Effects of strong magnetic fields in heavy ion collisions

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Workshop on “Chirality, Vorticity and Magnetic Field in Heavy ion Collisions”, March 19th - 23nd, Florence

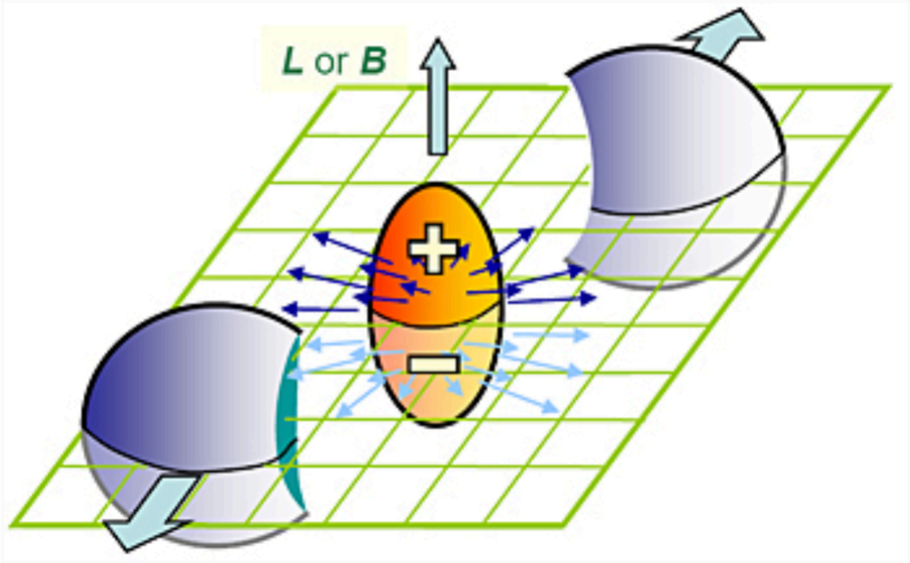
Outline

- Strong electromagnetic fields
- Chiral magnetic effect and chiral kinetic theory
- Chirality production in strong electromagnetic fields with Schwinger's formula
- Summary

1. Strong electromagnetic fields

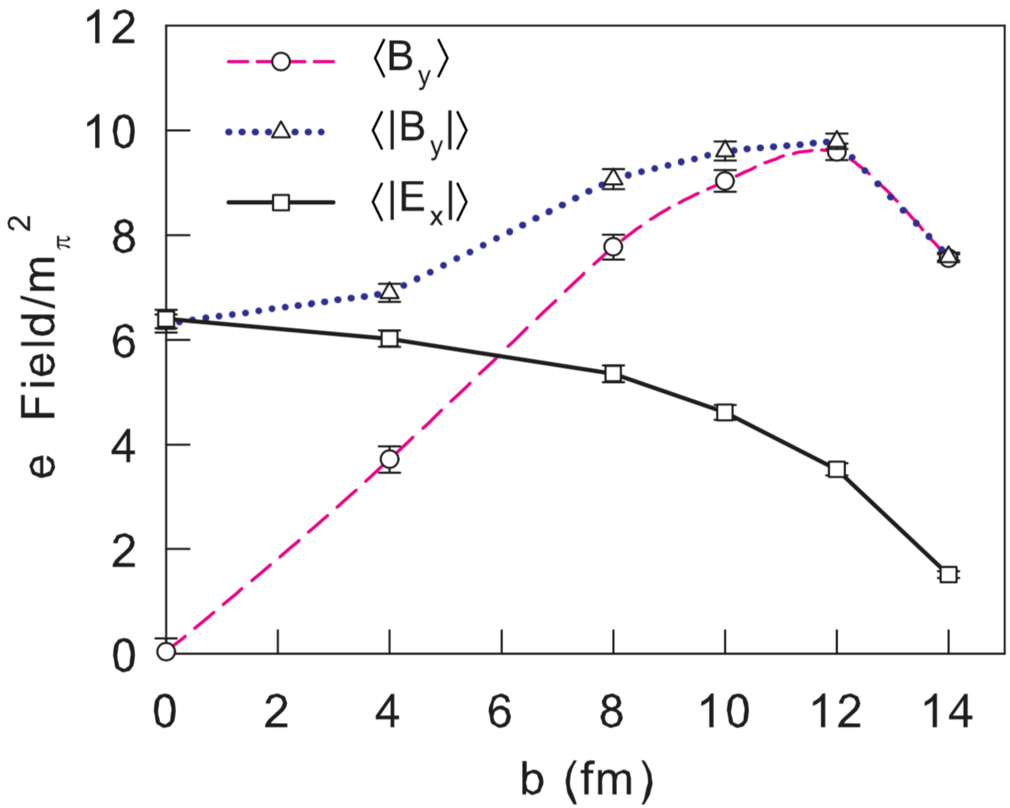
Strong Magnetic fields

- Relativistic heavy ion collisions



- B ~ 10^18 – 10^19 G, or a few m_pi^2:
  - Typical neutron star: ~10^12 G
  - Magnetars: ~10^15G

