Realistic Shell-Model Calculations for Double Beta-Decay

Nunzio Itaco

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Probing fundamental symmetries and interactions by low energy excitations with SPES RIBs

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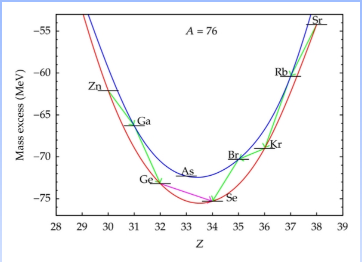
Double β-decay

Double β-decay is the rarest process yet observed in nature.



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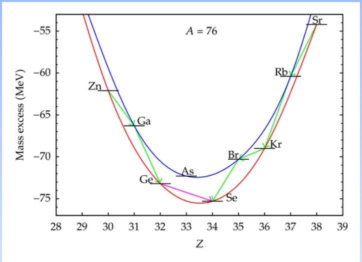

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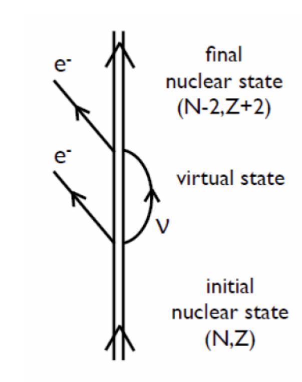

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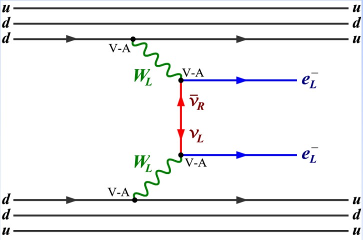
Neutrinoless double β-decay

The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, triggered by the search of "new physics" beyond the Standard Model.

- Its detection
  - would correspond to a violation of the conservation of the leptonic number
  - may provide more informations on the nature of neutrinos (neutrino as a Majorana particle, determination of its effective mass, ..).

Neutrinoless double β-decay

The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME



$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu} \left|M^{0\nu}\right|^2 \langle m_\nu \rangle^2$$

- $G^{0\nu}$ is the so-called phase-space factor, obtained by integrating over she single electron energies and angles, and summing over the final-state spins
- $\langle m_\nu \rangle = \mid \sum_k m_k U_{ek}^2 \mid$ effective mass of the Majorana neutrino, U_{ek} being the lepton mixing matrix

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Calculating nuclear matrix elements

Nuclear matrix elements may play a major role in several processes whose interest goes beyond the realm of Nuclear Physics

- $0\nu\beta\beta$ decay $\Rightarrow$

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 \langle m_\nu \rangle^2$$

- Dark matter $\Rightarrow$

$$\frac{d\sigma}{dp^2} = \frac{2}{(2J_i+1)\pi v^2}\sum |\langle i|\mathcal{L}_\chi|f\rangle|^2$$

- fundamental symmetries $\Rightarrow$

$$\langle i|\mathcal{L}_{leptons-nucleons}|f\rangle = \langle i|\int dx j^\mu(x) J_\mu(x)|f\rangle$$

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The calculation of the NME

The nuclear matrix element (NME) is expressed as

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_{F} - M^{0\nu}_{T} \ ,$$

where

$$M^{0\nu}_{GT} =< 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_{GT}(r_{mn}) \vec{\sigma}_m \cdot \vec{\sigma}_n \mid 0^+_i >$$

$$M^{0\nu}_{F} =< 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_{F}(r_{mn}) \mid 0^+_i >$$

$$M^{0\nu}_{T} =< 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_{T}(r_{mn}) \left[3 \left(\vec{\sigma}_m \cdot \hat{r}_{mn}\right) \left(\vec{\sigma}_n \cdot \hat{r}_{mn}\right) - \vec{\sigma}_m \cdot \vec{\sigma}_n\right] \mid 0^+_i >$$

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The calculation of the NME

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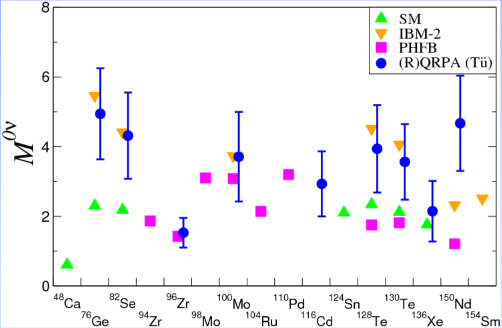

- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

Realistic Shell-Model Calculations

Shell model ⇒ well-established approach to obtain a microscopic description of both collective and single-particle properties of nuclei

Napoli-Caserta group

- L. Coraggio (INFN-NA)
- L. De Angelis (INFN-NA)
- T. Fukui (INFN-NA)
- A. Gargano (INFN-NA)
- N. I. (UNICAMPANIA and INFN-NA)
- F. Nowacki (IPHC-CNRS Strasbourg)

Realistic Shell-Model Calculations

Our final goal is to compute the $0\nu\beta\beta$-decay NME for ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe.

