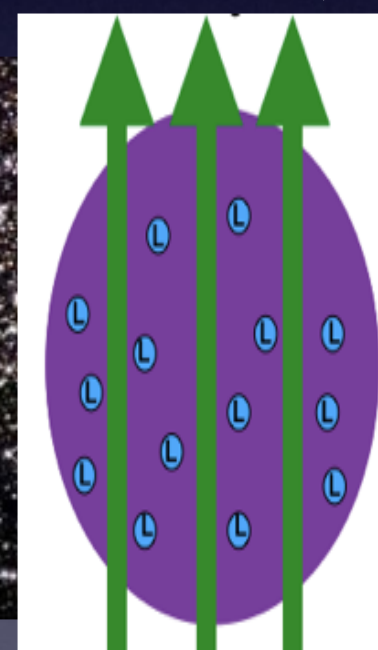
A reality check for chiral magnetic transport & holography

Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, GGI Florence, Italy

March 19th, 2018

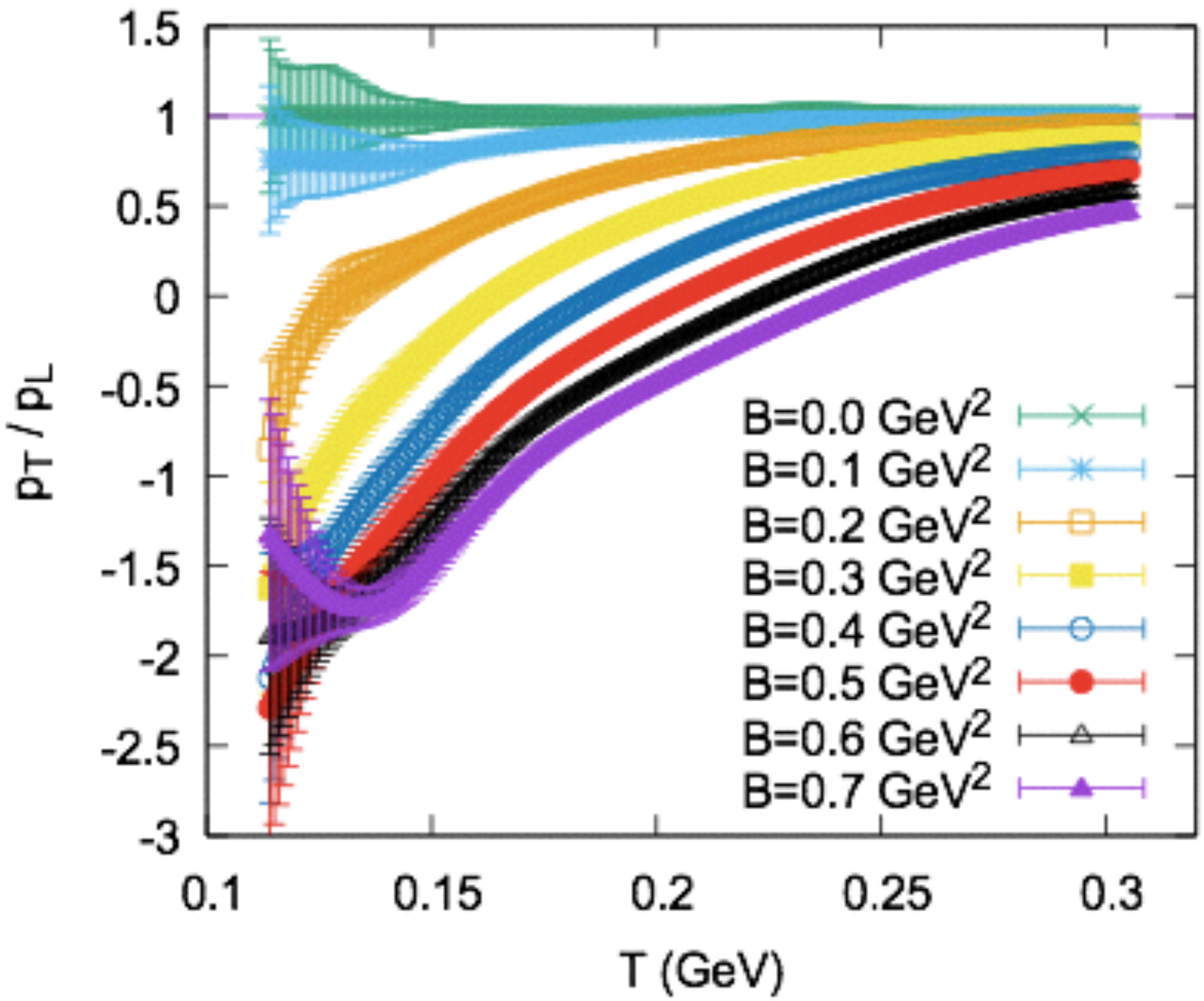

Matthias Kaminski

University of Alabama

Scale invariance in LQCD with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]

preliminary results



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$

longitudinal pressure: $p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$

F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*

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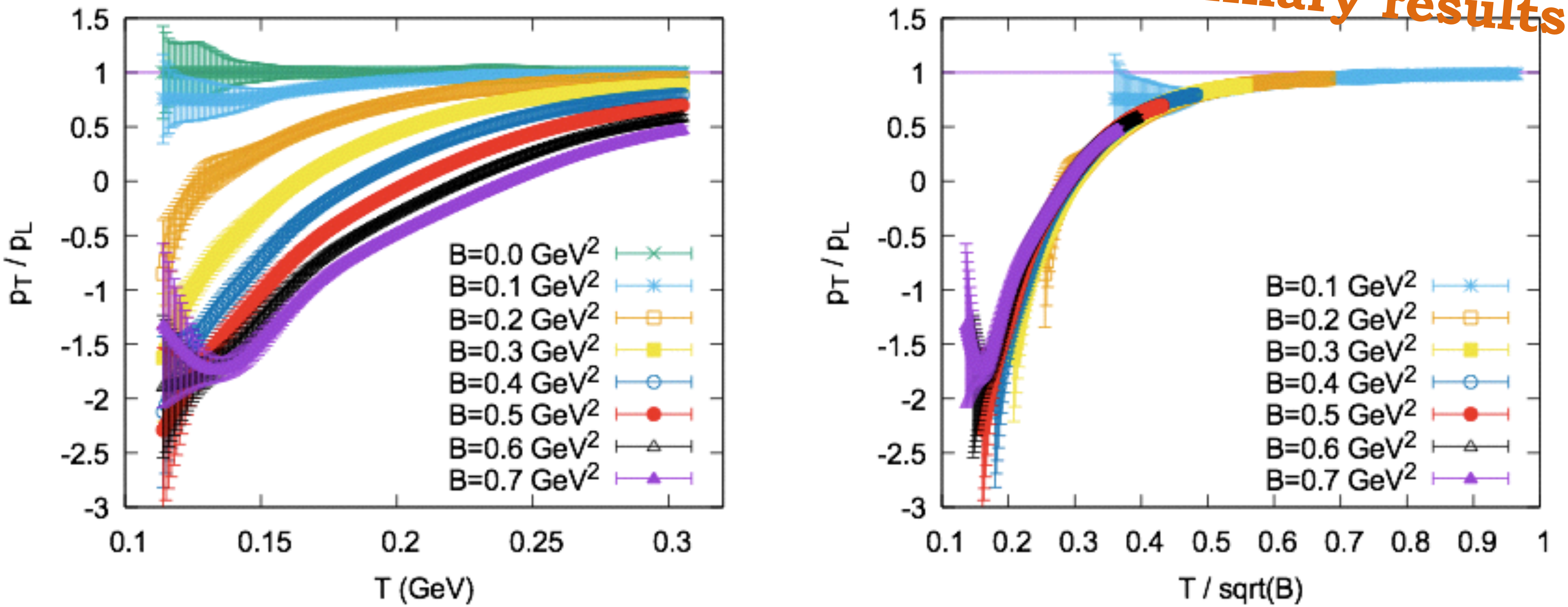

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Good agreement with N=4 Super-Yang-Mills (from holography)

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]

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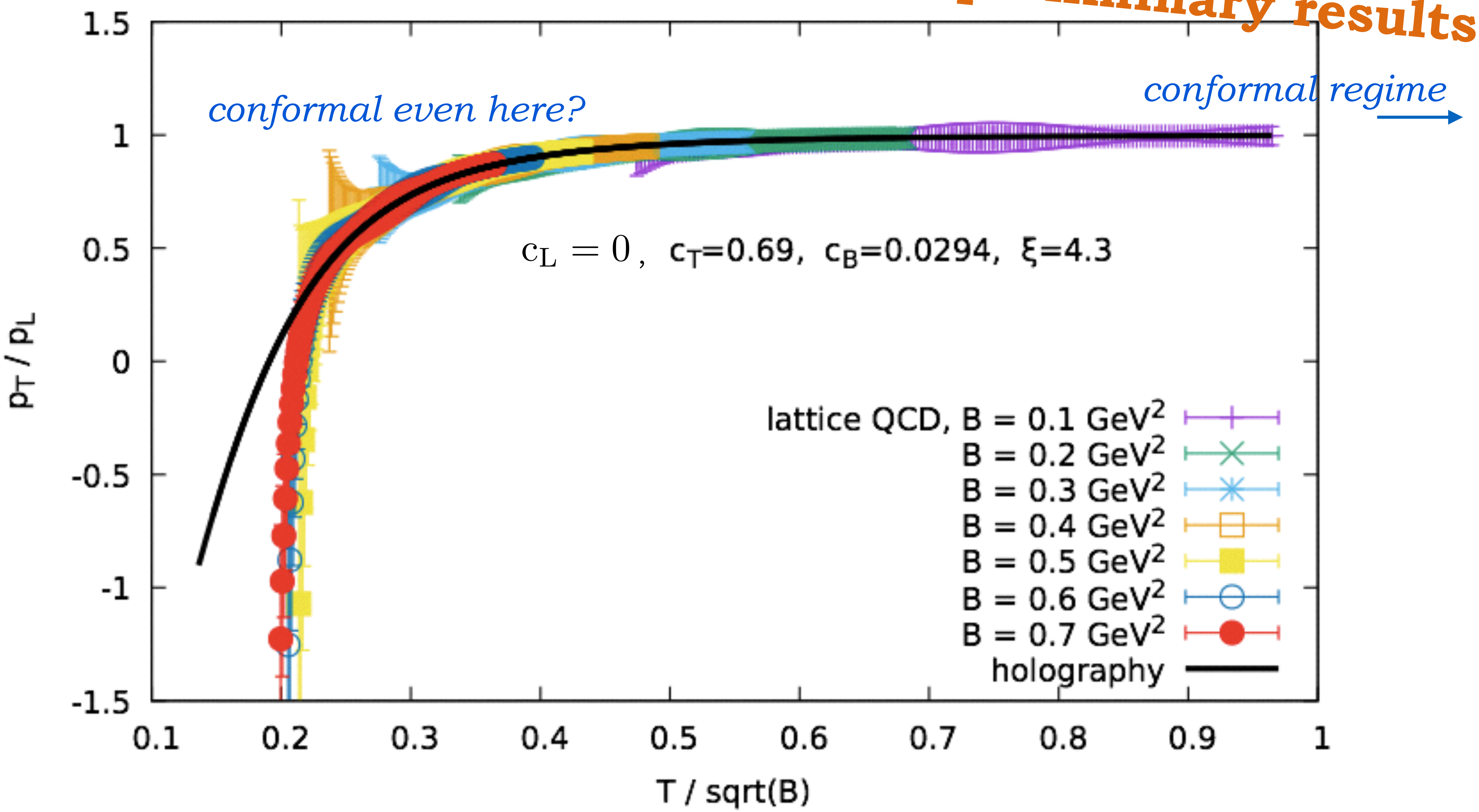

renormalization scale: $$\mu = \sqrt{c_T T^2 + c_L \Lambda_{\mathrm{H}}^2 + c_B |B|}$$

How does the renormalization scale enter?

[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]
[Fuini, Yaffe, JHEP (2015)]

Total action: $$S = S_{\mathrm{QCD}}(e, B) + S_{\mathrm{EM}}(e, B)$$

QCD action coupled to external magnetic field (through covariant derivative)

action for external magnetic field; not included in code (not part of the dynamics)

Electric charge is renormalization scale dependent: $$e^2(\mu) = Z_e(\mu)\, e_0^2, \qquad Z_e(\mu) = 1 + 2b_1 e^2 \log\frac{\mu}{\Lambda}, \qquad \mu = \sqrt{c_T T^2 + c_L \Lambda_{\mathrm{H}}^2 + c_B |B|}$$

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this free energy is renormalization scale dependent

hence this pressure is renormalization scale dependent

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How to compare QCD to Super-Yang-Mills

SYM action: $$S = S_{\mathrm{SYM}}(e, \mathcal{B}) + S_{\mathrm{EM}}(e, \mathcal{B})$$

SYM field content: fermions, scalar particles, vector field

SYM properties: conformal symmetry, supersymmetry, ...

SYM appears to be entirely different from QCD!