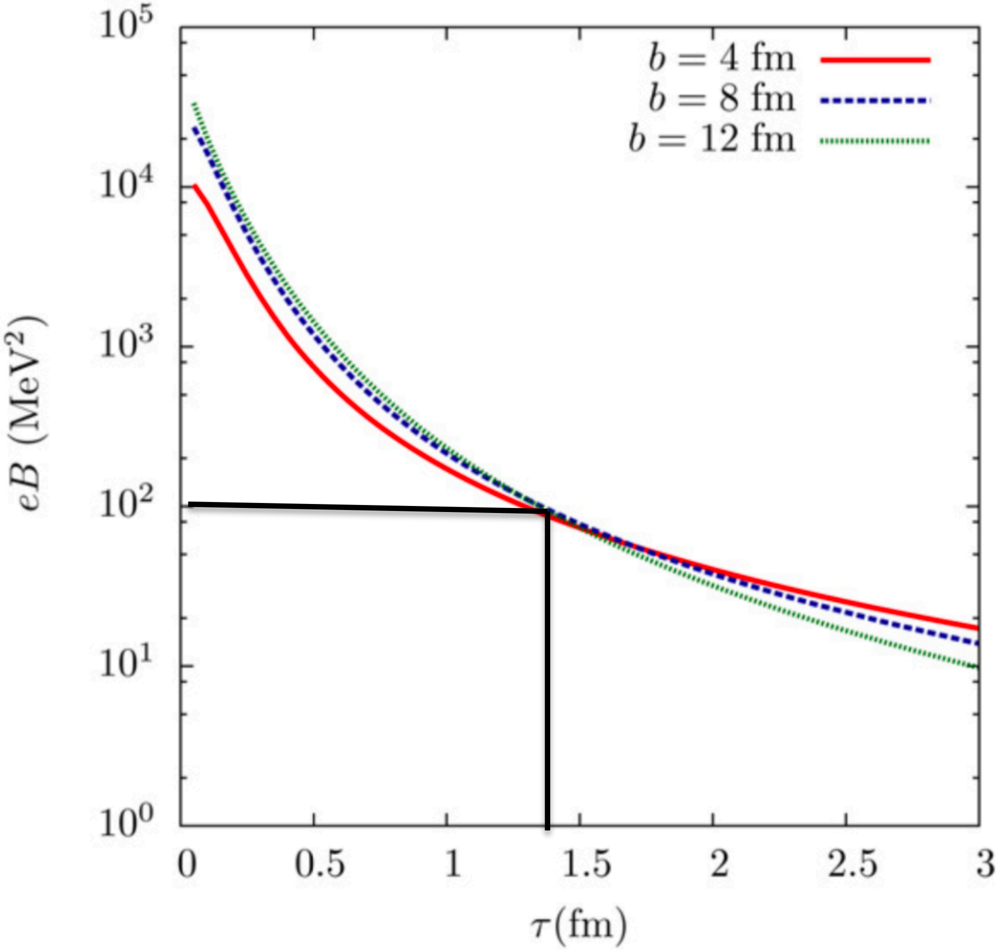
Strong electromagnetic fields



- Event by event cases (200GeV)
- Estimations from Lienard-Weichart potential
- MC-Glauber Model + Woods-Saxon nuclear distributions

A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015; H. Li, X.l. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013

Magnetic fields decay in vacuum



Kharzeev, McLerran, Warringa, Nucl. Phys. A(2008)

Relativistic Magneto-hydrodynamics

- Total energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{EM}, \; T^{\mu\nu}_{EM} = -F^{\mu\lambda}F^{\nu}_{\lambda} + \frac{1}{4}g^{\mu\nu}F^2,$$

- Relativistic hydrodynamics + Maxwell equations

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu F^{\mu\nu} = j^\nu,$$

$$\partial_\mu j^\mu = 0, \qquad \epsilon_{\mu\nu\alpha\beta}\partial^\nu F^{\alpha\beta} = 0.$$

X.G. Huang, M. Huang, D. H. Rischke, A. Sedrakian PRD 2009; V. Roy, SP, L. Rezzolla, D.H. Rischke PRD 2015; SP, V. Roy, L. Rezzolla, D.H. Rischke PRD 2016.

1D ideal Longitutidnal Bjorken MHD

- Ideal Magneto-hydrodynamic limit:

  Electric conductivity is infinite $\mathbf{E}+\mathbf{v}\times\mathbf{B}=0$

- Time evolution of : (τ: proper time)
  - fluid energy density:

$$\epsilon = \epsilon_0 \left(\tau_0/\tau\right)^{4/3},$$

  - Magnetic fields:

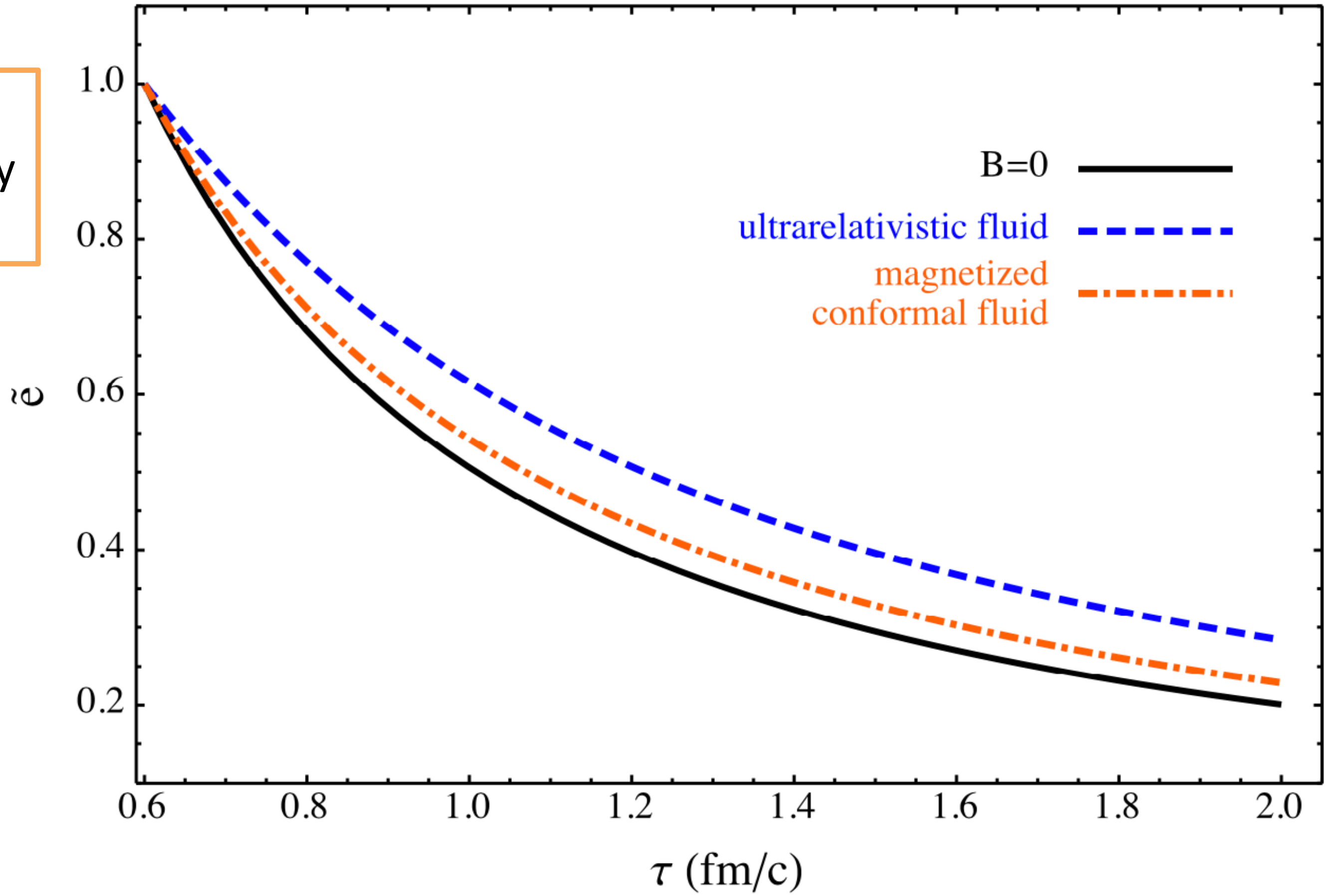
$$B_y = B_{y0}\left(\tau_0/\tau\right)$$

  V. Roy, SP, L. Rezzolla, D.H. Rischke PRD 2015;

- Frozen flux theorem:

Magnetization Effects

Normalized Fluid energy density



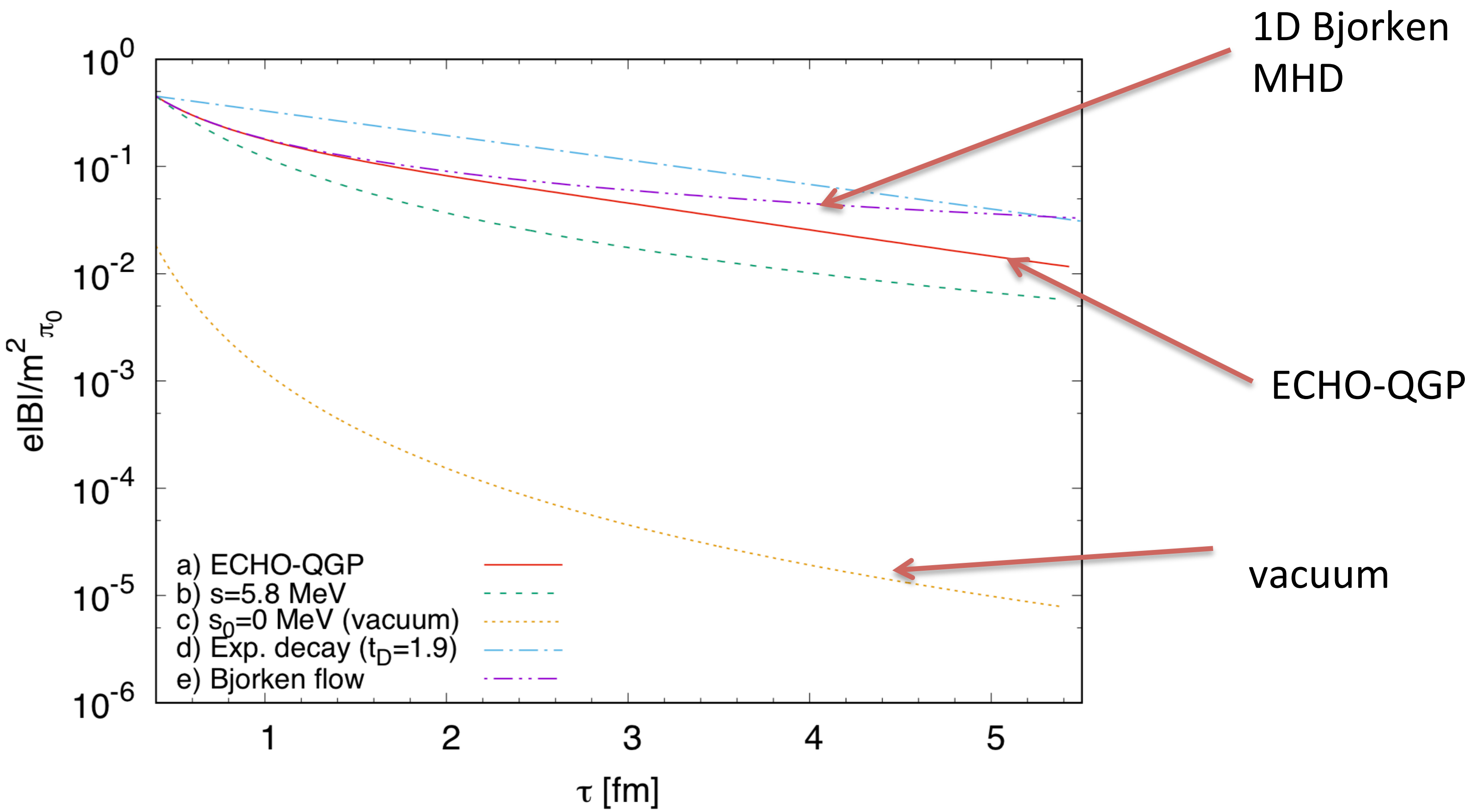
Set:

(Magnetic susceptibility)∗(initial B)/(initial fluid dentiy) = 0.5;

SP, V. Roy, L. Rezzolla, D.H. Rischke PRD 2016;

MHD simulations

- ECHO-QGP: 3+1D ideal MHD

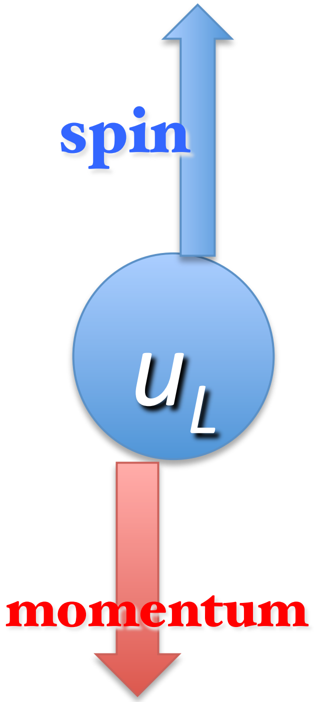


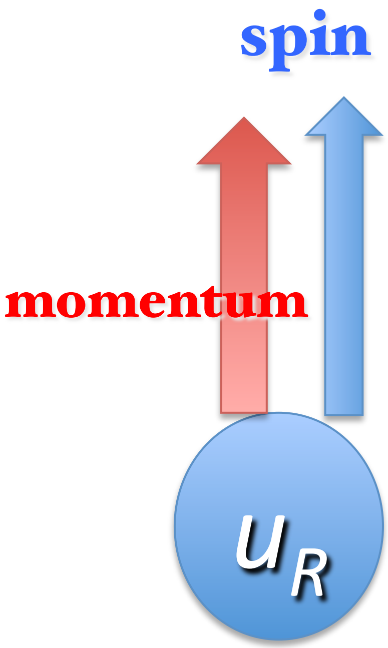
G. Inghirami, L.D. Zanna, A. , M. H. Moghaddam, F.Becattini, M. Bleicher, EPJC 2016

2. Chiral Magnetic Effect and Chiral kinetic theory

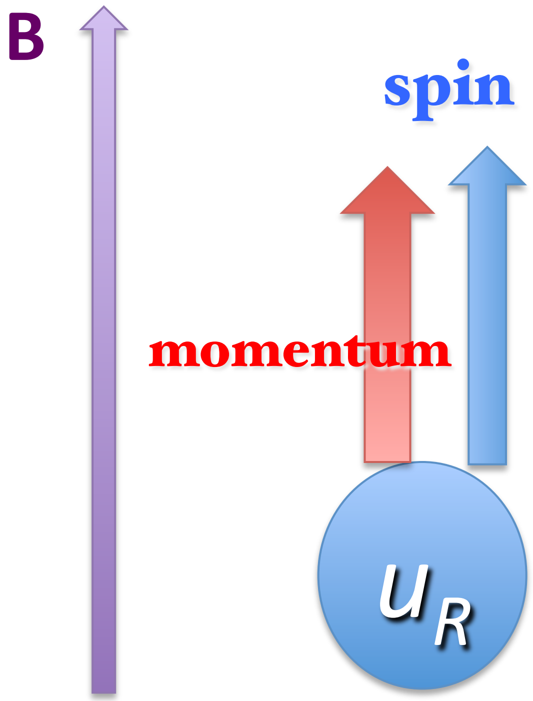
Chiral Magnetic Effects (I)

- Massless fermions – Chirality:





Chiral Magnetic Effect (II)



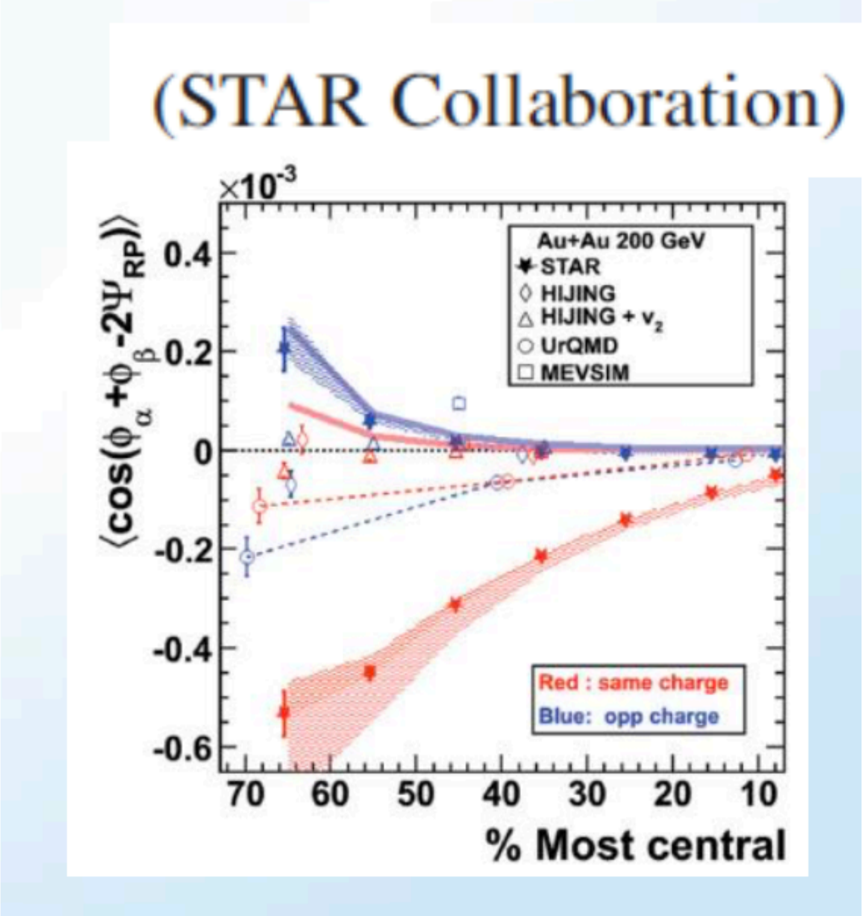
- Magnetic fields
- Nonzero axial chemical potential
  - Number of Left handed fermions ≠ Number of Right handed fermions
- Charge current

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B},$$

Ref: Kharzeev, Fukushima, Warrigna, PRD, PRL (2008,2009), etc. ...

Chiral Magnetic Effect (II)

- Heavy ion collisions: charge separation



Chiral kinetic equation (I)

- Hamiltonian formulism, effective theory

  Son, Yamamoto, PRL, (2012); PRD (2013)

- Path integration

  Stephanov, Yin, PRL (2012);

  Chen, Son, Stephanov, Yee, Yin, PRL, (2014);

  J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD (2014)

- Wigner function
  - hydrodynamics, equilibrium

    J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);

  - out-of-equilibrium, quantum field theory

    Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017) Cf. Talk by Di-Lun

  - Other studies: Cf. Talk by P.F. Zhuang

    A.P. Huang, S.Z. Su, Y. Jiang, J.F. Liao, P.F. Zhuang, arXiv:1801.03640

- Other Studies: Cf. Talks by N. Muller Talk and X.G. Huang
  - world-line formulism N. Muller, R, Venugopalan PRD 2017

Chiral kinetic theory (II)

- Wigner function out-of-equilibrium, quantum field theory

  Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017) **Cf. Talk by Di-Lun**

- Master equation: Dirac equation to kinetic theory
- **J.P. BLAIZOT, E. IANCU, Phys. Report. 2010**

- Out-of-equilibrium: arbitrary distribution function

- The physical interpretation of side-jump from QFT side

- Interaction effects

Non-linear electromagnetic response

- Chiral kinetic theory + relaxation time approaches
- Large chemical potential limit:

J.W. Chen, T. Ishii, SP, N. Yamamoto, PRD 2016

- Side-jump, with/without mapping to fluid:

Y. Hidaka, SP, D.L. Yang, PRD 2018 **Cf. Talk by Di-Lun**

- Studies from other group:

E. V. Gorbar, I. A. Shovkovy, S. Vilchinskii, I. Rudenok, A. Boyarsky, and O. Ruchayskiy PRD 2016; E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, PRB 2017;E. V. Gorbar, D. O. Rybalka, , I. A. Shovkovy, PRD 2017

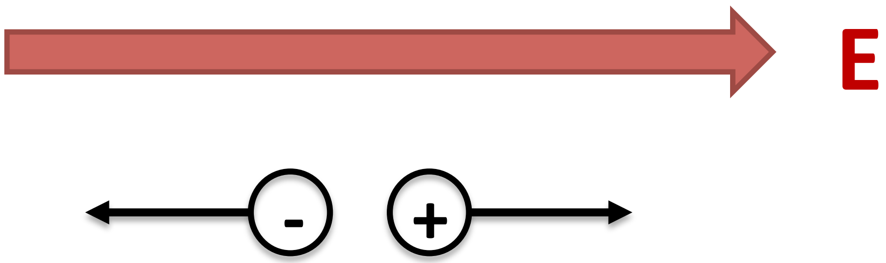
etc. …

3. Chirality production in strong electromagnetic fields with Schwinger's formula

In collaboration with Patrick Copinger, Kenji Fukshima, in preparation

Schwinger Pair Production

- Strong electric fields: Pair Production



- Strong electromagnetic fields: Pair Production +?

