On the Universality of the Chern-Simons Diffusion Rate

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QCD Chirality Workshop, March 21, 2018

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arXiv:1803.nnnnn

The Chern-Simons diffusion rate

Definition:

$$Q(x) = \frac{1}{32\pi^2} \mathrm{Tr} F \tilde{F}$$

Change in the Chern-Simons number

$$\Delta N_{CS} = \int d^4x \, Q(x)$$

Chern-Simons diffusion rate

$$\Gamma_{CS} = \frac{\langle (\Delta N_{CS})^2 \rangle}{Vt} = \int d^4x \langle Q(x) Q(0) \rangle$$

$$V = \text{volume}, \qquad t = \text{time}$$

Note: Minkowski correlator, real time physics, no reliable computational methods.

The Chern-Simons diffusion rate

On a state with temperature T (e.g. Quark-Gluon Plasma):

- Γ_{CS} relevant for Chiral Magnetic Effect [Fukushima-Kharzeev-Warringa 2008].
- Kubo formula:

$$\Gamma_{CS} = -\lim_{\omega \to 0} \frac{2T}{\omega} \mathrm{Im}\, G_R(\omega, \vec{k} = 0)$$

Thus: compute retarded correlator $G_R(\omega, \vec{k} = 0)$.

- Effective theory result [Moore-Tassler 2010]:

$$\Gamma_{CS} \sim c \cdot \lambda^5 \log \frac{1}{\lambda} \cdot T^4$$

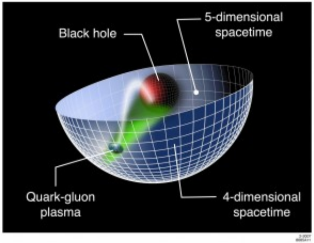
$$\lambda = g_{YM}^2 N_c \qquad \text{'t Hooft coupling}$$

Note: c is non-perturbative; result valid at $\alpha_s \ll 1$.

Holography in one slide

$$\langle e^{-\int C_0\, Q}\rangle_{FT} = e^{-S_{gravity}(C_0)}$$

[Maldacena, Witten, Gubser-Klebanov-Polyakov 1997]



- Q (operator) $\Leftrightarrow$ C (gravity field) and $C_0 = \lim_{r\to\infty} C$.
- C_0 determines C via equations of motion.
- Plug the solution in gravity action: $S_{gravity}(C_0)$.
- Perform functional derivatives w.r.t. C_0, getting n-point functions.
- Note: large N_c, large λ.

Holographic derivation

Apply to CS diff rate in $\mathcal{N} = 4$ SYM [Son-Starinets 2002]

- Background is $BH - AdS_5 \times S^5$, generated by N_c D3-branes:

$$ds^2 = e^{2A(r)}\left[-f(r)dt^2 + \frac{dr^2}{f(r)} + d\vec{x}_3^2\right] + \dots \qquad f(r_h) = 0$$

- D3-brane action contains

$$\int d^4x\, C\, F\tilde{F}$$

$\Rightarrow$ gravity field dual to Q is RR-potential C.

Holographic derivation

Apply to CS diff rate in $\mathcal{N} = 4$ SYM [Son-Starinets 2002]

- Action for C in 5d:

$$\int d^5x \sqrt{-g_5} \left[-\frac{1}{2} \partial_\mu C \partial^\mu C \right]$$

- Solve eq of motion for $C \Rightarrow$ Retarded correlator G_R.
- Use Kubo, result

$$\Gamma_{CS} = \frac{\lambda^2}{256\pi^3} T^4 = \frac{1}{2^7 \pi^5} \left(\frac{\lambda}{N_c} \right)^2 sT$$

$s =$ entropy density.

Holographic derivation

Comments:

- $\mathcal{N} = 4$ SYM a bit different from QCD.
- Other holographic results look different, eg:
  - $\mathcal{N} = 4$ SYM with magnetic field B [Basar-Kharzeev 2012]:

    $\Gamma_{CS} = \Gamma_{CS}(B = 0) \cdot f(B)$
  - $\mathcal{N} = 4$ SYM with anisotropy a [Bu 2014]:

    $\Gamma_{CS} = \Gamma_{CS}(a = 0) \cdot g(a)$
  - Witten model of holographic Yang-Mills [Craps et al 2012]:

    $\Gamma_{CS} = \frac{1}{2\pi} \frac{\lambda^3}{3^6 \pi^2} \frac{1}{M_{KK}^2} T^6$
  - ...
- Situation different from universal $\frac{\eta}{s} = \frac{1}{4\pi}$ [Kovtun-Son-Starinets 2004].

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Or does it?!

All existing calculations done in "wrapped brane models".