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Strategy:

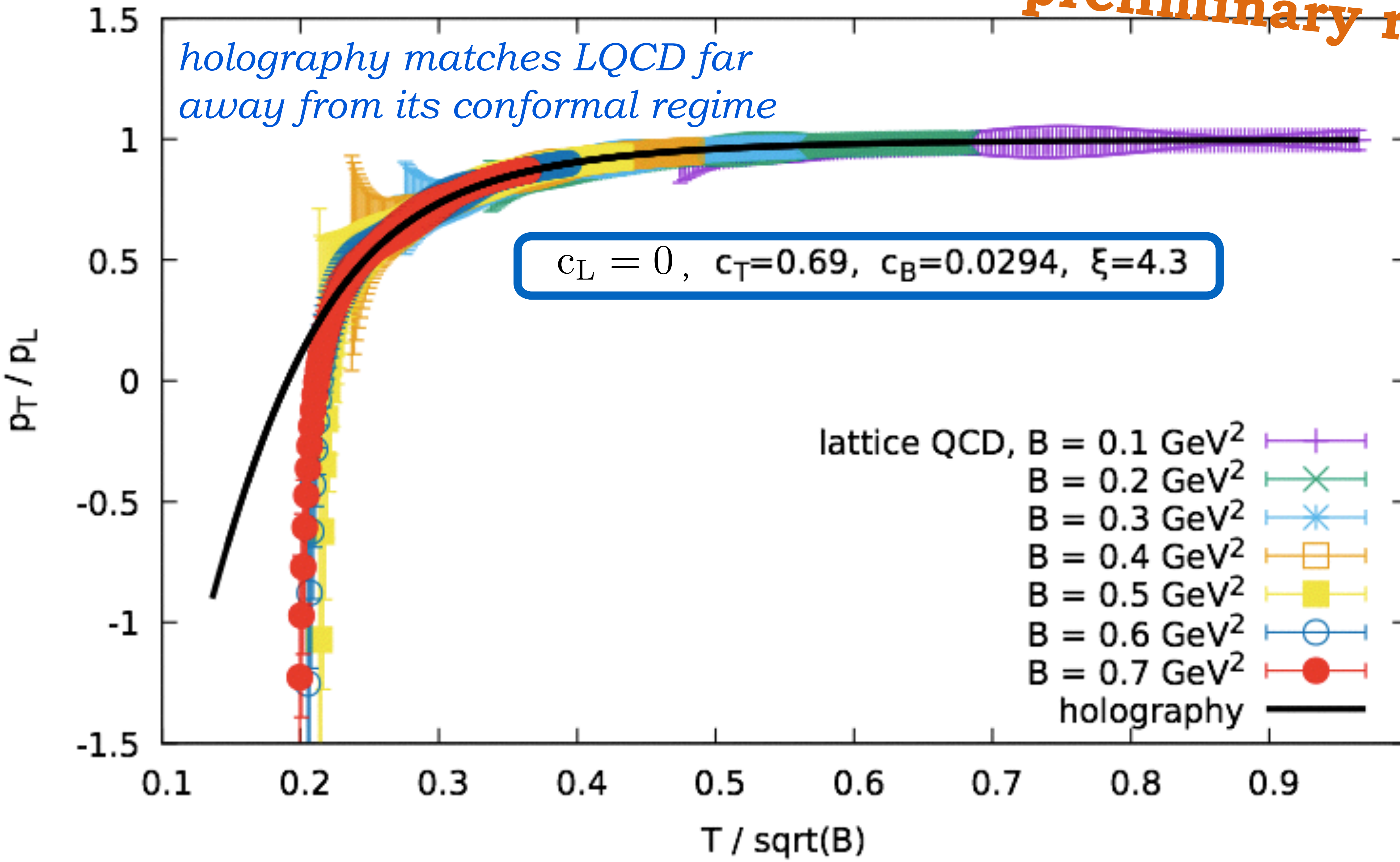
- compare thermodynamic quantities (macroscopic / effective); e.g. pressure
- match divergencies in the two theories, i.e. match beta functions
- measure magnetic fields in "same units"
- compare two theories at same renormalization scale

SYM magnetic field $\mathcal{B}$ vs. QCD magnetic field B: $$B = \xi \mathcal{B}$$

Good agreement with N=4 Super-Yang-Mills (from holography)

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]

preliminary results

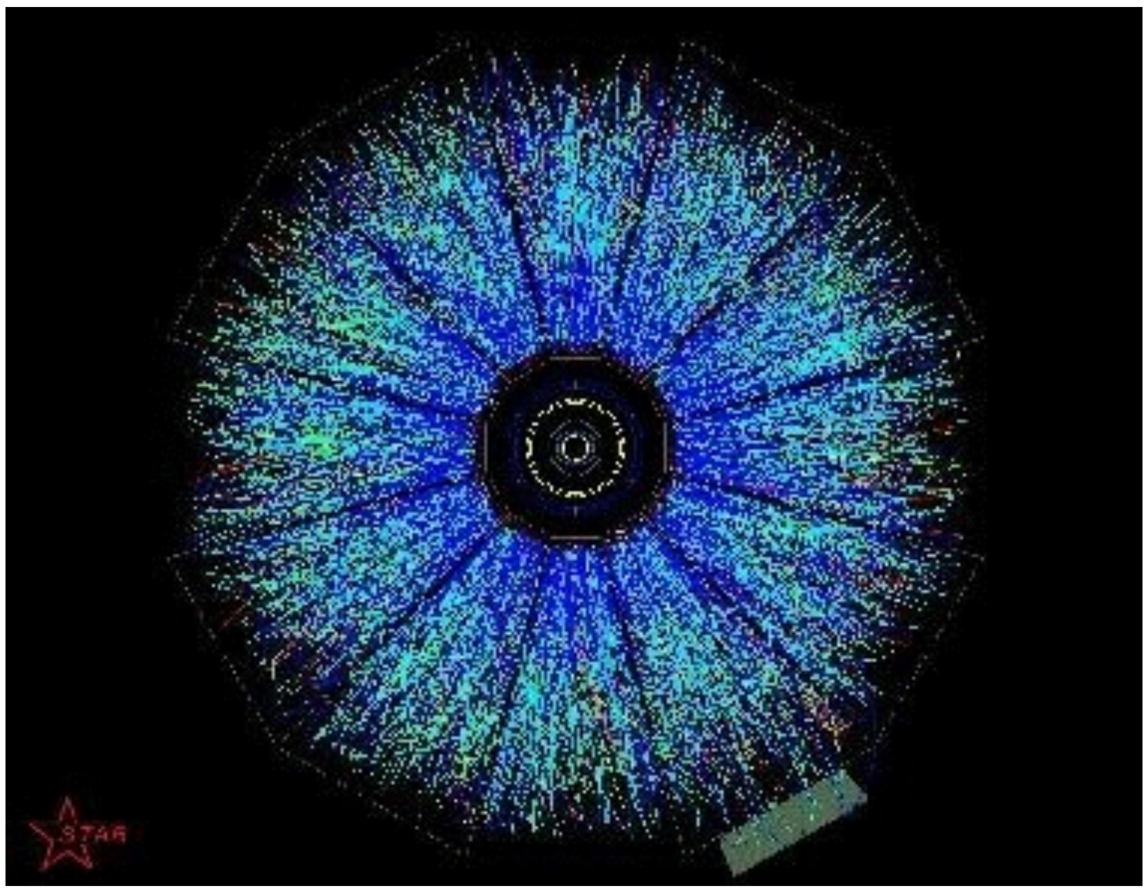


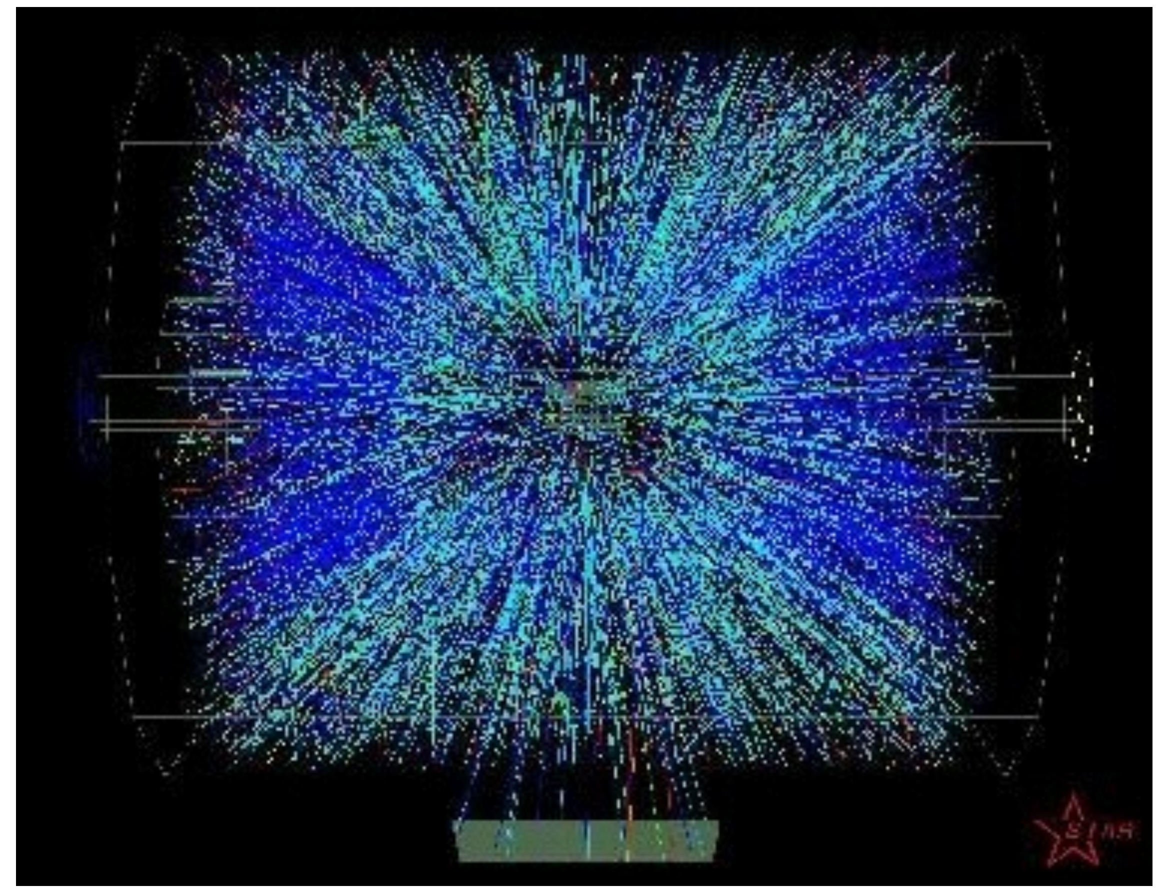
"ideal" renormalization scale:

- ***maximum overlap in LQCD data***
- ***maximum overlap with holography***
- ***minimum number of "fit parameters"***

$$\mu = \sqrt{c_T T^2 + c_L \Lambda_{\mathrm{H}}^2 + c_B |B|}$$

Motivation: Chiral transport effects in charged anisotropic plasma within a magnetic field at strong coupling





Front and side view of collision between gold ions at Brookhaven National Lab's Relativistic Heavy Ion Collider, captured by the Solenoidal Tracker at RHIC (STAR detector).

Method: use effective field theory (EFT) and holography in parallel (as effective descriptions)

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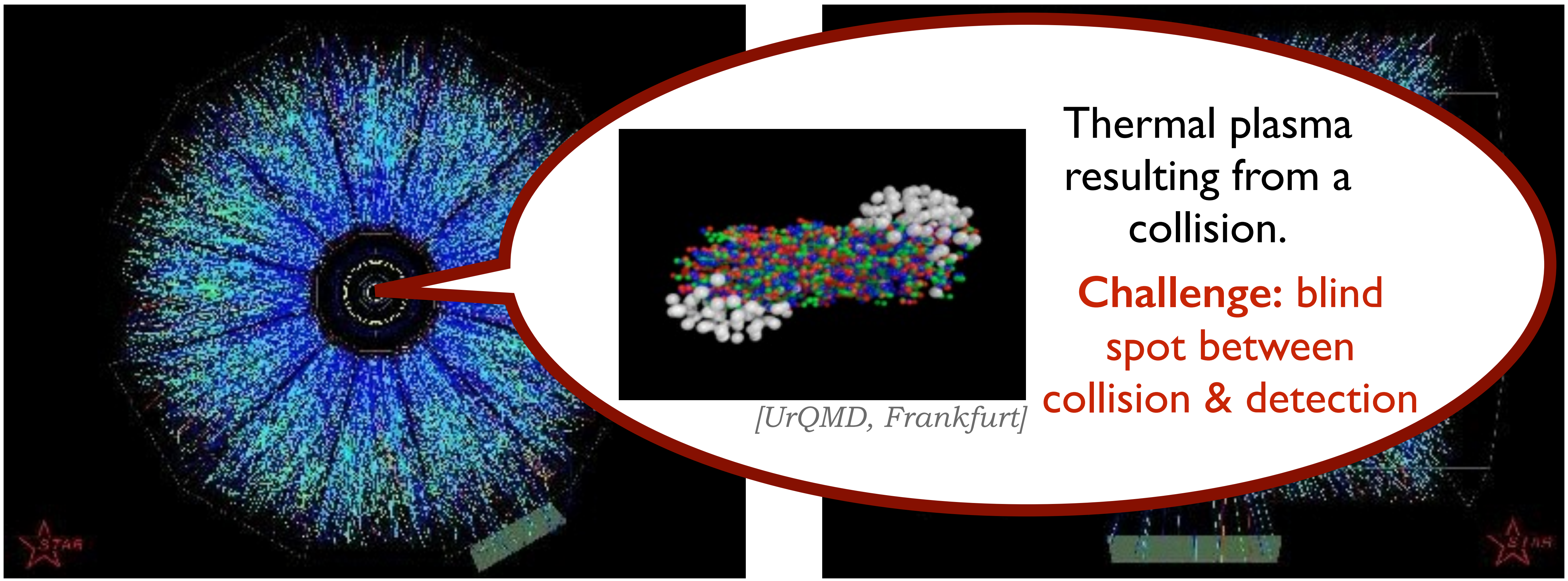

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How to choose a holographic model?

The same way, we choose a hydrodynamic model:

- match symmetries (and anomalies)
- include interesting operators

depends on the physical question

- match magnetic properties

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Einstein-Maxwell-Chern-Simons gravity has dual with:

cf. talk by K. Landsteiner

- chiral anomaly, breaking a U(1) axial symmetry
- axial current and energy momentum tensor

chiral magnetic transport

- thermodynamics match well (in external B field)

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

dual to $\mathcal{N}$=4 Super-Yang-Mills theory coupled to U(1)

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Successful example: holographic model discovering a chiral vortical effect (2008)

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]

[Banerjee et al; JHEP (2011)]

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Physical question:
What is the equilibrium state of a theory with chiral anomaly + external magnetic field ?