Chiral anomaly

- Chiral anomaly

$$\partial_\mu j_5^\mu = 2im\overline{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Axial charge current | **Pseudo-scalar** | **~ E.B**

- Pseudo-scalar term vanishes in massless limit
- Question: Finite mass corrections

World-line Formulism (I)

- Schwinger proper time:

$$\frac{i}{A+i\varepsilon} = \int_0^\infty dT e^{iT(A+i\varepsilon)}$$

- Spinor Feynman propagator at background fields:

$$G_A(x,y) =$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i}{\not{D} - m + i\varepsilon}$$

$$= \int_0^\infty dT e^{-im^2T} < y|(\not{D} + m) e^{-i\hat{H}_A T}|x >,$$

$$\hat{H}_A = -(\hat{p}^\mu - eA^\mu)^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}.$$

M. D. Schwartz, Quantum Field theory and the standard model;

Christian Schubert: lecture note on the Worldline Formalism

World-line Formulism (II)

- Spinor Feynman propagator at background fields:

$G_A(x,y) =$ 

$$= \int_0^\infty dT e^{-im^2 T} < y|(\not{D}+m)e^{-i\hat{H}_A T}|x >,$$

T: Schwinger proper time

- Path integral: (Homogenous, Constant E,B)

$$< y|e^{-i\hat{H}_A T}|x > = \int \mathcal{D}x \exp\left\{ i\int_0^T d\tau \left[-\frac{1}{4}\dot{x}^2 + eA^\mu \dot{x}_\mu \right] \right\} \exp\left\{ i\int_0^T d\tau \left(-\frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu} \right) \right\}$$

Cf. Talk by N. Muller

M. D. Schwartz, Quantum Field theory and the standard model;

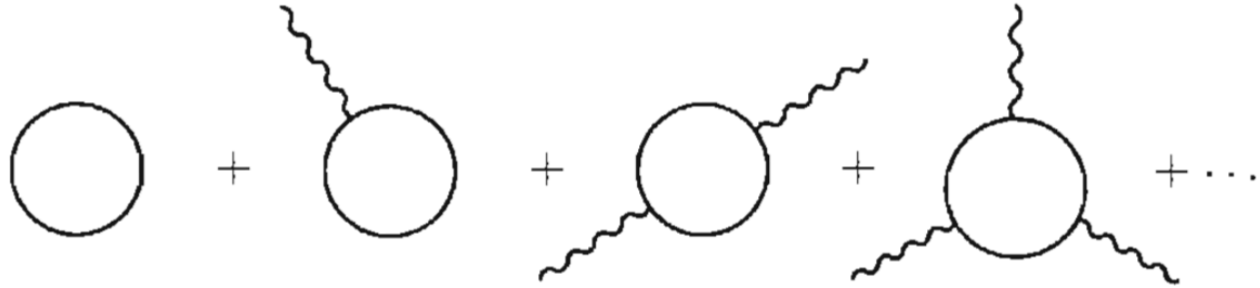
Christian Schubert: lecture note on the Worldline Formalism

World-line Formulism (II)

- Euler-Heisenberg Lagrangian

Review: G. V. DUNNE, arXiv:hep-th/0406216

$$\mathcal{L}_{eff}[A] =$$



- Vacuum persistence:

Schwinger pair production rate

(n=1 world-line instanton)

M. D. Schwartz, Quantum Field theory and the standard model;
Christian Schubert: lecture note on the Worldline Formalism

“Well-known” Results

- Homogenous Constant E,B at z direction

- Using world-line formulism (or original Schwinger’s methods):

$$< \partial_\mu j_5^\mu > = -2im < \overline{\psi}\gamma^5\psi > + \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} = 0$$

J. Schwinger, Phys. Rev. 82,5 (1951);
M. D. Schwartz, Quantum Field theory and the standard model;

$$< \overline{\psi}\gamma^5\psi > = i\frac{1}{4\pi^2}\frac{EB}{m}$$

?

