Relativistic MHD for astrophysical plasmas: reconnection and dynamo-chiral action

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Introduction

Outline

The baryonic matter in the Universe is almost invariably found in the form of plasma. Since in astrophysics we are dealing with macroscopic scales, usually the plasma dynamics is treated by using the *magnetohydrodynamics* (MHD) approximation.

In high-energy astrophysics we often employ (general) relativistic MHD, accounting for:

- high-speed velocities ($v \to c$),
- extremely hot gases ($c_s \to c/\sqrt{3}$),
- huge magnetic fields ($c_A \to c$),
- strong gravity around compact objects ($\sqrt{GM/R} \to c$).

Global MHD multidimensional simulations of the dynamics of such systems are already challenging, typically non-ideal effects are neglected (though numerical viscosity and resistivity are invariably present).

Here we present three case studies where non-ideal effects have to be considered in the numerical modeling of relativistic plasmas:

- fast reconnection models in resistive current sheets (Del Zanna et al. 2016),
- mean-field dynamo in disks around Kerr black holes (Bugli et al. 2014),
- covariant and $3+1$ formalism for dynamo-chiral MHD (Del Zanna et al. in prep).

The covariant equations for matter and fields

The equations for general relativistic hydrodynamics are those for baryon number (or equivalently mass) conservation and energy-momentum conservation

$$\nabla_\mu N^\mu = 0,$$
$$\nabla_\mu T^{\mu\nu} = 0,$$

supplemented by the second law of thermodynamics for the entropy current

$$\nabla_\mu \mathcal{S}^\mu \geq 0.$$

In relativistic MHD $T^{\mu\nu}$ is the *total* (matter and fields) energy-momentum tensor of the system, and the above equations are unchanged. The electromagnetic field obeys

$$\nabla_\mu F^{\mu\nu} = -I^\nu, \quad (\nabla_\nu I^\nu = 0)$$
$$\nabla_\mu F^{\star\mu\nu} = 0,$$

where $F^{\mu\nu}$ is the Faraday tensor and $F^{\star\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\kappa}F_{\lambda\kappa}$ its dual ($c \to 1, 4\pi \to 1$).

If we split the energy-momentum tensor and introduce the Lorentz force, we find

$$\nabla_\mu T^{\mu\nu}_{\mathrm{m}} = -\nabla_\mu T^{\mu\nu}_{\mathrm{f}} = -I_\mu F^{\mu\nu},$$

where $T^{\mu\nu}_{\mathrm{m}}$ and $T^{\mu\nu}_{\mathrm{f}}$ are the *matter* and *field* contributions, the latter given by

$$T^{\mu\nu}_{\mathrm{f}} = F^{\mu\lambda}F^{\nu}_{\lambda} - \frac{1}{4}g^{\mu\nu}F^{\lambda\kappa}F_{\lambda\kappa}.$$

Decomposition with u^μ and the ideal MHD condition

If dissipative effects are neglected and we introduce the *fluid* velocity u^μ, we have

$$N^\mu = nu^\mu,$$
$$T_{\rm m}^{\mu\nu} = eu^\mu u^\nu + p\Delta^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu},$$
$$\mathcal{S}^\mu = su^\mu,$$

where baryon density, energy density, kinetic pressure, and entropy density are

$$n = -N^\mu u_\mu, \quad e = T_{\rm m}^{\mu\nu} u_\mu u_\nu, \quad p = \tfrac{1}{3}\Delta_{\mu\nu} T_{\rm m}^{\mu\nu}, \quad s = -\mathcal{S}^\mu u_\mu.$$

The Faraday tensor and its dual can also be split according to u^μ

$$F^{\mu\nu} = u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\lambda\kappa} b_\lambda u_\kappa,$$
$$F^{\star\mu\nu} = u^\mu b^\nu - u^\nu b^\mu - \epsilon^{\mu\nu\lambda\kappa} e_\lambda u_\kappa,$$

where $e^\mu = F^{\mu\nu}u_\nu$ and $b^\mu = F^{\star\mu\nu}u_\nu$ are the electric and magnetic fields in the comoving frame ($e^\mu u_\mu = b^\mu u_\mu = 0$). In order to close the system we need to use a relativistic Ohm's law, which in the limit of a perfectly conducting plasma is

$$e^\mu = 0.$$

Thus the ideal MHD condition simply translates in the vanishing of the comoving electric field, and only b^μ enters the ideal MHD equations.

The equations of ideal GRMHD

The field component of $T^{\mu\nu}$ and the dual of the Faraday tensor simplify to

$$
\begin{aligned}
T_{\mathrm{f}}^{\mu\nu} &= \tfrac{1}{2}b^2u^\mu u^\nu + \tfrac{1}{2}b^2\Delta^{\mu\nu} - b^\mu b^\nu = b^2u^\mu u^\nu + \tfrac{1}{2}b^2 g^{\mu\nu} - b^\mu b^\nu, \\
F^{\star\mu\nu} &= u^\mu b^\nu - u^\nu b^\mu.
\end{aligned}
$$

The system of ideal GRMHD equations in conservative form is then

$$
\begin{aligned}
&\nabla_\mu(\rho u^\mu) = 0, \\
&\nabla_\mu[(e+p+b^2)u^\mu u^\nu + (p+\tfrac{1}{2}b^2)g^{\mu\nu} - b^\mu b^\nu] = 0, \\
&\nabla_\mu(u^\mu b^\nu - u^\nu b^\mu) = 0,
\end{aligned}
$$

in the unknowns $\rho = nm$, e, p, u^μ, b^μ, to be closed with an EoS $p = \mathcal{P}(\rho, e)$.

