Modeling Magnetised Neutron Stars in General Relativity

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TEONGRAV - Nodo Firenze
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Neutron Stars

Gravity - General Relativity

High compactness:
$GM/Rc^2 \sim 0.1$ with
R~15km and $M \sim 1.5\ M_\odot$

Nuclear Physics

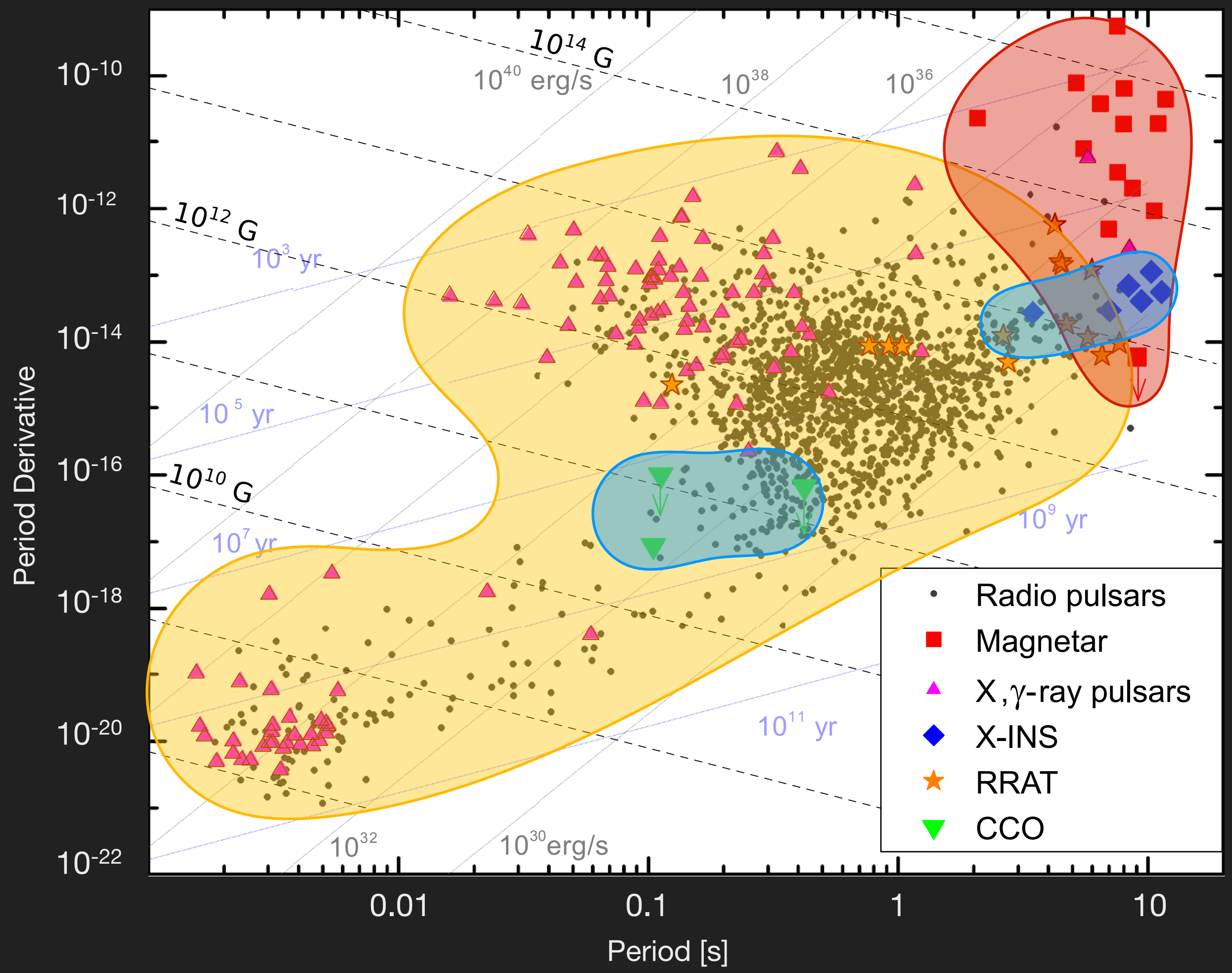
Unknown EoS:
supranuclear densities $\rho > 10^{14}$ g/cm^3

Condensed matter

Superfluidity
Superconductivity
Crust

High Energy Astrophysics

Fast rotation $P \sim 10^{-3} \div 10$ s
Strong B-field $> 10^9$G,
GRB, Pulsar Winds, GW
particle accelerator



Rotation powered

RadioPulsars, X / ɣ-ray pulsar, RRATs

Magnetic powered

Magnetars

Thermal powered

X-ray Isolated NSs, CCOs

Accretion powered

Low Mass X-ray Binaries
High Mass X-ray Binaries

Different manifestations of NSs:
- strength of the magnetic field
- morphology of the magnetic field

❖ Introduction

Magnetars

Slowly rotating young NS with
$B=10^{14}\div10^{15}$G @ surface

$$\begin{array}{lcl} P \sim 2-12\ \mathrm{s} & \multirow{2}{*}{$\Rightarrow$} & \tau \sim P/2\dot{P} \sim 10^4\ \mathrm{yr} \\ \dot{P} \sim 10^{-11} & & B \propto \sqrt{P\dot{P}} \sim 10^{14}\ \mathrm{G} \end{array}$$

High X-ray luminosity up to 10^{46} erg

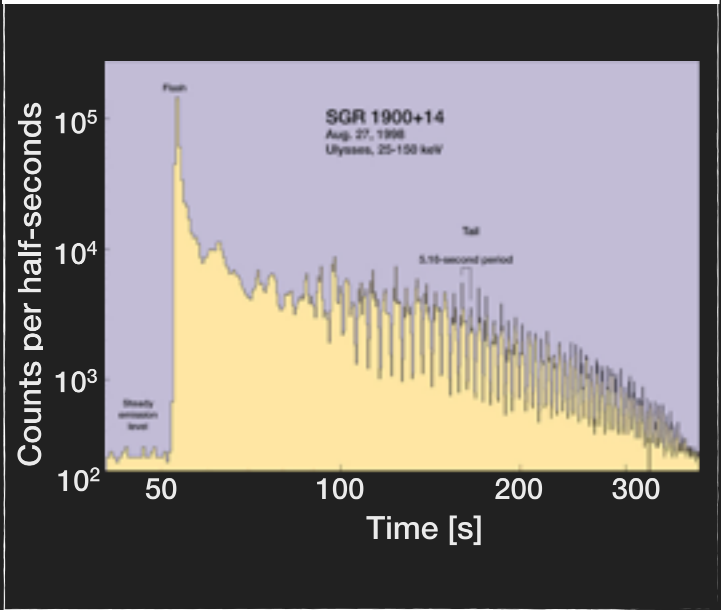
✘ rotational energy ~ 10^{44} erg
✔ magnetic energy ~10^{48} erg