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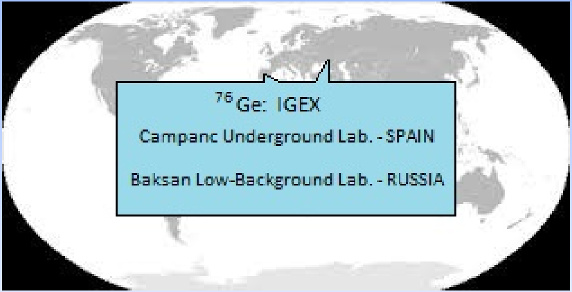

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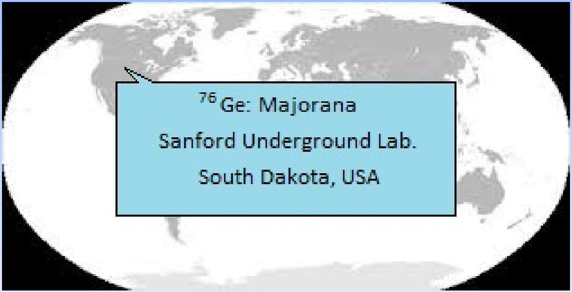

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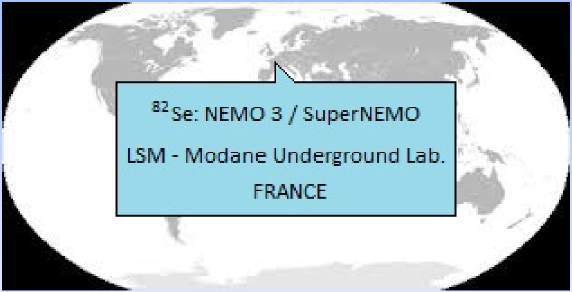

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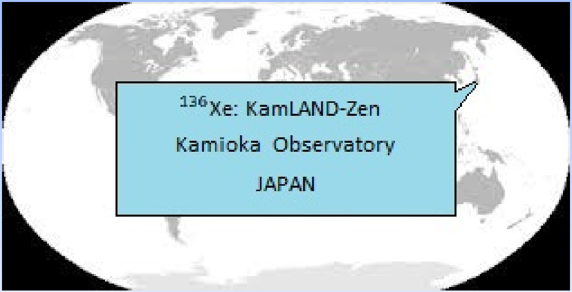

Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} \;\; (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian

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Workflow for a realistic shell-model calculation

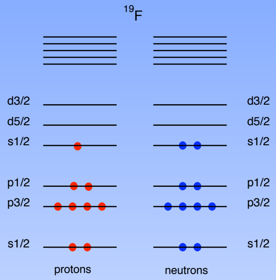
1. Choose a realistic *NN* potential (*NNN*)
2. Determine the model space better tailored to study the system under investigation
3. Derive the effective shell-model hamiltonian and operators by way of a many-body theory
4. Calculate the physical observables (energies, e.m. transition probabilities, ...)

Effective shell-model hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$



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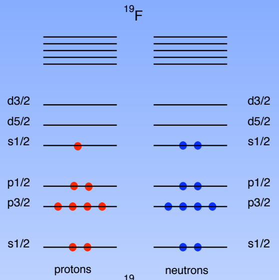
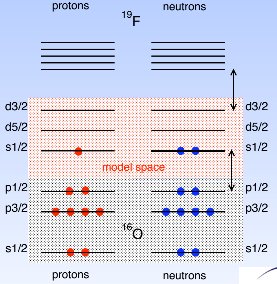
Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \ldots a_n^\dagger]_i|c\rangle \Rightarrow P = \sum_{i=1}^{d}|\Phi_i\rangle\langle\Phi_i|$$

Model-space eigenvalue problem

$$H_{\rm eff}P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$





The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \quad \begin{array}{c} \mathcal{H} = X^{-1}HX \\ \Longrightarrow \end{array} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

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The shell-model effective hamiltonian