In the laboratory fixed frame and in a Minkowskian spacetime, we have

$$
u^\mu = (\Gamma, \Gamma\mathbf{v}), \quad b^\mu = (\Gamma(\mathbf{v}\cdot\mathbf{B}), \mathbf{B}/\Gamma + \Gamma(\mathbf{v}\cdot\mathbf{B})\mathbf{v}), \quad b^2 = B^2/\Gamma^2 + (\mathbf{v}\cdot\mathbf{B})^2.
$$

E is now a derived quantity and the sourceless Maxwell equations are for **B** only

$$
\mathbf{E} = -\mathbf{v}\times\mathbf{B}, \qquad \frac{\partial\mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{B}), \quad \boldsymbol{\nabla}\cdot\mathbf{B} = 0,
$$

the *induction equation* and the *solenoidal condition*, as in non-relativistic MHD.

The ECHO code for GRMHD

For multi-dimensional simulations of relativistic plasmas the Firenze group developed the shock-capturing Eulerian Conservative High Order code (Del Zanna et al. 2007), solving the (G)RMHD system of conservation laws (here for a flat metric):

$$\frac{\partial}{\partial t}(\rho\Gamma) + \boldsymbol{\nabla}\cdot(\rho\Gamma\boldsymbol{v}) = 0,$$

$$\frac{\partial}{\partial t}\left(w\Gamma^2\boldsymbol{v} + \boldsymbol{E}\times\boldsymbol{B}\right) + \boldsymbol{\nabla}\cdot\left(w\Gamma^2\boldsymbol{v}\boldsymbol{v} - \boldsymbol{E}\boldsymbol{E} + \boldsymbol{B}\boldsymbol{B} + (p + u_{\mathrm{em}})\,\boldsymbol{I}\right) = 0,$$

$$\frac{\partial}{\partial t}\left(w\Gamma^2 - p + u_{\mathrm{em}}\right) + \boldsymbol{\nabla}\cdot\left(w\Gamma^2\boldsymbol{v} + \boldsymbol{E}\times\boldsymbol{B}\right) = 0,$$

$$\frac{\partial\boldsymbol{B}}{\partial t} + \boldsymbol{\nabla}\times\boldsymbol{E} = 0,$$

where $w = e + p$, $u_{\mathrm{em}} = \frac{1}{2}(E^2 + B^2)$ and $\boldsymbol{E} = -\boldsymbol{v}\times\boldsymbol{B}$, $p = \mathcal{P}(\rho, e)$.

Extensions and sub-versions of ECHO (`www.astro.unifi.it/echo/`):

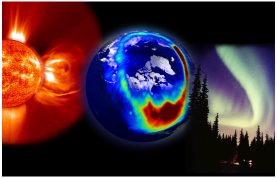
- X-ECHO (Bucciantini & Del Zanna 2011) - GRMHD evolution in a variable spacetime metric (under the *extended* conformally flat condition),
- XNS - equilibrium configurations for magnetized rotating neutron stars,
- ECHO-QGP (Del Zanna et al. 2013; Inghirami et al. 2016) - viscous RHD and ideal RMHD for heavy-ion collisions.

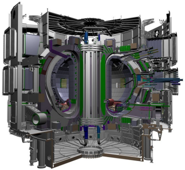
Resistivity and fast reconnection models

Reconnection in solar, space, and laboratory plasmas

Magnetic reconnection is one of the most efficient mechanisms to convert the energy of a magnetically dominated plasma into heat and particle acceleration.

Energy release is typically violent and occurs on very rapid timescales. It is responsible for solar flares, geomagnetic storms, and instabilities in fusion reactors.





Within classical MHD the magnetic field evolution is governed by the induction equation with a *resistivity* coefficient η (flux freezing in ideal MHD with $\eta = 0$):

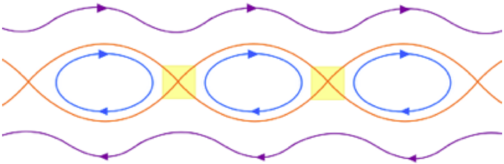
$$\boxed{\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}}$$

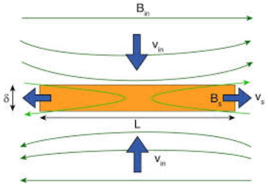
Fluid advection and Ohmic diffusion occur on very different timescales:

$$\tau_A = \frac{L}{v_A}, \quad \tau_D = \frac{L^2}{\eta}, \qquad \text{Lundquist number } S = \frac{L\, v_A}{\eta} = \frac{\tau_D}{\tau_A} \gg 1$$

Spontaneous reconnection (tearing instability)

The linear stability of current sheets was investigated by Furth et al. (1963); Coppi et al. (1976). In resistive MHD the equilibrium is unstable to the *tearing mode* leading to the formation of X-points where reconnection occurs and plasmoids.