EFT: Hydrodynamics - definitions

[Landau, Lifshitz]

universal **effective field theory (EFT)**, expansion in derivatives of temperature, chemical potential and velocity

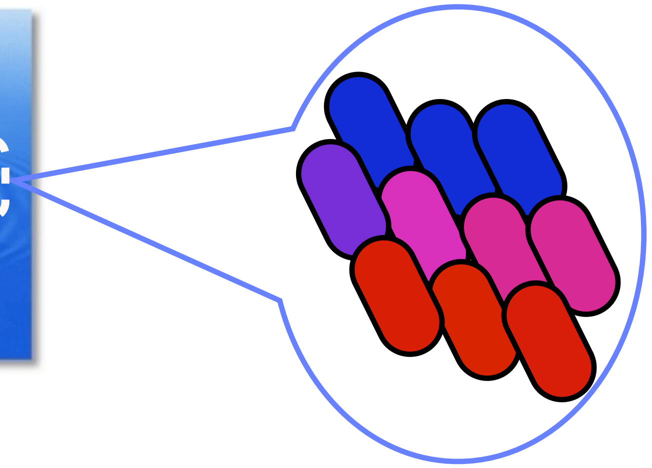
- fields $T(x),\ \mu(x),\ u^\nu(x)$

  temperature   *chemical potential*   *fluid velocity*

- conservation equations

$$\nabla_\nu J^\nu = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$



- constitutive equations

Conserved current $$\langle J^\mu \rangle = n u^\mu + \nu^\mu$$

charge density

Energy momentum $$\langle T_{\mu\nu} \rangle = \epsilon u_\mu u_\nu + P(\overbrace{g_{\mu\nu} + u_\mu u_\nu}^{\Delta_{\mu\nu}}) + \ldots$$

EFT result I: strong B thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly in **thermodynamic frame:**

Energy momentum tensor:

$$B \sim \mathcal{O}(1)$$

$$\langle T^{\mu\nu}_{\mathrm{EFT}} \rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

Axial current:

$$\langle J^{\mu}_{\mathrm{EFT}} \rangle = \left(n_0, 0, 0, \underline{\xi_B^{(0)} B} \right) + \mathcal{O}(\partial)$$

based on previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

[Israel; Gen.Rel.Grav. (1978)]

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Strong B thermodynamics with anomaly in **thermodynamic frame:**

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equilibrium heat current

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"magnetic pressure shift"

Axial current:

$$\langle J^{\mu}_{\rm EFT}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

equilibrium charge current

➡ new contributions to thermodynamic equilibrium observables

based on previous work:

[Kovtun; JHEP (2016)]

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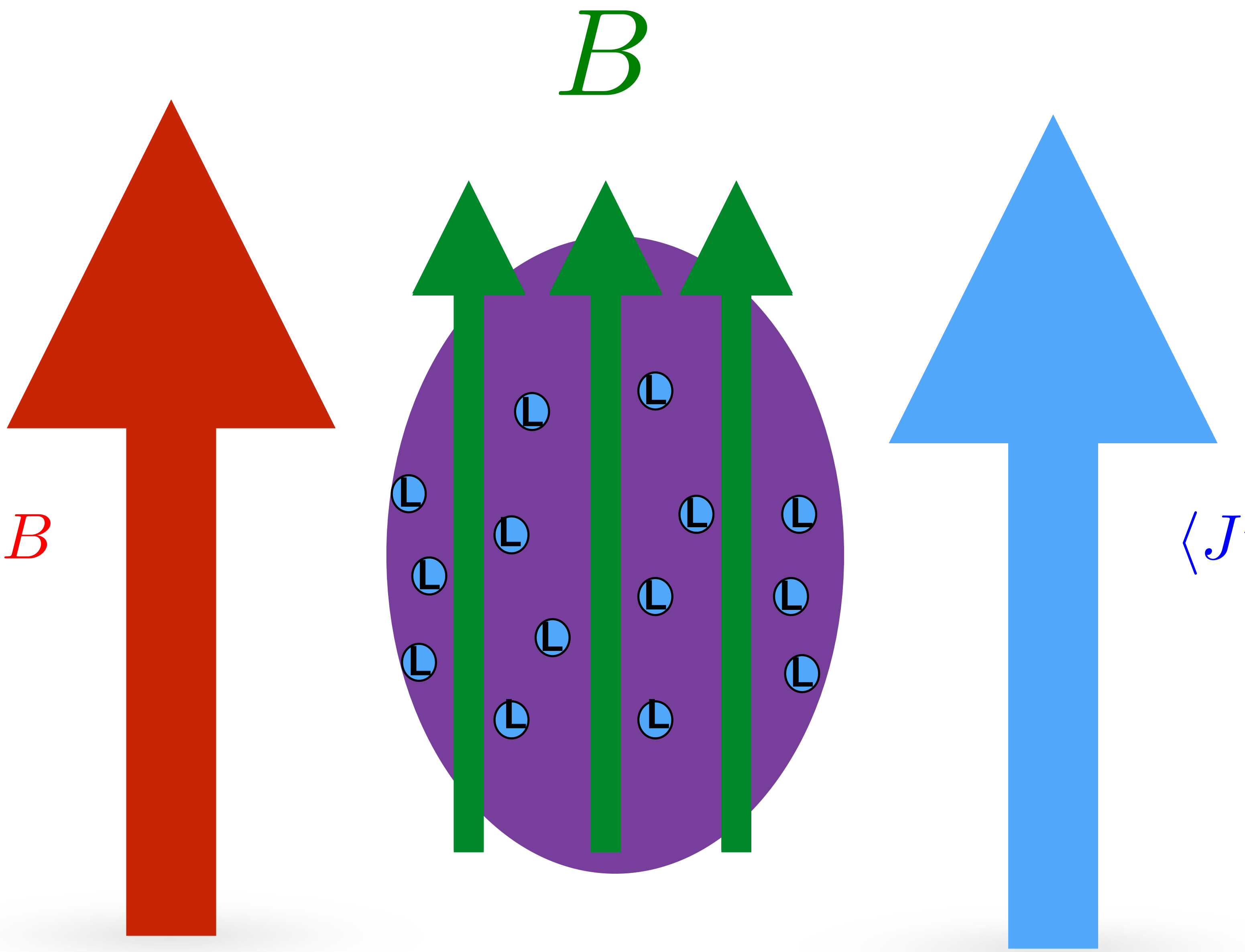
Currents in equilibrium

$$\langle T^{0z} \rangle = \xi_V^{(0)} B$$

heat current



$$\langle J^{z} \rangle = \xi_B^{(0)} B$$

axial current

Holographic result: thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]

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- **external magnetic field**
- **charged plasma**
- anisotropic plasma

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Thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ -4\,c_4 & 0 & 0 & 8\,w_4 - u_4 \end{pmatrix}$$

$$\langle J^\mu\rangle = (\rho,\, 0,\, 0,\, p_1)\ .$$

with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

$$\langle T^{\mu\nu}_{\text{EFT}}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)}B \\ 0 & P_0 - \chi_{BB}B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB}B^2 & 0 \\ \xi_V^{(0)}B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^\mu_{\text{EFT}}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)}B\right) + \mathcal{O}(\partial)$$

➡ **agrees in form with strong *B* thermodynamics from EFT**

Physical question:

What is the near-equilibrium transport behavior of a theory with chiral anomaly + external magnetic field ?

EFT result II: weak B hydrodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

spin 1 modes under SO(2) rotations around *B*

$$\omega = \quad - ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

former momentum diffusion modes

$\mathfrak{s}_0 = s_0/n_0$

$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$

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$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

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spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i \Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

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former sound modes

➡ **a chiral magnetic wave**
[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2 B T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]
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spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - iD_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

$$w_0 = \epsilon_0 + P_0$$
$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$
$$c_s^2 = (\partial P/\partial\epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6w_0} + c_s^2\frac{w_0\,\sigma}{2n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3CT_0\mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\frac{1-c_s^2}{w_0}\xi_V^{(0)}, \qquad v_0 = \frac{2B\,T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

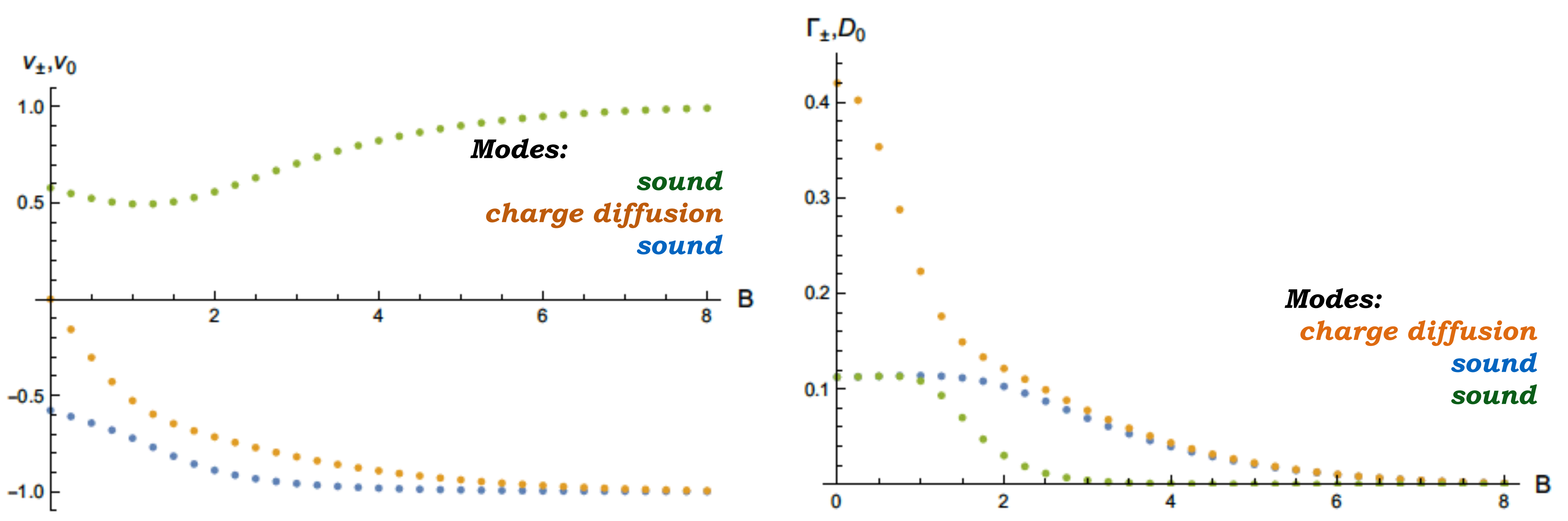
known from entropy current argument
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes *[Ammon, Kaminski et al.; JHEP (2017)]*

- Weak B: **holographic results are in "agreement" with hydrodynamics.**
- Strong B: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at ...**

the speed of light **and without attenuation**



confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

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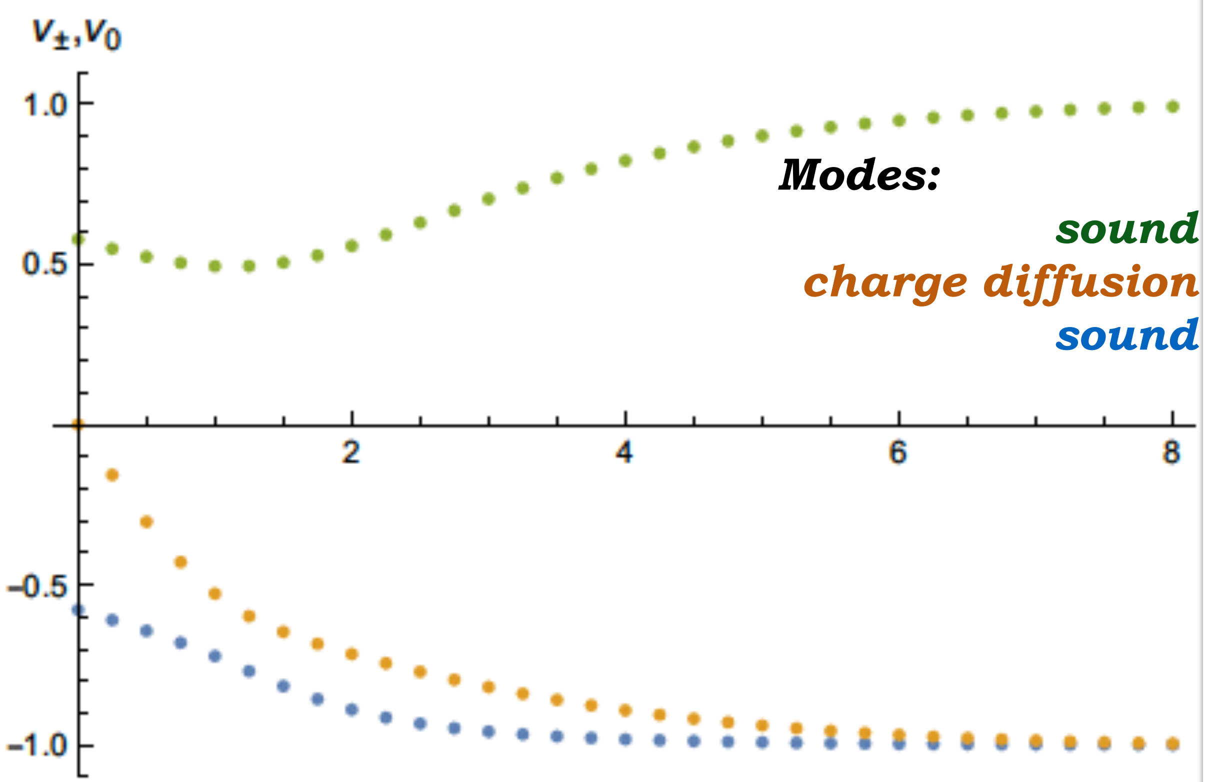

