Chiral Kinetic Theory in Wigner Function Formalism

Pengfei Zhuang
(Tsinghua University, Beijing)

- *Covariant approach*
- *Application: non-equilibrium CME*
- *Equal-time approach*
- *Application: off-shell oscillations*

Thanks to Xingyu Guo, Anping Huang, Yin Jiang, Jinfeng Liao and Shuzhe Shi

Classical and Quantum Kinetic Theories

- *Classical kinetic equations*

$$\left(p^{\mu}\partial_{\mu}+\partial^{\mu}m^{2}\partial_{\mu}^{p}\right)f(x,p)=C \quad + \quad \left(p^{\mu}p_{\mu}-m^{2}\right)f(x,p)=0$$ 【on-shell condition】

- *Quantum kinetic equations*

$$W(x,p)=\int d^{4}y e^{ipy}\left\langle \psi(x+\frac{y}{2})e^{iQ\int_{-1/2}^{1/2}ds A(x+sy)\cdot y}\overline{\psi}(x-\frac{y}{2})\right\rangle$$

Quantum transport equations + Constraint equations 【off-shell equations】

- *Applications in heavy ion collisions*
  - *Many classical transport codes like AMPT, BAMPS and UrQMD*
  - *Quantum transport + on-shell condition for chirality:*

M.Stephanov and Y.Yin, PRL109, 162001 (2012);
D.Son and N.Yamamoto. PRD87, 085016(2013);
J.Chen, S.Pu, Q.Wang and X.Wang, PRL110, 262301(2013);
Y.Hidaka, S.Pu and D.Yang. PRD95, 091901(2017); and ……

- *Off-shell effect (beyond quasiparticle approximation, non-fermion gas)*

P.Zhuang and U.Heinz, Ann.Phys.245, 311(1996)

Covariant Kinetic Equations in Chiral Symmetry Restoration Phase

A.Huang, S.Shi, Y.Jiang, J.Liao and PZ, arXiv: 1801.03640

Dirac equation for massless fermions in electromagnetic fields

$$\gamma^\mu \mathcal{D}_\mu \psi(x) = 0$$

Covariant kinetic equation

$$\gamma^\mu K_\mu W(x,p) = 0$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2} D_\mu,$$

$$\Pi_\mu = p_\mu - iQ\hbar \int_{-1/2}^{1/2} ds s F_{\mu\nu}(x - i\hbar s \partial_p)\partial^\nu$$

$$D_\mu = \partial_\mu - Q \int_{-1/2}^{1/2} ds F_{\mu\nu}(x - i\hbar s \partial_p)\partial^\nu$$

Spin decomposition

$$W = \frac{1}{4}\left(F + i\gamma_5 P + \gamma^\mu V_\mu + \gamma^\mu \gamma^5 A_\mu + \frac{1}{2}\sigma^{\mu\nu} S_{\mu\nu}\right)$$

16 transport equations and 16 constraint equations

D.Vasak, M.Gyulassy and H.Elze, Ann.Phys.173, 462(1987)

Introducing chiral components:

$$J_\mu^\chi = \frac{1}{2}\left(V_\mu - \chi A_\mu\right), \qquad \chi = \pm 1$$

Coupled equations for V_μ and A_μ $\rightarrow$ decoupled equations for J_μ^+ and J_μ^- :

constraint equations
$$\begin{cases} \Pi^\mu J_\mu^\chi = 0 \\ \hbar \varepsilon^{\mu\nu\rho\sigma} D_\rho J_\sigma^\chi = -2\chi\left(\Pi^\mu J_\chi^\nu - \Pi^\nu J_\chi^\mu\right) \end{cases}$$

transport equations $\quad D^\mu J_\mu^\chi = 0$

Semi-classical Expansion

A.Huang, S.Shi, Y.Jiang, J.Liao and PZ, arXiv: 1801.03640

To the zeroth order in $\hbar$ (classical limit):

classical constraint equations $\begin{cases} p^\mu J_\mu^{\chi(0)} = 0 \\ p_\mu J_\nu^{\chi(0)} - p_\nu J_\mu^{\chi(0)} = 0 \end{cases}$

solution $J_\mu^{\chi(0)} = p_\mu f_\chi^{(0)} \delta(p^2)$, $\quad f_\chi^{(0)}(x,p)$: particle distribution

classical transport equation (Vlasov equation): $p^\mu \left(\partial_\mu - QF_{\mu\nu}\partial_p^\nu\right) f_\chi^{(0)} = 0$