If measured on top of the only available scale, the sheet width a, the instability growth rate $\gamma = 1/\tau$ is far too slow to explain solar flares ($\tau \sim \tau_A \sim 10^3$s, $S \sim 10^{12}$):

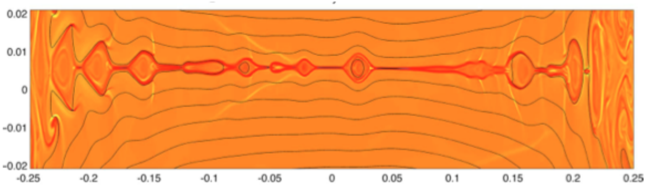
$$\boxed{\gamma\,\bar{\tau}_A \simeq 0.6\,\bar{S}^{-1/2}, \quad k_{\max} a \simeq 1.4\,\bar{S}^{-1/4}} \quad (\bar{\tau}_A = a/v_A, \quad \bar{S} = a\,v_A/\eta)$$

The same scaling is found for the reconnection rate of steady-state incompressible models (Sweet-Parker, SP) in thin current sheets configurations.

Tearing instability of a SP sheet (plasmoid instability)

Until 10 years ago, standard MHD reconnection was basically dead, only sub-MHD Hall/kinetic effects were considered as viable mechanisms to produce faster rates.

Revival of one-fluid resistive MHD: discovery of the *plasmoid instability* of steady-state Sweet-Parker (SP) current sheets (Loureiro et al. 2007; Lapenta 2008; Samtaney et al. 2009; Bhattacharjee et al. 2009; Cassak et al. 2009; Huang & Bhattacharjee 2010).



The tearing mode applied to a SP current sheet with $a = LS^{-1/2}$ leads to

$$\gamma\,\tau_A \sim S^{1/4} \gg 1, \quad k_{\max} L \sim S^{3/8} \gg 1$$

for $S > S_c \sim 10^4$, once we normalize with the *macroscopic* τ_A and L.

Relativistic resistive MHD simulations confirmed this scenario (Watanabe & Yokoyama 2006; Zenitani et al. 2010; Zanotti & Dumbser 2011; Takahashi et al. 2013, Takamoto 2013).

The *ideal* tearing instability

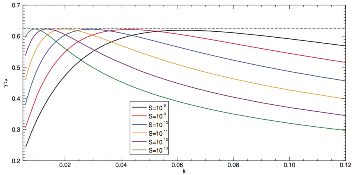
For a generic dependence of the aspect ratio with S, the growth rate is

$$a/L \sim S^{-\alpha} \Rightarrow \gamma \sim \bar{\tau}_A^{-1} \bar{S}^{-1/2} = \tau_A^{-1} S^{-1/2} S^{3/2\alpha}$$

thus, there is a critical value for an *ideal* tearing mode:

$$\boxed{\alpha = 1/3 \Rightarrow \gamma \sim \tau_A^{-1}}$$

For $S = 10^{12}$ the threshold $a/L \sim S^{-1/3} = 10^4$ is 100 times larger than the SP one. In a dynamical thinning scenario reconnection occurs on *ideal* timescales, well before the SP configuration can be realized (Pucci & Velli, 2014, Landi et al. 2015, Tenerani et al. 2015).



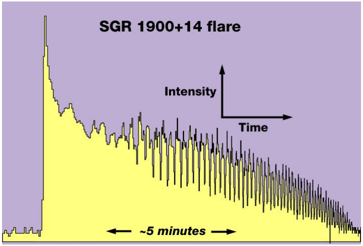
By solving the rescaled tearing instability equations, the dispersion relation $\gamma(k)$ for a varying S now clearly shows the expected ideal limit, provided S is large enough:

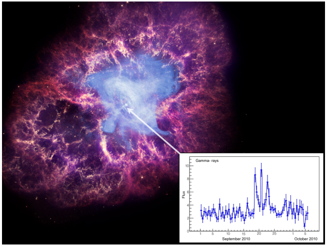
$$\boxed{\gamma_{\max}\tau_A \simeq 0.6, \quad k_{\max} a \simeq 1.4\, S^{-1/6}}$$

Reconnection in relativistic plasmas

In astrophysical sources it may explain several high-energy phenomena:

- the SGR events of magnetars (Lyutikov 2003; Elenbaas et al. 2016)
- jet launching in AGN/microquasar systems (Romanova & Lovelace 1992)
- jet launching in GRB engines (Drenkhahn & Spruit 2002)
- energy conversion in pulsar winds (Coroniti 1990, Sironi & Spitkovsky 2011)
- gamma-ray flares observed in the Crab Nebula (Cerutti et al. 2013, 2014)





In (Del Zanna et al. MNRAS 460, 3753, 2016) the standard and *ideal* tearing instability has been investigated, both analytically and numerically, for the relativistic MHD case.