Evolution and dissipation of the internal magnetic field

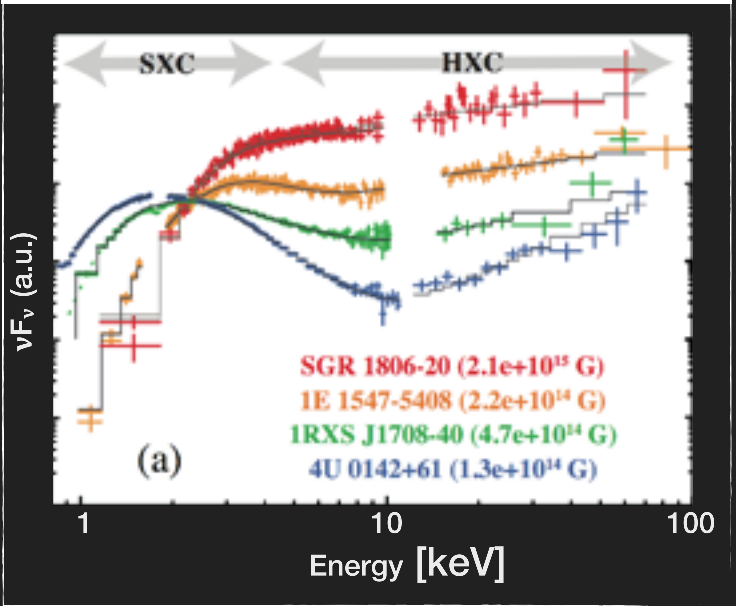
Phenomenology:

- steady state emission $L_X \sim 10^{35}$-10^{36} erg s^{-1}
- blackbody + high energy tail
- resonant cyclotron scattering
- bursting activity with L ~10^{33} - 10^{44} erg s^{-1} (in 0.1-100 s)
  - Short bursts L_x~10^{41} erg s^{-1}, few sec.
  - Giant Flares L_x >10^{44} erg s^{-1}, few min.

Bursting Activity

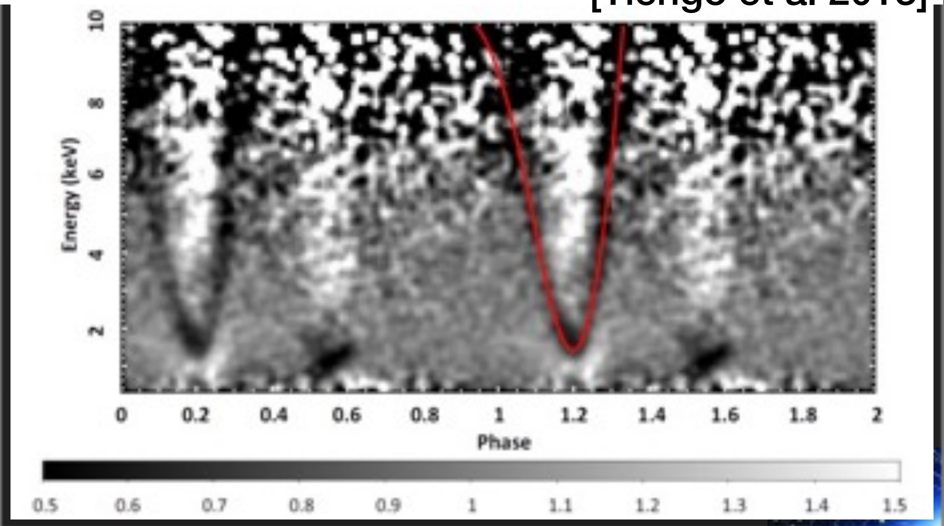


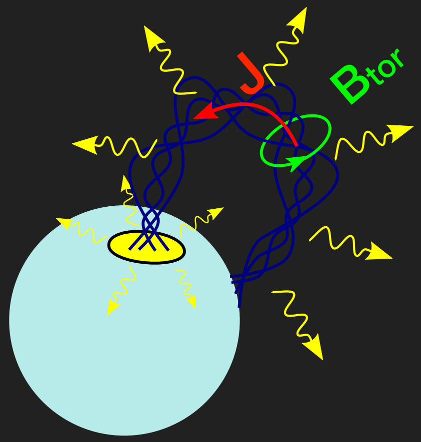
Persistent emission



Absorbtion Lines

[Tiengo et al 2013]





❖ Introduction

Why should we care?

Magnetospheric physics

Future X-ray polarimeters as XIPE and IXPE → detailed infos about magnetospheric environments (geometry of the magnetic field, currents characteristics)

$$B_Q = \frac{m^2}{\hbar e} \approx 4.4 \times 10^{13}\ \mathrm{G}$$

→ QED effects on photon propagation, i.e. vacuum polarisation [observed in a XINS, Mignani et al 2016]

Gravitational waves

Strong magnetic fields $\Longrightarrow$ large quadrupole deformation

ellipticity $\epsilon_B \propto B^2$ frequency $f_{GW} = 2/P$ strain $h \propto \frac{I\epsilon_B}{D} f_{GW}^2$

B~10^{16} G and rapid rotation at birth (P~1ms) → Emission of observable GWs within D~20 Mpc with aLigo & Virgo [Dall'Osso et al. 2009]

Gamma Ray Bursts

Most energetic explosion of modern Universe (~10^{50} erg in 0.1 -10 s)

- supernovae
- NS merging

→ Black Hole + accretion disk

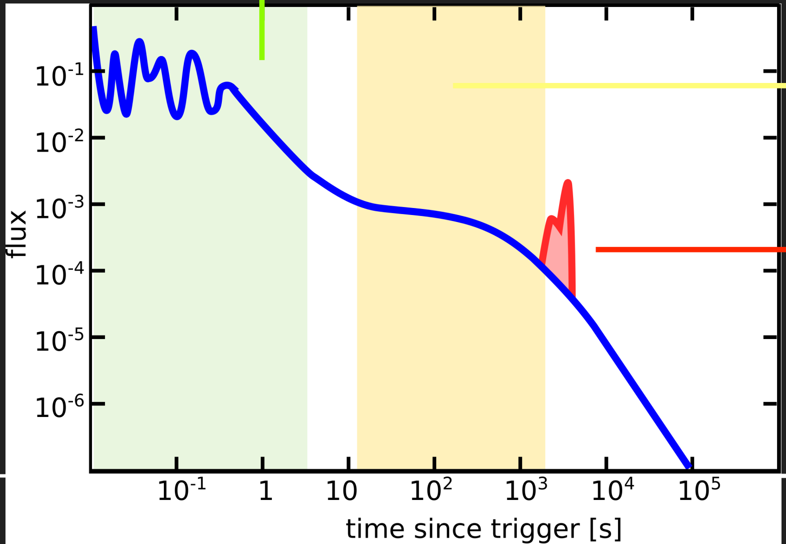
millisecond magnetar

❖ Introduction

The Birth of a Magnetar

Proto-Magnetar model for GRBs

[see e.g. Metzger et al 2011]



Prompt emission

- neutrino cooling drives baryon loaded wind
- high radiative efficiency of the jet

X-ray Plateau

- requires long lived central engine
- pulsar-like energy injection
- relativistic magnetically driven wind with ~10^{49}-10^{50} erg/s

Rebrightening

Reactivation of the inner engine

Magnetar signature:
extended activity
(X-ray plateau)
observed by SWIFT

Magnetic spin-down
VS
GW spin-down

$$\dot{\omega} = \frac{\boxed{K_{\rm d}}}{2}\omega^3 - \frac{\boxed{K_{\rm GW}}}{4}\omega^5$$

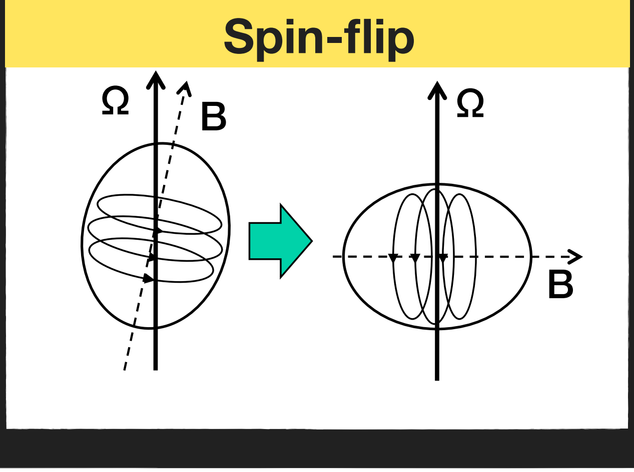
$$\boxed{K_{\rm d} = (B_{\rm d}^2 R^6)/3Ic^3}$$

$$\boxed{K_{\rm GW} = (2/5)f(\chi)(G/c^5)I\epsilon_B^2}$$

$\epsilon_B \propto B^2$

Gravitational Wave emission

- efficiency depends on the geometry of the B-field
- prolate deformation → dissipative mechanisms drives spin flip → maximize GWs emission