"Universality" of the result

"Wrapped brane models"

Dp-brane action

$$S = -\tau_p \int d^{p+1}x \, e^{-\phi} \sqrt{-det(g + 2\pi\alpha' F)} + \tau_p \int \sum_n C_n \wedge e^{2\pi\alpha' F}$$

with

$$\begin{aligned} \tau_p &= \text{brane tension} = \text{charge} \\ \phi &= \text{dilaton} \\ g &= \text{metric} \\ F &= \text{YM gauge field on the brane} \\ C_n &= \text{RR potentials} \end{aligned}$$

"Universality" of the result

$$\text{Wrap Dp-brane on } (p-3)\text{-cycle } \Omega_{p-3}$$
$$\Downarrow$$
$$\text{4d gauge theory in IR}$$

Expanding at low energies

$$\mathcal{L} = -\frac{1}{4g_{YM}^2}\mathrm{Tr}F^2 - \frac{\theta_{YM}}{32\pi^2}\mathrm{Tr}F\tilde{F}$$

with

$$\begin{aligned} \frac{1}{g_{YM}^2} &= \tau_p(2\pi\alpha')^2 \int_{\Omega_{p-3}} d^{p-3}x\, e^{\frac{p-7}{4}\phi}\sqrt{det(g_E)} \\ \theta_{YM} &= \tau_p(2\pi\alpha')^2 \int_{\Omega_{p-3}} C_{p-3} \end{aligned}$$

Thus: Gravity field dual to Q is $\int_{\Omega_{p-3}} C_{p-3} \equiv \tilde{C}$.

"Universality" of the result

Action of $\tilde{C}$ in 5d from reduction of 10d action of dC_{p-3}:

$$\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\,H\left[-\frac{1}{2}\partial_\mu\tilde{C}\partial^\mu\tilde{C}\right]$$

with

$$H=\frac{g_{YM}^4}{(8\pi^2)^2}$$

$$\Downarrow^1$$

Chern-Simons diffusion rate has "universal" form

$$\Gamma_{CS}=\frac{\alpha_s^2}{(2\pi)^3}sT$$

[1] [Son-Starinets 2002, Gursoy-Iatrakis-Kiritsis-Nitti-O'Bannon 2013]

"Universality" of the result

Comments:

- Checked also in $\mathcal{N} = 4$ SYM with flavors and $\mathcal{N} = 1$ models.
- Can calculate first $1/\lambda$ correction: Γ_{CS} decreases [Bu 20014].
  Is holographic result an upper bound on Γ_{CS}?
- Problem in extending result to other models: identification of coupling λ and gravity field dual to Q.

Inclusion of the anomaly

Anomaly

$$\partial_\mu J^\mu_A = -qQ$$

holographically reproduced by Stuckelberg action [Klebanov et al 2002]

$$\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\left[-\frac{1}{2}\left(\partial_\mu C + qA_\mu\right)\left(\partial^\mu C + qA^\mu\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

A^μ: gravity field dual to J^μ_A; q: anomaly coefficient.

From dimensional reduction of main holographic models

(D3/D7, Maldacena-Nunez, Klebanov-Strassler, Witten-Sakai-Sugimoto)

Inclusion of the anomaly

Repeat holographic calculation of G_R.

Only two possible solutions:

- $q = 0$ (no anomaly), or
- $\Gamma_{CS}(q \neq 0) = 0$.

It's a general result (mild assumptions).

Inclusion of the anomaly

Why expected:

- Via anomaly $\partial_\mu J^\mu_A = -qQ$, $\Gamma_{CS} \sim \langle QQ \rangle \sim \langle \partial J_A \partial J_A \rangle$.
- $\mathcal{Q}_A = \int d^3x\, J^t_A$ not conserved (anomaly), thus

$$\Gamma_{CS} \sim \langle \mathcal{Q}_A(t \to \infty)\mathcal{Q}_A(0)\rangle_R = 0$$

Decay rate Γ:

- With only gapped modes

$$\langle \mathcal{Q}(t)\mathcal{Q}(0)\rangle_R \sim e^{-\Gamma t}$$

- Holography: Γ from quasi-normal modes of gravity field A^μ dual to J^μ_A on black hole spacetime.
- General expected behavior at small q from equations of motion and numerics in [Jimenez Alba-Landsteiner-Melgar 2014]:

$$\Gamma \sim q^2$$

Conclusions:

- Holography seems to point towards a large coupling universality of Γ_{CS} in terms of s, T, α_s.
- Can use the result for estimates in the QGP, e.g. if
  - $\alpha_s(T_c) \sim 1/2$
  - $s(T_c) \sim 10\,T_c$ [Borsanyi et al 2013, Bazavov et al 2014]

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$$\Gamma_{CS}(T_c) \sim 10^{-2}\,T_c^4 .$$

- With anomaly $\Gamma_{CS} = 0$.
- Decay rate goes as $\Gamma \sim q^2$.

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Thank you for your time!

The Chern-Simons diffusion rate

Γ_{CS} relevant for Chiral Magnetic Effect

[Fukushima-Kharzeev-Warringa 2008]

Axial anomaly:

$$\partial_\mu J^\mu_A = -2Q$$

- ΔN_{CS} generates a $\Delta_{chirality} \Rightarrow \mu_A \neq 0$ (chemical potential).
- $\Delta_{chirality} + \vec{B} \Rightarrow \vec{J}^{em} = \sigma_{CME}\vec{B}$, $\sigma_{CME} = \frac{e^2}{2\pi}\mu_A$.