Anomalous Transport and Holography

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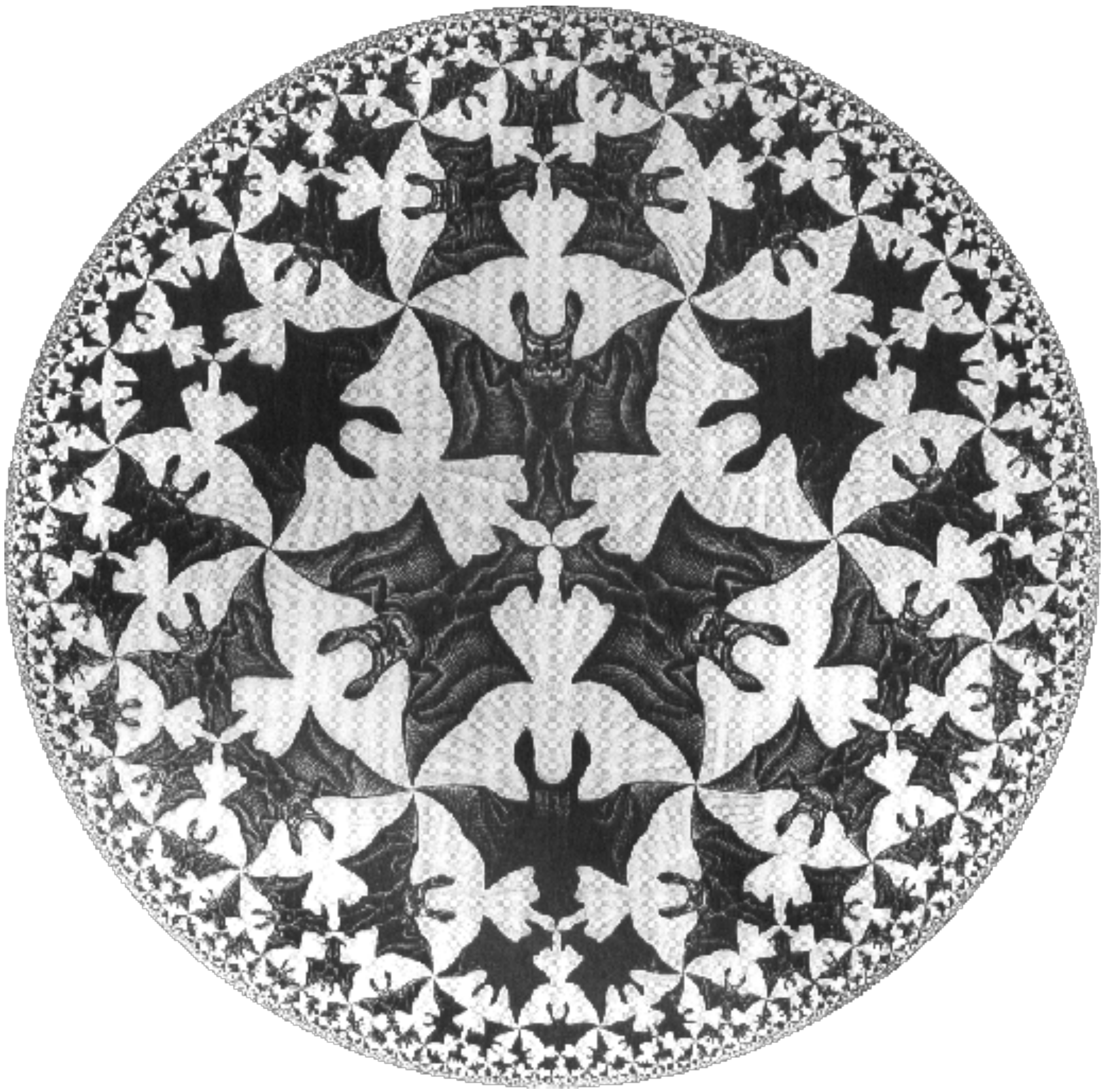
Outline

- Holography and currents
- Symmetry Breaking I & II
- More Symmetry Breaking (Poincare)
- Quenching
- Questions