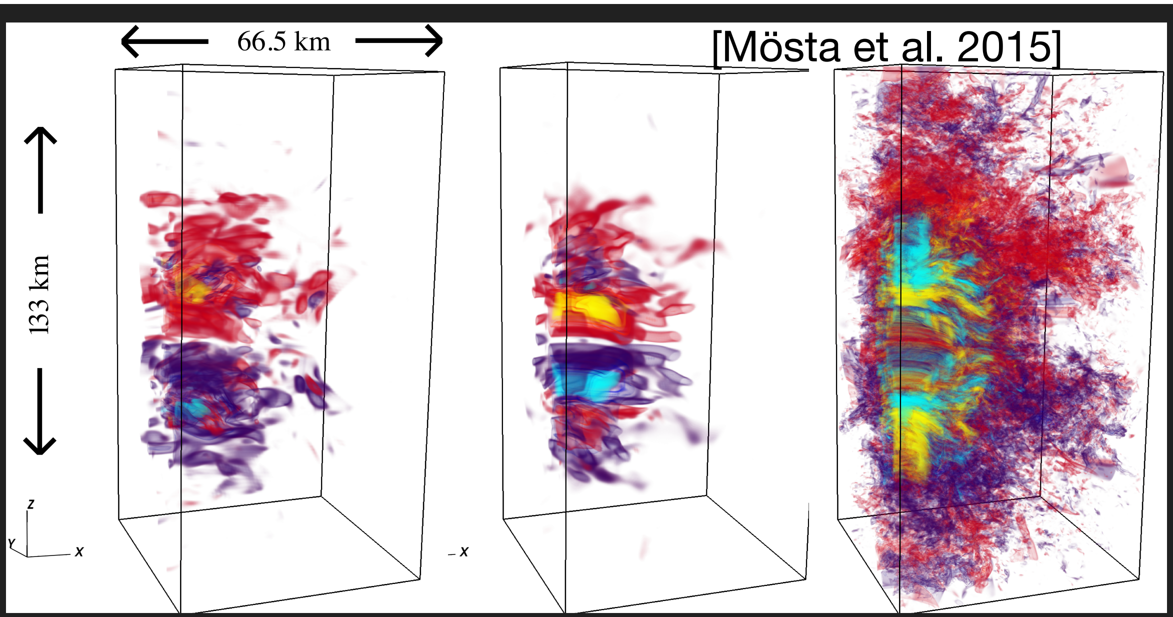


Cutler 2002

❖ Introduction

The morphology of the magnetic field

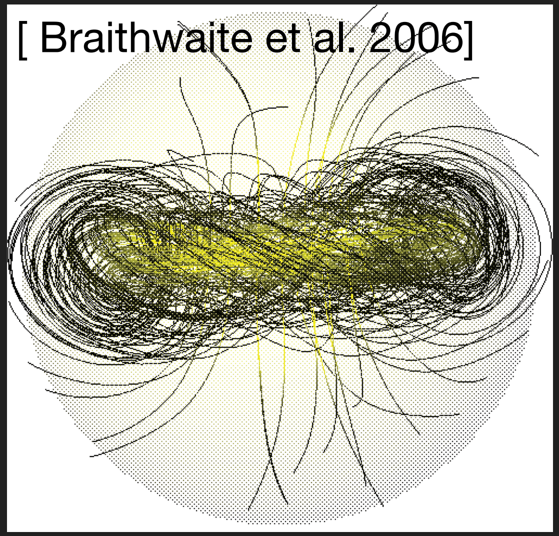
Dynamo



[Mösta et al. 2015]

- rapid differential rotation with P~10 ms
- large toroidal magnetic fields

Twisted Torus



[Braithwaite et al. 2006]

- relaxation of a random field with non vanishing helicity in a stratified star

- Alfvén crossing time ~ 0.1s @ B~10^{14}G
- Kelvin-Helmoltz ~ 100s

Equilibrium models of magnetized NSs

Magnetic field morphology
& currents distributions

↔

Properties of NS structure
(mass, radius, deformation)

- lack of a full parameter space investigation
- limited set of explored current distributions

- starting configuration for stability analysis
- computation of synthetic emission
- quasi-stationary evolution

❖ Mathematical Framework

Governing equations

NS ➜ ideal magnetised plasma (young NSs)

Einstein equation

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T^{\mu\nu} = [\rho(1+\epsilon)+p]u^{\mu}u^{\nu} + pg^{\mu\nu} + F^{\mu}{}_{\lambda}F^{\nu\lambda} - \frac{1}{4}(F^{\lambda\kappa}F_{\lambda\kappa})g^{\mu\nu}$$

Continuity equation

$$\nabla_{\mu}(\rho u^{\mu}) = 0$$

Energy conservation

$$\nabla_{\mu}T^{\mu\nu} = 0$$

Maxwell equations

$$\nabla_{\mu}\,{}^{*}F^{\mu\nu} = 0$$

$$\nabla_{\mu}F^{\mu\nu} = -j^{\nu}$$

GRMHD equations

Euler equation $\rho h a_{\mu} + u_{\mu}u^{\nu}\partial_{\nu}p + \partial_{\mu}p = L_{\mu}$

3+1 formalism

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- α Lapse function - β Shift Vector

Notation

- ρ baryonic density
- ϵ specific internal energy
- p pressure
- $F^{\mu\nu}$ Faraday tensor
- j^{ν} 4-current
- L_{μ} Lorentz force

❖ Mathematical Framework

Governing Equations

- Basic requirements: **Stationarity** ($\partial_t=0$) and **Axisymmetry** ($\partial_\varphi=0$)

CFC Approximation

- Conformally flat metric

  [e.g. Wilson & Mathews 2003]

$$ds^2 = -\alpha^2 dt^2 + \underbrace{\psi^4}_{\text{conformal factor}}\underbrace{[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\,(d\phi - \omega dt)^2]}_{\text{flat metric}}$$

- Einstein equation reduces to a set of 8 PDEs
- Hierarchical scheme
- Numerically stable form
- Consistency with full GR (10^{-4}) even at mass shedding and up to B~10^{19}G

[Oron 2002, Bucciantini et al. 2011, Pili et al 2014]

E.M. field

$$\nabla\cdot\mathbf{B}=0,$$
$$\nabla\times(\alpha\mathbf{E}+\beta\times\mathbf{B})=0,$$
$$\nabla\cdot\mathbf{E}=\rho_e,$$
$$\nabla\times(\alpha\mathbf{B}-\beta\times\mathbf{E})=\alpha\mathbf{J}-\rho_e\beta.$$

$$\mathbf{B}=\frac{\nabla\Psi}{R}\times\mathbf{e}_{\hat\varphi}+\frac{\mathcal{I}}{\alpha R}\mathbf{e}_{\hat\varphi}$$
$$\mathbf{E}=\frac{\nabla\Phi+\omega\nabla\Psi}{\alpha}$$
with $R = \psi^2 r\sin\theta$