Folded-diagram expansion

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

$\Rightarrow$ Recursive equation for H_{eff} $\Rightarrow$ iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\mathrm{eff}} = \hat{Q} - \hat{Q}'\int\hat{Q} + \hat{Q}'\int\hat{Q}\int\hat{Q} - \hat{Q}'\int\hat{Q}\int\hat{Q}\int\hat{Q}\cdots,$$

generalized folding

The perturbative approach to the shell-model $H^{\rm eff}$

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

The $\hat{Q}$-box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the $\hat{Q}$-box

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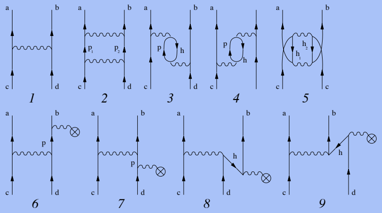

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Effective operators

Φ_α = eigenvectors obtained diagonalizing $H_{\rm eff}$ in the reduced model space $\Rightarrow |\Phi_\alpha\rangle = P|\Psi_\alpha\rangle$

$$\langle\Phi_\alpha|\hat{O}|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\hat{O}|\Psi_\beta\rangle$$

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Effective operator $\hat{O}_{\rm eff}$: definition

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$\hat{O}_{\rm eff}$ can be derived consistently in the MBPT framework

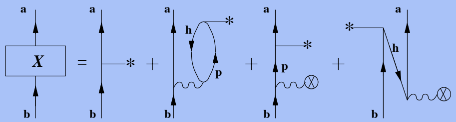
$$\hat{O}_{\rm eff} = (P + \hat{Q}_1 + \hat{Q}_1\hat{Q}_1 + \hat{Q}_2\hat{Q} + \hat{Q}\hat{Q}_2 + \cdots)(\chi_0 + +\chi_1 + \chi_2 + \cdots) \ ,$$

K. Suzuki and R. Okamoto, Prog. Theor. Phys. **93** *, 905 (1995)*

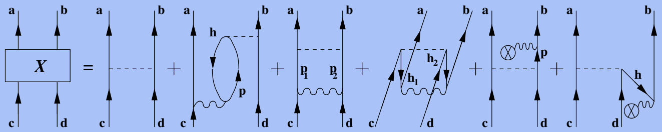
The shell-model effective operators

We arrest the χ series at χ_0, and expand it perturbatively:

One-body operator



Two-body operator



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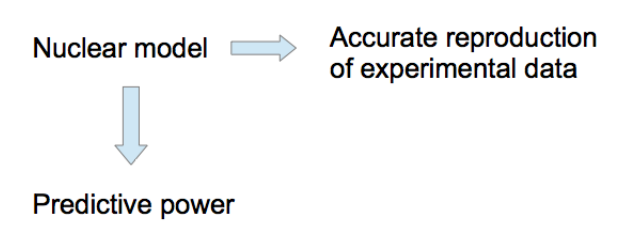
Nuclear models and predictive power



Realistic shell-model calculations for
^{130}Te, ^{136}Xe, ^{76}Ge and ^{82}Se

$\Downarrow$

Test our approach calculating observables related to the GT strengths and $2\nu\beta\beta$ decay and comparing the results with data.

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Realistic Shell-Model Calculations

- ^{76}Ge, ^{82}Se: four proton and neutron orbitals outside double-closed ^{56}Ni $\rightarrow 0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- ^{130}Te, ^{136}Xe: five proton and neutron orbitals outside double-closed ^{100}Sn $\rightarrow 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

PHYSICAL REVIEW C
covering nuclear physics

Volume 95, Issue 6
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HIGHLIGHTED ARTICLES

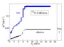
Editors' Suggestion

PDF HTML

Calculation of Gamow-Teller and two-neutrino double-β decay properties for ^{130}Te and ^{136}Xe with a realistic nucleon-nucleon potential

L. Coraggio, L. De Angelis, T. Fukui, A. Gargano, and N. Itaco

Phys. Rev. C **95**, 064324 (2017) – Published 23 June 2017



The authors tackle the important subject of nuclear matrix elements governing double-β decay in a first-principles shell-model calculation. They derive the shell-model effective interaction and the Gamow-Teller transition operator from a realistic nucleon-nucleon interaction. The procedure is tested on the two-neutrino double-β decays of ^{130}Te and ^{136}Xe for which experimental data exist. This test precedes an application to the neutrinoless double-β decay of the same nuclei.

