Nonequilibrium axial charge production in expanding color fields

Naoto Tanji (ECT*)
Axial charge production in Glasma

The space-time distribution of axial charges is indispensable for the understanding of CME.

Possible axial charge production mechanisms:

- **Quark production in Glasma**
- **Sphaleron transition in QGP/Glasma**

Moore, Tassler (2011)
Mace, Schlichting, Venugopalan (2016)
Axial charge production in Glasma

The space-time distribution of axial charges is indispensable for the understanding of CME. Possible axial charge production mechanisms:

- **Quark production in Glasma**
- **Sphaleron transition in QGP/Glasma**

Challenges:

- Nonequilibrium
- Nonperturbative
- Quantum dynamics of quarks
- Expanding geometry

Real-time lattice simulations of quantum fermion fields and classical gauge fields
CGC initial conditions

Classical YM eqs. coupled to large-x color sources

\[ D_\mu F^{\mu\nu} = \delta^{\nu+}\delta(x^-)\rho_{(1)}(x_\perp) + \delta^{\nu-}\delta(x^+)\rho_{(2)}(x_\perp) \]

Solution at \( \tau = 0^+ \) in the FS gauge \( A^\tau = 0 \)

\[ E_z(\tau = 0^+, x_\perp) = -ig \left[ \alpha^{i(1)}, \alpha^{i(2)} \right] \]
\[ B_z(\tau = 0^+, x_\perp) = -ig \epsilon^{ij} \left[ \alpha^{i(1)}, \alpha^{j(2)} \right] \]

with

\[ \alpha^{i(n)}(x_\perp) = \frac{i}{g} V^{\dagger}_{(n)} \partial^i V_{(n)} \]
\[ V_{(n)}(x_\perp) = \exp \left[ -ig \nabla_{\perp}^{-2} \rho_{(n)} \right] \]


Numerical solution for \( \tau > 0 \) in the MV model

Kapp, McLerran (2006)
CGC initial conditions

Classical YM eqs. coupled to large-x color sources

\[ D_\mu F^{\mu\nu} = \delta^{\nu+} \delta(x^{-}) \rho(1)(x_{\perp}) + \delta^{\nu-} \delta(x^{+}) \rho(2)(x_{\perp}) \]

Solution at \( \tau = 0^+ \) in the FS gauge \( A^\tau = 0 \)

\[ E_z(\tau = 0^+, x_{\perp}) = -ig \left[ \alpha^{i(1)}, \alpha^{i(2)} \right] \]
\[ B_z(\tau = 0^+, x_{\perp}) = -ig e^{ij} \left[ \alpha^{i(1)}, \alpha^{j(2)} \right] \]

with

\[ \alpha^{(n)}(x_{\perp}) = \frac{i}{g} V^{\dagger}(n) \partial^i V(n) \]
\[ V(n)(x_{\perp}) = \exp \left[ -ig \nabla_{\perp}^2 \rho(n) \right] \]


Numerical solution for \( \tau > 0 \) in the MV model

We consider flux-tube-like configurations with a Gaussian profile

\[ \langle ( - ) \rangle \]
Quark fields

Up to the initial surface $\tau = 0^+$, the Dirac equation under the CGC classical gauge fields can be solved analytically.

Gelis, Kajantie, Lappi (2006); Gelis, Tanji (2016)

The evolution for $\tau > 0$ can be described by solving the Dirac equation for the mode functions

$$\left(i\gamma^0 \partial_\tau + \frac{i}{\tau} \gamma^3 D_\eta + i\gamma^i D_i - m\right) \psi_{p\perp,\nu,s,c}(x) = 0$$

on a real-time lattice in the expanding geometry.

To realize the chiral anomaly on the lattice, we employ the Wilson fermion extended to the expanding geometry.

Tanji, Berges (2018)
Adler-Bell-Jackiw anomaly equation

\[ \partial_\mu j^\mu_5 = 2m \langle \bar{\psi} i \gamma_5 \psi \rangle + \frac{g^2}{4\pi^2} E^a \cdot B^a \]

Axial current \( j^\mu_5 = \langle \bar{\psi} \gamma^\mu \gamma_5 \psi \rangle \)

The Wilson fermion exactly satisfies

\[ \partial_\mu j^\mu_5 = 2m \langle \bar{\psi} i \gamma_5 \psi \rangle + \langle \bar{\psi} i \gamma_5 W \psi \rangle \]

where \( W \psi \) is the Wilson term added to the Dirac equation to suppress doublers.

The axial anomaly is realized if

\[ \langle \bar{\psi} i \gamma_5 W \psi \rangle \approx \frac{g^2}{4\pi^2} E^a \cdot B^a \]

which has been confirmed numerically in non-expanding systems.