Φ - electric potential

Ψ - magnetic flux function

$\mathcal{I}$ - toroidal magnetization

Ω - NS rotation at spatial ∞

MHD / force free regime $\boxed{\mathbf{E}=-\mathbf{v}\times\mathbf{B}}$ $\xrightarrow[v^\phi=\frac{\Omega-\omega}{\alpha}]{\text{Pure rotational flow}}$ $\boxed{\partial_i\Phi=-\Omega\partial_i\Psi}$

Stationarity with arbitrary B-field ⇒ Ω=const $\longrightarrow$ $\boxed{\Phi=-\Omega\Psi+C}$

❖ **Mathematical Framework**

Governing Equations

Euler equation → $\frac{\partial_i p}{\rho h} + \partial_i \ln \alpha - \partial_i \ln \Gamma = \frac{L_i}{\rho h}$

- Integrability condition (Lorentz force) → $L_i = \rho h\, \partial_i \mathcal{M}$ Magnetisation function
- Barotropicity: $p = p(\rho)$ ⟶ $\frac{\partial_i p}{\rho h} = \partial_i \ln h$

  i.e. polytropic:

  $p = K_a \rho^{1+1/n}$

  $h = 1 + (1+n) K_a \rho^{1/n}$

$$\ln \frac{h}{h_c} + \ln \frac{\alpha}{\alpha_c} - \ln \frac{\Gamma}{\Gamma_c} = \mathcal{M} - \mathcal{M}_c$$

c - quantities evaluated at the center of the star

Lorentz factor: $\Gamma = (1 - v_\phi v^\phi)^{-1/2}$

$\mathbf{B} \cdot \mathbf{L} = 0$ (from MHD condition $\mathbf{E} \cdot \mathbf{B} = 0$) $\Rightarrow$ $\mathcal{M} = \mathcal{M}(\Psi)$

$L_\phi \propto (\mathbf{D}\mathcal{I} \times \mathbf{D}\Psi)_\phi = 0$ (from axisymmetry) $\Rightarrow$ $\mathcal{I} = \mathcal{I}(\Psi)$

Grad-Shafranov Eq.

$$\nabla \cdot \left[\frac{\alpha}{R^2} \left(1 - v^2\right) \nabla \Psi \right] + \frac{\mathcal{I}}{\alpha R^2} \frac{d\mathcal{I}}{d\Psi} + \alpha \rho h \frac{d\mathcal{M}}{d\Psi} = 0$$

Force-free limit ρ→0

$$\nabla \cdot \left[\frac{\alpha}{R^2} \underline{\left(1 - v^2\right)} \nabla \Psi \right] + \frac{\mathcal{I}}{\alpha R^2} \frac{d\mathcal{I}}{d\Psi} = 0$$

Light Cylinder v=1: $R = R_L = \alpha^{-1}(\Omega - \omega)$

require an additional boundary condition

The XNS code

Equations to solve:

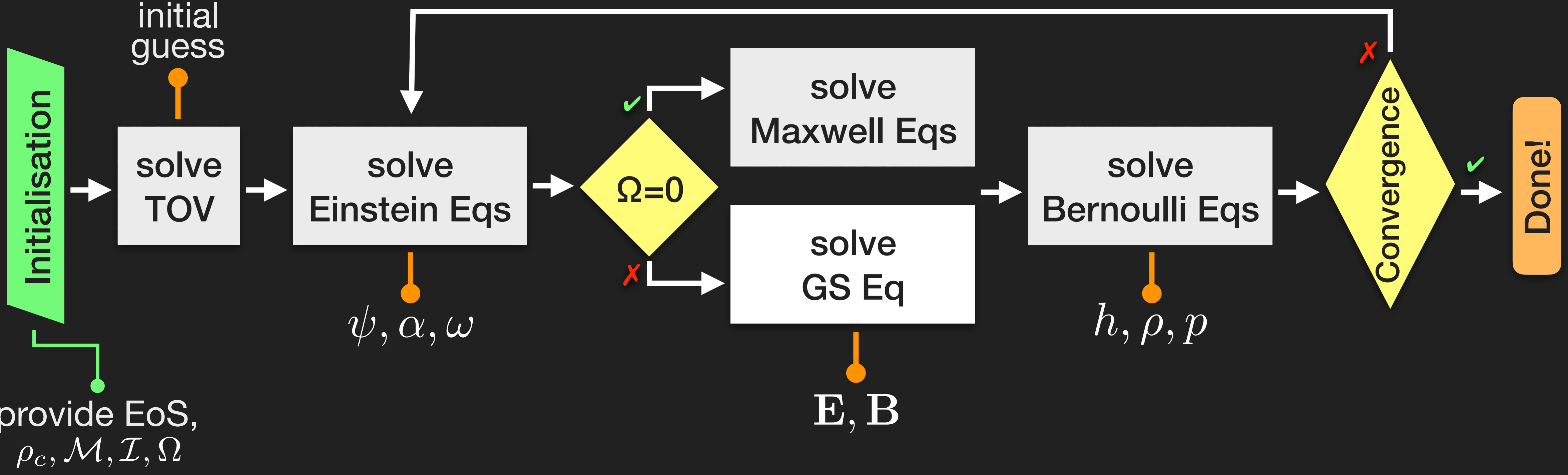
$$\Delta q = h q^p$$

$$\Delta_* X_\phi = H_\phi$$

Semi-spectral method

$$q(r,\theta) = \sum_{l=0}^{\infty} A_l Y_l(\theta) \qquad X_\phi(r,\theta) = \sum_{l=0}^{\infty} [C_l(r) Y_l'(\theta)]$$

- II order radial discretisation → direct inversion of tridiagonal matrices



Initialisation

initial guess

solve TOV

solve Einstein Eqs

ψ, α, ω

Ω=0

✓ solve Maxwell Eqs

✗ solve GS Eq

$\mathbf{E}, \mathbf{B}$

solve Bernoulli Eqs

h, ρ, p

Convergence

✗

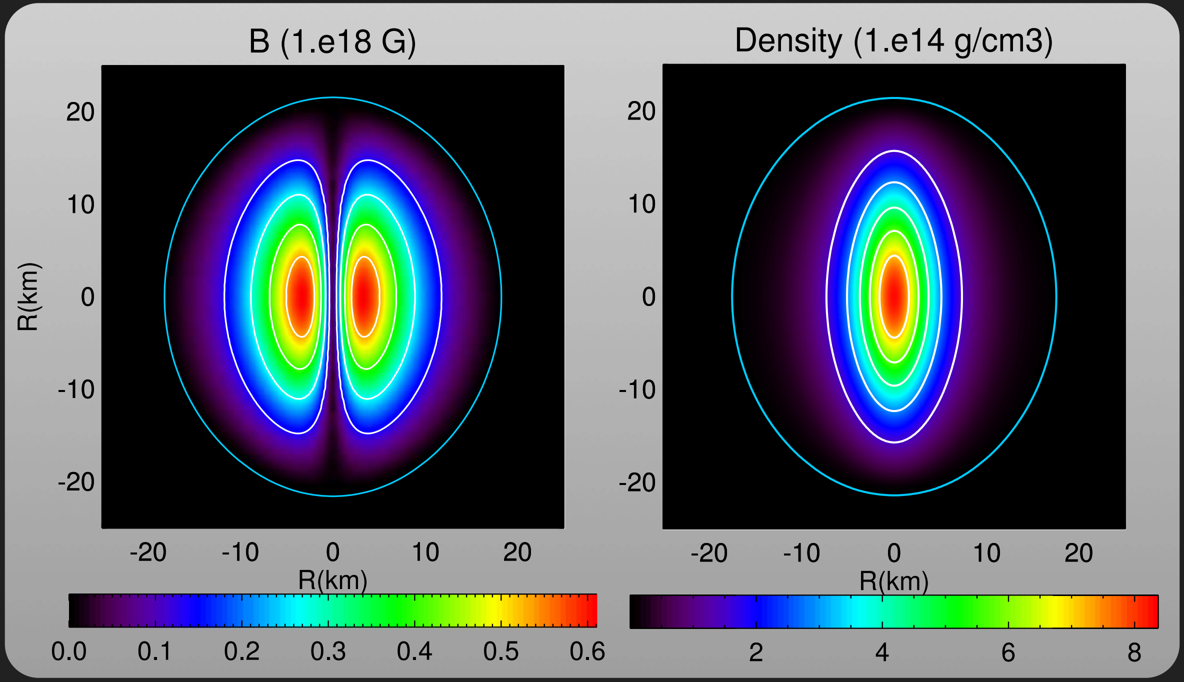
✓ Done!