Puzzle: Mass corrections (II)

- Taking m->0 at the very beginning: Weyl fermions

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

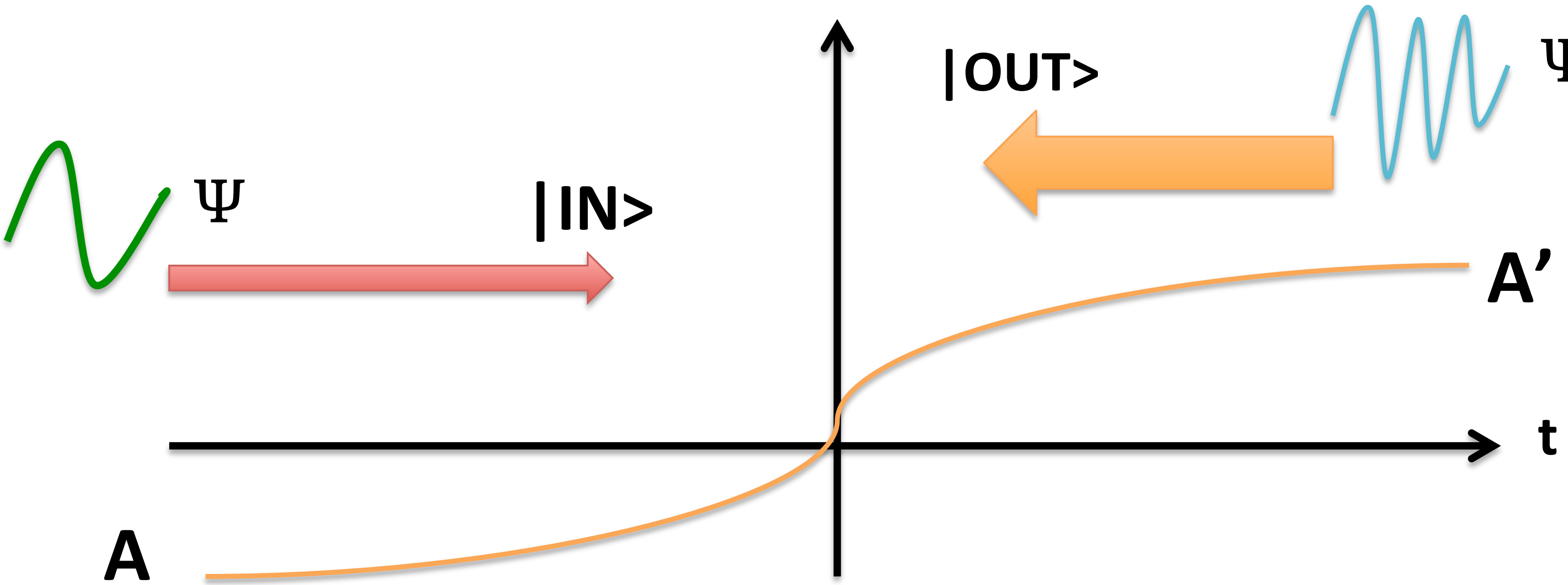
- After all the calculations, taking m->0.

$$< \partial_\mu j_5^\mu > = -2im < \overline{\psi}\gamma^5\psi > -\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} = 0$$

IN and OUT States

- Homogenous Constant Ez,Bz field:

$$A^z(t) = eE_z t, \quad H = H(A(t)),$$



Unstable vacuum

- |0, IN> is NOT equal to |0, OUT>
- $$|<0, out \mid 0, in>|^2 \neq 1$$
- Schwinger Pair Production Rate:

$$P_0 = 1 - |<0, out \mid 0, in>|^2 = \frac{e^2 E_z B_z}{4\pi^2} \coth\left(\frac{B_z}{E_z}\pi\right) \exp\left(-\frac{m^2\pi}{|eE_z|}\right)$$

(n=1 world-line instanton)

Expectation Value: IN-IN states

- Transition amplitude

$$< 0, out|\partial_\mu j_5^\mu|0, in >$$

- Expectation value

$$< 0, in|\partial_\mu j_5^\mu|0, in >$$

Review: F. Gelis, N. Tanji 2015; N. Tanji 2009

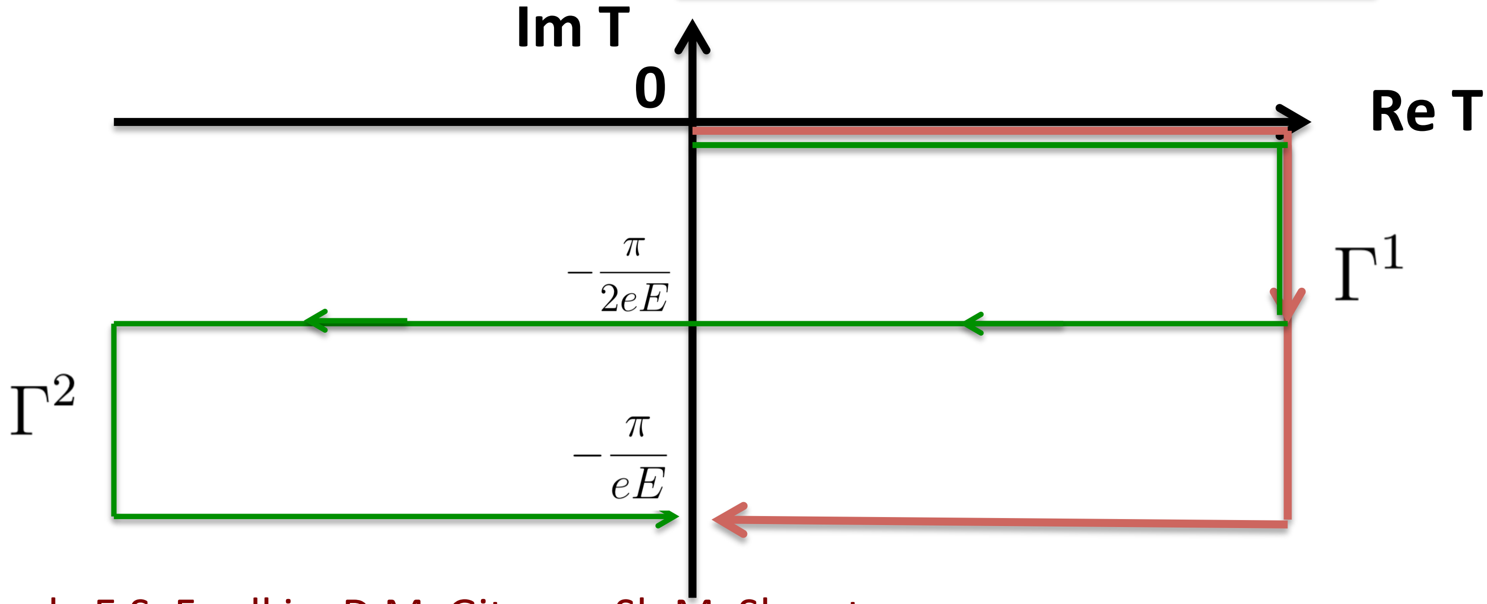
Textbook: E.S. Fradkin, D.M. Gitman, Sh.M. Shvartsman: Quantum Electrodynamic with Unstable Vacuum, 1991