The relativistic MHD case

Consider now a relativistic plasma where both magnetic and thermal energy can be comparable to the rest mass energy, so that in general

$$\sigma_0 = B_0^2/\rho_0 \sim 1, \qquad \beta_0 = 2p_0/B_0^2 \sim 1$$

The conservative form of resistive relativistic MHD in Minkowski spacetime is

$$\begin{aligned}
&\partial_t(\rho\Gamma) + \boldsymbol{\nabla}\cdot(\rho\Gamma\boldsymbol{v}) = 0\\
&\partial_t(w\Gamma^2\boldsymbol{v} + \boldsymbol{E}\times\boldsymbol{B}) + \boldsymbol{\nabla}\cdot(w\Gamma^2\boldsymbol{v}\boldsymbol{v} - \boldsymbol{E}\boldsymbol{E} - \boldsymbol{B}\boldsymbol{B} + (p + u_{\rm em})\boldsymbol{I}) = 0\\
&\partial_t(w\Gamma^2 - p + u_{\rm em}) + \boldsymbol{\nabla}\cdot(w\Gamma^2\boldsymbol{v} + \boldsymbol{E}\times\boldsymbol{B}) = 0\\
&\partial_t\boldsymbol{B} + \boldsymbol{\nabla}\times\boldsymbol{E} = 0\\
&\partial_t\boldsymbol{E} - \boldsymbol{\nabla}\times\boldsymbol{B} = -\boldsymbol{J}
\end{aligned}$$

with $\Gamma = 1/\sqrt{1-v^2}$, $w = e + p$, $u_{\rm em} = \frac{1}{2}(E^2 + B^2)$, $p = \mathcal{P}(\rho, e)$. The electric field is not simply provided by $\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} = 0$ but in the resistive case must be evolved by Maxwell's equations, with the electric current provided by the relativistic Ohm law

$$e^\mu = \eta j^\mu \Rightarrow \boldsymbol{J} = (\boldsymbol{\nabla}\cdot\boldsymbol{E})\boldsymbol{v} + \eta^{-1}\,\Gamma[\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} - (\boldsymbol{E}\cdot\boldsymbol{v})\boldsymbol{v}]$$

IMEX (IMplicit-EXplicit) Runge-Kutta high-order methods to treat *stiff* terms $\propto \eta^{-1}$ employed in the ECHO code (Del Zanna et al. 2007, 2014).

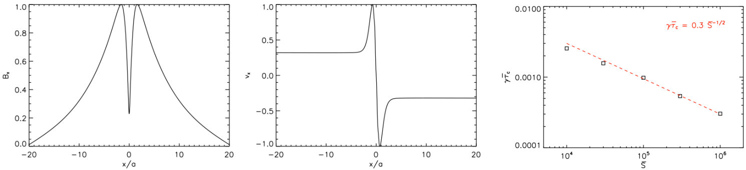
The tearing instability in relativistic MHD: linear analysis

We consider 2D, incompressible, linear perturbations of force-free current sheet

$$\boldsymbol{B}_0 = B_0[\tanh(x/a)\hat{\mathbf{y}} + \mathrm{sech}(x/a)\hat{\mathbf{z}}]$$

and retrieve exactly the same equations of the classical MHD with the only exception $\rho_0 \to w_0 + B_0^2$ as the plasma inertial term:

$$\partial_t \boldsymbol{B}_1 = \boldsymbol{\nabla} \times (\boldsymbol{v}_1 \times \boldsymbol{B}_0) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B}_1,$$
$$\partial_t (w_0 \boldsymbol{v}_1 + \boldsymbol{E}_1 \times \boldsymbol{B}_0) = -\boldsymbol{\nabla}(p_1 + \boldsymbol{B}_0 \cdot \boldsymbol{B}_1) + (\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{B}_1 + (\boldsymbol{B}_1 \cdot \boldsymbol{\nabla})\boldsymbol{B}_0,$$



The maximum growth rate then depends of the relativistic Alfvén speed as

$$\gamma_{\max}\bar{\tau}_c \simeq 0.6\, c_A\, \bar{S}^{-1/2}, \quad c_A = B_0/\sqrt{w_0 + B_0^2} = 0.5 \quad (\sigma_0 = \beta_0 = 1)$$

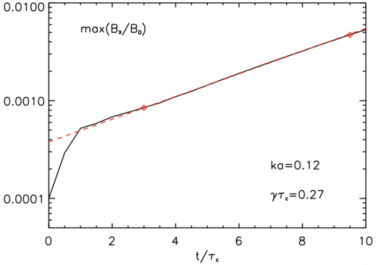
where $\bar{\tau}_c = a/c$ and $\bar{S} = ac_A/\eta$, here from 10^4 to 10^6. A similar study was previously performed for lower S in the *force-free electrodynamics* regime (Komissarov et al. 2007).

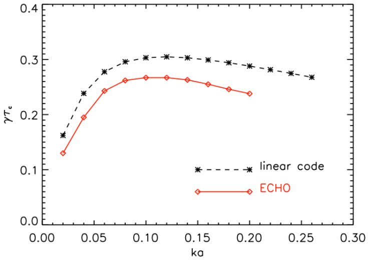
The *ideal* tearing instability in relativistic MHD

Let us study the tearing instability for the critical (inverse) aspect ratio

$$a/L = S^{-1/3} = 0.01, \quad S = Lc_A/\eta = 10^6$$

Single-mode runs show a clear linear phase and the predicted dispersion relation.