RECALL: weak B hydrodynamic poles

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

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former sound modes

confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

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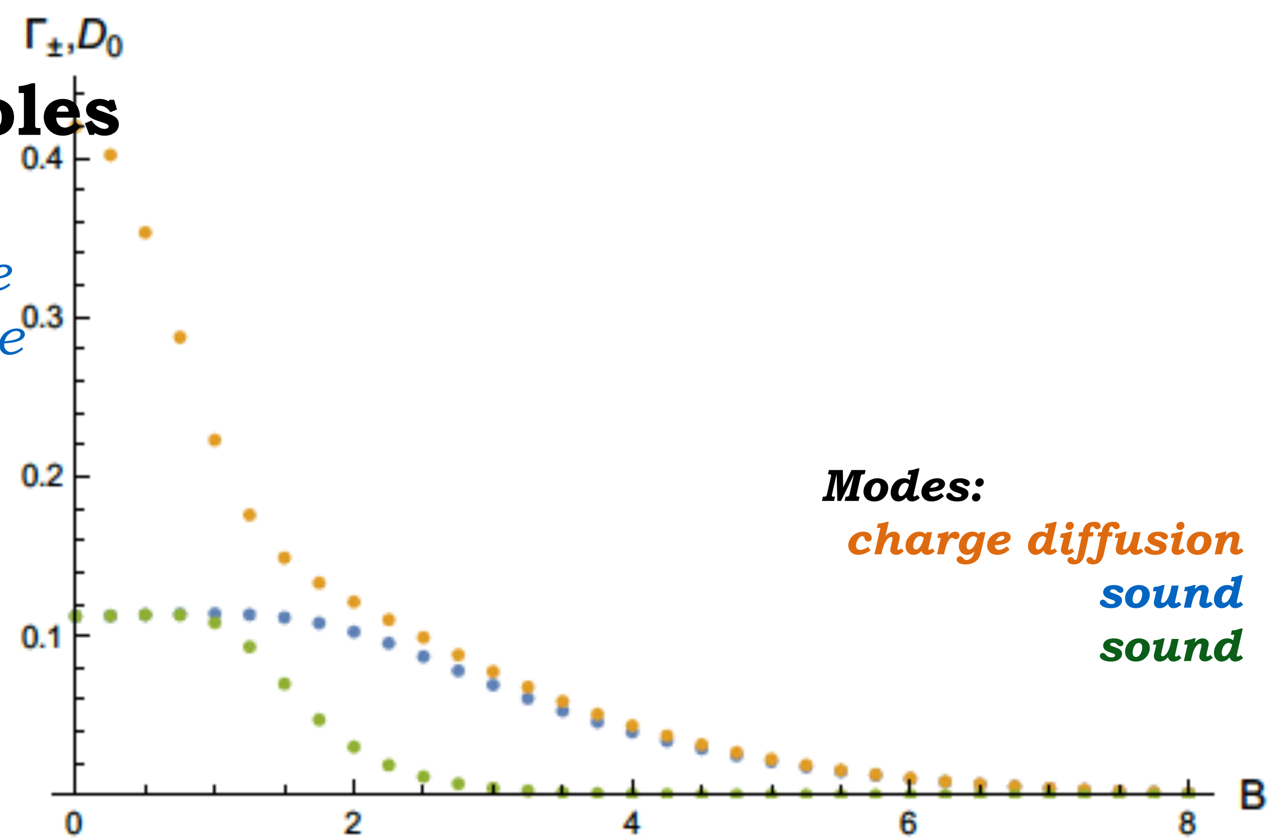

confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

Caveat: weak B hydrodynamics comparison

Spin-1 modes

No knowledge of anisotropic (B-dependent) transport coefficients

except zero charge: [Finazzo, Critelli, Rougemont, Noronha; PRD (2016)]

— take B=0 values of this model instead

weak B hydro prediction:

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - ik^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2 \sigma}{\epsilon_0 + P_0}$$

calculate from holography

We find agreement between hydrodynamic prediction and holographic model for small values of B, increasing deviations for larger B.

Real part of spin-1 modes matches exactly even at large B!

Things for which there was no time …

➡ transport coefficients and correlators

[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; work in progress]

➡ magnetohydrodynamics (dynamic B)

[Hernandez, Kovtun; JHEP (2017)]

[Grozdanov, Hofman, Iqbal; PRD (2017)]

[Hattori, Hirono, Yee, Yin; (2017)]

➡ axial *and* vector current

cf. talk by U. Gursoy

[Landsteiner, Megias, Pena Benitez; PRD (2014)]

[Ammon, Grieninger, Jimenez-Alba, Macedo, Melgar; JHEP (2016)]

➡ far-from-equilibrium: perform such holographic calculations in time-dependent metric backgrounds: "holographic thermalization"

[Janik; PRD (2006)]

[Chesler, Yaffe; PRL (2011)]

[Fuini, Yaffe; (JHEP) 2015)]

[Cartwright, Kaminski; work in progress]

cf. talks by P. Zhuang and D.-L. Yang

Method summary: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory,
for example **QCD**

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↓ *model*

(Hard) problem in "similar" model theory

⟷ *holography (gauge/gravity correspondence)*

Simple problem in a particular gravitational theory

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Solve problems in effective field theory (EFT), e.g.:

- hydrodynamic approximation of original theory
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➡ Holography is good at predictions that are **qualitative** or **universal**.

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➡ **Understand holography as an effective description.**

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CONSISTENCY CHECK

Solve problems in effective field theory (EFT), e.g.:

- hydrodynamic approximation of original theory

REALITY CHECK: MODEL APPROPRIATE?

- hydrodynamic approximation of model theory

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➡ **Understand holography as an effective description.**

Conclusions

- LQCD data shows conformal behavior matched by SYM
  cf. talk by M. D'Elia
- holography in parallel with hydrodynamics (effective field theory) is a successful program
- transport properties of plasma change qualitatively with B, charge, and anomaly coefficient
- strong B results (fully backreacted) at any $\mu,\ T,\ \omega,\ k$
- **Outlook:**

  ✳ construct holographic & effective description far from equilibrium (excentricities/flow, transport, ridge, ...)

  ✳ compare to QCD (e.g. lattice) and experiments

Collaborators

Friedrich-Schiller University of Jena, Germany

Prof. Dr. Martin Ammon

Dr. Julian Leiber

Sebastian Grieninger

Regensburg University, Germany

Prof. Dr. Andreas Schäfer

Goethe University, Frankfurt, Germany

Dr. Gergely Endrödi

University of Washington, Seattle, USA

Prof. Dr. Laurence Yaffe

University of Alabama, Tuscaloosa, USA

Dr. Jackson Wu

Roshan Koirala

Casey Cartwright

APPENDIX

Recent update: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

strong B: $\omega = \pm \frac{B_0 n_0}{w_0} - \frac{i B_0^2}{w_0}(\sigma_\perp \pm i\tilde{\sigma}) - i D_c k^2$

parity-odd

weak B: $\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$

Agreement in form

Exact agreement in real part!

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Anisotropic transport coefficients

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Agreement in form

Exact agreement in real part!

Spin-0 modes

strong B: $\omega = \pm k v_s - i \frac{\Gamma_{s,\parallel}}{2} k^2 \, ,$

$\omega = -i D_\parallel k^2 \, ,$

$$D_\parallel = \frac{\sigma_\parallel \, w_0^2}{n_0^2 \chi_{11} + w_0^2 \chi_{33} - 2 n_0 w_0 \chi_{13}}$$

parity-odd

weak B: $\omega_0 = v_0 \, k - i D_0 \, k^2 + \mathcal{O}(\partial^3)$

$\omega_+ = v_+ \, k - i \Gamma_+ \, k^2 + \mathcal{O}(\partial^3)$

$\omega_- = v_- \, k - i \Gamma_- \, k^2 + \mathcal{O}(\partial^3)$

Agreement in form

$$D_0 = \frac{w_0^2 \, \sigma}{\tilde{c}_P n_0^3 T_0} \qquad v_0 = \frac{2 B \, T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$

$$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$$

Recent update: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

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strong B: $\omega = \pm \frac{B_0 n_0}{w_0} - \frac{iB_0^2}{w_0}(\sigma_\perp \pm i\tilde{\sigma}) - iD_c k^2$

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weak B: $\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - ik^2 \frac{\eta}{\epsilon_0 + P_0} + k\frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}$

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$\omega_- = v_- k - i\Gamma_- k^2 + \mathcal{O}(\partial^3)$

$$D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0} \qquad v_0 = \frac{2BT_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

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$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$

EFT calculation I: strong B thermodynamics

[Ammon, Kaminski et al. (2017)]

For any theory with chiral anomaly

$$\partial_\mu {J_A}^\mu = C\,\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}\, F_{\rho\sigma}$$

Axial current with strong external B field:

$B \sim \mathcal{O}(1)$

$$\langle J^\mu_{\rm EFT}\rangle = n_0 u^\mu + \xi_B B^\mu + \mathcal{O}(\partial)$$

Energy momentum tensor with strong external B field:

$$\langle T^{\mu\nu}_{\rm EFT}\rangle = \epsilon_0 u^\mu u^\nu + P_0 \Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu$$
$$+ M^{\mu\alpha} g_{\alpha\beta} F^{\beta\nu} + u^\mu u^\alpha \left(M_{\alpha\beta}F^{\beta\nu} - F_{\alpha\beta}M^{\beta\nu}\right) + \mathcal{O}(\partial)$$

$$q^\mu = \xi_V B^\mu\,, \qquad M^{\mu\nu} = \chi_{BB}\epsilon^{\mu\nu\alpha\beta}B_\alpha u_\beta$$

based on previous work: [Kovtun; JHEP (2016)]
[Jensen, Loganayagam, Yarom; JHEP (2014)]
[Israel; Gen.Rel.Grav. (1978)]

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in thermodynamic frame:

$$q^\mu = \underline{\xi_V B^\mu}\,, \qquad M^{\mu\nu} = \chi_{BB}\epsilon^{\mu\nu\alpha\beta}\underline{B_\alpha u_\beta}$$

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu$$

based on previous work: [Kovtun; JHEP (2016)]
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Example: hydrodynamic correlators in 2+1

Simple (non-chiral) example in 2+1 dims:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1, 0, 0)$$

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sources $\quad A_t,\ A_x \propto e^{-i\omega t + ikx} \qquad u^\mu = (1,0,0)$

fluctuations $\quad n = n(t,x,y) \propto e^{-i\omega t + ikx} \quad$ (fix T and u)

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one point functions $\quad \nabla_\mu j^\mu = 0$

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

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$$\langle j^y \rangle = 0$$

susceptibility: $\chi = \dfrac{\partial n}{\partial \mu}$

Einstein relation: $D = \dfrac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\langle j^x j^x \rangle = \dfrac{\delta \langle j^x \rangle}{\delta A_x} = \dfrac{i\omega^2 \sigma}{\omega + iDk^2}$

$\Longrightarrow$ hydrodynamic poles in spectral function

General hydrodynamic correlators

sources:

$$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon\, h_{\mu\nu} + \mathcal{O}(\varepsilon^2)$$

$$A_\mu = \hat{A}_\mu + \varepsilon\, a_\mu + \mathcal{O}(\varepsilon^2)$$

fluctuations:

$$T(t,x_3) = T_0 + \varepsilon\, T_1(t,x_3)\,, \quad u^\nu(t,x_3) = u_0^\nu + \varepsilon\, u_1^\nu(t,x_3)\,, \quad \mu(t,x_3) = \mu_0 + \varepsilon\, \mu_1(t,x_3)$$

note also: $\epsilon(t,x_3) = \epsilon_0 + \varepsilon\, \frac{\partial \epsilon}{\partial T} T_1(t,x_3) + \varepsilon\, \frac{\partial \epsilon}{\partial \mu} \mu_1(t,x_3)$

plugging this into hydro constitutive and conservation equations leads to linear but coupled system of equations for fluctuations

$$DX = S$$

differentiation matrix acting on vector of fluctuations

vector depending on sources

EFT calculation: chiral hydrodynamics with magnetic field

For any theory with chiral anomaly

$$\partial_\mu {J_A}^\mu = C\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}\, F_{\rho\sigma}$$

[Son,Surowka; PRL (2009)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
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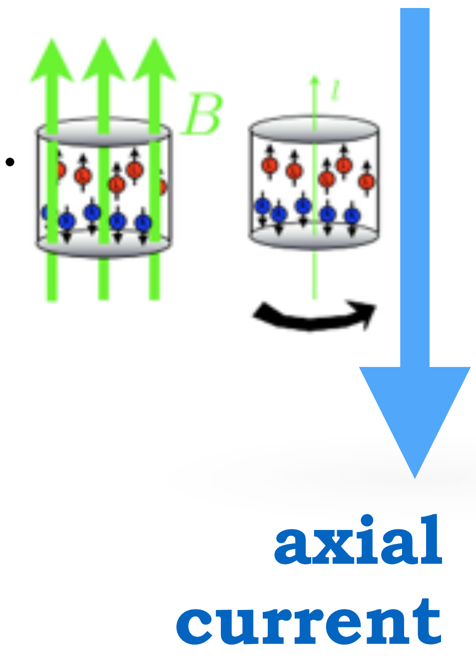
Axial current with weak external B field: $\boxed{B \sim \mathcal{O}(\partial)}$

$$\langle {J_A}^\mu \rangle = n u^\mu + \sigma E^\mu - \sigma T \Delta^{\mu\nu} \nabla_\nu \left(\frac{\mu}{T}\right) + \xi_B B^\mu + \xi_V \Omega^\mu + \dots$$



Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \epsilon u^\mu u^\nu + P \Delta^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu + \tau^{\mu\nu}$$