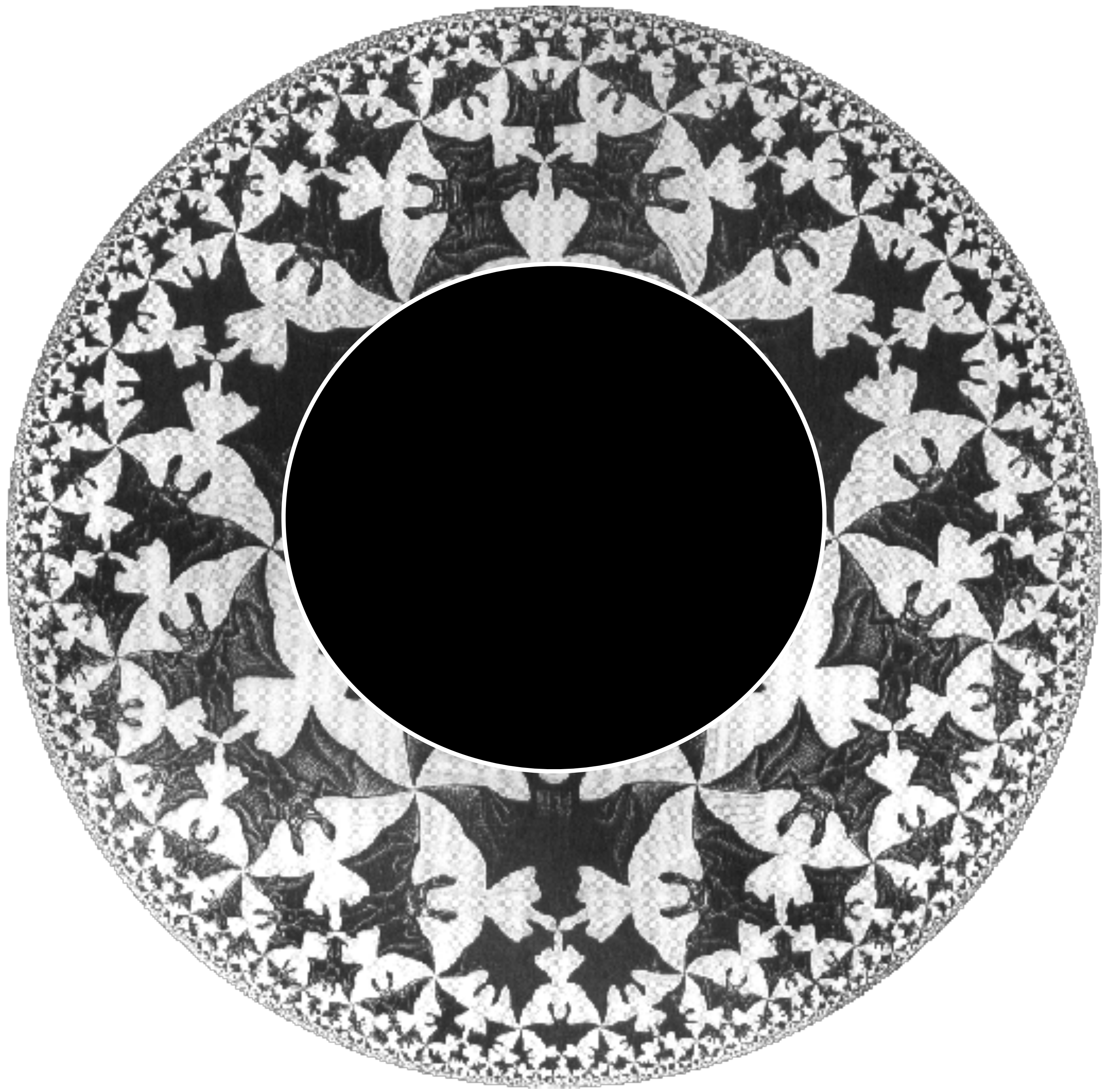
Holography: $ds^2 = r^2(-f(r)dt^2 + d\vec{x}^2) + \frac{dr^2}{f(r)r^2}$



[Maldacena] [Witten] [Gubser, Klebanov, Polyakov]

Holography:

$$ds^2 = r^2(-f(r)dt^2 + d\vec{x}^2) + \frac{dr^2}{f(r)r^2}$$



[Maldacena] [Witten] [Gubser, Klebanov, Polyakov]

AdS/CFT

$$\int_{\Phi|_\partial=\Phi_0} D\Phi e^{iS[\Phi]} = e^{iZ[\Phi_0]}$$

$$\frac{\delta^n Z[\Phi_0]}{\delta\Phi_0^1(x_1)\cdots\delta\Phi_0^n(x_n)} = \langle O_1(x_1)\ldots O_n(x_n)\rangle$$

Path integral (string theory) on AdS is hard. In practice resort to semi classical limit:

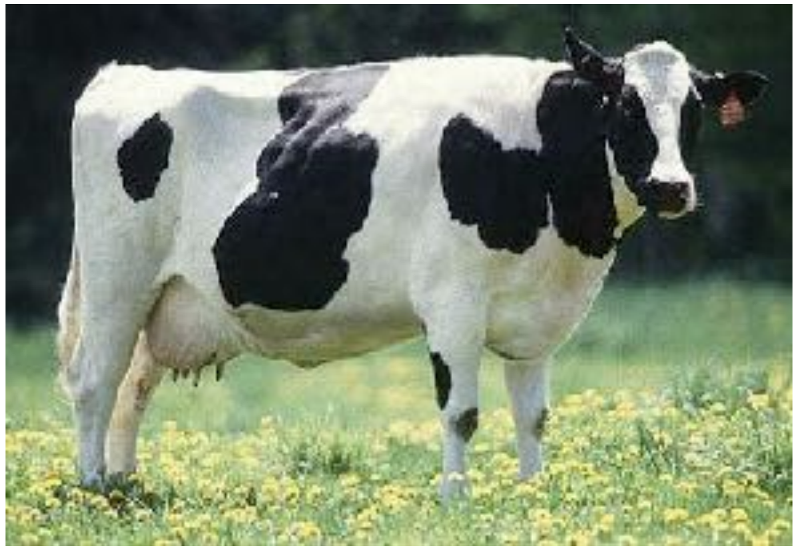
$$S_{grav}[\Phi_0] = Z[\Phi_0]$$

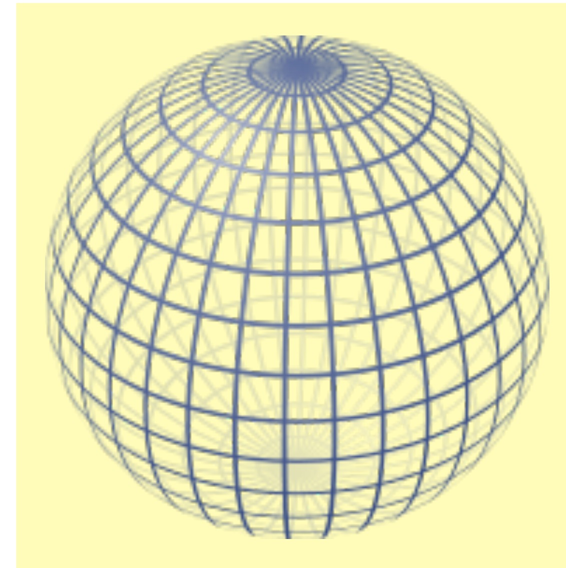
AdS/CFT

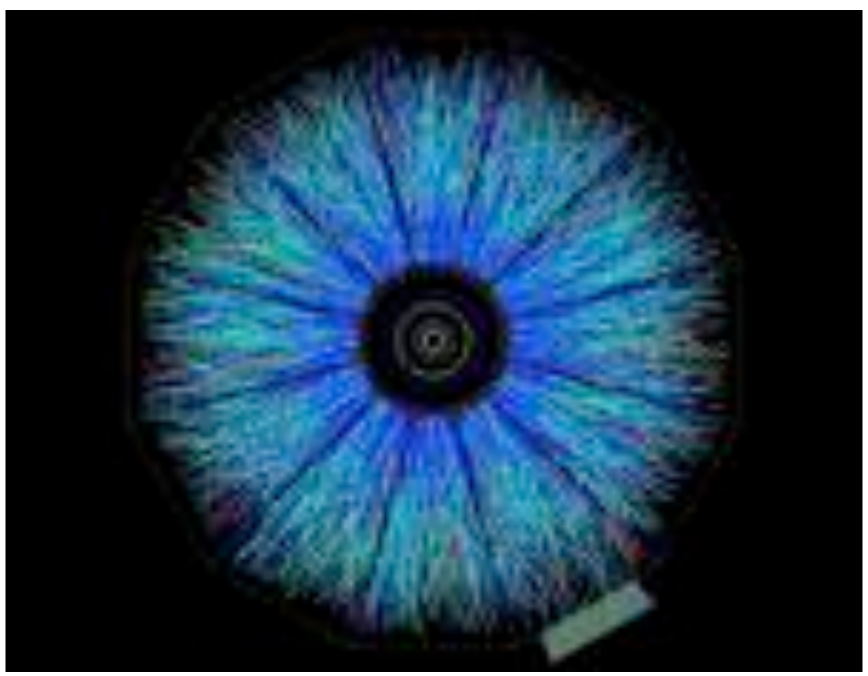
Dictionary

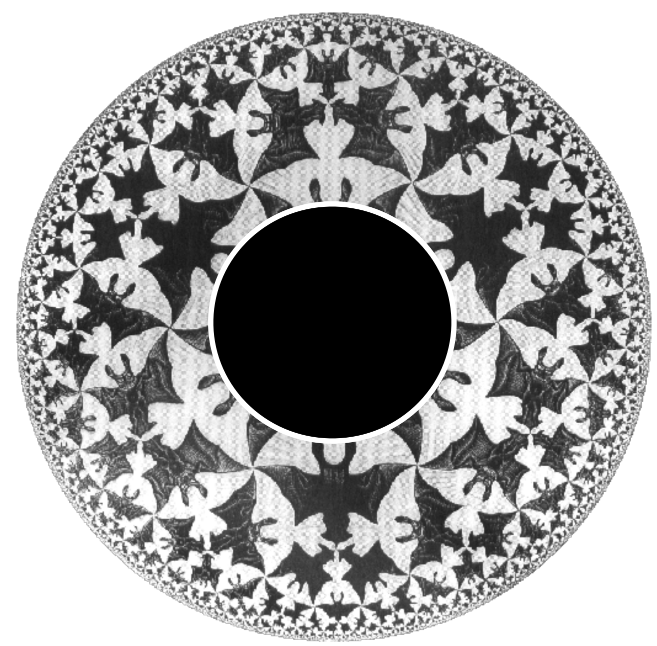
AdS	Field Theory
five dimensional	four dimensional
r-direction	RG flow
strongly coupled	weakly coupled
gravity	no gravity
metric	energy momentum tensor
gauge field	current
scalar field	scalar operator

AdS = spherical (hyperbolic) cow of sQGP









$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Policastro, Son, Starinets]
[Kovtun, Son, Starinets]

CME in Holography

- Anomalies = Chern-Simons terms

$$S = \int d^5x\sqrt{-g}\frac{1}{4}\left(F_R^2 + F_L^2\right) + \frac{\alpha}{3}\left[A_R \wedge F_R \wedge F_R - A_L \wedge F_L \wedge F_L\right]$$

- Change to V-A basis

$$S = \int \frac{\alpha}{3}\left(A \wedge F_A \wedge F_A + A \wedge F_V \wedge F_V + V \wedge F_A \wedge F_V\right)$$

- Eliminate explicit V-dependence by adding boundary term

$$S_{ct} = c_1 \int_\partial V \wedge A \wedge F_V$$

- Chose coefficient such that

V-A $\neq$ R-L

$$S_{CS} = \int \frac{\alpha}{3} A \wedge \left[F_A \wedge F_A + 3F_V \wedge F_V\right]$$

[Rebhan, Stricker, Schmitt]
[Gynther, K.L., Pena-Benitez, Rebhan]