To the first order in $\hbar$:

constraint equations $\begin{cases} p^\mu J_\mu^{\chi(1)} = 0 \\ \varepsilon^{\mu\nu\rho\sigma}\left(\partial_\rho - QF_{\rho\theta}\partial_p^\theta\right) J_\sigma^{\chi(0)} = -2\chi\left(p^\mu J_{\chi(1)}^\nu - p^\nu J_{\chi(1)}^\nu\right) \end{cases}$

general solution $\quad J_\mu^{\chi(1)} = p_\mu f_\chi^{(1)}\delta(p^2) + K_\mu^\chi \delta(p^2) + \chi Q F_{\mu\nu} p^\nu f_\chi^{(0)} \delta'(p^2)$

$K_\mu^\chi = \dfrac{\chi}{2p\cdot n}\varepsilon^{\mu\nu\rho\sigma} p_\nu n_\rho \left(\partial_\sigma - QF_{\sigma\theta}\partial_p^\theta\right) f_\chi^{(0)}$ for any 4-vector n_μ satisfying $n_\mu n^\mu = 1$

$p^2\delta'(p^2) = -\delta(p^2)$

general transport equation for $f_\chi = f_\chi^{(0)} + \hbar f_\chi^{(1)}$:

$$
\begin{aligned}
0 = \delta\left(p^2 - \hbar\frac{\chi Q}{p\cdot n} p_\lambda \widetilde{F}^{\lambda\nu} n_\nu\right) \\
\times \left\{ p\cdot\nabla + \hbar\frac{\chi}{2\,(p\cdot n)^2}\left[(\partial_\mu n_\sigma)p^\sigma - QF_{\mu\alpha}n^\alpha\right]\epsilon^{\mu\nu\lambda\rho} n_\nu p_\lambda \nabla_\rho \right. \\
\left. -\hbar\frac{\chi}{2p\cdot n}\epsilon^{\mu\nu\lambda\rho}\left(\partial_\mu n_\nu\right) p_\lambda \nabla_\rho + \hbar\frac{\chi Q}{2p\cdot n} p_\lambda \left(\partial_\sigma \widetilde{F}^{\lambda\nu}\right) n_\nu \partial_p^\sigma \right\} f_\chi
\end{aligned}
$$

1) Complete solution to $\hbar^1$
2) General non-equilibrium distribution f_χ
3) General electromagnetic fields
4) Frame dependence n_μ
5) on-shell but shifted shell

3D Anomalous Transport

$$f_\chi(x,\vec{p}) = \int dp_0 f_\chi(x,p)$$

Integrating the transport equations over p_0 (with the help of the on-shell condition) in the rest frame with $n_\mu = (1,0,0,0)$:

$$\Big\{(1+\hbar Q\mathbf{B}\cdot\mathbf{b}_\chi)\,\partial_t + \Big(\widetilde{\boldsymbol{v}} + \hbar Q(\widetilde{\boldsymbol{v}}\cdot\mathbf{b}_\chi)\mathbf{B} + \hbar Q\widetilde{\mathbf{E}}\times\mathbf{b}_\chi\Big)\cdot\nabla_\mathbf{x} + Q\Big(\widetilde{\mathbf{E}} + \widetilde{\boldsymbol{v}}\times\mathbf{B} + \hbar Q(\widetilde{\mathbf{E}}\cdot\mathbf{B})\mathbf{b}_\chi\Big)\cdot\nabla_\mathbf{p}\Big\} f_\chi(x,\mathbf{p}) = 0.$$

$$\widetilde{\mathbf{E}} = \mathbf{E} - \frac{1}{Q}\nabla_\mathbf{x}E_\mathbf{p}, \quad E_\mathbf{p} = |\mathbf{p}|\left(1-\hbar Q\mathbf{B}\cdot\mathbf{b}_\chi\right),$$

$$\widetilde{\boldsymbol{v}} = \frac{\partial E_\mathbf{p}}{\partial\mathbf{p}} = \widehat{\mathbf{p}}\,(1+2\hbar Q\mathbf{B}\cdot\mathbf{b}_\chi) - \hbar Q b_\chi\mathbf{B}, \quad \mathbf{b}_\chi = \chi\frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

The same results as obtained by M.Stephanov and Y.Yin, PRL109, 162001 (2012);
D.Son and N.Yamamoto. PRD87, 085016(2013);
J.Chen, S.Pu, Q.Wang and X.Wang, PRL110, 262301(2013);
Y.Hidaka, S.Pu and D.Yang. PRD95, 091901(2017); and ……