Definitions and properties:

$$\tau^{\mu\nu} = -\eta \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}\nabla_\lambda u^\lambda g_{\alpha\beta}\right) - \zeta \Delta^{\mu\nu}\nabla_\lambda u^\lambda$$

$$E^\mu = F^{\mu\nu} u_\nu$$

$$B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda} u_\nu F_{\rho\lambda}$$

$q^\mu = \xi_V B^\mu + \xi_3 \omega^\mu$ $\quad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ $\quad u^\mu u_\mu = -1$ $\quad u_\mu \tau^{\mu\nu} = 0,\ u_\mu \nu^\mu = 0,$ and $u_\mu q^\mu = 0.$

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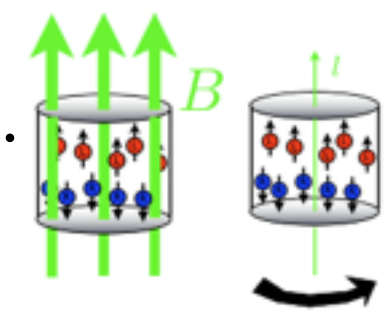
(ideal) charge flow — *conductivity term* — *charge diffusion* — *chiral magnetic conductivity term* — *chiral vortical conductivity term*



axial current

Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \underbrace{\epsilon u^\mu u^\nu + P \Delta^{\mu\nu}}_{\text{ideal fluid}} + \underbrace{u^\mu q^\nu + u^\nu q^\mu}_{\text{heat current}} + \tau^{\mu\nu}$$

Definitions and properties:

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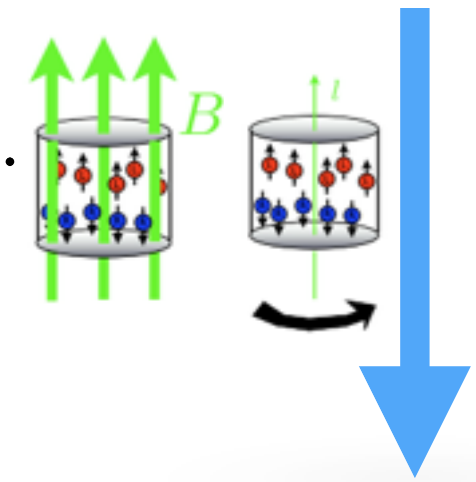
- nu^μ: *(ideal) charge flow*
- σE^μ: *conductivity term*
- $-\sigma T \Delta^{\mu\nu} \nabla_\nu (\mu/T)$: *charge diffusion*
- $\xi_B B^\mu$: *chiral magnetic conductivity term*
- $\xi_V \Omega^\mu$: *chiral vortical conductivity term*



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- $u^\mu q^\nu + u^\nu q^\mu$: *heat current*

Definitions and properties:

$$\tau^{\mu\nu} = -\eta \Delta^{\mu\alpha} \Delta^{\nu\beta} \left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3} \nabla_\lambda u^\lambda g_{\alpha\beta} \right) - \zeta \Delta^{\mu\nu} \nabla_\lambda u^\lambda$$

$$E^\mu = F^{\mu\nu} u_\nu$$

$$B^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} u_\nu F_{\rho\lambda}$$

$$q^\mu = \xi_V B^\mu + \xi_3 \omega^\mu \qquad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \qquad u^\mu u_\mu = -1 \qquad u_\mu \tau^{\mu\nu} = 0,\ u_\mu \nu^\mu = 0, \text{ and } u_\mu q^\mu = 0.$$

EFT calculation: chiral hydrodynamics with magnetic field

For any theory with chiral anomaly

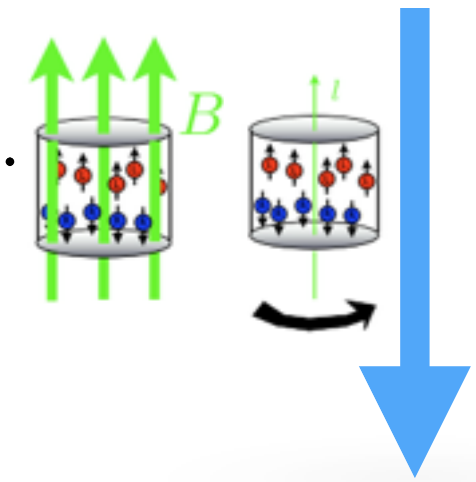
$$\partial_\mu {J_A}^\mu = C\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}\, F_{\rho\sigma}$$

[Son, Surowka; PRL (2009)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]

Axial current with weak external B field: $\boxed{B \sim \mathcal{O}(\partial)}$

$$\langle {J_A}^\mu \rangle = n u^\mu + \sigma E^\mu - \sigma T \Delta^{\mu\nu} \nabla_\nu \left(\frac{\mu}{T}\right) + \xi_B B^\mu + \xi_V \Omega^\mu + \ldots$$

(ideal) charge flow — *conductivity term* — *charge diffusion* — *chiral magnetic conductivity term* — *chiral vortical conductivity term*



axial current

Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \underbrace{\epsilon u^\mu u^\nu + P \Delta^{\mu\nu}}_{\text{ideal fluid}} + \underline{u^\mu q^\nu + u^\nu q^\mu} + \tau^{\mu\nu}$$

heat current

chiral effects measured in Weyl semi metals
e.g. *[Huang et al; PRX (2015)]*

neutron stars?
[Kaminski et al.; PLB (2014)]

Now calculate hydrodynamic 1- and 2-point functions and determine their poles!

[Landau, Lifshitz]
[Kadanoff; Martin]

Landau levels at large B

Weak coupling arguments *[Kharzeev, Yee; PRD (2011)]*

fermions in lowest Landau level carry all chirality; line of arguments leads to speed of light for chiral magnetic wave at large B

$$v_0 \to 1 \quad \text{for} \quad B \gg T^2$$

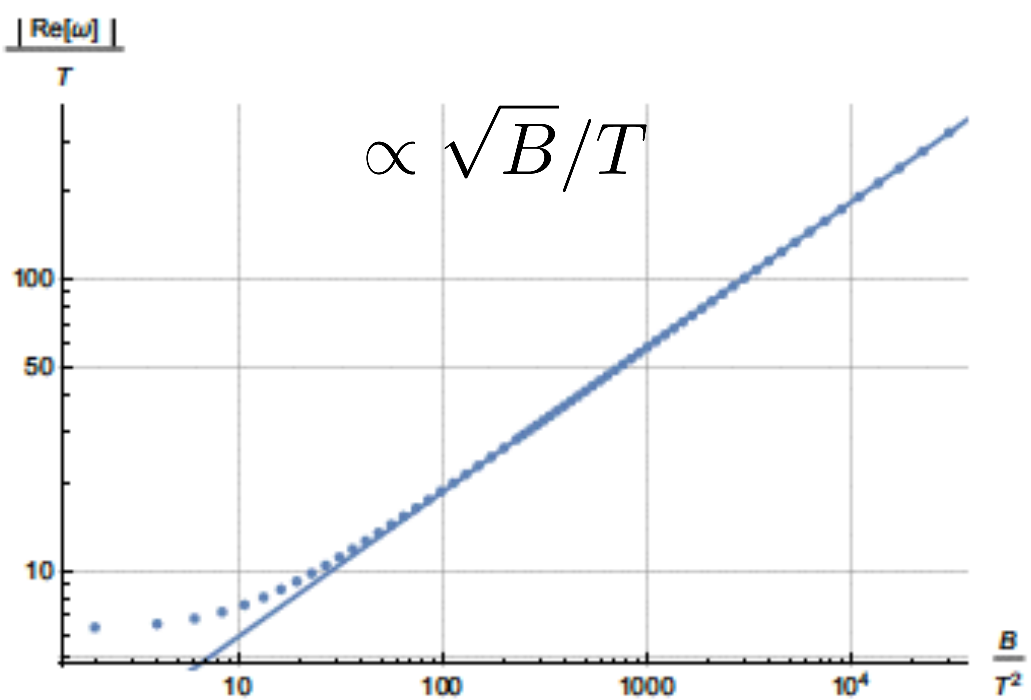
$$E_n = \sqrt{2eB(n + \frac{1}{2} - s_z) + p_z^2}$$

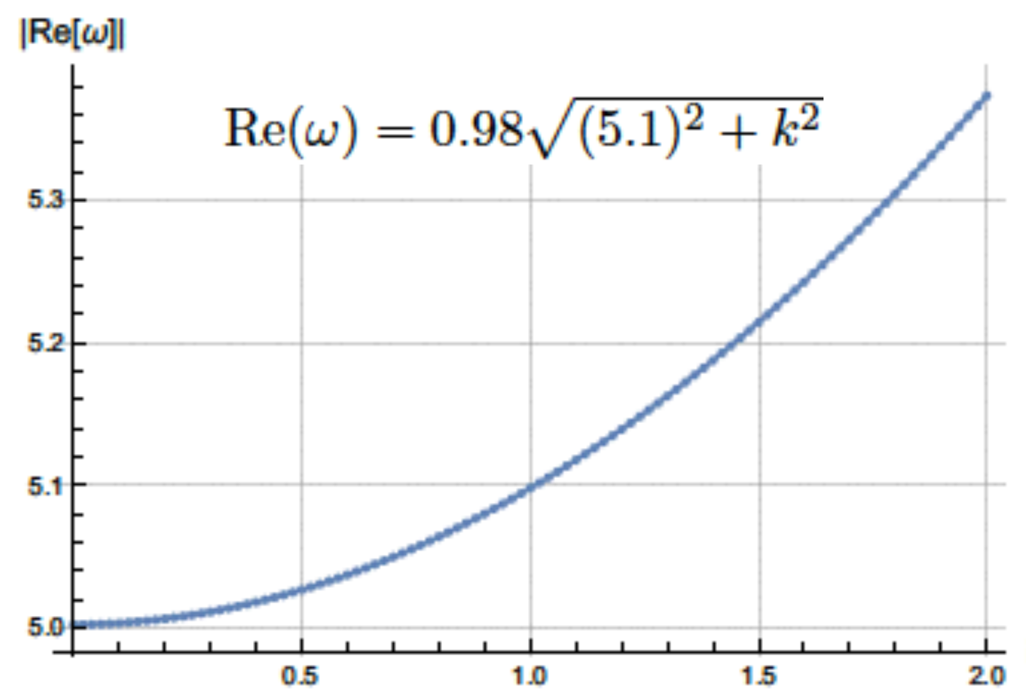
energy of fermion in n-th Landau level

Strong coupling calculation

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

lowest Landau level: $v_0 \to 1$
next to lowest Landau level:



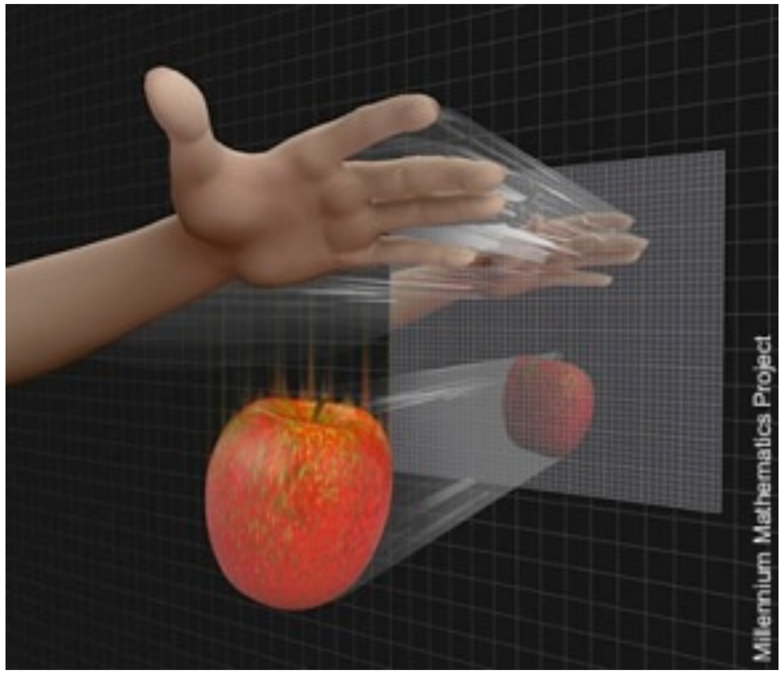


➡ **Landau levels also dominate at strong coupling (note critical CS-coupling)**

Holography concepts

Gauge/Gravity Correspondence based on
holographic principle *['t Hooft (1993)]*

$$S_{max}(\text{volume}) \propto \text{surface area}$$



Holography concepts

Gauge/Gravity Correspondence based on
holographic principle *['t Hooft (1993)]*

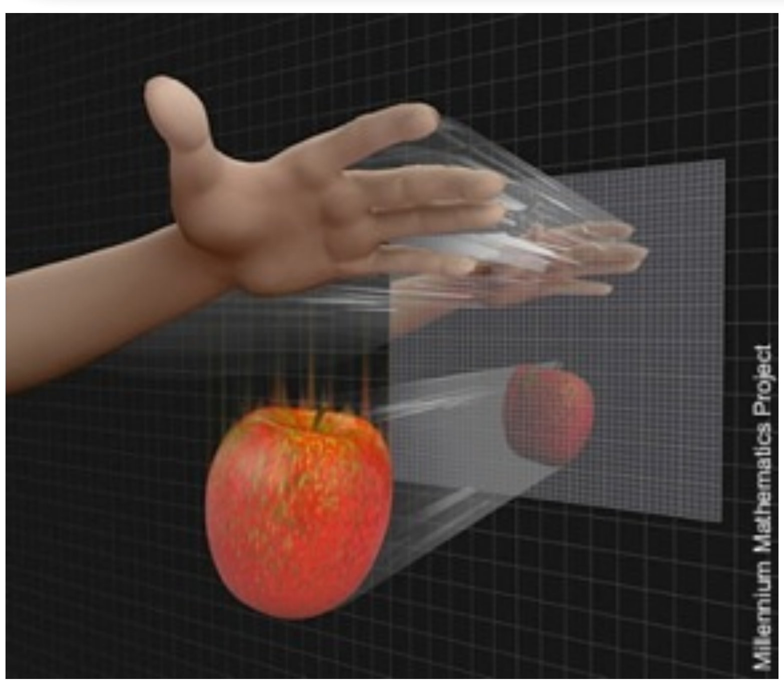
$$S_{max}(\text{volume}) \propto \text{surface area}$$

String theory gives one example (AdS/CFT).