IN-IN states vs World-line Formulism

- Feynman Propagator: Homogenous Constant Ez,Bz

$$< 0, out|S(x,y)|0, in >= (\not{D} + m) \int_0^\infty g(x,y,T)dT$$

$$< 0, in|S(x,y)|0, in >= (\not{D} + m) \left[\theta(x_3 - y_3) \int_{\Gamma^1} + \theta(y_3 - x_3) \int_{\Gamma^2}\right] g(x,y,T)dT$$



Textbook: E.S. Fradkin, D.M. Gitman, Sh.M. Shvartsman: Quantum Electrodynamic with Unstable Vacuum, 1991

Mass corrections

$$< 0, in|\overline{\psi}\gamma^5\psi|0, in > = i\frac{e^2 EB}{4\pi^2}\frac{1}{m}(1 - e^{-m^2\pi/eE})$$

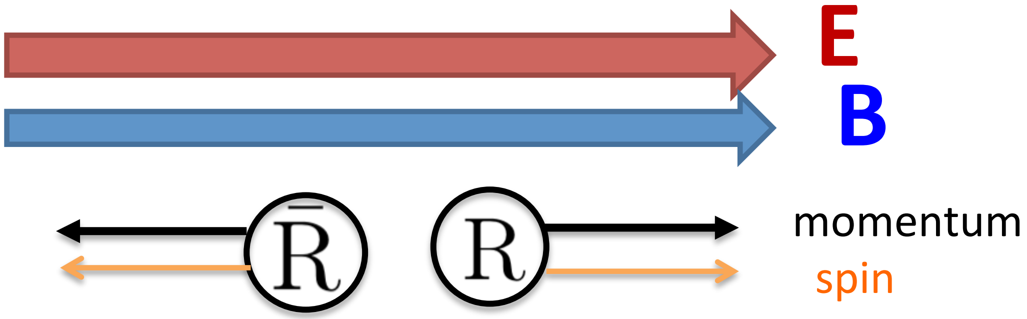
$$(m \to 0) \Rightarrow 0$$

$$< 0, in|\partial_\mu j_5^\mu|0, in > = -\frac{e^2}{2\pi^2}EBe^{-m^2\pi/eE}$$

$$(m \to 0) \Rightarrow -\frac{e^2}{2\pi^2}EB$$

Physical Picture

- Strong electromagnetic fields: Chirality Production



$$\frac{1}{2}\partial_t n_5 = \text{Schwinger Pair Production rate}$$

K. Fukushima, D.Kharzeev, H. Warringa PRL 2010

Other quantities

- Expectation value of charge current:

$$< 0, in|j^{\mu}|0, in >= \delta_3^{\mu} \frac{e^2}{2\pi^2} EBt \coth\left(\frac{B}{E}\pi\right) \exp\left(-\frac{\pi m^2}{eE}\right)$$

- Expectation value of axial charge current:

$$< 0, in|j_5^{\nu}|0, in >= -\delta_0^{\nu} \frac{e^2}{2\pi^3} EBt \exp\left(-\frac{\pi m^2}{eE}\right)$$

Chirality production

- Strong electromagnetic fields:
  Chirality production + Schwinger pair production

- Expectation values in unstable vacuum

- Mass corrections to time derivatives of axial charge

4. Summary

Summary

- Magnetic fields will delay slowly in the media.
- Chiral Magnetic Effect and Chiral kinetic theory
- Chirality production in strong electromagnetic fields with Schwinger's formula

Thank you!

Backup

Lienard-Weichart formula

- Estimations from classic electromagnetic dynamics

$$\vec{E}(\vec{r},t) = \frac{e}{4\pi}\sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{R}_i - R_i\vec{v}_i}{(R_i - \vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2), \qquad (1)$$

$$\vec{B}(\vec{r},t) = \frac{e}{4\pi}\sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{v}_i \times \vec{R}_i}{(R_i - \vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2), \qquad (2)$$

- Position of charged particles:

e.g. MC-Glauber Model + Woods-Saxon nuclear distributions