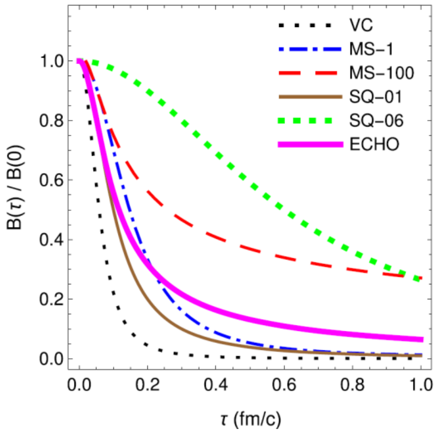
Non-equilibrium CME

A.Huang, Y.Jiang, S.Shi, J.Liao and PZ, PLB777, 177(2018)

Strong electromagnetic fields before thermalization in heavy ion collisions
→ possible non-equilibrium CME

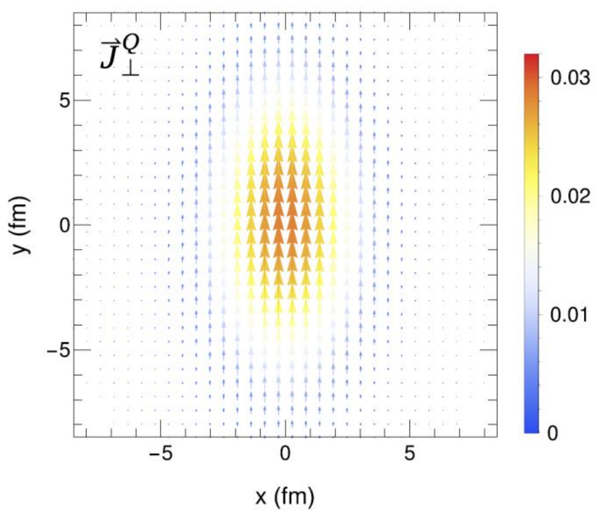
Suppose $\vec{E}=0$ *and* $\vec{B}=B(t)\vec{e}_y$,

$$\begin{cases} \left(\partial_t+\dot{\vec{x}}\cdot\vec{\nabla}+\dot{\vec{p}}\cdot\vec{\nabla}_p\right)f_\chi=-\frac{f_\chi-f_{\chi\,th}}{\tau_R} \\ \dot{\vec{x}}=\frac{1}{1+Q\vec{B}\cdot\vec{b}_\chi}\frac{\vec{p}}{|\vec{p}|}\left(1+2Q\vec{B}\cdot\vec{b}_\chi\right) \\ \dot{\vec{p}}=\frac{1}{1+Q\vec{B}\cdot\vec{b}_\chi}Q\frac{\vec{p}}{|\vec{p}|}\times\vec{B} \end{cases}$$

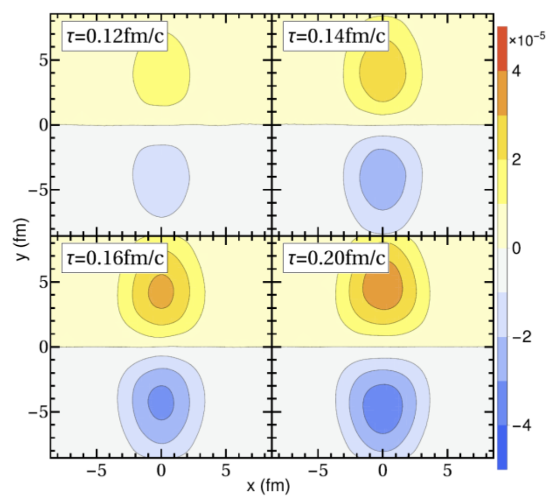


+ initial distribution + initial imbalance $(1\pm\lambda$ *for* $\chi=\pm)$ *+ B(t)*

Transverse charge current



Net charge density



Equal-time Kinetic Theory

3D distribution $f(x,\vec{p}) = \int dp_0 f(x,p)$ is not equivalent to the 4D distribution $f(x,p)$!
They are equivalent to each other only in on-shell case.
How to obtain a quantum kinetic theory?

- *Covariant Wigner function*

$$W(x,p) = \int d^4 y e^{ipy} \left\langle \psi(x+\frac{y}{2}) e^{iQ\int_{-1/2}^{1/2} ds A(x+sy)\cdot y} \bar{\psi}(x-\frac{y}{2}) \right\rangle$$

Advantage: covariant theory.
Disadvantage: Only for quasiparticles, it can be solved as an initial value problem.