N=4 Super-Yang-Mills in 3+1 dimensions (CFT) $\Leftrightarrow$ Typ II B Supergravity in (4+1)-dimensional Anti de Sitter space (AdS)

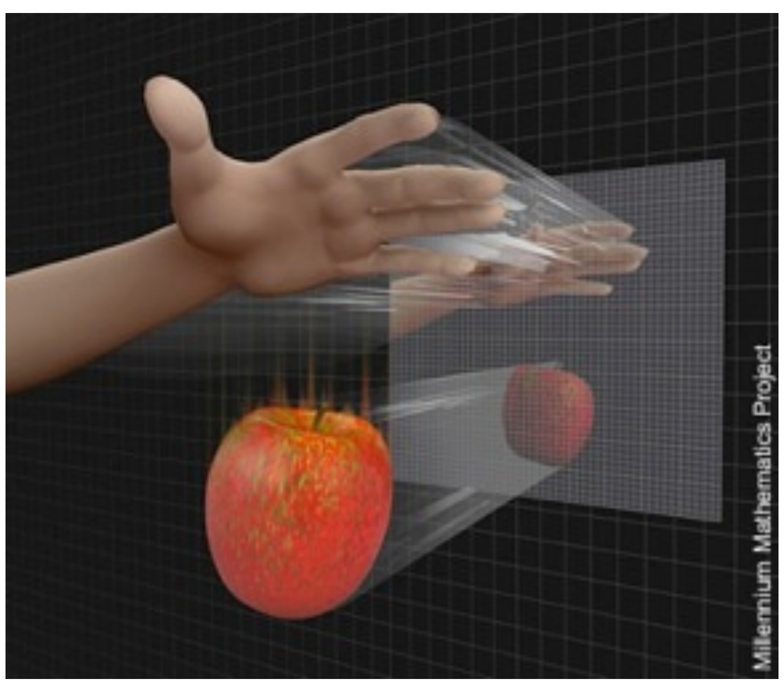
[Susskind (1995)]
[Maldacena (1997)]

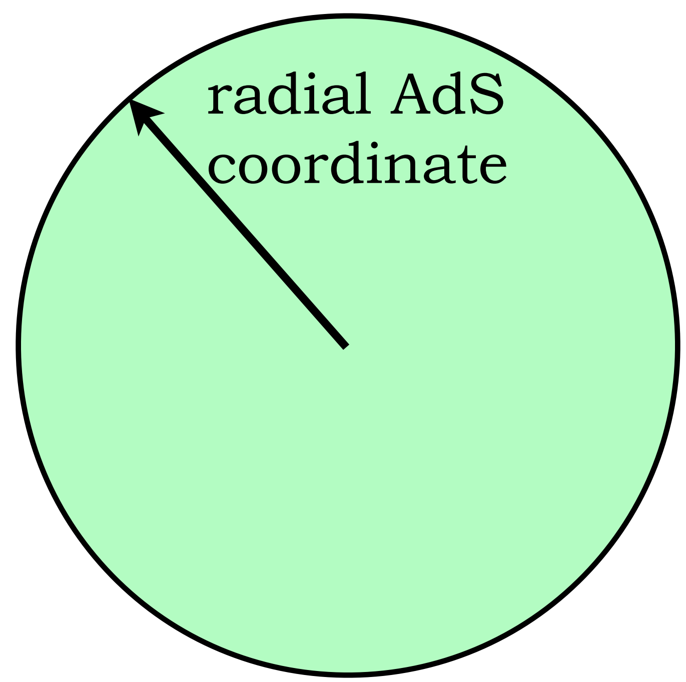
Many other examples are conjectured and tested