CME in Holography

Currents

$$\mathcal{J}^\mu_V = \sqrt{-g}F_V^{\mu r} + 4\alpha\epsilon^{\mu\nu\rho\lambda}A_\nu F^V_{\rho\lambda}$$
$$\mathcal{J}^\mu_A = \sqrt{-g}F_A^{\mu r} + 4\frac{\alpha}{3}\epsilon^{\mu\nu\rho\lambda}A_\nu F^A_{\rho\lambda}$$

$$J^\mu_V = \sqrt{-g}F_V^{\mu r}$$
$$J^\mu_A = \sqrt{-g}F_A^{\mu r}$$

Anomalies

$$\partial_\mu \mathcal{J}^\mu_V = 0$$
$$\partial_\mu \mathcal{J}^\mu_A = -\frac{\alpha}{3}\epsilon^{\mu\nu\rho\lambda}\left(F^A_{\mu\nu}F^A_{\rho\lambda} + 3F^V_{\mu\nu}F^V_{\rho\lambda}\right)$$

- Variation of effective action
- Wess-Zumino consistency condition
- Not invariant under axial gauge trafo
- Not unique

$$\partial_\mu J^\mu_V = -2\alpha\,\epsilon^{\mu\nu\rho\lambda}F^A_{\mu\nu}F^V_{\rho\lambda}$$
$$\partial_\mu J^\mu_A = -\alpha\,\epsilon^{\mu\nu\rho\lambda}\left(F^A_{\mu\nu}F^A_{\rho\lambda} + F^V_{\mu\nu}F^V_{\rho\lambda}\right)$$

- Not Variation of effective action
- Not invariant under axial gauge taro
- Unique

AdS has rediscovered theory of *covariant* and *consistent* anomaly [Bardeen, Zumino]

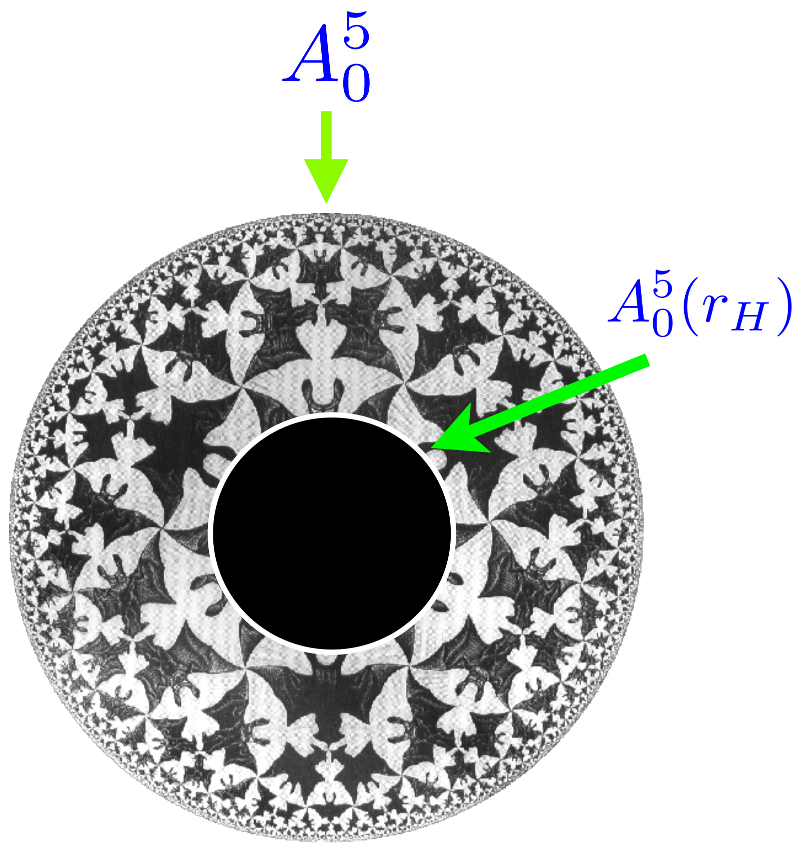
CME in Holography

CME and CSE

$$\vec{\mathcal{J}} = \frac{\mu_5 - A_0^5}{2\pi^2}\vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu}{2\pi^2}\vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu_5 - \frac{A_0^5}{3}}{2\pi^2}\vec{B}_5$$



- Equilibrium $H \to H - \mu_5 Q_5 - \mu Q$

$$A_0^5 = \mu_5$$

$$V_0 = \mu$$

- Bloch theorem [Yamamoto]

Symmetry breaking - I

CME and CSE

$$\vec{\mathcal{J}} = \frac{\mu_5 - A_0^5}{2\pi^2}\vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu}{2\pi^2}\vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu_5 - \frac{A_0^5}{3}}{2\pi^2}\vec{B}_5$$

- Holographic mass term
- Breaks axial symmetry at tree level
- Soft: only in IR
- Counterterms not affected
- Fate of CME et al.?