- *Equal-time Wigner function*

$$W(x,\vec{p}) = \int d^3\vec{y} e^{-i\vec{p}\cdot\vec{y}} \left\langle \psi(t,\vec{x}+\frac{\vec{y}}{2}) e^{-iQ\int_{-1/2}^{1/2} ds \vec{A}(\vec{x}+s\vec{y})\cdot\vec{y}} \psi^+(t,\vec{x}-\frac{\vec{y}}{2}) \right\rangle = \int dp_0 W(x,p)\gamma_0$$

Advantage: a well defined initial value problem.
Disadvantage: How to derive a complete 3D kinetic theory?

I.Bialynicki-Birula, P.Gornicki and J.Rafelski, PRD44, 1825(1991)
P.Zhuang and U.Heinz, Ann.Phys.245, 311(1996); PRD57, 6525(1998)
S.Ochs and U.Heinz, Ann. Phys. 266, 351(1998)

Covariant Kinetic Equations in Chiral Symmetry Breaking Phase

X.Guo and PZ, arXiv: 1711.02924

- NJL model with SU(2) symmetry and $U_A(1)$ symmetry

$$\mathcal{L} = \bar\psi\big(i\gamma^\mu \mathcal{D}_\mu - m_0\big)\psi + G\sum_{a=0}^{3}[(\bar\psi\tau_a\psi)^2 + (\bar\psi i\gamma_5\tau_a\psi)^2],$$
$$\mathcal{D}_\mu = \partial_\mu + iQA_\mu, \quad Q = diag(Q_u, Q_d)$$

2 order parameters:

$$\sigma(x) = 2G\langle\bar\psi\psi\rangle, \qquad \pi(x) = 2G\langle\bar\psi i\gamma_5\tau_3\psi\rangle$$

Dirac equation:

$$\big(i\gamma^\mu\mathcal{D}_\mu - (m_0 - \sigma) + i\gamma_5\tau_3\pi\big)\psi = 0$$

- Covariant equation in chiral symmetry breaking phase:

$$\big(\gamma^\mu K_\mu + \gamma_5\tau_3 K_5 - M\big)W(x,p) = -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{W(x,p) - W^{th}(x,p)}{\theta}$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2}\mathrm{D}_\mu, \quad K_5 = \Pi_5 + \frac{i\hbar}{2}\mathrm{D}_5, \quad M = M_1 + iM_2$$

$$\Pi_\mu = p_\mu - iQ\hbar\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{d}s s F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu, \quad \Pi_5 = \sin(\frac{\hbar}{2}\nabla)\pi(x) \qquad \nabla \equiv \partial_x\cdot\partial_p$$

$$\mathrm{D}_\mu = \partial_\mu - Q\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{d}s F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu, \quad \mathrm{D}_5 = \cos(\frac{\hbar}{2}\nabla)\pi(x)$$

$$M_1 = m_0 - \cos(\frac{\hbar}{2}\nabla)\sigma(x), \quad M_2 = \sin(\frac{\hbar}{2}\nabla)\sigma(x)$$

- p_0-integrating the covariant equation → equal-time transport equation for $W(x,\vec{p}) = \int dp_0 W(x,p)\gamma_0$ + constraint equation for $W^{(1)}(x,\vec{p}) = \int dp_0\, p_0 W(x,p)\gamma_0$

Equal-time Transport and Constraint Equations

X.Guo and PZ, arXiv: 1711.02924

After spin and flavor decomposition

$$W(x,\mathbf{p}) = \sum_{q=u,d} \tau_q \left(f_{0q} + \gamma_5 f_{1q} - i\gamma_0\gamma_5 f_{2q} + \gamma_0 f_{3q} + \gamma_5\gamma_0\gamma\cdot\mathbf{g}_{0q} + \gamma_0\gamma\cdot\mathbf{g}_{1q} - i\gamma\cdot\mathbf{g}_{2q} - \gamma_5\gamma\cdot\mathbf{g}_{3q}\right)/8$$

32 transport and 32 constraint equations:

$$\hbar(\hat{D}_q f_{0q} + \hat{\mathbf{D}}_q\cdot\mathbf{g}_{1q}) + 2\hat{\sigma}_o f_{3q} + 2\hat{\pi}_{oq} f_{2q} = (f_{0q} - f_{0q}^{th})/\theta,$$
$$\hbar(\hat{D}_q f_{1q} + \hat{\mathbf{D}}_q\cdot\mathbf{g}_{0q}) + 2(m_0 - \hat{\sigma}_e) f_{2q} - 2\hat{\pi}_{eq} f_{3q} = (f_{1q} - f_{1q}^{th})/\theta,$$
$$\hbar\hat{D}_q f_{2q} + 2\hat{\mathbf{\Pi}}_q\cdot\mathbf{g}_{3q} - 2(m_0 - \hat{\sigma}_e) f_{1q} - 2\hat{\pi}_{oq} f_{0q} = (f_{2q} - f_{2q}^{th})/\theta,$$
$$\hbar\hat{D}_q f_{3q} - 2\hat{\mathbf{\Pi}}_q\cdot\mathbf{g}_{2q} + 2\hat{\sigma}_o f_{0q} + 2\hat{\pi}_{eq} f_{1q} = (f_{3q} - f_{3q}^{th})/\theta,$$
$$\hbar(\hat{D}_q\mathbf{g}_{0q} + \hat{\mathbf{D}}_q f_{1q}) - 2\hat{\mathbf{\Pi}}_q\times\mathbf{g}_{1q} + 2\hat{\sigma}_o\mathbf{g}_{3q} - 2\hat{\pi}_{oq}\mathbf{g}_{2q} = (\mathbf{g}_{0q} - \mathbf{g}_{0q}^{th})/\theta,$$
$$\hbar(\hat{D}_q\mathbf{g}_{1q} + \hat{\mathbf{D}}_q f_{0q}) - 2\hat{\mathbf{\Pi}}_q\times\mathbf{g}_{0q} + 2(m_0 - \hat{\sigma}_e)\mathbf{g}_{2q} - 2\hat{\pi}_{eq}\mathbf{g}_{3q} = (\mathbf{g}_{1q} - \mathbf{g}_{1q}^{th})/\theta,$$
$$\hbar(\hat{D}_q\mathbf{g}_{2q} + \hat{\mathbf{D}}_q\times\mathbf{g}_{3q}) + 2\hat{\mathbf{\Pi}}_q f_{3q} - 2(m_0 - \hat{\sigma}_e)\mathbf{g}_{1q} - 2\hat{\pi}_{oq}\mathbf{g}_{0q} = (\mathbf{g}_{2q} - \mathbf{g}_{2q}^{th})/\theta,$$
$$\hbar(\hat{D}_q\mathbf{g}_{3q} + \hat{\mathbf{D}}_q\times\mathbf{g}_{2q}) - 2\hat{\mathbf{\Pi}}_q f_{2q} + 2\hat{\sigma}_o\mathbf{g}_{0q} + 2\hat{\pi}_{eq}\mathbf{g}_{1q} = (\mathbf{g}_{3q} - \mathbf{g}_{3q}^{th})/\theta$$

$$\int dp_0 p_0 F_q = \hbar\hat{\mathbf{D}}_q\cdot\mathbf{g}_{2q}/2 - \hat{\Pi}_q f_{3q} + \hat{\pi}_{oq} f_{1q} + (m_0 - \hat{\sigma}_e) f_{0q},$$
$$\int dp_0 p_0 P_q = \hbar\hat{\mathbf{D}}_q\cdot\mathbf{g}_{3q}/2 - \hat{\Pi}_q f_{2q} - \hat{\pi}_{eq} f_{0q} + \hat{\sigma}_o f_{1q},$$
$$\int dp_0 p_0 V_{0q} = \hat{\mathbf{\Pi}}_q\cdot\mathbf{g}_{1q} - \hat{\Pi}_q f_{0q} + \hat{\pi}_{eq} f_{2q} + (m_0 - \hat{\sigma}_e) f_{3q},$$
$$\int dp_0 p_0 A_{0q} = -\hat{\mathbf{\Pi}}_a\cdot\mathbf{g}_{0q} - \hat{\Pi}_q f_{1q} + \hat{\pi}_{oq} f_{3q} + \hat{\sigma}_o f_{2q},$$
$$\int dp_0 p_0 \mathbf{V}_q = \hbar\hat{\mathbf{D}}_q\times\mathbf{g}_{0q}/2 + \hat{\mathbf{\Pi}}_q f_{0q} + \hat{\Pi}_q\mathbf{g}_{1q} - \hat{\pi}_{oq}\mathbf{g}_{3q} - \hat{\sigma}_o\mathbf{g}_{2q},$$
$$\int dp_0 p_0 \mathbf{A}_q = -\hbar\hat{\mathbf{D}}_q\times\mathbf{g}_{1q}/2 - \hat{\mathbf{\Pi}}_q f_{1q} + \hat{\Pi}_q\mathbf{g}_{0q} - \hat{\pi}_{eq}\mathbf{g}_{2q} - (m_0 - \hat{\sigma}_e)\mathbf{g}_{3q},$$
$$\int dp_0 p_0 S_q^{0i}\mathbf{e}_i = \hbar\hat{\mathbf{D}}_q f_{3q}/2 - \hat{\mathbf{\Pi}}_q\times\mathbf{g}_{3q} + \hat{\Pi}_q\mathbf{g}_{2q} - \hat{\pi}_{eq}\mathbf{g}_{0q} - \hat{\sigma}_o\mathbf{g}_{1q},$$
$$\int dp_0 p_0 S_{jkq}\epsilon^{ijk}\mathbf{e}_i = \hbar\hat{\mathbf{D}}_q f_{2q} - 2\hat{\mathbf{\Pi}}_q\times\mathbf{g}_{2q} - 2\hat{\Pi}_q\mathbf{g}_{3q} + 2\hat{\pi}_{oq}\mathbf{g}_{1q} + 2(m_0 - \hat{\sigma}_e)\mathbf{g}_{0q}$$