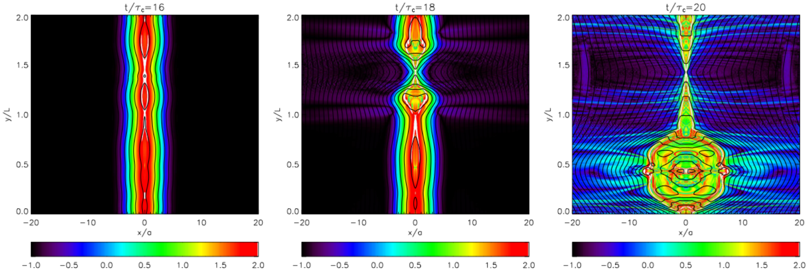
We thus find that the *ideal* tearing mode effectively grows, independently on S, as

$$\boxed{\gamma_{\max} \simeq 0.6 c_A/L \sim c/L}$$

that is on light-crossing times, as requested to explain explosive events in high-energy astrophysical sources.

The fully nonlinear case

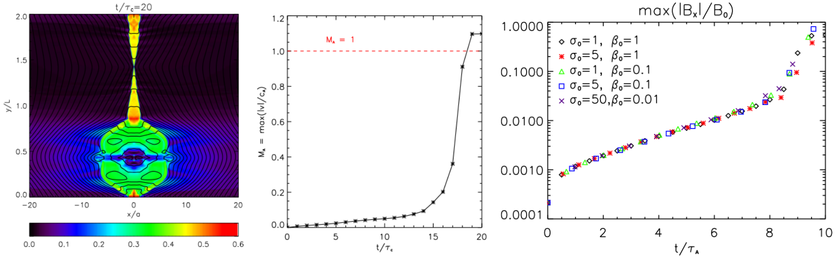
In the fully nonlinear and multi-mode case secondary reconnection events occur and the initial ~ 5 islands of the tearing instability start to merge.



Colors refer to to $|\nabla \times \boldsymbol{B}|$ in log scale. The final evolution is very rapid and we end up with a situation with an X-point, two symmetric exhausts, and a major plasmoid where additional instabilities occur.

We do not clearly see the plasmoid instability because we have adopted periodical boundaries along the current sheet.

MHD shocks and Petschek fast reconnection



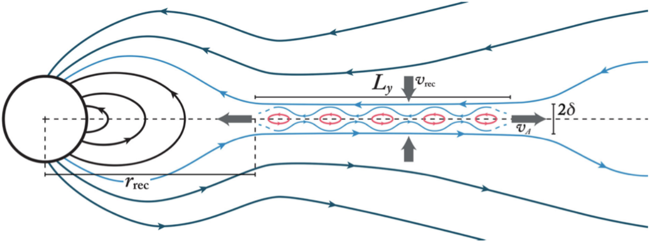
We find a quasi-stationary Petschek scenario for relativistic plasmas (Lyubarsky 2005):

- channels delimited by slow shocks originating from the X-point,
- fast magnetosonic jets propagating in the exhausts and feeding the plasmoid,
- maximum velocity in funnels does not exceed the external c_A (here 0.5),
- we measure $\mathcal{R} \simeq 0.05 - 0.06$, matching the expected fast reconnection rate:

$$\mathcal{R} \equiv M_A = \frac{|v_x|}{c_A} = \frac{\pi}{4 \ln S},$$

- universal growth of perturbations in t/τ_A for various σ_0 and β_0, up to $c_A = 0.98$.

Application to magnetars giant flares



The (standard) tearing instability in current sheets above large coronal loops has been recently employed to model giant flares (SGRs) in magnetars (Elenbaas et al. 2016).

The observed e-folding and peak times in the gamma-ray light curves are

$$\tau_e \sim 0.1 - 1\,\mathrm{ms}, \quad \tau_{peak} \sim 1 - 10\,\mathrm{ms}$$

Our model for fast reconnection predicts, *independently on S* (thus on microphysics!):

$$\tau_e \simeq \frac{1}{\gamma_{max}} \simeq \frac{L}{0.6 c_A} \simeq 0.2\,\mathrm{ms}, \quad (L \simeq 5R_\star = 50\,\mathrm{km},\ c_A \simeq c)$$

provided a thinning process has shrunk the current sheet down to $\delta/L \sim S^{-1/3}$. A similar mechanism may operate at Crab Nebula's termination shock (Olmi et al. 2016).

Mean-field dynamo in GRMHD

Mean-field dynamo in classical MHD

We know that in stellar interiors and protostellar disks the magnetic field is amplified by mean-field dynamo processes: *correlated* small-scale fluctuations in velocity and magnetic field provide a mean electromotive force and amplify seed magnetic fields.

Within classical MHD consider small-scale turbulent fluctuations in the fields $\boldsymbol{v}$ e $\boldsymbol{B}$:

$$\boldsymbol{v}(\boldsymbol{x},t) = \boldsymbol{v}_0(\boldsymbol{x},t) + \delta\boldsymbol{v}(\boldsymbol{x},t), \quad \boldsymbol{B}(\boldsymbol{x},t) = \boldsymbol{B}_0(\boldsymbol{x},t) + \delta\boldsymbol{B}(\boldsymbol{x},t)$$

[assumption of kinematic regime $\Rightarrow$ $\boldsymbol{v}_0(\boldsymbol{x},t)$ fixed].