- In Holography charged scalar

$$S = \int d^5x\sqrt{-g}\left(|D_A\phi|^2 + m^2|\phi|^2\right) \quad , \quad \phi \approx \frac{M}{r} + \cdots$$

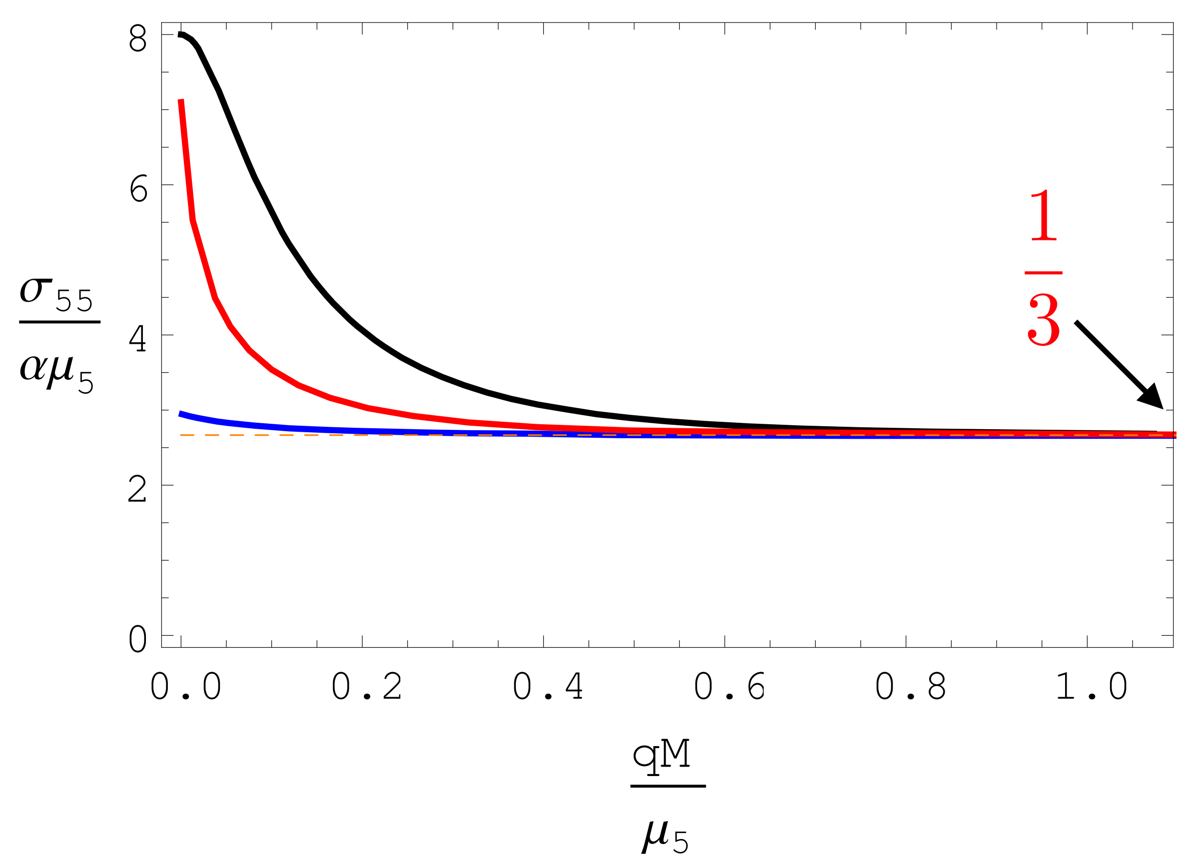
- Intuition: no chiral transport for large mass (strong breaking)
- BUT: *covariant* or *consistent* current ?

