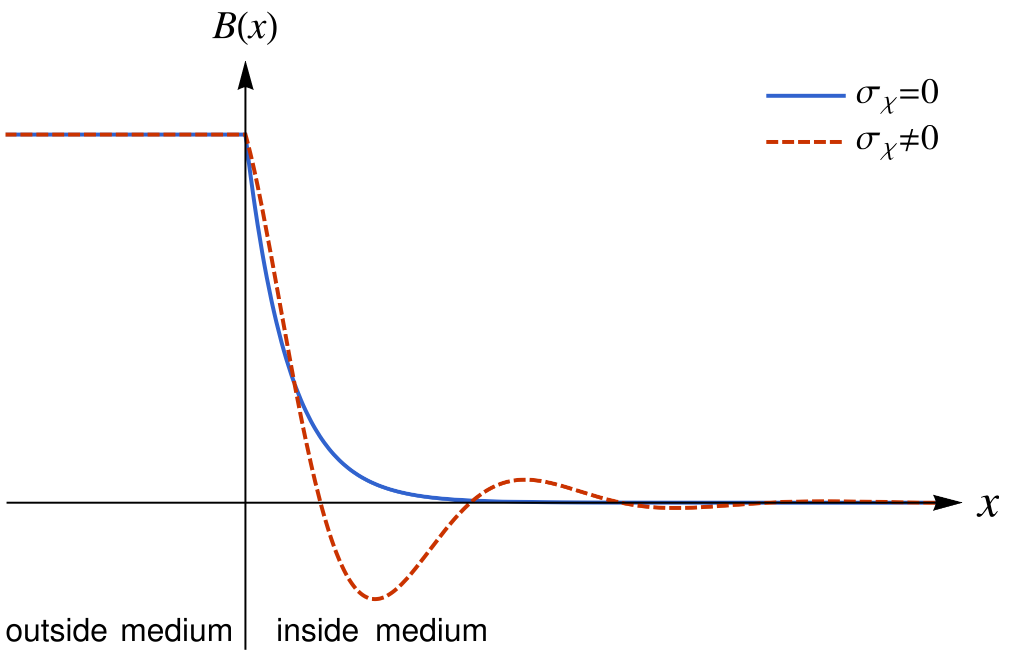


FIG. 1. Meissner effect in a chiral medium.

Meissner effect appears if $\boxed{\sigma_\chi^2 < 4\sigma_e \sigma_m}$

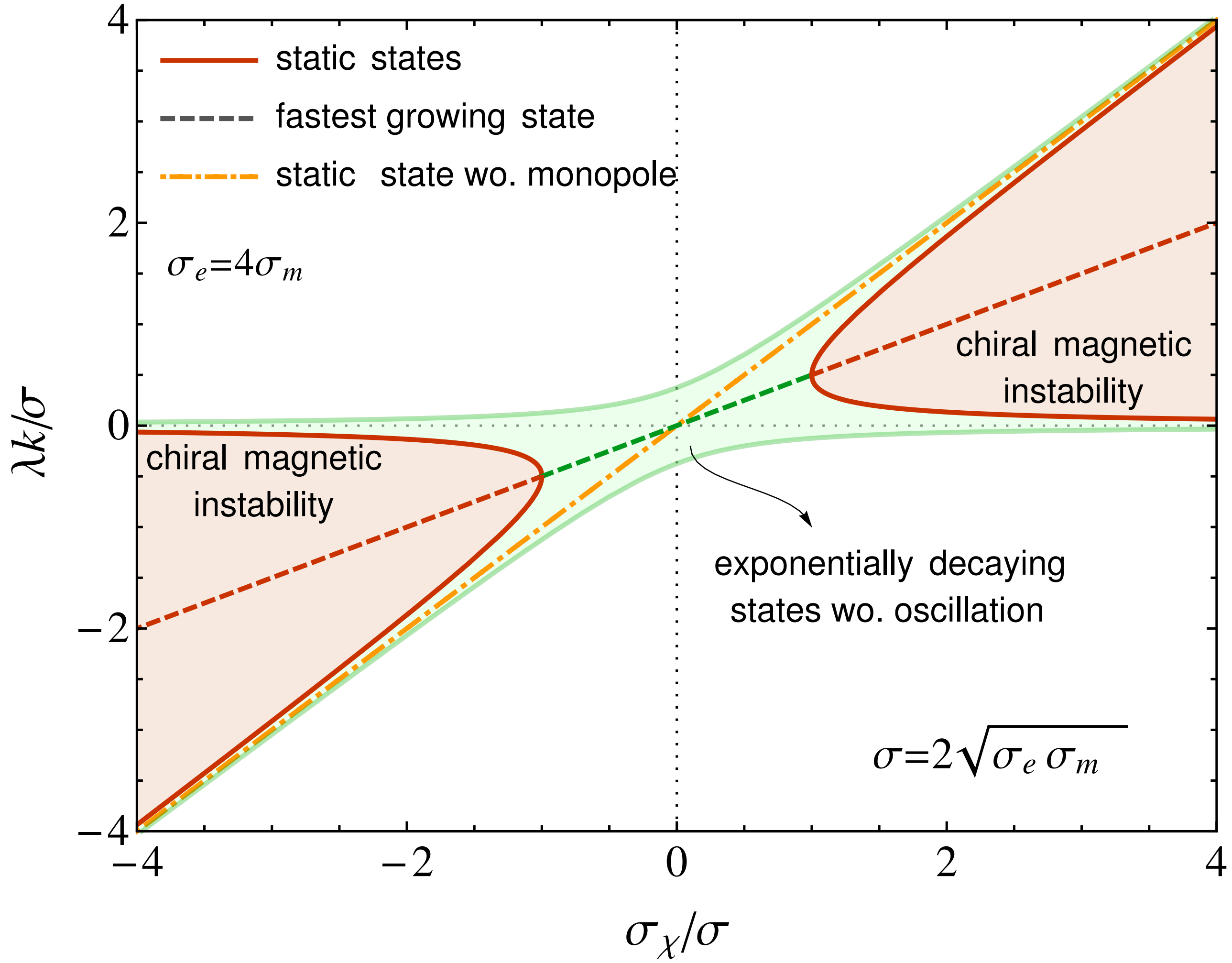
London penetration depth $L = (\sigma_e \sigma_m - \sigma_\chi^2/4)^{-1/2}$