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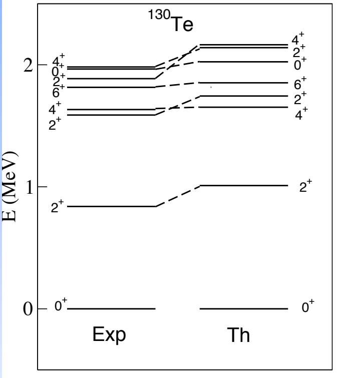
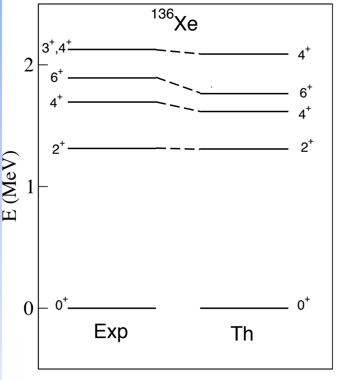
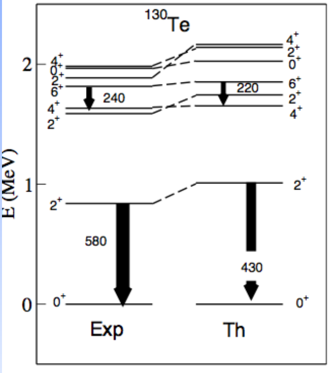
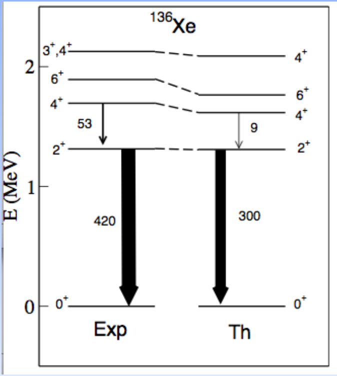
Spectroscopic properties: ^{100}Sn core



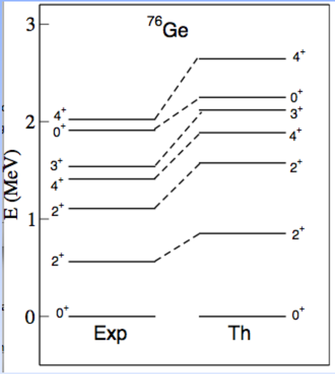


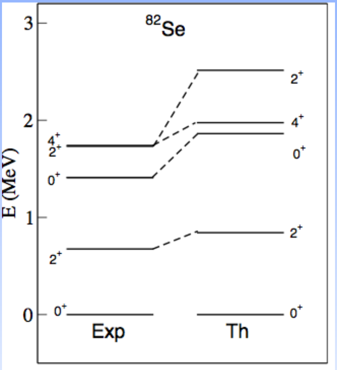
Spectroscopic properties: ^{100}Sn core (B(E2)s in e^2fm^4)





Spectroscopic properties: ^{56}Ni core





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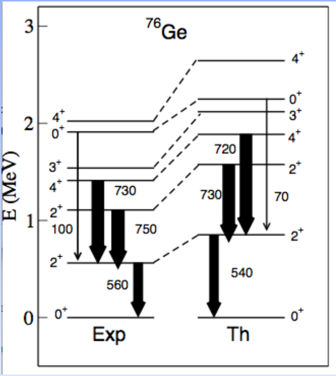

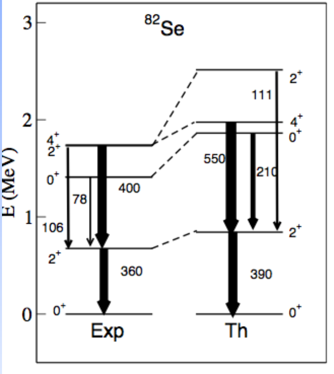

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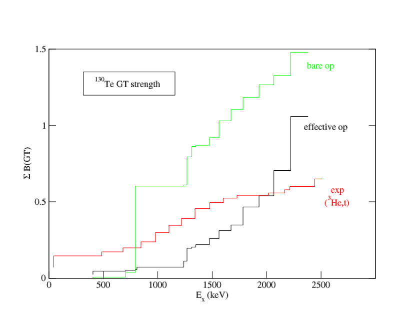
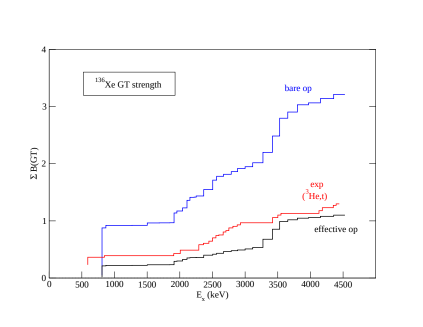
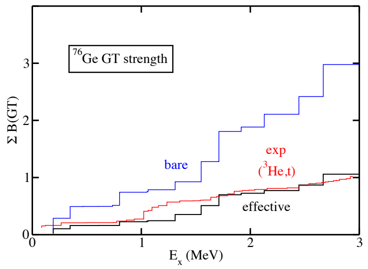
GT$^-$ strength distribution: ^{100}Sn core