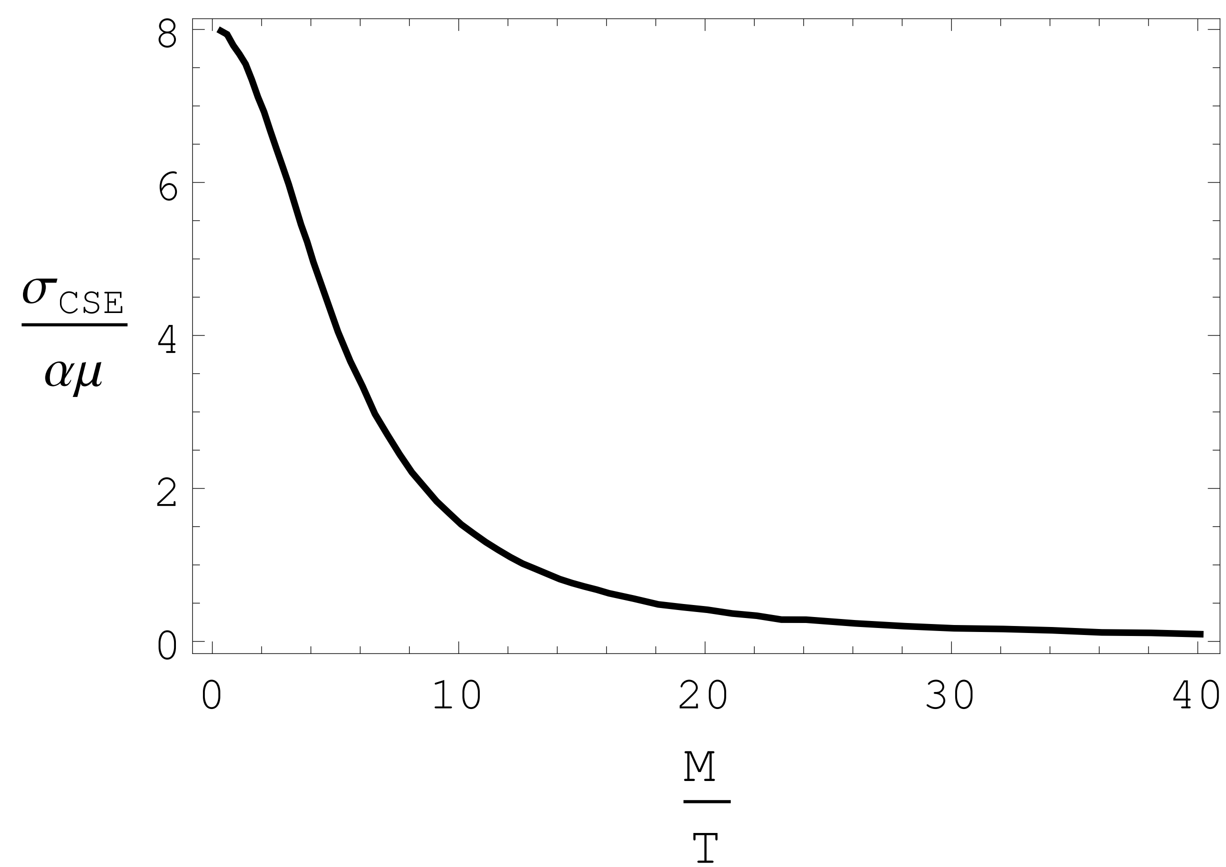
Symmetry breaking - I

CME and CSE

CME =0 all the way (Bloch!)

[Jimenez-Alba, K.L., Liu, Sun]





Consistent currents vanish !

Same for spontaneous breaking [Amado, Lisker, Yarom]

However lattice no change in CSE [Buividovich, Puhr]

???

Symmetry breaking - II

- In Holography "linear axion" background (massless scalar)

$$S = \int d^5x \sqrt{-g} \left(|\partial\phi|^2 \right) \quad , \quad \phi \approx kx + \cdots$$

- Background breaks translations eoms are homogeneous
- Graviton has mass

$$T^{0i} = T^{i0}$$

- Charge (Momentum) = Current (Energy-current)

1. Intuition: Momentum density, broken symmetry
2. Intuition: Energy current is dissipationless

Symmetry breaking - II

- Include also gravitational anomaly

$$S = \int d^5 x A \wedge \mathrm{tr}(R \wedge R)$$

- Higher derivative term: extrinsic curvature component

$$\delta S = \int_\partial d^4 x \theta \mathrm{tr}[R^{(4)} \wedge R^{(4)}) + D(K \wedge DK)]$$

- Asymptotic AdS: extrinsic curvature component vanishes
- On BH horizon gives rise to T^2 terms in CVE etc..
- K is tensor in 4 dim sense

$$\mathcal{J} \to \mathcal{J} - K \wedge DK$$

Symmetry breaking - II

- Can extrinsic curvature term be seen in UV?

$$ds^2 = -fr^2dt^2 + \frac{dr^2}{r^2 f} + r^2 d\vec{x}^2 \qquad f = 1 + \frac{k^2}{r^2} - (1 - \frac{k^2}{4})\frac{r_H^4}{r^4}$$

Unusual power!

- Extrinsic curvature as additional variable

$$\delta S_{on-shell} = \int_\partial \sqrt{-g}(t^{\mu\nu}\delta g_{\mu\nu} + u^{\mu\nu}\delta K_{\mu\nu})$$

- Energy momentum tensor (Ward identity)

$$\Theta^{\mu\nu} = t^{\mu\nu} + u^{\mu\lambda}K_\lambda^\nu$$

- New term is due to gravitational Chern-Simons term

Symmetry breaking - II

- CME and CVE without new term

$$T^{0i} = \left(8\alpha\mu^2 + 32\lambda T^2 - 4\lambda k^2\right) B^i$$
$$J^i = \left(8\alpha\mu^2 + 32\lambda T^2\right) 2\omega^i$$

- Impossible in unitary theory
- CVE = 2 CME for energy current by Kubo formulas

- Including the new term

$$T^{0i} = \left(8\alpha\mu^2 + 32\lambda T^2\right) B^i$$

- All is well!
- Energy current being dissipation wins!

[Copetti, Fernandez-Pendas, K.L., Megias]

[Copetti, Fernandez-Pendas]

Quenching the CME

- Natural question: anomaly induced transport **far** from equilibrium physics

- Possible importance for Heavy Ion Collisions (magnetic field has already decayed in hydrodynamic regime)

- Holography allows both: study fast time evolution (quenches and anomalous transport [Lin, Yee]

- Study CME via gravitational Chern-Simons term

See also talks of Zhuang, Yang

What to look for

- "Minimal" setup: inject energy
- Equilibrium: energy — temperature $T_0 \longrightarrow T$
- CME in energy-momentum tensor
- First near equilibrium = hydro

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu + p\eta_{\mu\nu} + \hat{\sigma}_B (u_\mu B_\nu + u_\nu B_\mu) \,,$$

$$J_\mu = \rho u_\mu + \sigma_B B_\mu \,,$$

$$J^X_\mu = \rho_X u_\mu + \sigma_{B,X} B_\mu$$

- Landau frame

$$\hat{\sigma}_B = 0$$

$$\sigma_B = 24\alpha\mu - \frac{\rho}{\epsilon + p} \left(12\alpha\mu^2 + 32\lambda\pi^2 T^2\right)$$

$$\sigma_{B,X} = -\frac{\rho_X}{\epsilon + p} \left(12\alpha\mu^2 + 32\lambda\pi^2 T^2\right)$$