Holography concepts II

strongly coupled quantum field theory ⟷ *correspondence* ⟷ weakly curved gravity

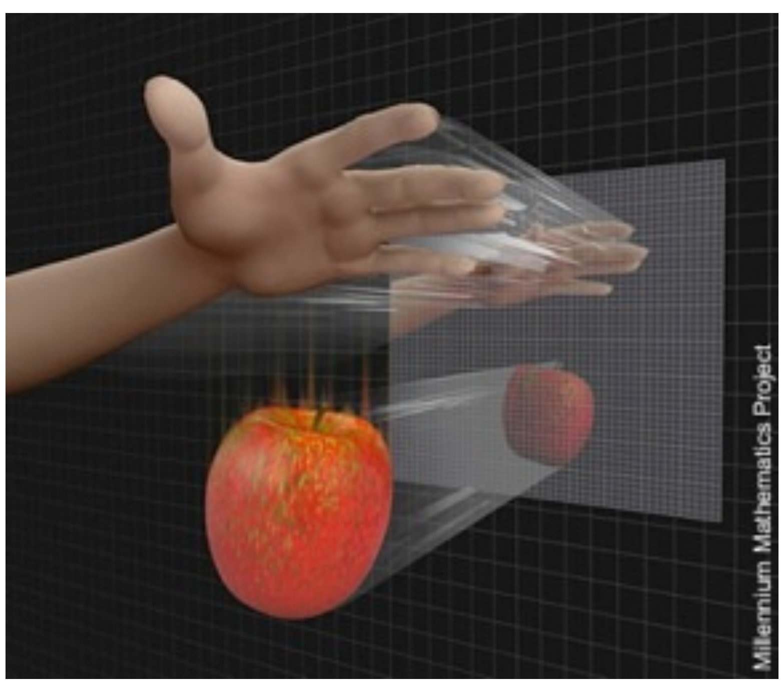


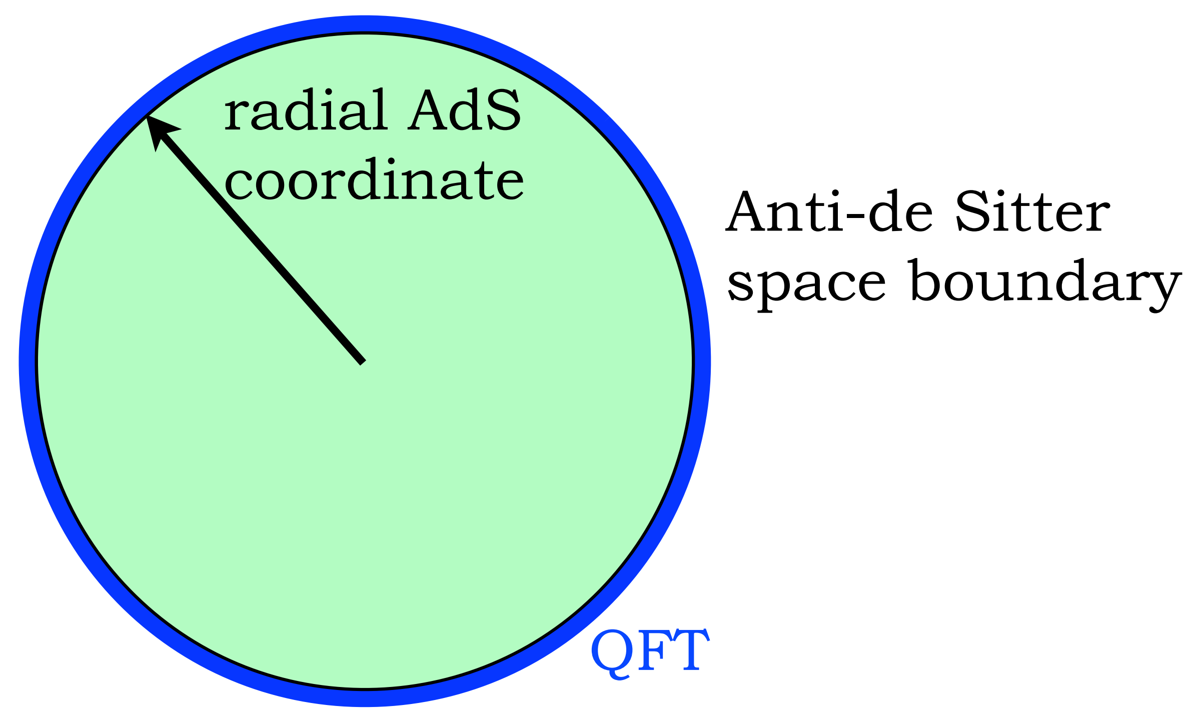


Anti-de Sitter
space boundary

Holography concepts II

strongly coupled quantum field theory ⟷ *correspondence* ⟷ weakly curved gravity

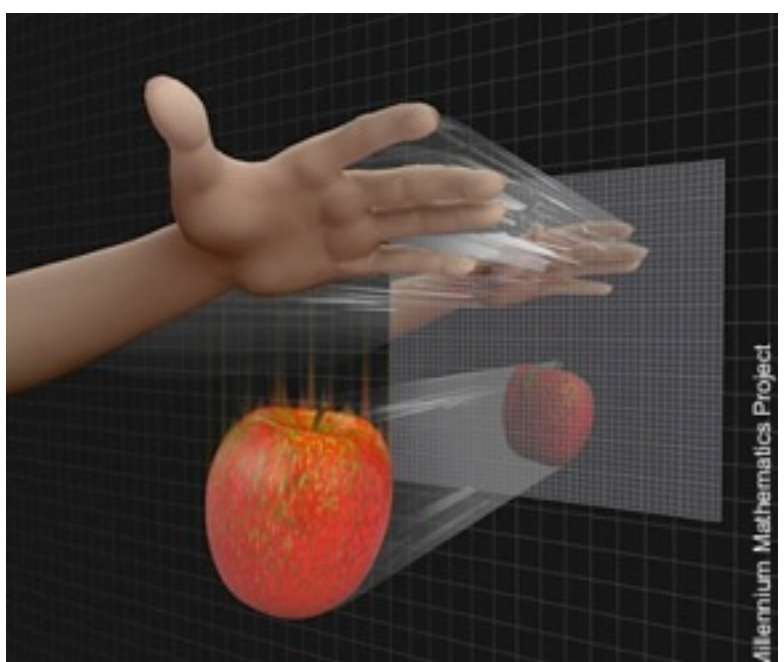


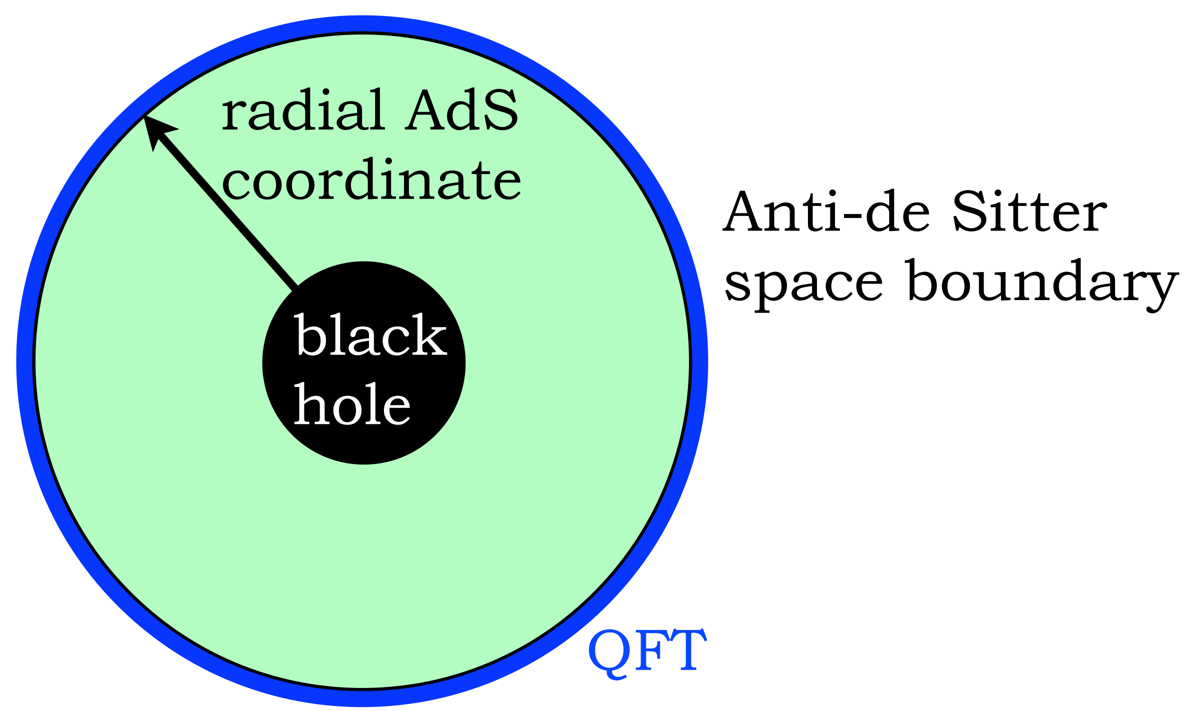


Holography concepts II

strongly coupled quantum field theory ⟷ *correspondence* ⟷ weakly curved gravity

QFT temperature ⟷ Hawking temperature



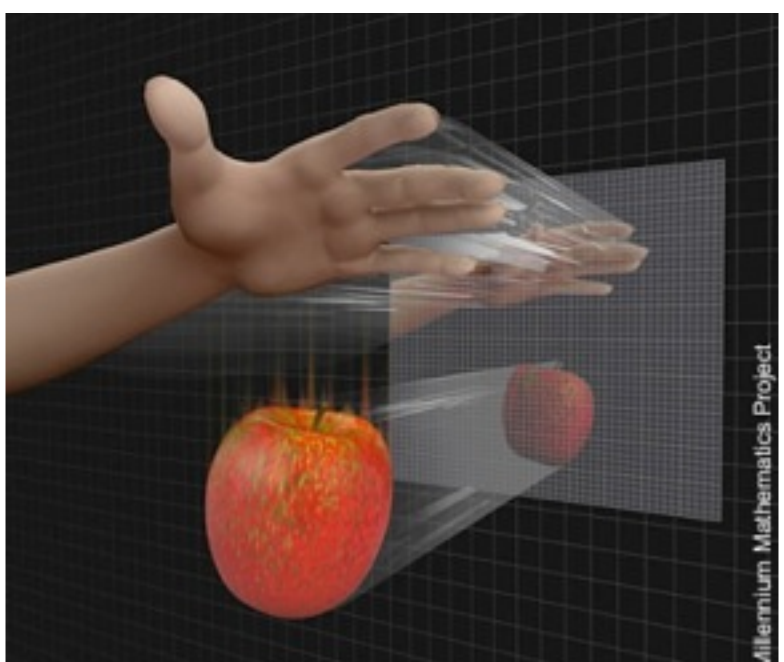


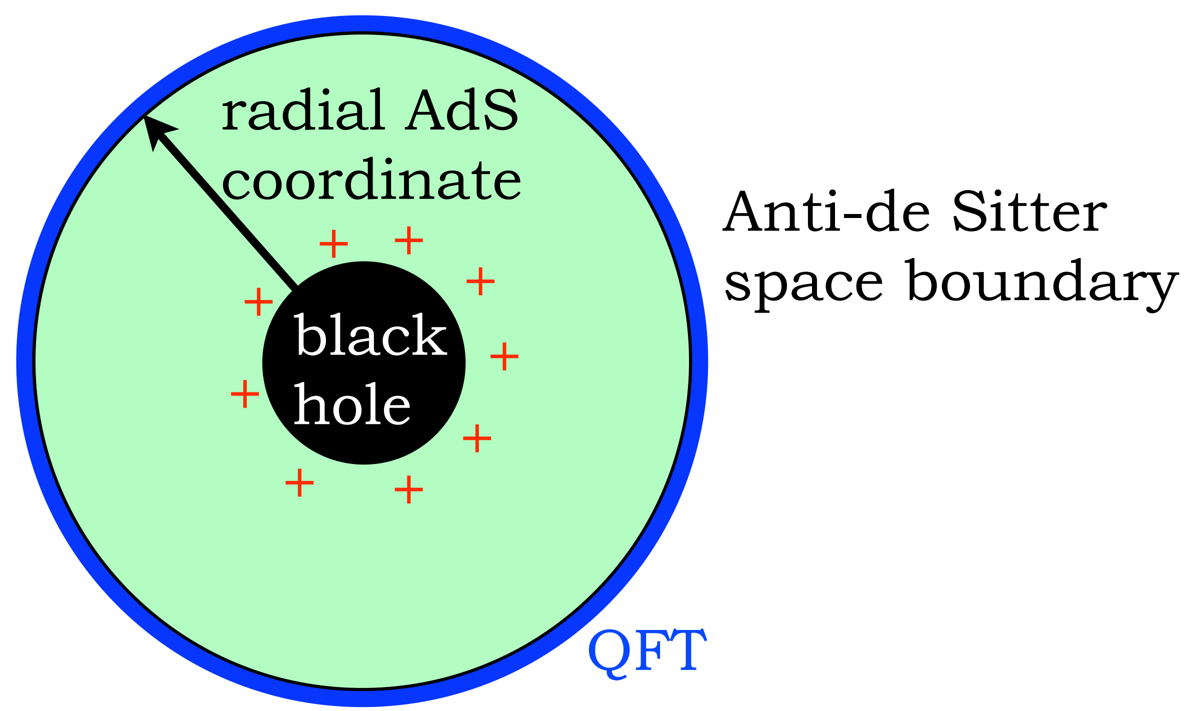
Holography concepts II

strongly coupled quantum field theory ⟷ *correspondence* ⟷ weakly curved gravity

QFT temperature ⟷ Hawking temperature

conserved charge ⟷ charged black hole



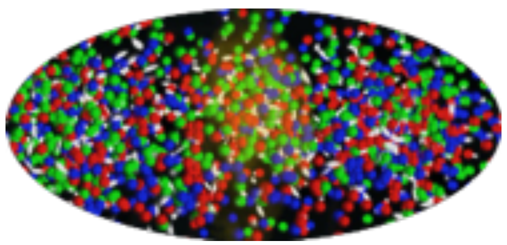


Outlook: Holography far-from equilibrium

thermalization

examples:
quench, heavy ion collision

time



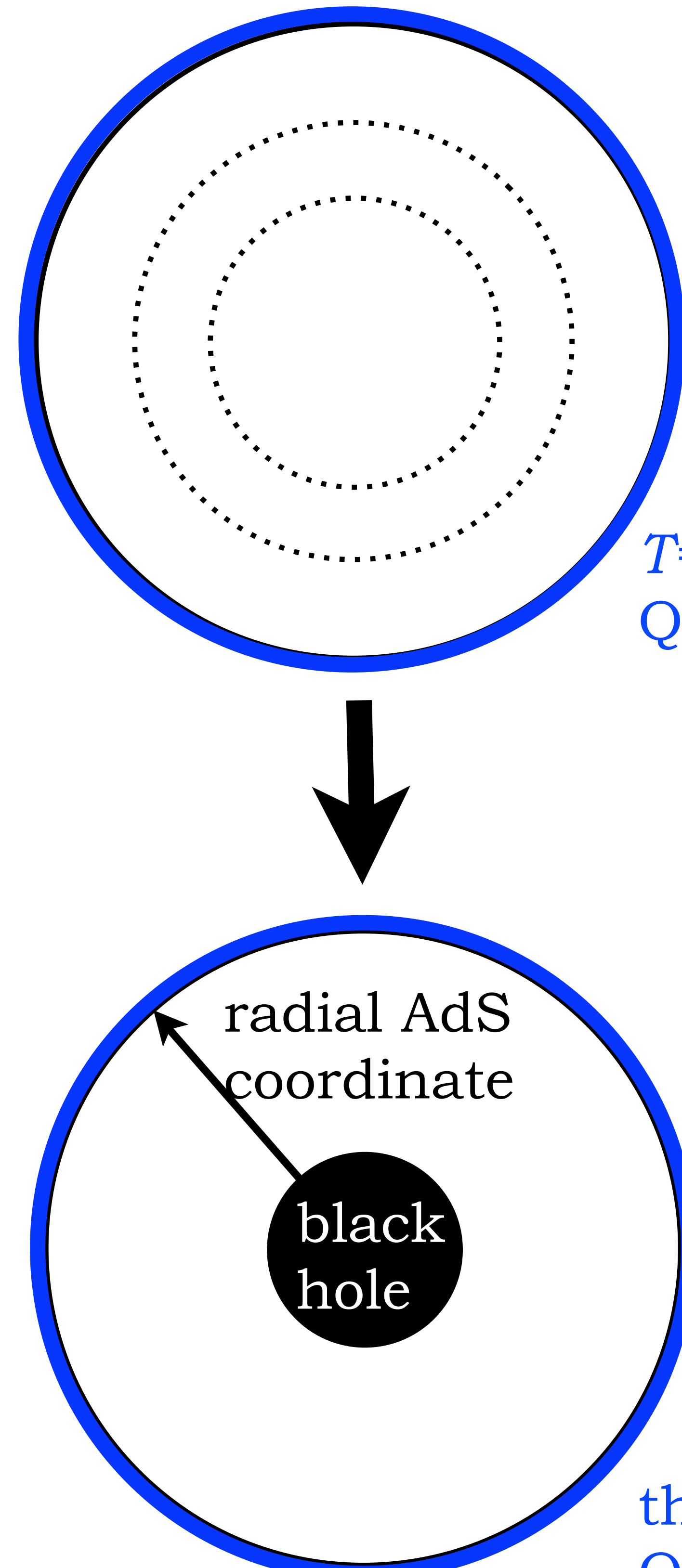
T=0 many body system

thermal
QFT

plasma at *T>0*

correspondence

horizon
formation



T=0
QFT

boundary of
Anti de Sitter
space

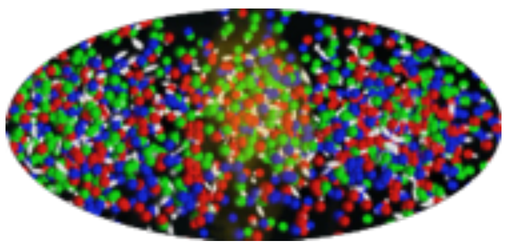
thermal
QFT

Outlook: Holography far-from equilibrium

thermalization

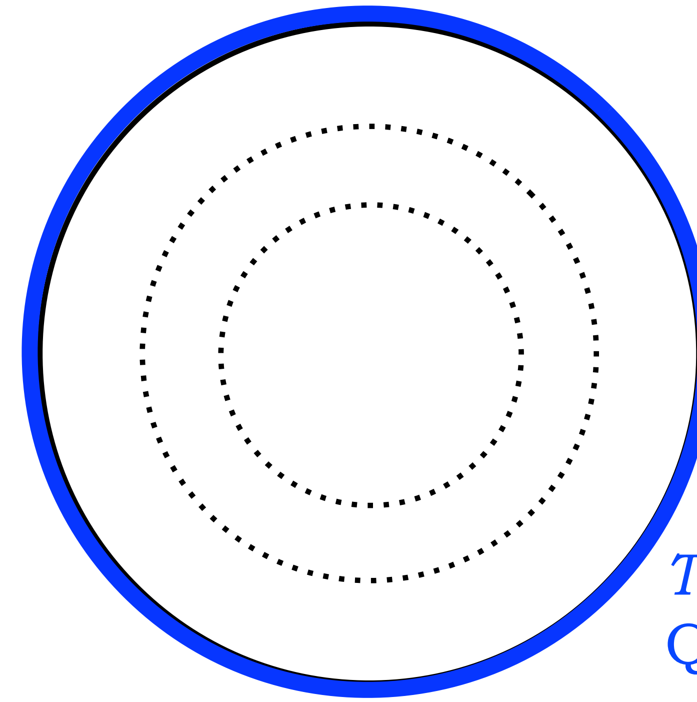
examples:
quench, heavy ion collision

time



T=0 many body system

horizon
formation



T=0
QFT

correspondence

boundary of
nti de Sitter
pace

Investigate:

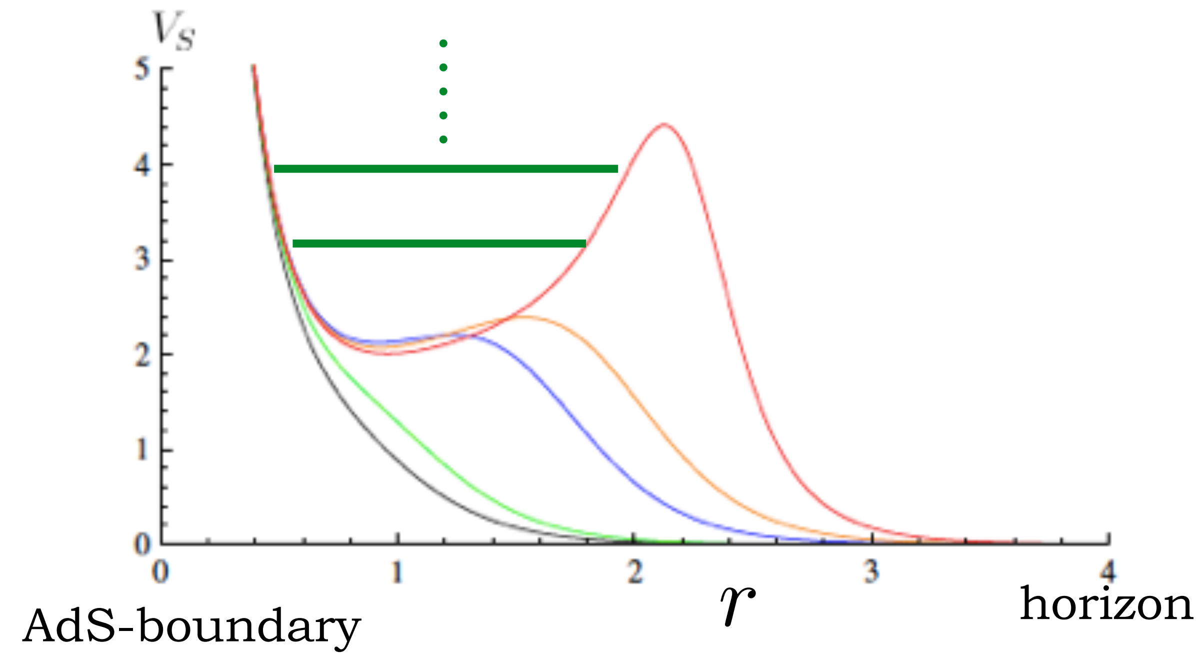
- **evolution of electromagnetic fields**
- **transport far from equilibrium**
- **initial excentricities versus flow harmonics**
- **dynamical evolution of “the ridge”**

thermal
QFT

What are quasi-normal modes in gravity?