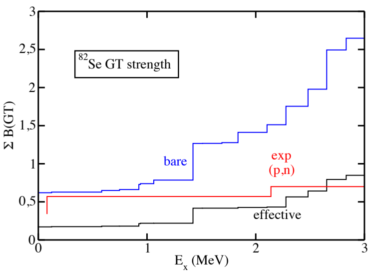




$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma} B_{exp}(GT) \Rightarrow B_{th}(\mathrm{GT}) = \frac{\left|\langle \Phi_f | \sum_j \vec{\sigma}_j \vec{\tau}_j | \Phi_i \rangle\right|^2}{2J_i + 1}$$

GT$^-$ strength distribution: ^{56}Ni core





$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma} B_{exp}(GT) \Rightarrow B_{th}(\mathrm{GT}) = \frac{\left|\langle \Phi_f | \sum_j \vec{\sigma}_j \vec{\tau}_j | \Phi_i \rangle\right|^2}{2J_i + 1}$$

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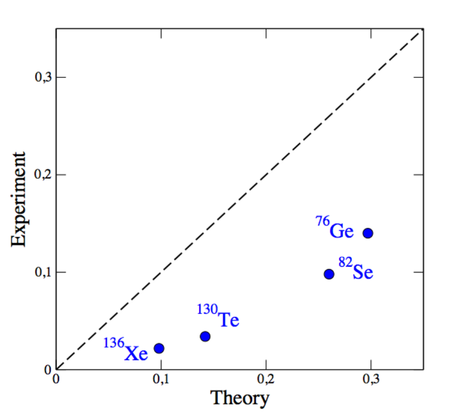
INFN Istituto Nazionale di Fisica Nucleare

$2\nu\beta\beta$ nuclear matrix elements

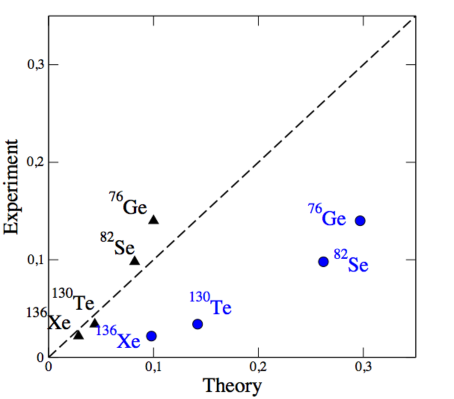
$$M_{2\nu}^{\rm GT} = \sum_n \frac{\langle 0_f^+ || \vec{\sigma}\tau^- || 1_n^+ \rangle \langle 1_n^+ || \vec{\sigma}\tau^- || 0_i^+ \rangle}{E_n + E_0}$$



Blue dots: bare GT operator

$2\nu\beta\beta$ nuclear matrix elements

$$M_{2\nu}^{\rm GT} = \sum_n \frac{\langle 0_f^+ || \vec{\sigma}\tau^- || 1_n^+ \rangle \langle 1_n^+ || \vec{\sigma}\tau^- || 0_i^+ \rangle}{E_n + E_0}$$



Blue dots: bare GT operator
Black triangles: effective GT operator

Conclusions and perspectives

- RSM calculations provide a satisfactory description of observed GT-strength distributions and $2\nu2\beta$ NME

Conclusions and perspectives

- RSM calculations provide a satisfactory description of observed GT-strength distributions and $2\nu2\beta$ NME

- $2\nu\beta\beta$
  - Role of real three-body forces and two-body currents (present collaboration with Pisa group)
  - Evaluation of the contribution of three-body correlations (blocking effect)
- $0\nu\beta\beta$
  - Derivation of the DGT effective operator to calculate the DGT strength distribution (experiments @ RNCP Osaka, RIBF RIKEN, LNS Catania)
  - Derivation of the two-body effective operator

Realistic Shell-Model Calculations for Double Beta-Decay

Nunzio Itaco

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Probing fundamental symmetries and interactions by low energy excitations with SPES RIBs