What to look for

- Energy current = Momentum density

$$T_{0i} = T_{i0}$$

- Momentum density = conserved charge

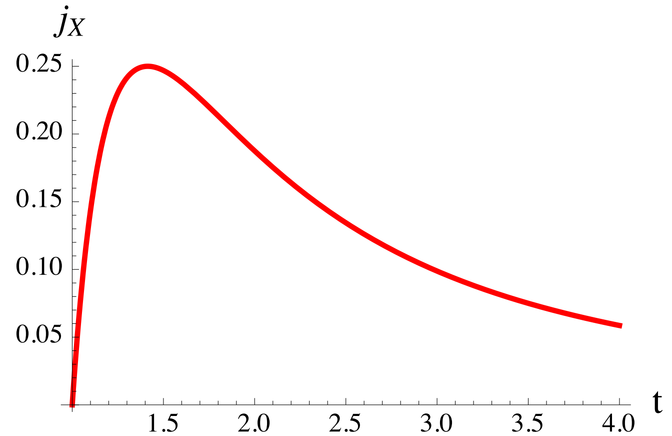
$$32\lambda\pi^2 T_0^2 \vec{B} = (\epsilon + p)\vec{v}$$

- Monitor response in tracer U(1) current

$$\vec{J}_X = 32\frac{\rho_X}{\epsilon + p}(T_0^2 - T^2)\pi^2\lambda\vec{B}$$

- Removing constants: benchmark near equilibrium curve

$$j_X = \frac{T^2/T_0^2 - 1}{T^4/T_0^4}$$



Holographic quench

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(\mathcal{R} - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{4q^2}F_X^2 - \frac{1}{2}(\partial\phi)^2 + \lambda\epsilon^{MNOPQ}A_M R^A_{BNO}R^B_{APQ}\right)$$

- Tracer U(1) in decoupling limit
- Holographic quench $\quad \phi_0(t,\vec{x}) = \frac{1}{2}\eta\left(1+\tanh\frac{t}{\tau}\right)$

$$ds^2 = \frac{1}{z^2}\left(-f(t,z)e^{-2\delta(t,z)}dt^2 + \frac{dz^2}{f(t,z)} + d\vec{x}^2\right)$$

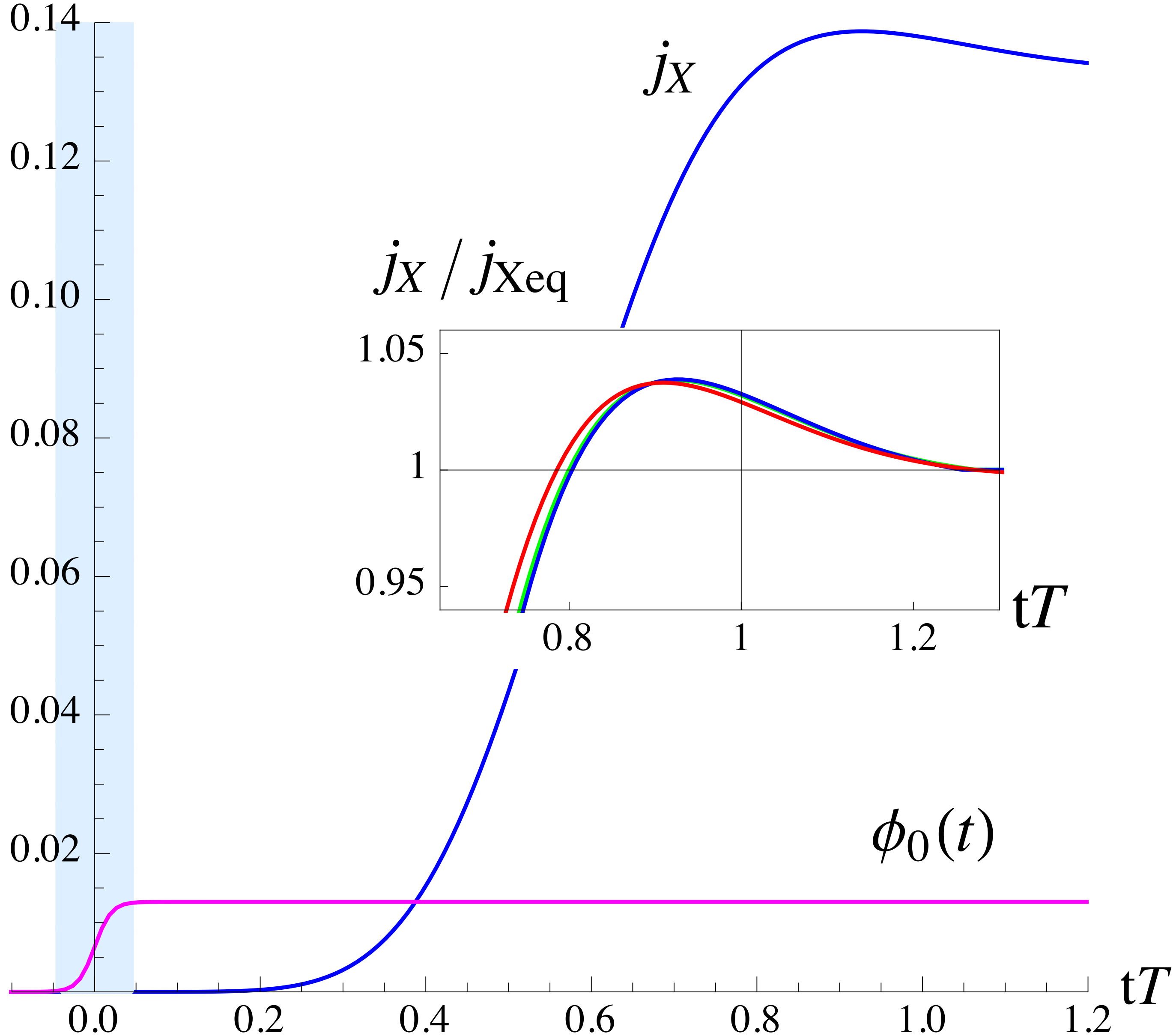
$$F_{xy} = B \qquad\qquad F_{X,0z} = \rho_X\, z\, e^{-\delta(t,z)}$$