provide EoS,
$\rho_c, \mathcal{M}, \mathcal{I}, \Omega$

freely available at:
www.arcetri.astro.it/science/ahead/XNS/

❖ Equilibrium models

Toroidal Magnetic fields



$$\mathcal{I} = K_m(\alpha^2 R^2 \rho h)^m$$

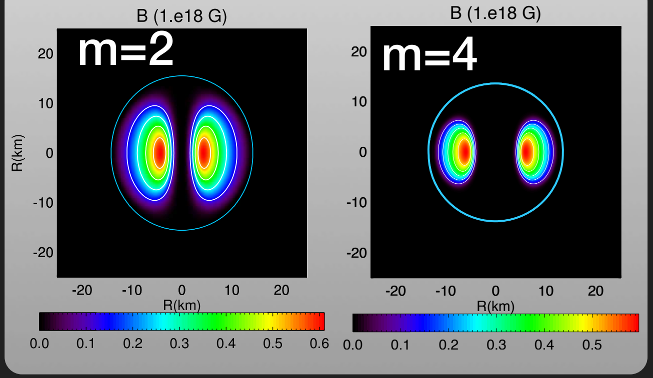
$$\mathcal{M} = -\frac{mK_m^2}{2m-1}(\alpha^2 R^2 \rho h)^{2m-1}$$

K_m - magnetisation constant

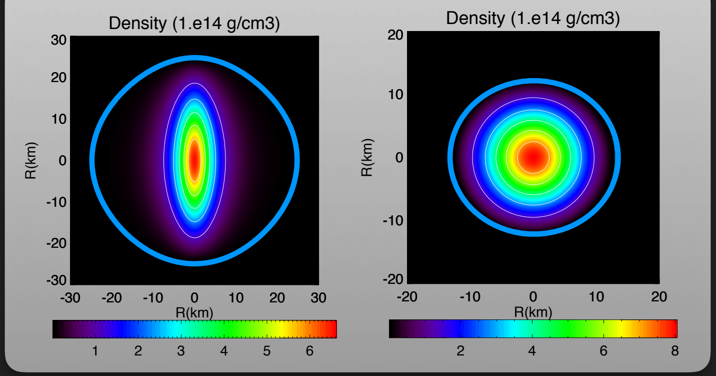
m - magnetisation index

- prolate deformation
- inflation of low-density outer layers
- maximum B_{max} allowed @ fixed mass
- m regulates the distribution of currents

current distribution



adding rotation



Surface ellipticity

- r_e equatorial radius
- r_p polar radius

$$e_s = \frac{r_e}{r_p} - 1$$

Mean deformation

- $\mathcal{I}_{zz}, \mathcal{I}_{xx}$ moments of inertia

$$\bar{e} = \frac{\mathcal{I}_{zz} - \mathcal{I}_{xx}}{\mathcal{I}_{zz}}$$

Quadrupole deformation

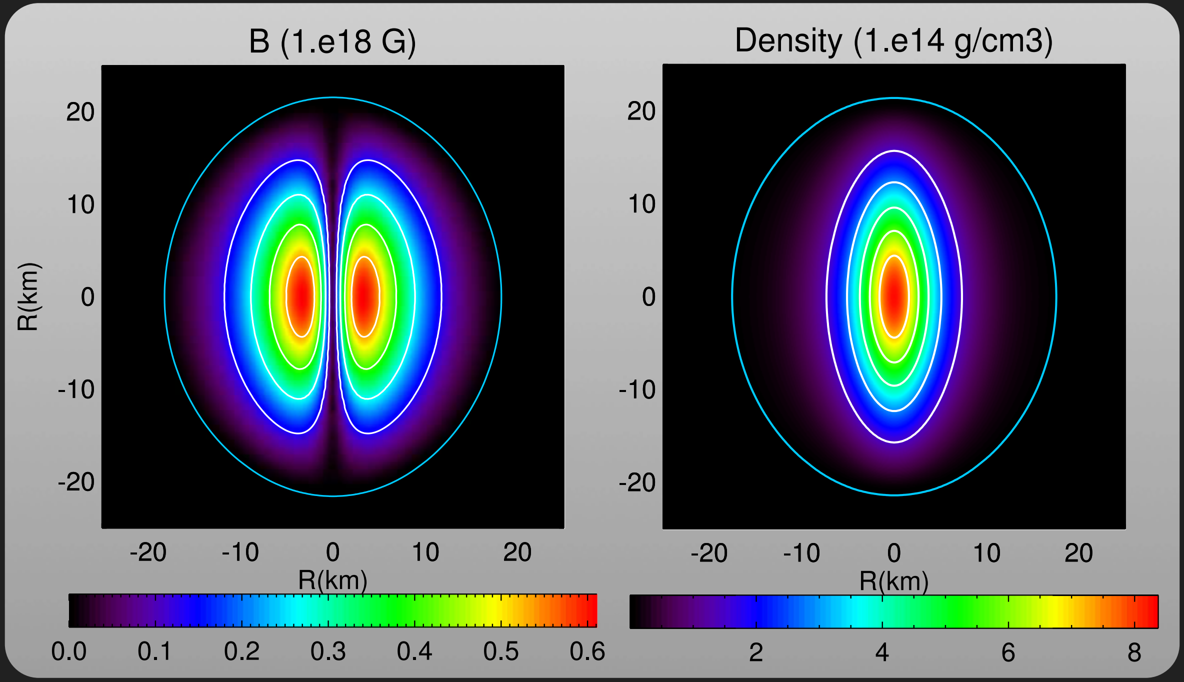
- $\mathcal{I}_{zz}$ quadrupole moment
- $\mathcal{J}$ angular momentum