Karsten, Smit (1981)

TANJI, Mueller, Berges (2016);
Mueller, Hebenstreit, Berges (2016);
Mace, Mueller, Schlichting, Sharma (2017)
**Anomaly equation in the expanding geometry**

ABJ anomaly equation in the $\tau$-$\eta$ coordinates

\[
\frac{1}{\tau} \partial_{\tau} (\tau j_5^\tau) + \partial_{i} j_5^i + \frac{1}{\tau} \partial_{\eta} j_5^\eta = 2m \langle \bar{\psi} i \gamma_5 \psi \rangle + \frac{g^2}{4\pi^2} \mathbf{E}^a \cdot \mathbf{B}^a
\]

boost-invariant background \quad m \approx 0

Axial charge density per unit transverse area and unit rapidity

\[
\frac{dN_5}{d^2 x_\perp d\eta} = \tau j_5^\tau (x) = - \int_0^\tau \tau' \partial_i j_5^i \, d\tau' + \frac{g^2}{4\pi^2} \int_0^\tau \tau' \mathbf{E}^a \cdot \mathbf{B}^a \, d\tau'
\]
Anomaly equation in the expanding geometry

ABJ anomaly equation in the \( \tau-\eta \) coordinates

\[
\frac{1}{\tau} \partial_\tau (\tau j_5^\tau) + \partial_i j_5^i + \frac{1}{\tau} \partial_\eta j_5^\eta = 2m\langle \bar{\psi}i\gamma_5\psi \rangle + \frac{g^2}{4\pi^2} E^a \cdot B^a
\]

boost-invariant background \( m \approx 0 \)

Axial charge density per unit transverse area and unit rapidity

\[
\frac{dN_5}{d^2 x_\perp d\eta} = \tau j_5^\tau (x)
\]

\[
= - \int_0^\tau \tau' \partial_i j_5^i d\tau' + \frac{g^2}{4\pi^2} \int_0^\tau \tau' E^a \cdot B^a d\tau'
\]

diffusion

source term

• In a uniform system or at very early times, the diffusion term is negligible. Then the axial charge density can be computed solely from the gauge fields.

• Otherwise, one needs to solve the Dirac equation.

Lappi, Schlichting (2017)
Uniform Glasma

Take the limit of the flux tube width \( \rightarrow \infty \)

\[ Q : \text{typical energy scale of the Glasma} \]

Time evolution of the field strength

- Similar behavior to that with the MV initial condition.
- In this uniform system, the decay of the fields is a purely nonlinear effect.

Time evolution of \( E \cdot B \)
Verification of the anomaly relation

\[ \frac{dN_5}{d^2x_\perp d\eta} \approx \frac{g^2}{4\pi^2} \int_0^\tau \tau' E^a \cdot B^a \, d\tau' \]

For \( Q = 1 \text{ GeV} \), \( \frac{dN_5}{d^2x_\perp d\eta}/Q^2 = 0.04 \)

1 excess of right-quarks over left-quarks per flavor in a box with 1fm^2 transverse area and one unit of rapidity.
Glasma flux tubes

The profile of flux tubes in the transverse plane

- Two flux tubes to satisfy the periodic b.c.
- Distorted Gaussian to have both $E$ and $B$. 

$$g^2 E \cdot B$$

$$Q_T = 0^+$$
Glasma flux tubes

The profile of flux tubes in the transverse plane

\[ Q \tau = 0.5 \]

\[ g^2 E \cdot B \]
Glasma flux tubes

The profile of flux tubes in the transverse plane

\[ Q \tau = 1 \]

\[ g^2 E \cdot B \]
Glasma flux tubes

The profile of flux tubes in the transverse plane

\[ Q_\tau = 2 \]

\[ g^2 E \cdot B \]
Glasma flux tubes

The profile of flux tubes in the transverse plane

\[ Q \tau = 3 \]

\[ g^2 E \cdot B \]
Glasma flux tubes

Transverse profiles of $E \cdot B$ for different times

Time evolution of the space-averaged field strength

Time evolution of the space-averaged $E \cdot B$
Glasma flux tubes

Verification of the space-averaged anomaly relation

$$\frac{dN_5^x}{d^2 x_\perp d\eta} \approx \frac{g^2}{4\pi^2} \int_0^\tau \tau' \mathbf{E}^\alpha \cdot \mathbf{B}^\alpha d\tau'$$

$m/Q = 0.01$

$N_x = N_y = 64, \ N_\eta = 256$
Verification of the space-averaged anomaly relation

\[ \overline{\frac{dN_5}{d^2x_\perp d\eta}} \approx \frac{g^2}{4\pi^2} \int_0^\tau \tau' \overrightarrow{E^a} \cdot \overrightarrow{B^a} d\tau' \]

The monotonic increase corresponds to the decaying $E \cdot B \sim 1/\tau$.

$c.f.$ Uniform case

$m/Q = 0.01$

$N_x = N_y = 64, \; N_\eta = 256$
Local anomaly budget

\[
\frac{dN_5}{d^2 x_\perp \, d\eta} + \int_0^\tau \tau' \partial_i j_5^i \, d\tau' \approx \frac{g^2}{4\pi^2} \int_0^\tau \tau' E^a \cdot B^a \, d\tau'
\]

For \( Q_\tau \gtrsim 1 \) the diffusion term takes some fraction of the anomaly budget.
The axial charge production in the longitudinally expanding geometry can be described by the real-time lattice simulations with the Wilson fermion.

The classical gauge fields having nonzero $E \cdot B$ exhibit nontrivial behaviors.

Because the axial charge density is related with the time integral of $E \cdot B$, it depends on the time history and it can remain even after $E \cdot B$ dies out.

In inhomogeneous gauge fields, we need solving the Dirac equation to properly compute the axial charge production including its diffusion dynamics.

- Real-time simulations of CME in the expanding system by applying a U(1) magnetic field.
- More realistic configurations?