- To linear order in B

$$J^\mu_X = \lim_{z\to 0}\sqrt{-g}\,F_X^{\mu z}$$

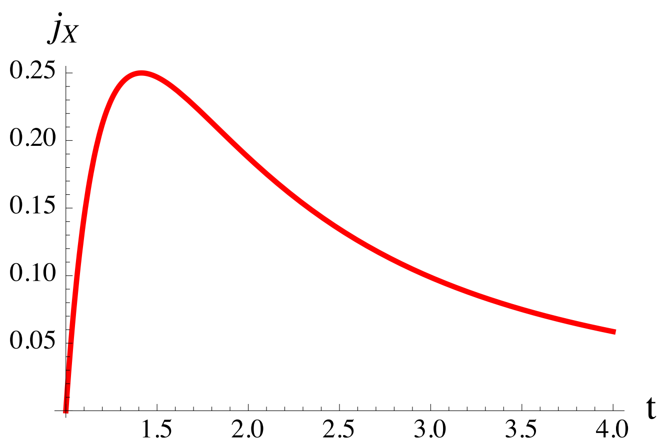
Holographic quench

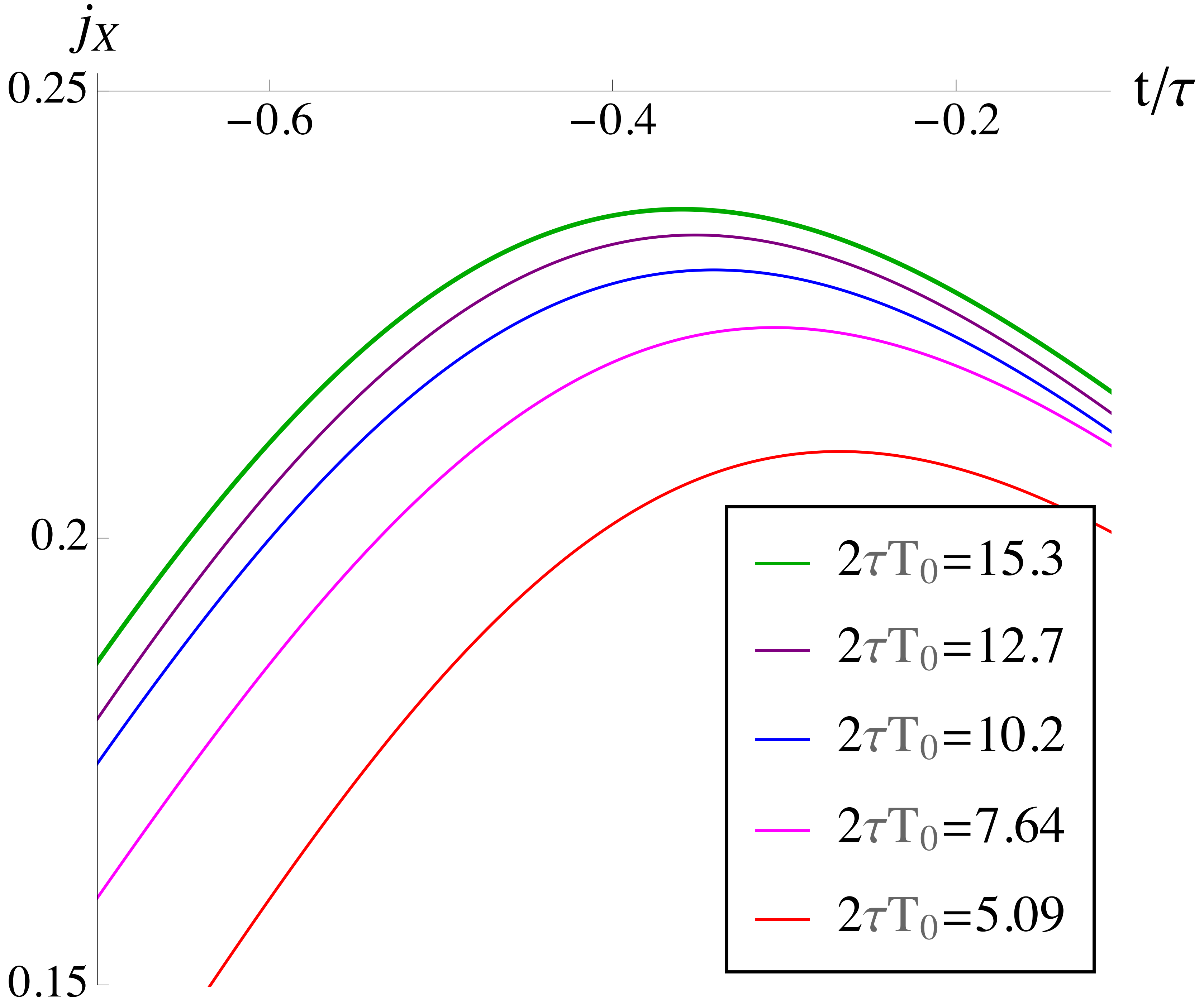
- Fast quenches $2\tau T < 1$



Holographic quench

- Very slow quenches





Summary

- Holography is efficient discovery tool for transport
- Fate of anomalous transport under symmetry breaking
- New questions:
  - Why do consistent currents vanish?
  - What is the extrinsic curvature in field theory?
- Anomalous transport far from equilibrium (QGP)