$$e_q = \frac{3}{2}\frac{\mathcal{I}_{zz}}{\mathcal{J}/\Omega}$$

$$e_q \sim 0.5\bar{e}$$

❖ Equilibrium models

Toroidal Magnetic fields



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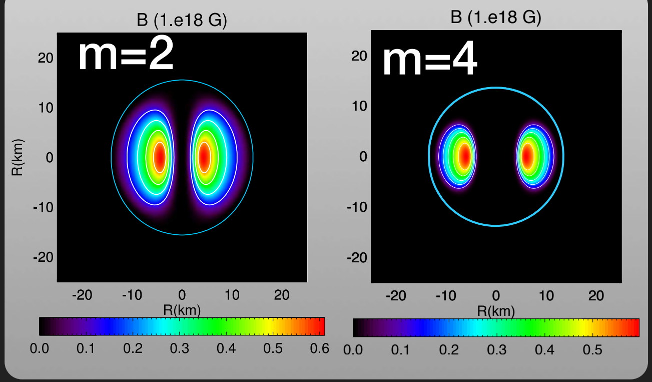
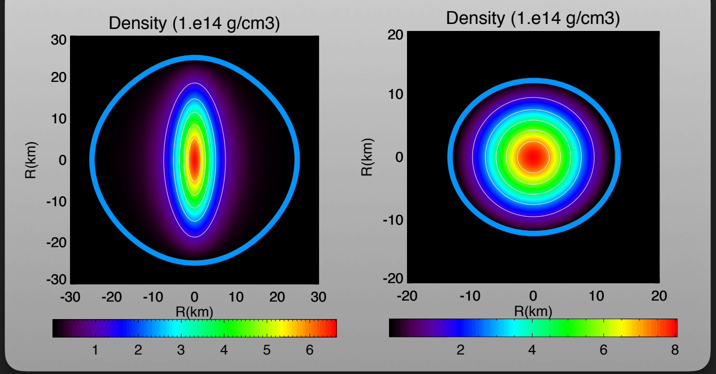
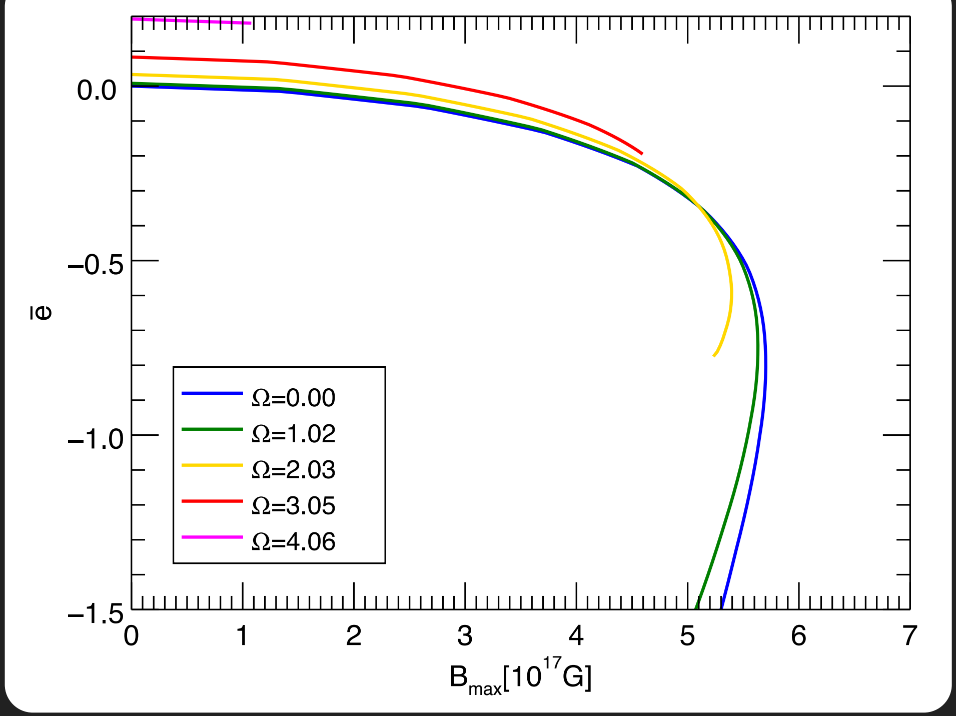


adding rotation





Bilinear regime: B≲ $2\text{x}10^{17}$G, Ω≲$3\text{x}10^{3}\text{s}^{-1}$ →

$$\bar{e} \simeq -\frac{d_B}{m} B_{17}^2 + d_\Omega\, \Omega_{\text{ms}}^2 \qquad e_s \simeq -\frac{s_B}{m} B_{17}^2 + s_\Omega\, \Omega_{\text{ms}}^2$$

@M=1.55M_{sun} : d_B=$9.4\text{x}10^{-3}$ d_Ω=0.31

❖ Equilibrium models

Global Relations

H - magnetic energy

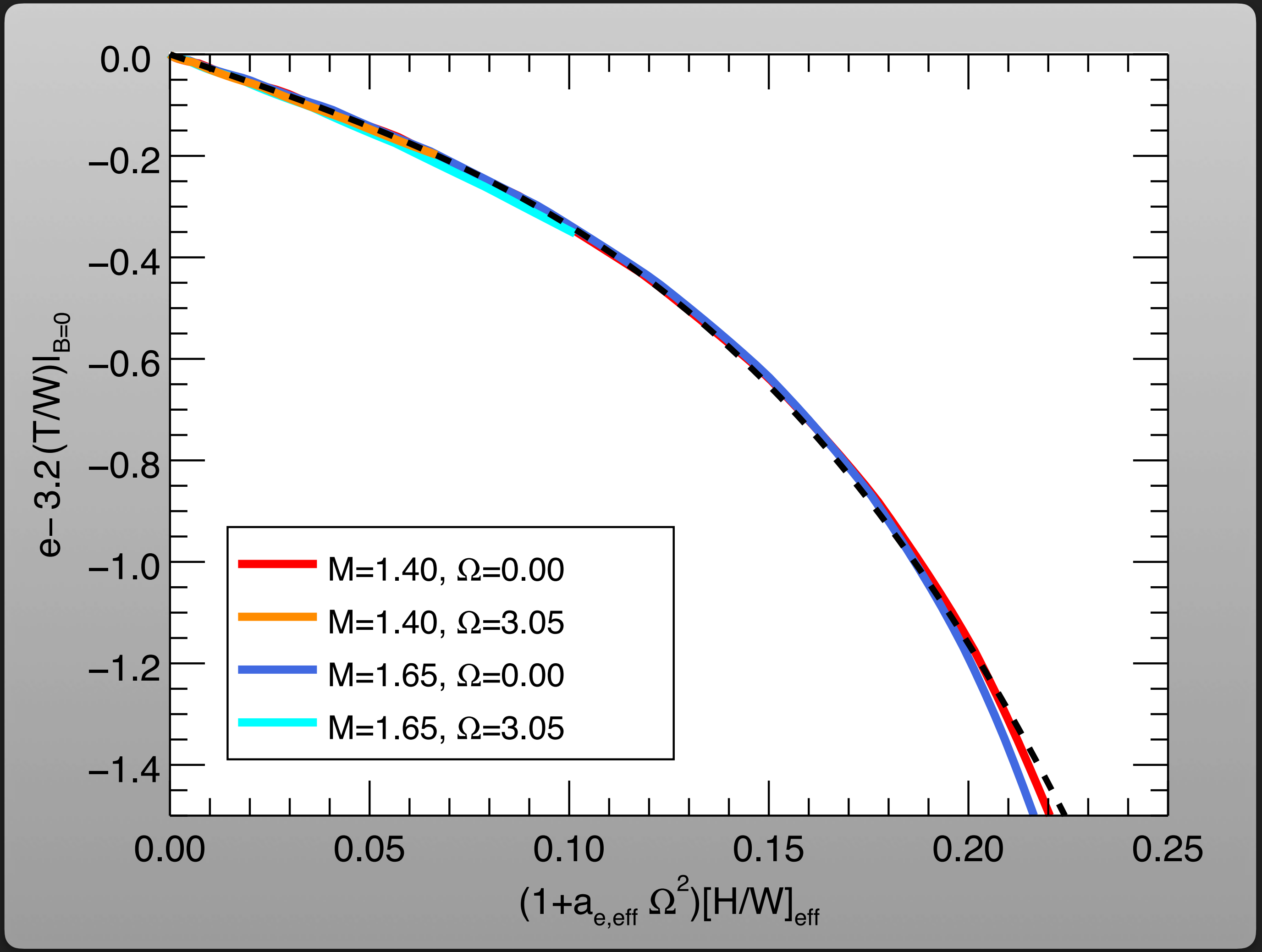
T - rotational energy

W - gravitational binding energy

Global relation

- $\left[\frac{H}{W}\right]_{\mathrm{eff}} = \left[0.84 + \frac{0.16}{m}\right] \frac{1.55\mathrm{M}_\odot}{M} \frac{H}{W}$
- $a_{\bar{e}\ \mathrm{eff}} = -\left(3.94 - 2.98 \frac{M}{1.55\mathrm{M}_\odot}\right)$