Same is true of electric field. Moreover, one can introduce $\boldsymbol{A}$ such that $\boldsymbol{E} = \mathrm{curl}\, \boldsymbol{A}$.
Everything is dual!

ADIABATIC SOLUTION OF MCS WITH MONOPOLES

$$-\nabla^2 \boldsymbol{B} + \partial_t^2 \boldsymbol{B} = -(\sigma_e + \sigma_m)\partial_t \boldsymbol{B} - \sigma_e \sigma_m \boldsymbol{B} + \sigma_\chi(t) \boldsymbol{\nabla} \times \boldsymbol{B}$$

Dispersion: $$\omega_{k\lambda}(t) = -\frac{i(\sigma_e + \sigma_m)}{2} + \lambda_1 \frac{i}{2}\sqrt{(\sigma_e + \sigma_m)^2 + 4(\sigma_\chi \lambda k - \sigma_e \sigma_m - k^2)} \qquad \lambda_1 = \pm 1$$



Instability (exponential increase of $\mathcal{H}$) occurs for *Im* ω>0:

$$\frac{\sigma_\chi}{2} - \frac{1}{2}\sqrt{\sigma_\chi^2 - 4\sigma_e\sigma_m} \leq k \leq \frac{\sigma_\chi}{2} + \frac{1}{2}\sqrt{\sigma_\chi^2 - 4\sigma_e\sigma_m}\,.$$

⇒ only possible if $\sigma_\chi^2 > 4\sigma_e\sigma_m$.

👉 chiral instability is incompatible with the Meissner effect

BTW, the Fastest Growing State model captures the essential qualitative features.

FASTEST GROWING STATE WITH MONOPOLES

Use FGS model to illustrate the general features the chiral evolution.

$$\omega_{k\lambda}(t) = -\frac{i(\sigma_e + \sigma_m)}{2} + \lambda_1 \frac{i}{2}\sqrt{(\sigma_e + \sigma_m)^2 + 4(\sigma_\chi \lambda k - \sigma_e \sigma_m - k^2)}$$

The growth rate is largest for $k_\star = \dfrac{\sigma_\chi \lambda}{2}$,

Amplitude $\quad a_\star(t) = a_\star(0) e^{\frac{1}{2}\gamma(t)} \qquad \gamma(t) = \int_0^t \left[\sqrt{(\sigma_e - \sigma_m)^2 + \sigma_\chi^2(t')} - (\sigma_e + \sigma_m)\right] dt'$