$$\hat{D}_q = \partial_t + Q_q\int_{-1/2}^{1/2} ds\mathbf{E}(\mathbf{x} + is\hbar\nabla_p)\cdot\nabla_p,$$
$$\hat{\mathbf{D}}_q = \nabla + Q_q\int_{-1/2}^{1/2} ds\mathbf{B}(\mathbf{x} + is\hbar\nabla_p)\times\nabla_p,$$
$$\hat{\Pi}_q = iQ_q\hbar\int_{-1/2}^{1/2} dss\mathbf{E}(\mathbf{x} + is\hbar\nabla_p)\cdot\nabla_p,$$
$$\hat{\mathbf{\Pi}}_q = \mathbf{p} - iQ_q\hbar\int_{-1/2}^{1/2} dss\mathbf{B}(\mathbf{x} + is\hbar\nabla_p)\times\nabla_p,$$
$$\hat{\sigma}_e = \cos(\frac{\hbar}{2}\nabla\cdot\nabla_p)\sigma(x),$$
$$\hat{\sigma}_o = \sin(\frac{\hbar}{2}\nabla\cdot\nabla_p)\sigma(x),$$
$$\hat{\pi}_{eq} = sgn(Q_q)\cos(\frac{\hbar}{2}\nabla\cdot\nabla_p)\pi(x),$$
$$\hat{\pi}_{oq} = sgn(Q_q)\sin(\frac{\hbar}{2}\nabla\cdot\nabla_p)\pi(x),$$

Self-consistent coupling between Wigner function and chiral and pion condensates:

$$\sigma(x) = G\int\frac{d^3\mathbf{p}}{(2\pi)^3}(f_{3u}(x,\mathbf{p}) + f_{3d}(x,\mathbf{p})),$$
$$\pi(x) = -G\int\frac{d^3\mathbf{p}}{(2\pi)^3}(f_{3u}(x,\mathbf{p}) - f_{3d}(x,\mathbf{p})).$$

Constraint and Transport Equations in Classical Limit (I)

X.Guo and PZ, arXiv: 1711.02924

- Expanding the 32 + 32 equations in $\hbar$.

- Relations among the classical components

$$
\begin{aligned}
f^{\pm}_{1q} &= \pm\frac{\mathbf{p}}{E_p}\cdot\mathbf{g}^{\pm}_{0q},\\
f^{\pm}_{2q} &= \pm\frac{\pi}{E_p}f^{\pm}_{0q},\\
f^{\pm}_{3q} &= \pm\frac{m_0-\sigma}{E_p}f^{\pm}_{0q},\\
\mathbf{g}^{\pm}_{1q} &= \pm\frac{\mathbf{p}}{E_p}f^{\pm}_{0q},\\
\mathbf{g}^{\pm}_{2q} &= \frac{\mathbf{p}\times\mathbf{g}^{\pm}_{0q}+\pi\mathbf{g}^{\pm}_{3q}}{m_0-\sigma},\\
\mathbf{g}^{\pm}_{3q} &= \pm\frac{E_p^2(m_0-\sigma)\mathbf{g}^{\pm}_{0q}-(m_0-\sigma)(\mathbf{p}\cdot\mathbf{g}^{\pm}_{0q})\mathbf{p}\mp E_p\pi\mathbf{p}\times\mathbf{g}^{\pm}_{0q}}{E_p m^2}.
\end{aligned}
$$

On-shell (quasi-particle) condition:

$$E_p = \sqrt{m^2+\mathbf{p}^2}$$

Only 2 independent distributions: f_0 and $\vec{g}_0$.