$$\boxed{\begin{gathered}\bar{e} \simeq 3.2 \frac{T}{W}\bigg|_{B=0} + \mathcal{F}\left(\left[1 + a_{\bar{e},\mathrm{eff}}\Omega^2_{\mathrm{ms}}\right]\left[\frac{H}{W}\right]_{\mathrm{eff}}\right) \\ \text{with} \quad \mathcal{F}(x) = -2.71x - 0.068(10x)^{3.2}\end{gathered}}$$



Bilinear regime (H→0, T→0)

$C_{\bar{e}}$, K_{e_s} : EoS dependency

W_0 = W(H→0, T→0)

$$\bar{e} \simeq \frac{C_{\bar{e}}}{W_0}\left[T - 1.3\frac{H}{M/\mathrm{M}_\odot}\right] \qquad e_s \simeq \frac{K_{e_s}}{W_0}\left[T - 0.23\frac{H}{M/\mathrm{M}_\odot}\right]$$

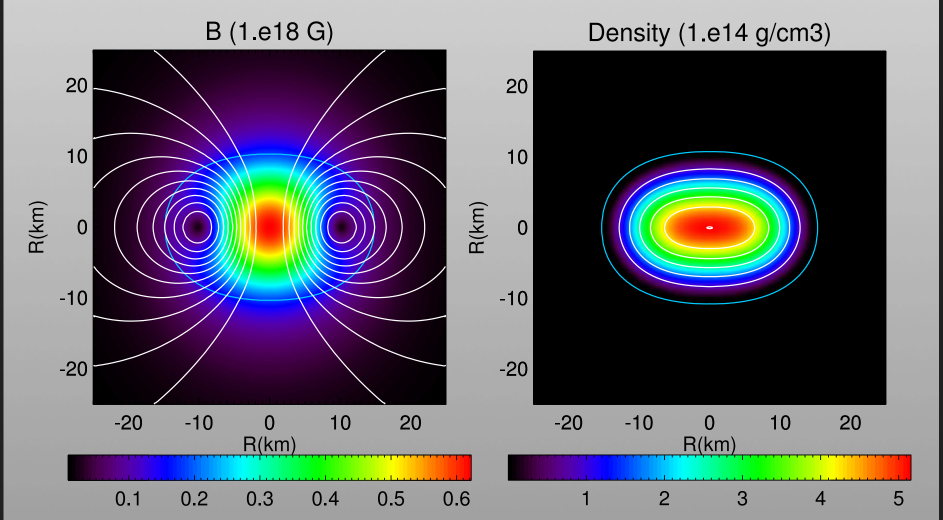
@ M=1.4 M_{Sun} same deformation coefficients by Frieben & Rezzolla 2012

❖ Equilibrium models

Purely Poloidal B-fields

$$\mathcal{M}(\Psi) = k_{\rm pol}\Psi\left(1 + \frac{\xi}{\nu+1}\Psi^{\nu}\right) \qquad \mathcal{I} = 0$$

k_{pol} - magnetisation
ν - magnetisation index
ξ - non-linear current



Pili et al. 2014

Bilinear regime $B \lesssim 2\text{x}10^{17}$G ($\mu \lesssim 2\text{x}10^{34}$erg/G), $\Omega \lesssim 3\text{x}10^{3}\text{s}^{-1}$

$$\bar{e} = d_\Omega\,\Omega^2_{\rm ms} + d_{\rm B}\,B^2_{17}$$

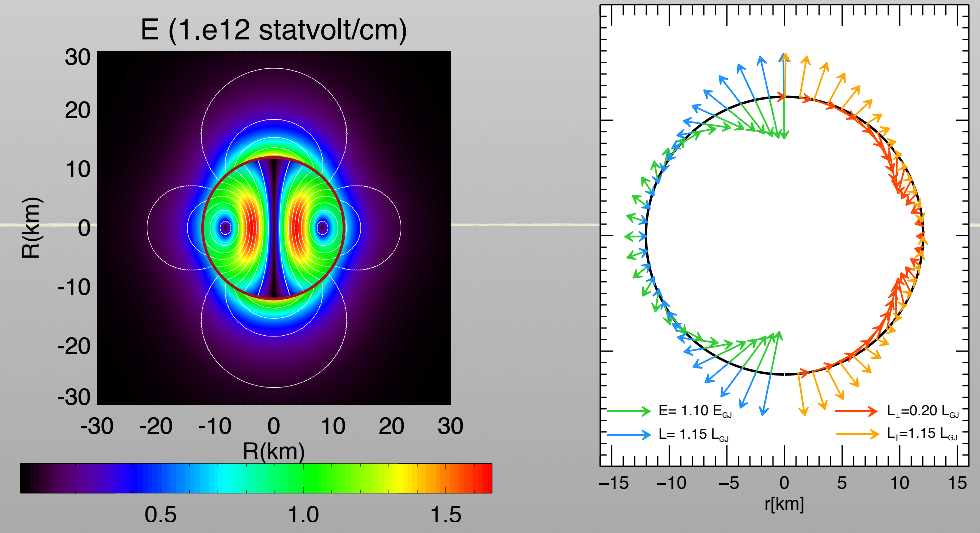
$$\bar{e} = d_\Omega\,\Omega^2_{\rm ms} + d_\mu\,\mu^2_{35}$$

d_B= 5.4x10^{-3},
d_μ=0.14, d_Ω=0.3

Adding rotation:

- Inside the star : $\Phi = -\Omega\Psi + C$ C sets the global electric charge

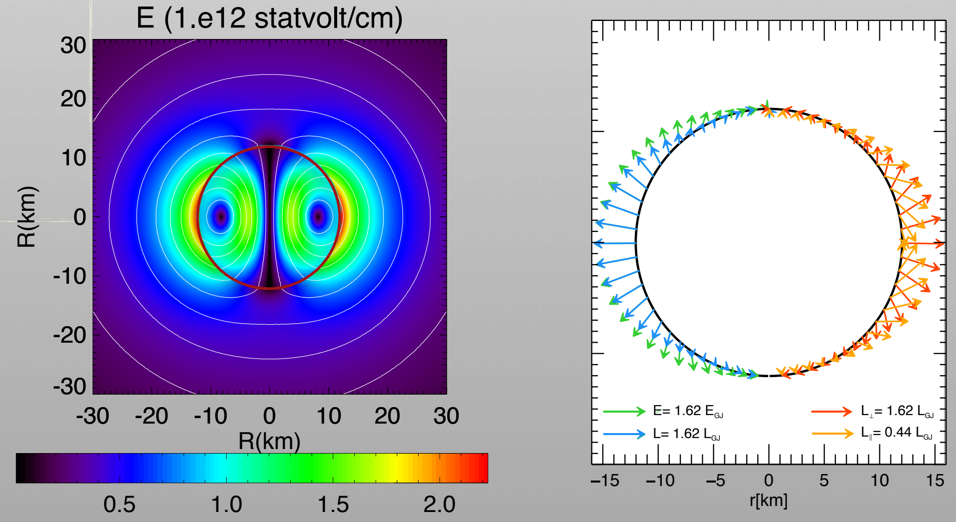
Globally neutral NS



- surface charge density < 0 ($\Omega \,/\!/\, B$)
- extraction of e$^-$ from polar caps
- prolate crustal stresses

$$\boxed{\sigma_e = E^r_{\rm out} - E^r_{\rm in}}$$

Vanishing polar electric field (Q=3x10^{24} statC)