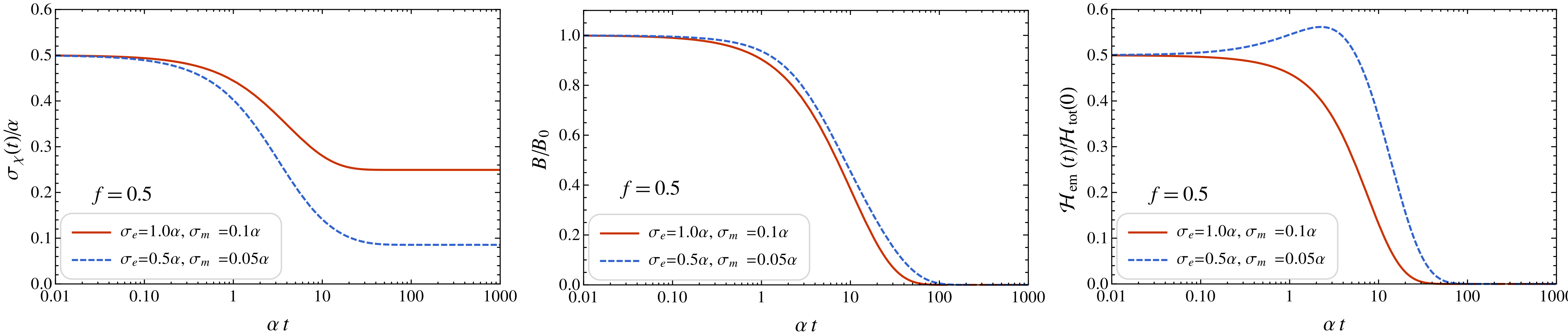
Magnetic helicity $\quad \mathcal{H}_{\rm em}(t) = f \mathcal{H}_{\rm tot}(0) e^{\gamma(t)}$ $\quad f$ is the initial fraction of the helicity stored in B

One can derive a closed equation governing time-evolution of the chiral conductivity

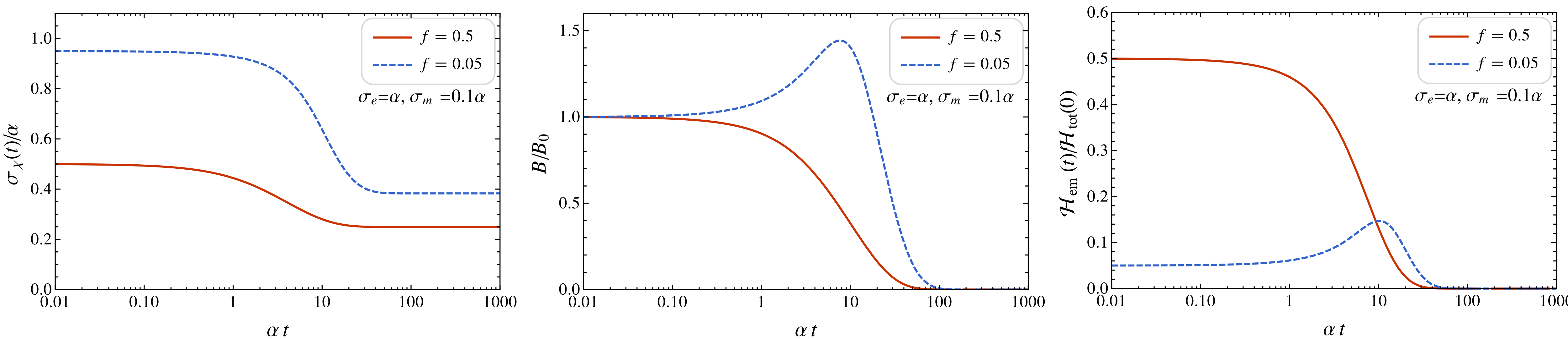
$$\boxed{\dot{\sigma}_\chi = -\left[f + F(1-f) - F(\sigma_\chi)\right]\left(\sqrt{(\sigma_e - \sigma_m)^2 + \sigma_\chi^2} - \sigma_e + \sigma_m\right)}$$

$$F(\sigma_\chi) = \frac{1}{\sigma_\chi}\left\{\sigma_\chi^2 + 2\sigma_m\left[\sqrt{\sigma_\chi^2 + (\sigma_e - \sigma_m)^2} + \sigma_e - \sigma_m\right] - 2\sigma_\chi \sigma_m \ln\left[\sigma_\chi + \sqrt{\sigma_\chi^2 + (\sigma_e - \sigma_m)^2}\right]\right\} ,$$