The resistive induction equation for the mean magnetic field reads:

$$\partial_t \boldsymbol{B}_0 = \boldsymbol{\nabla} \times (\boldsymbol{v}_0 \times \boldsymbol{B}_0) + \eta_r \nabla^2 \boldsymbol{B}_0 + \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}}$$

$$\boldsymbol{\mathcal{E}} = \langle \delta\boldsymbol{v} \times \delta\boldsymbol{b} \rangle \simeq \alpha \boldsymbol{B}_0 - \beta \boldsymbol{\nabla} \times \boldsymbol{B}_0$$

$$\Downarrow$$

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \alpha \boldsymbol{\nabla} \times \boldsymbol{B} + (\eta_r + \beta)\nabla^2 \boldsymbol{B} \quad [\boldsymbol{E}' = -\alpha \boldsymbol{B} + (\eta_r + \beta)\boldsymbol{J}]$$

- Differential rotation $\Rightarrow$ Generation of toroidal field (Ω effect)
- Toroidal field $\Rightarrow$ Generation of poloidal field (α effect)

$$\boldsymbol{B}_P \Rightarrow \boldsymbol{B}_T \Rightarrow \boldsymbol{B}_P \Rightarrow \dots$$

Exponentially growing $\alpha - \Omega$ dynamo modes, damped by resistivity.

Amplification of magnetic fields in relativistic plasmas

The amplification of ordered magnetic fields on large scales is supposed to be crucial for many high-energy astrophysical processes too:

- jets from AGNs and micro-quasars
- Blandford-Znajek mechanism from Kerr Black Holes (AGNs, GRBs)
- formation of magnetars with $B \sim 10^{15-17}$ G (GRBs?)

For relativistic plasmas we need to generalize the classical dynamo model to GRMHD.

The Maxwell equations in covariant form are

$$\nabla_\mu F^{\mu\nu} = -I^\nu, \quad \nabla_\mu F^{*\mu\nu} = 0$$

$$F^{\mu\nu} = u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\lambda\kappa} b_\lambda u_\kappa, \quad I^\mu = q_0 u^\mu + j^\mu$$

A fully covariant formulation of Ohm's law for resistive plasma with dynamo action was proposed by Bucciantini & Del Zanna (2013)

$$\boxed{e^\mu = \eta j^\mu + \xi b^\mu}$$

$$(\xi \equiv -\alpha_{dyn}, \quad \eta = \eta_r + \beta_{dyn})$$

Implementation within $3+1$ GRMHD

In the GRMHD ECHO code (Del Zanna et al., 2007) the usual $3+1$ split is adopted

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\,(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

Maxwell's equations are solved in the form

$$\gamma^{-1/2}\partial_t\left(\gamma^{1/2}\boldsymbol{B}\right) + \boldsymbol{\nabla}\times(+\alpha\boldsymbol{E} + \boldsymbol{\beta}\times\boldsymbol{B}) = 0, \qquad (\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0)$$

$$\gamma^{-1/2}\partial_t\left(\gamma^{1/2}\boldsymbol{E}\right) + \boldsymbol{\nabla}\times(-\alpha\boldsymbol{B} + \boldsymbol{\beta}\times\boldsymbol{E}) = -(\alpha\boldsymbol{J} - q\boldsymbol{\beta}), \qquad (\boldsymbol{\nabla}\cdot\boldsymbol{E} = q)$$

Ohm's law in the $3+1$ language is

$$\Gamma[\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} - (\boldsymbol{E}\cdot\boldsymbol{v})\boldsymbol{v}] = \eta(\boldsymbol{J} - q\boldsymbol{v}) + \xi\Gamma[\boldsymbol{B} - \boldsymbol{v}\times\boldsymbol{E} - (\boldsymbol{B}\cdot\boldsymbol{v})\boldsymbol{v}]$$

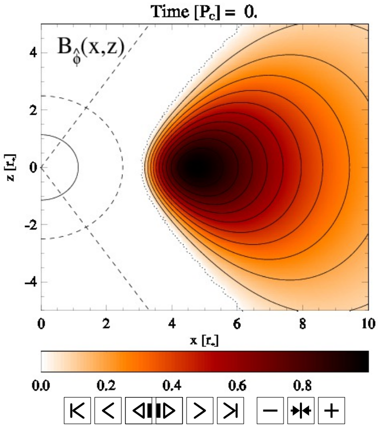
The equation for the electric field, using the sources q and $\boldsymbol{J}$, is

$$\gamma^{-1/2}\partial_t(\gamma^{1/2}\boldsymbol{E}) + \boldsymbol{\nabla}\times(-\alpha\boldsymbol{B} + \boldsymbol{\beta}\times\boldsymbol{E}) + (\alpha\boldsymbol{v} - \boldsymbol{\beta})\boldsymbol{\nabla}\cdot\boldsymbol{E} =$$
$$-\alpha\Gamma\,\eta^{-1}\{[\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} - (\boldsymbol{E}\cdot\boldsymbol{v})\boldsymbol{v}] - \xi[\boldsymbol{B} - \boldsymbol{v}\times\boldsymbol{E} - (\boldsymbol{B}\cdot\boldsymbol{v})\boldsymbol{v}]\}$$

Resistive terms $\propto \eta^{-1}$ can evolve on time scales $\tau_\eta \ll \tau_h$, though dynamo terms do not add extra complexity. We employ IMEX methods (Pareschi & Russo 2005; Palenzuela et al. 2009; Bucciantini & Del Zanna 2013; Dionysopoulou et al. 2013; Del Zanna et al. 2014).