- Conservation laws → Physics of the classical components

f_0 : number density, $\vec{g}_1$: number current , $\vec{g}_0$: spin density ($\vec{J}=\vec{r}\times\vec{p}f_0+\frac{\hbar}{2}\vec{g}_0$), f_1 : helicity density

f_3 : mass density , f_2 : pion condensate , $\vec{g}_3$: magnetic moment, $\vec{g}_2$:

I.Bialynicki-Birula, P.Gornicki and J.Rafelski, PRD44, 1825(1991)

Constraint and Transport Equations in Classical Limit (II)

X.Guo and PZ, arXiv: 1711.02924

From

$$\sigma(x) = G\int \frac{d^3\mathbf{p}}{(2\pi)^3}(f_{3u}(x,\mathbf{p}) + f_{3d}(x,\mathbf{p})),$$
$$\pi(x) = -G\int \frac{d^3\mathbf{p}}{(2\pi)^3}(f_{3u}(x,\mathbf{p}) - f_{3d}(x,\mathbf{p})).$$

$\vec{p}$-integrating the classical relations for f_3 and f_2 leads to the gap equations for the order parameters $\sigma(x)$ and $\pi(x)$:

$$\pi(x) = 0,$$
$$(m_0 - \sigma(x))\left(1 + 2G\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{f_{0u}^{+}(x,\mathbf{p}) - f_{0u}^{-}(x,\mathbf{p} + f_{0d}^{+}(x,\mathbf{p}) - f_{0d}^{-}(x,\mathbf{p})}{E_p}\right) - m_0 = 0,$$

Conclusion: At classical level, chiral symmetry can be spontaneously broken, but $U_A(1)$ symmetry is protected !

Transport Equations to the First Order in $\hbar$ (I)

X.Guo and PZ, arXiv: 1711.02924

The classical transport equations for f_{0q} and $\vec{g}_{0q}$ ($q = u, d$):

$$\left(D_a \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}_a \mp \frac{\nabla m^2 \cdot \nabla_p}{2E_p}\right) f_{0a}^{\pm} = 0,$$

$$\left(D_a \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}_a \mp \frac{\nabla m^2 \cdot \nabla_p}{2E_p}\right) \mathbf{g}_{0a}^{\pm}$$

$$= \frac{q_a}{E_p^2}\left[\mathbf{p} \times \left(\mathbf{E} \times \mathbf{g}_{0a}^{\pm}\right) \mp E_p \mathbf{B} \times \mathbf{g}_{0a}^{\pm}\right] - \frac{1}{2E_p^4}\left(\partial_t m^2 \mathbf{p} \mp E_p \nabla m^2\right) \times \left(\mathbf{p} \times \mathbf{g}_{0a}^{\pm}\right)$$

Bargmann-Michel-Telegdi equation in phase space

V.Bargmann, L.Michel and V.Telegdi, Phys. Rev. Lett. **2**, 435(1959).

Homogeneous solution:

$$\mathbf{g}_{0a}^{\pm} = \frac{q_a}{m^2}\left(\mathbf{B} \pm \frac{\mathbf{p}}{E_p} \times \mathbf{E}\right)$$

Transport Equations to the First Order in $\hbar$ (II)

X.Guo and PZ, arXiv: 1711.02924

- *From the 32 transport equations to the order in $\hbar$, we obtain also the quantum correction to pion condensate:*

$$\pi^{(1)} = \frac{G}{2m_0}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\sum_{a=u,d} q_a\left[(\mathbf{B}\times\nabla_p)\cdot\mathbf{g}_{0a}+\mathbf{E}\cdot\nabla_p f_{1a}\right].$$

In homogeneous case

$$\begin{aligned}\pi^{(1)} &= -\frac{G}{2m_0}\left(q_u^2-q_d^2\right)\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{2E_p^2+m^2}{E_p^3}\mathbf{E}\cdot\mathbf{B}\\ &= -\frac{G}{4\pi^2 m_0}\left(q_u^2-q_d^2\right)\frac{\Lambda^3}{m^2\sqrt{\Lambda^2+m^2}}\mathbf{E}\cdot\mathbf{B},\end{aligned}$$

Quantum effect: $U_A(1)$ symmetry is broken only at quantum level, due to the quark spin.

the same result in effective field theory, see G.Cao and X.Huang, PLB757, 1(2016), *nonequilibrium production, see the talk by* Naoto Tanji *in this workshop*