EXAMPLES OF EVOLUTION WITH MONOPOLES



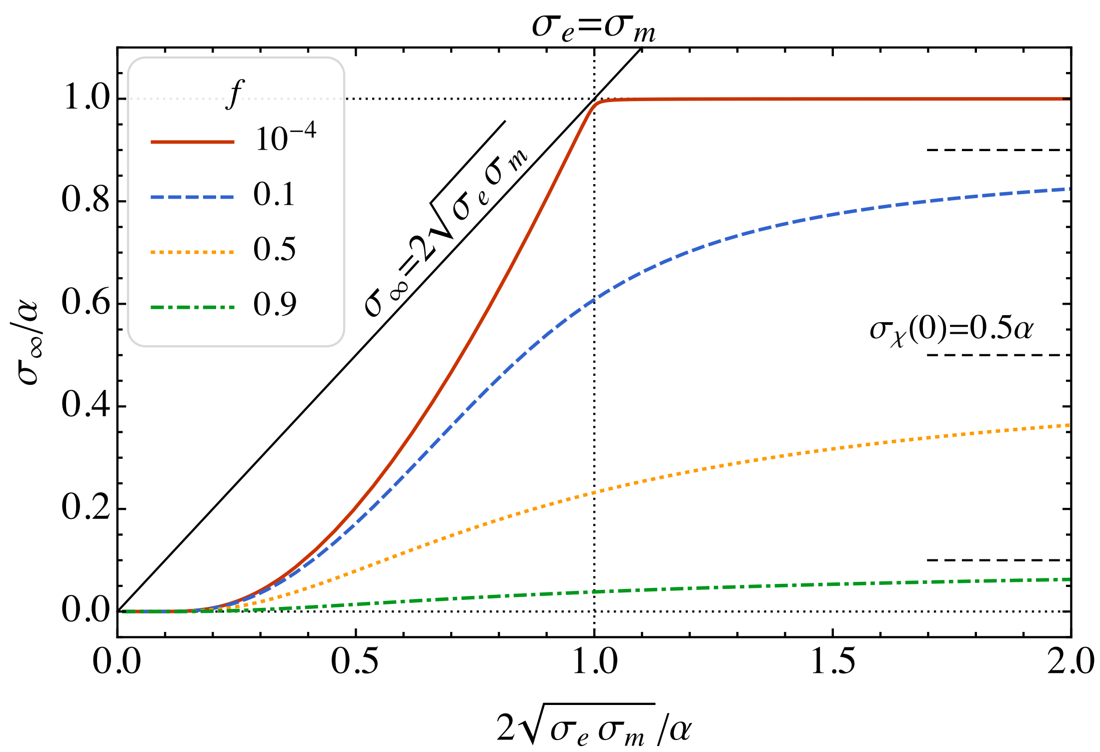
(a) $f = 0.5,\ \sigma_e = 10\sigma_m$



(c) $\sigma_e = 1\alpha, \sigma_m = 0.1\alpha$

Notice that σ_χ does not vanish at $t\to\infty$ unlike no-monopole case. This is because part of magnetic helicity leaks out and evolution stops when the system runs out of the magnetic helicity, but the medium helicity is finite.

FINAL VALUES OF CHIRAL CONDUCTIVITY



At $t\to\infty$ σ_χ approaches a constant that satisfies

$$\sigma_\chi^2(t\to\infty) < 4\sigma_e\sigma_m$$

😲 Final state is a stable and exhibits the Meisner effect!

Stability = exponential decay of small magnetic helicity fluctuations

WHEN MAGNETIC HELICITY IS STABLE

Using FGS one can derive a more stringent stability condition.

Instability of magnetic helicity = period of growth → stability condition is $\dot{\mathcal{H}}_{\rm em} < 0.$

$$\mathcal{H}_{\rm em} = \mathcal{H}_{\rm tot}(0)\left[f + F(1-f) - F(\sigma_\chi)\right].$$

→ $F'(\sigma_\chi)\dot{\sigma}_\chi > 0.$

$$\dot{\sigma}_\chi = -\left[f + F(1-f) - F(\sigma_\chi)\right]\left(\sqrt{(\sigma_e-\sigma_m)^2+\sigma_\chi^2} - \sigma_e + \sigma_m\right)$$

→ $\dot{\sigma}_\chi < 0$

→ $F'(\sigma_\chi) < 0$

$$F(\sigma_\chi) = \frac{1}{\sigma_\chi}\left\{\sigma_\chi^2 + 2\sigma_m\left[\sqrt{\sigma_\chi^2+(\sigma_e-\sigma_m)^2}+\sigma_e-\sigma_m\right] - 2\sigma_\chi\sigma_m \ln\left[\sigma_\chi + \sqrt{\sigma_\chi^2+(\sigma_e-\sigma_m)^2}\right]\right\},$$

→ $\sigma_\chi^2 \leq 4\sigma_e\sigma_m$

In particular,

$$\boxed{\sigma_\chi^2(0) \leq 4\sigma_e\sigma_m\,.}$$

SUMMARY

1. Focus of this talk: dual QED with linear response.

2. There is a superconducting phase; Meissner effect for both $\boldsymbol{E}$ and $\boldsymbol{B}$

3. Magnetic current dissipates helicity but conserves energy

4. Chiral evolution ends up always in a superconducting state satisfying $\boxed{\sigma_\chi^2 < 4\sigma_e\sigma_m}$

5. A superconducting state is always stable: a small fluctuation of magnetic helicity decays exponentially with time.

6. Superfluid component may explain very low viscosity of QGP

Close CMP analogue: spin ice, but has quadratic response

🤔 Interesting: what is the role of the Dirac quantization $eg=N/2$?