Kinematic dynamo in accretion tori

As a first astrophysical application: $\alpha - \Omega$ kinematic dynamo in thick accretion tori around maximally rotating Kerr Black Holes with the ECHO code (Bugli et al. 2014).



$$C_\xi = \frac{\xi R}{\eta} = 5$$

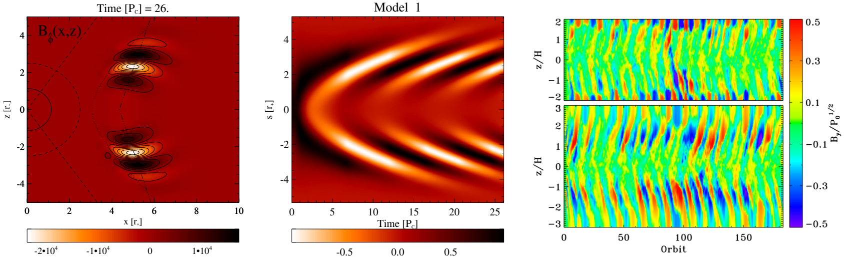
$$C_\Omega = \frac{\Delta\Omega R^2}{\eta} = 400$$

$$\gamma P_c = 0.39$$

$$\tau / P_c = 8.4$$

$$(P_c = 76.5 G M_{\rm BH} / c^3)$$

Butterfly diagrams



By following the paths taken by maxima vs time *butterfly* diagrams can be created. Changing the sign of ξ we invert the direction of migration of magnetic fields, and a situation as observed in the solar cycle can be reproduced.

Periodical reactivation of Blandford-Znajeck effect may be explained by dynamo in disk, since the frequency of the accreting field is related to the microphysics and not to P_c.

Similar results are found in 3D MHD simulations of stratified disks with shearing box local models, due to fully developed turbulence and MRI (Davis et al. 2010).

Dynamo-Chiral resistive relativistic MHD

The Chiral Magnetic Effect

The Chiral Magnetic Effect (CME) in a plasma of unbalanced left- and right-handed relativistic fermions is the quantum phenomenon of electric charge separation leading to a (macroscopic) current along the magnetic field (Vilenkin 1980, Kharzeev 2014).

This may be important in condensed matter or quark-gluon plasmas, mainly for

- heavy-ion collisions (Kharzeev 2004, Voloshin 2004, Huang 2016),
- the early Universe (Vilenkin & Leahy 1982, Tashiro et al. 2012),
- possibly neutron star interiors (Dvornikov & Semikov 2015, Sigl & Leite 2016).

Introducing a CME conductivity, we then assume the presence of an additional current

$$\boldsymbol{J}_{\rm CME} = \sigma_{\rm CME}\boldsymbol{B}, \quad \sigma_{\rm CME} \propto \mu_A,$$

where μ_A is the chiral chemical potential. The related *axial charge* evolves in time as

$$\partial_t n_A + \boldsymbol{\nabla}\cdot\boldsymbol{J}_A \propto \boldsymbol{E}\cdot\boldsymbol{B},$$

so regions of non-ideal MHD (also reconnection sites?) may be sources of CME.

CME is known to lead to small-scale magnetic field amplification of dynamo-type and to turbulent cascade (Rogachevskii 2017, Brandenburg 2017, Schober et al. 2018).

Analogy between CME and mean field dynamo

Let us consider the mean-field MHD, where the Ohm law takes the form

$$\boldsymbol{E}' \equiv \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = -\alpha_{\mathrm{dyn}} \boldsymbol{B} + \eta \boldsymbol{J},$$

where α_{dyn} is the turbulent dynamo coefficient and η the combined Ohmic and turbulent resistivity. This can also be rewritten in terms of conduction coefficients as

$$\boldsymbol{J} = \sigma_E \boldsymbol{E}' + \sigma_B \boldsymbol{B},$$

where $\sigma_E = 1/\eta$ and $\sigma_B = \alpha_{\mathrm{dyn}}/\eta$.

This form is exactly the same as that for chiral-MHD, where the first contribution is the usual conduction electric current and the second that due to CME.

We propose that the corresponding covariant form applies to the fields in the comoving frame of the fluid as

$$\boxed{j^\mu = \sigma_E e^\mu + \sigma_B b^\mu}$$

where we recall that

$$q_0 u^\mu + j^\mu = I^\mu = \nabla_\nu F^{\mu\nu}, \qquad e^\mu = F^{\mu\nu} u_\nu, \qquad b^\mu = F^{*\mu\nu} u_\nu,$$

and that the same machinery employed to model resistive and mean-field dynamo effects in (G)RHMD (Bucciantini & Del Zanna 2013) can be applied as it is to CME.