- heuristically: the eigenmodes of black holes or black branes

$$H\,\phi = -\partial_r{}^2\phi + V_S\phi = E\phi$$



- the “ringing” of black holes

- quasi-eigensolutions to the linearized Einstein equations

Dual QFT: Quasi-normal modes are poles of correlators

black hole

Example: metric tensor fluctuations

QNMs of $\phi := {h_x}^y$ are poles of $\langle T_{xy} T_{xy} \rangle$

[Kovtun, Starinets; JHEP 2005]

Fourier transformation of gravity field:

$$h_{xy}(t) \propto e^{-i\omega t} h_{xy}(\omega)$$

Resonance and decay are encoded in QNM frequency:

$$e^{-i\omega t} = e^{-i(\mathrm{Re}\omega)\, t} e^{(\mathrm{Im}\omega)\, t}$$

resonance frequency (mass of the associated quasiparticle)

damping (decay width of the quasiparticle)

Holographic calculation: QNMs

- start with **gravitational background** (metric, matter content)

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed)

- impose **boundary conditions** that are
  (i) in-falling at horizon

  (ii) vanishing at AdS-boundary

- numerical implementations
  (a) spectral methods (generalized eigenvalue problem)

$$\left(A[\bar{g}_{mn}, \bar{A}_m, \partial_z, k] + \omega\, B[\bar{g}_{mn}, \bar{A}_m, \partial_z, k]\right) \begin{pmatrix} h_{mn}(z) \\ a_m(z) \end{pmatrix} = 0$$

  (b) shooting (spin-2 only)

Holographic calculation: QNMs

- start with **gravitational background** (metric, matter content)

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed $\phi(t) \propto e^{-i\omega t}\phi(\omega)$)

- impose **boundary conditions** that are
  in-falling at horizon:

and
  vanishing at AdS-boundary: $\lim_{u \to u_{bdy}} \phi(u) = 0$

Holographic calculation: QNMs

- start with **gravitational background** (metric, matter content)

  Example: (charged) Reissner-Nordstrom black brane in 5-dim AdS

  [Janiszewski, Kaminski; PRD (2015)]

  $$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2 \qquad f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

  $$A_t = \mu - \frac{Q}{L r^2}$$

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed $\phi(t) \propto e^{-i\omega t}\phi(\omega)$)

  Example: metric tensor fluctuation

  $$\phi := h_x{}^y \qquad 0 = \phi'' - \frac{f(u) - u f'(u)}{u f(u)}\phi' + \frac{\omega^2 - f(u)k^2}{4 r_H^2 u f(u)^2}\phi \qquad u = \left(\frac{r_H}{r}\right)^2$$

- impose **boundary conditions** that are
  in-falling at horizon: $\phi = (1-u)^{\pm\frac{i\tilde{\omega}}{2(2-\tilde{q}^2)}}\left[\phi^{(0)} + \phi^{(1)}(1-u) + \phi^{(2)}(1-u)^2 + \ldots\right]$

and
  vanishing at AdS-boundary: $\lim\limits_{u \to u_{bdy}} \phi(u) = 0$

Fluctuation equations: a glimpse

- one decoupled spin-2 equation (for 1 field)

$$0 = -\frac{1}{3 z^2 u(z) v(z)^4 w(z)^2}$$
$$h_{12}(z) \Big(v(z)^4 \big(-w(z)^2 (-2 z^4 c(z) \mathrm{Av}'(z) P'(z) + z^4 \mathrm{Av}'(z)^2 + 3 i k z^2 c'(z) + 3 i k z c(z) + z^4 c(z)^2 P'(z)^2 + 12 u(z) + 3 i \omega z - 24) - 3 i z^2 w(z) w'(z) (k c(z) + \omega) + 3 k^2 z^2 + z^4 u(z) P'(z)^2\big) -$$
$$2 B^2 z^4 w(z)^2 + 6 z v(z)^3 w(z)^2 v'(z) (i k z c(z) + 4 u(z) + i \omega z) - 12 z^2 u(z) v(z)^2 w(z)^2 v'(z)^2\Big) -$$
$$\frac{h_{12}'(z) \left(3 z v(z)^4 w(z)^2 (-2 i k z c(z) - z u'(z) - u(z) - 2 i \omega z) + 6 z^2 u(z) v(z)^3 w(z)^2 v'(z) - 3 z^2 u(z) v(z)^4 w(z) w'(z)\right)}{3 z^2 u(z) v(z)^4 w(z)^2} + h_{12}''(z)$$

- 8 coupled spin-1 equations (for 6 fields) — one third visible here

$$v(z)^4 (3 z w(z) (-u(z) (w(z) (z \mathrm{a1}'(z) \mathrm{Av}'(z) - z c'(z) h_{13}'(z) + h_{v1}'(z) + z h_{v1}''(z)) + z w'(z) (h_{v1}'(z) - 2 c(z) h_{13}'(z))) - i z \mathrm{a1}(z) w(z) \mathrm{Av}'(z) (k c(z) + \omega) + z w(z) (c(z) (h_{v1}'(z) (w(z)^2 (-c'(z)) - 2 i k) + h_{13}'(z) (-u'(z) - i \omega)) + c(z)^2 w(z)^2 c'(z) h_{13}'(z) - i \omega h$$

$$-3 i B \omega z^2 \mathrm{a1}(z) v(z)^2 w(z)^2 + v(z)^4 (3 z w(z) (-u(z) (w(z) (z \mathrm{a2}'(z) \mathrm{Av}'(z) - z c'(z) h_{23}'(z) + h_{v2}'(z) + z h_{v2}''(z)) + z w'(z) (h_{v2}'(z) - 2 c(z) h_{23}'(z))) - i z \mathrm{a2}(z) w(z) \mathrm{Av}'(z) (k c(z) + \omega) + z w(z) (c(z) (h_{v2}'(z) (w(z)^2 (-c'(z)) - 2 i k) + h_{23}'(z) (-u'(z) - i \omega)) + c$$

$$w(z)^2 (h_{13}(z) (-v(z)^4 (c(z) (-2 z^4 \mathrm{Av}'(z) P'(z) + 6 i k z) + z^4 \mathrm{Av}'(z)^2 + z^4 c(z)^2 P'(z)^2 + 12 u(z) + 3 i \omega z - 24) + B^2 z^4 + 6 z v(z)^3 v'(z) (2 u(z) + i k z c(z))) - 3 z v(z)^2 (v(z)^2 (z u(z) \mathrm{a1}'(z) P'(z) + i z \mathrm{a1}(z) P'(z) (k c(z) + \omega) + i k z c(z) h_{13}'(z) + i k z h_{v1}'(z) - i k h_{v1}$$

$$3 i B k z^2 \mathrm{a1}(z) v(z)^2 w(z)^2 + w(z)^2 (3 i z v(z)^2 (v(z)^2 (i (u(z) (z \mathrm{a2}'(z) P'(z) + h_{23}'(z) + z h_{23}''(z)) + z (h_{23}'(z) (i k c(z) + u'(z) + 2 i \omega) + i k h_{v2}'(z))) - z \mathrm{a2}(z) P'(z) (k c(z) + \omega) + k h_{v2}(z)) - i B z^3 P'(z) (c(z) h_{13}(z) - h_{v1}(z)) - 2 k z v(z) h_{v2}(z) v'(z)) + h_{23}(z) (-v$$

$$-v(z)^3 w(z) (v(z)^3 (z w(z)^2 (-w'(z) (u(z) \mathrm{a1}'(z) + z^2 (c(z) h_{13}(z) - h_{v1}(z)) (c(z) P'(z) - \mathrm{Av}'(z))) + i \gamma z \mathrm{a2}(z) (k \mathrm{Av}'(z) + \omega P'(z))) + w(z)^3 (z (-u(z) \mathrm{a1}''(z) + z^2 h_{13}(z) \mathrm{Av}'(z) c'(z) - z^2 \mathrm{Av}'(z) h_{v1}'(z) + z h_{v1}(z) (-z \mathrm{Av}''(z) - \mathrm{Av}'(z) + z c'(z) P'(z)) +$$
$$-v(z)^3 w(z) (v(z)^3 (z w(z)^2 (-w'(z) (u(z) \mathrm{a2}'(z) + z^2 (c(z) h_{23}(z) - h_{v2}(z)) (c(z) P'(z) - \mathrm{Av}'(z))) - i \gamma z \mathrm{a1}(z) (k \mathrm{Av}'(z) + \omega P'(z))) + w(z)^3 (z (-u(z) \mathrm{a2}''(z) + z^2 h_{23}(z) \mathrm{Av}'(z) c'(z) - z^2 \mathrm{Av}'(z) h_{v2}'(z) + z h_{v2}(z) (-z \mathrm{Av}''(z) - \mathrm{Av}'(z) + z c'(z) P'(z)) +$$
$$v(z)^2 (w(z)^2 (z (\mathrm{a1}'(z) (z c(z) P'(z) - z \mathrm{Av}'(z)) + z c'(z) h_{13}'(z) + c(z) (h_{13}'(z) + z h_{13}''(z)) - h_{v1}'(z) - z h_{v1}''(z)) + 4 h_{v1}(z)) + i k z^2 (\mathrm{a1}(z) P'(z) + h_{13}'(z)) + 2 h_{13}(z) (c(z) w(z) (z w'(z) - 2 w(z)) + z (w(z)^2 c'(z) + i k)) + z w(z) w'(z) (z c(z) h_{13}'(z) -$$
$$z^2 (w(z)^2 (B \mathrm{a1}'(z) + 2 v'(z)^2 (h_{v2}(z) - c(z) h_{23}(z))) - B z^2 h_{13}(z) P'(z)) + v(z)^2 (w(z)^2 (z (\mathrm{a2}'(z) (z c(z) P'(z) - z \mathrm{Av}'(z)) + z c'(z) h_{23}'(z) + c(z) (h_{23}'(z) + z h_{23}''(z)) - h_{v2}'(z) - z h_{v2}''(z)) + 4 h_{v2}(z)) + i k z^2 (\mathrm{a2}(z) P'(z) + h_{23}'(z)) + 2 h_{23}(z) (c(z$$

- 10 coupled spin-0 equations (for 6 fields)

Chiral effects in vector/axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD *U(1)*)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral magnetic effect

Axial current (e.g. QCD axial *U(1)*)

$$J_A^\mu = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu$$

chiral vortical effect

chiral separation effect

Chiral effects in vector/axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

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$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral magnetic effect

Axial current (e.g. QCD axial *U(1)*)

$$J_A^\mu = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu$$

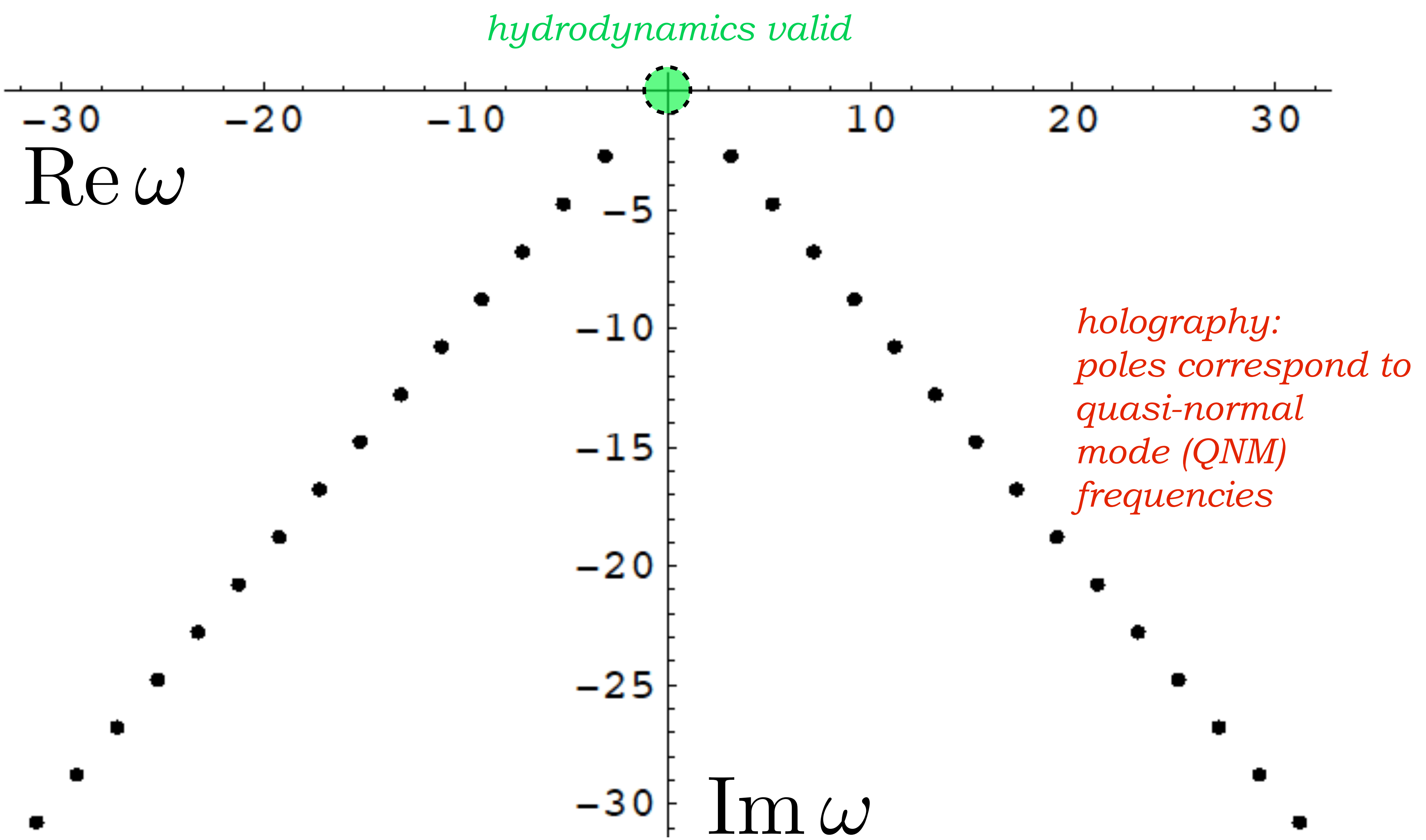
chiral vortical effect

chiral separation effect

Stepping stones: QNMs far beyond hydrodynamics

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of

$$\langle T_{xy} T_{xy} \rangle(\omega, k) = G^R_{xy,xy}(\omega, k) = -i \int d^4x \; e^{-i\omega t + ikz} \langle [T_{xy}(z), T_{xy}(0)] \rangle$$



[Starinets; JHEP (2002)]