- *In chiral limit with $m_0=\sigma=\pi=0$, we recover the chiral kinetic equation*

$$\begin{aligned}&\Big\{(1+\hbar Q\mathbf{B}\cdot\mathbf{b}_\chi)\,\partial_t+\left(\tilde{\boldsymbol{v}}+\hbar Q(\tilde{\boldsymbol{v}}\cdot\mathbf{b}_\chi)\mathbf{B}+\hbar Q\tilde{\mathbf{E}}\times\mathbf{b}_\chi\right)\cdot\nabla_{\mathbf{x}}\\ &+Q\left(\tilde{\mathbf{E}}+\tilde{\boldsymbol{v}}\times\mathbf{B}+\hbar Q(\tilde{\mathbf{E}}\cdot\mathbf{B})\mathbf{b}_\chi\right)\cdot\nabla_{\mathbf{p}}\Big\}\, f_\chi(x,\mathbf{p})=0.\end{aligned}$$

Full Transport Equations: Off-shell Oscillations

X.Guo and PZ, arXiv: 1711.02924

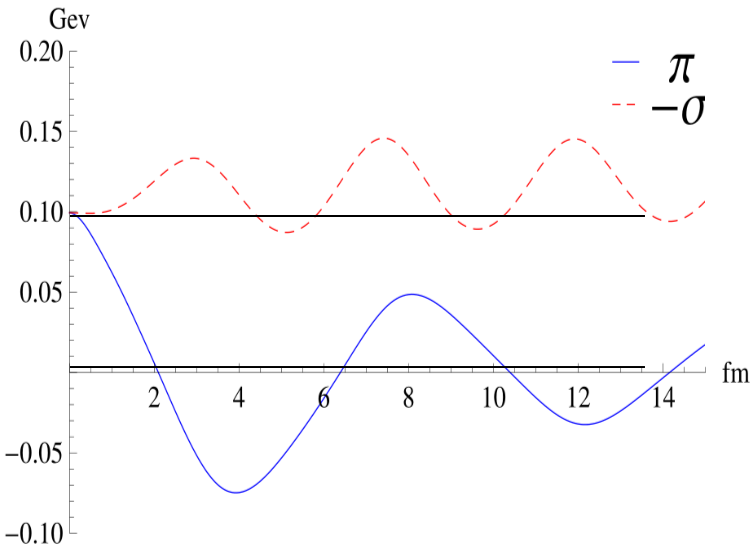
When the EM fields disappear in heavy ion collisions, is there any effect on the later evolution of the system ?

A homogeneous system
t < 0: EM fields
t > 0: no EM fields

initial condition: $\sigma(0) = \pi(0) = 0.1\ GeV$

quantum transport equations at t>0:

$$
\begin{aligned}
&\partial_t f_{0a} = 0,\\
&\partial_t f_{1a} + 2(m_0 - \sigma) f_{2a} - 2sgn(q_a)\pi f_{3a} = 0,\\
&\partial_t f_{2a} + 2\mathbf{p}\cdot\mathbf{g}_{3a} - 2(m_0 - \sigma) f_{1a} = 0,\\
&\partial_t f_{3a} - 2\mathbf{p}\cdot\mathbf{g}_{2a} + 2sgn(q_a)\pi f_{1a} = 0,\\
&\partial_t \mathbf{g}_{0a} - 2\mathbf{p}\times\mathbf{g}_{1a} = 0,\\
&\partial_t \mathbf{g}_{1a} - 2\mathbf{p}\times\mathbf{g}_{0a} + 2(m_0 - \sigma)\mathbf{g}_{2a} - 2sgn(q_a)\pi\mathbf{g}_{3a} = 0,\\
&\partial_t \mathbf{g}_{2a} + 2\mathbf{p} f_{3a} - 2(m_0 - \sigma)\mathbf{g}_{1a} = 0,\\
&\partial_t \mathbf{g}_{3a} - 2\mathbf{p} f_{2a} + 2sgn(q_a))\pi\mathbf{g}_{1a} = 0.
\end{aligned}
$$



Strong and long-lived quantum oscillations around the quasiparticle values, induced by off-shell effect !

Summary and Outlook

1) Quantum correction is systematically studied via semi-classical expansion. A general solution to the first order in $\hbar$ is obtained in chiral limit. The first order quantum correction comes from the quark spin.

2) Quarks are not on the shell in general case (chiral breaking phase) even at the first order in $\hbar$.

3) Equal-time approach is a well defined initial value problem for off-shell quarks, $W(x,\vec{p}) = \int dp_0\, W(x,p)\gamma_0$ and $W^{(1)}(x,\vec{p}) = \int dp_0\, p_0 W(x,p)\gamma_0$ are independent!

4) Off-shell induced oscillations extend the quantum anomaly.

5) Possible applications in heavy ion collisions.