Dynamo-Chiral resistive relativistic MHD equations

Let us now rewrite the equations in the rest frame of the observer (lab frame). We have

$$u^\mu = \Gamma(1, \boldsymbol{v}), \quad e^\mu = \Gamma(\mathbf{v}\cdot\mathbf{E}, \mathbf{E} + \mathbf{v}\times\boldsymbol{B}), \quad b^\mu = \Gamma(\mathbf{v}\cdot\mathbf{B}, \mathbf{B} - \mathbf{v}\times\boldsymbol{E}),$$

with $\Gamma = 1/\sqrt{1-v^2}$, so that if $I^\mu = (q, \boldsymbol{J})$, the conduction current transverse to u^μ is

$$j^\mu = I^\mu - q_0 u^\mu = (q - \Gamma q_0, \boldsymbol{J} - \Gamma q_0 \boldsymbol{v})$$

The time and spatial components of Ohm's law are then

$$q - \Gamma q_0 = \sigma_E \Gamma(\mathbf{v}\cdot\mathbf{E}) + \sigma_B \Gamma(\mathbf{v}\cdot\mathbf{B})$$

$$\boldsymbol{J} - \Gamma q_0 \boldsymbol{v} = \sigma_E \Gamma(\mathbf{E} + \mathbf{v}\times\boldsymbol{B}) + \sigma_B \Gamma(\mathbf{B} - \mathbf{v}\times\boldsymbol{E})$$

and combining the two, Ohm's law for Dynamo-Chiral resistive relativistic MHD is

$$\boxed{\boldsymbol{J} = q\boldsymbol{v} + \sigma_E \Gamma[\mathbf{E} + \mathbf{v}\times\boldsymbol{B} - (\mathbf{v}\cdot\mathbf{E})\boldsymbol{v}] + \sigma_B \Gamma[\mathbf{B} - \mathbf{v}\times\boldsymbol{E} - (\mathbf{v}\cdot\mathbf{B})\boldsymbol{v}]}$$

in which we can employ, directly from the Gauss constraint

$$q = \nabla\cdot\boldsymbol{E}$$

The above Ohm's law must be inserted in Maxwell equations and coupled to the conservation laws for mass and total momentum-energy, as seen before.

$3+1$ GRMHD equations for a curved spacetime

The Dynamo-Chiral form of Ohm's law applies unchanged to the full $3+1$ GRMHD system (Del Zanna et al. in prep). The equations for a curved manifold are:

$$
\begin{aligned}
&\partial_t(\sqrt{\gamma}D) + \partial_k[\sqrt{\gamma}(\alpha D v^k - \beta^k D)] = 0,\\
&\partial_t(\sqrt{\gamma}S_i) + \partial_k[\sqrt{\gamma}(\alpha S_i^k - \beta^k S_i)] = \sqrt{\gamma}(\tfrac{1}{2}\alpha S^{lm}\partial_i\gamma_{lm} + S_k\partial_i\beta^k - \mathcal{E}\partial_i\alpha),\\
&\partial_t(\sqrt{\gamma}\mathcal{E}) + \partial_k[\sqrt{\gamma}(\alpha S^k - \beta^k\mathcal{E})] = \sqrt{\gamma}(\alpha S^{lm}K_{lm} - S^k\partial_k\alpha),\\
&\partial_t(\sqrt{\gamma}B^i) + [ijk]\partial_j(\alpha E_k + [klm]\sqrt{\gamma}\beta^l B^m) = 0,\\
&\partial_t(\sqrt{\gamma}E^i) - [ijk]\partial_j(\alpha B_k - [klm]\sqrt{\gamma}\beta^l E^m) = -\sqrt{\gamma}(\alpha J^i - \beta^i q),
\end{aligned}
$$

with the two non-evolutionary constraints

$$\partial_k(\sqrt{\gamma}B^k) = 0, \qquad \partial_k(\sqrt{\gamma}E^k) = \sqrt{\gamma}\,q,$$

where $D = \rho\Gamma$ is the mass density, S^i the momentum flux, and $\mathcal{E}$ the energy density, all measured in the Eulerian frame. The *extrinsic curvature* term is

$$\alpha S^{lm}K_{lm} = \tfrac{1}{2}S^{lm}(\beta^k\partial_k\gamma_{lm} - \partial_t\gamma_{lm}) + S^l{}_m\partial_l\beta^m,$$

to be found from Einstein's equations (or assigned).

Summary

Non-ideal effects are commonly neglected in relativistic hydro/MHD simulations, here we have shown three cases where they are important:

- When magnetic fields are so strong such that $c_A \to c$, the reconnection process must be studied in the appropriate resistive relativistic MHD regime (Del Zanna et al. 2016). The tearing instability may occur on light-crossing times if $a/L \sim S^{-1/3}$ (*ideal* tearing).
- Turbulence is expected to provide a dynamo process in (general) relativistic plasmas too. We have derived an Ohm's law in both covariant and $3+1$ form and applied to disks around Kerr black holes (Bucciantini & Del Zanna 2013; Bugli et al. 2014).
- A unified set of equations for Dynamo-Chiral resistive relativistic MHD has been derived in both covariant and $3+1$ form, valid for curved manifolds, ready for numerical integration (Del Zanna et al. in prep).

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