- surface charge density > 0 ($\Omega \,/\!/\, B$)
- extraction of e$^-$ from the equatorial region
- oblate crustal stresses

Pili et al. 2017

Crust bending due
to the Lorentz force

f_r f_θ component of Lorentz force
$\Delta\rho$ g buoyancy
σ related to Young modulus E_Y

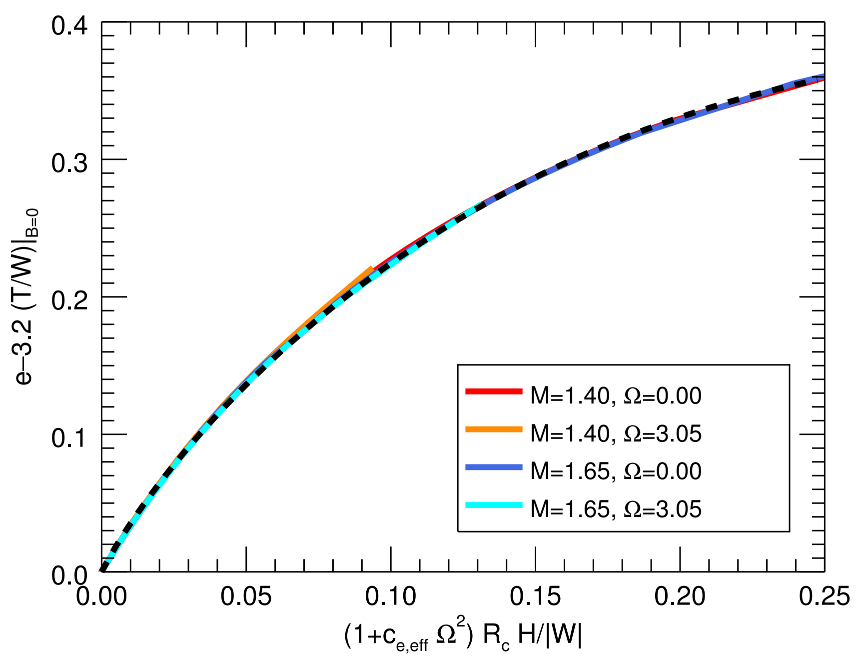
→

$\rho=10^{11}$g/cm^2, $E_{Young}=10^{28}$erg cm^{-3}

- $\Delta e_s \sim 10^{-9} \ll 8\text{x}10^{-6}$
- $\Delta\bar{e} \sim 10^{-11} \ll 1\text{x}10^{-5}$

❖ Equilibrium models

Global Relations and Current distribution



Global relation

- $\left[\frac{H}{W}\right]_{\rm eff} = \frac{1.55 M_\odot}{M}\frac{H}{W}$
- $c_{\bar{e},{\rm eff}} = -3.6 + 3.0\frac{M}{1.55{\rm M}_\odot}$

$$\bar{e} \simeq 3.2\frac{T}{W}\Big|_{B=0} + \mathcal{G}\left(\left[1 + c_{\bar{e},{\rm eff}}\Omega_{\rm ms}^2\right]\frac{H}{W}R_{14}\right)$$

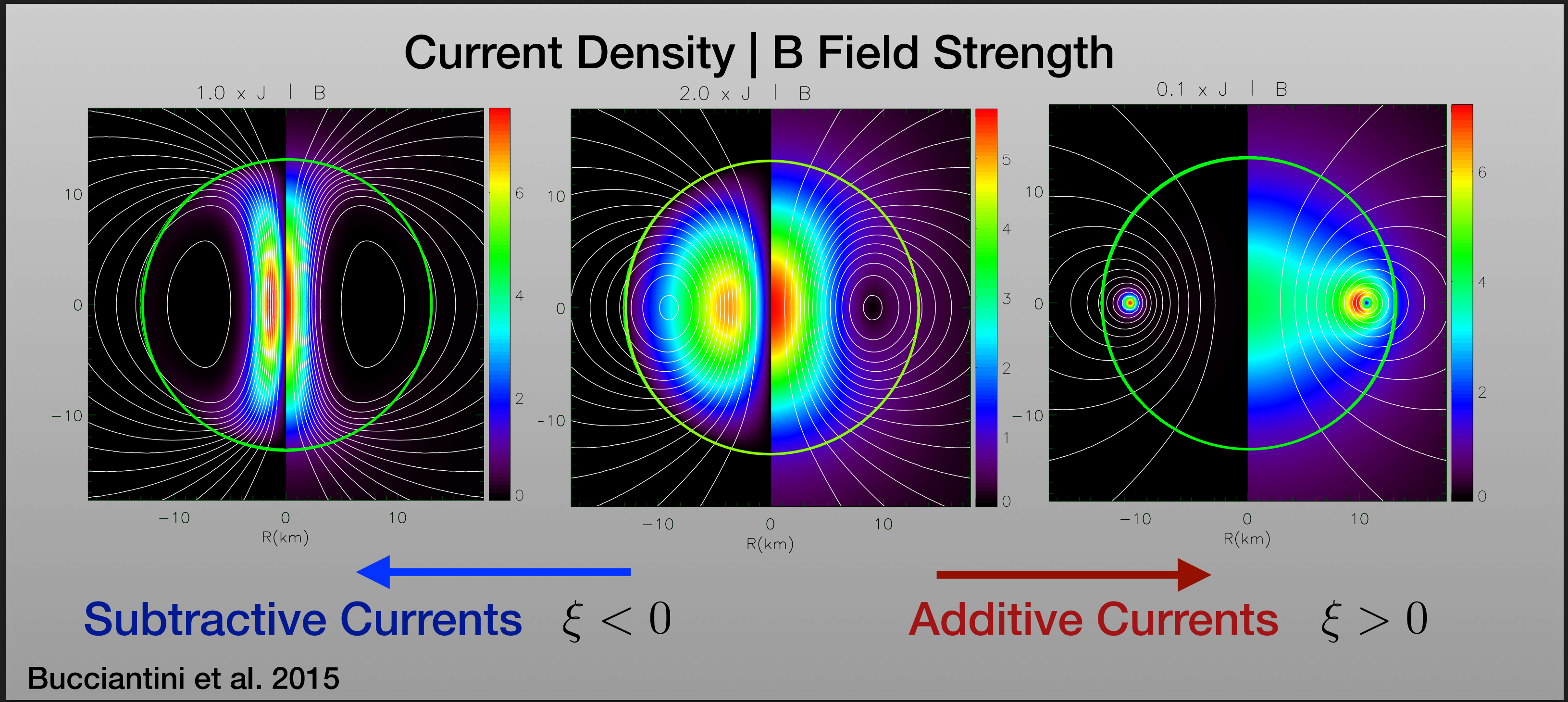
$$\mathcal{G} = 4.8\,x - 5.1\,x^{1.3}$$

Bilinear regime (H→0, T→0)

$C_{\bar{e}}$, K_{es} : EoS dependency

W_0 = W(H→0, T→0)

$$\bar{e} = \frac{C_{\bar{e}}}{W_o}[T + 1.8\frac{H}{M/M_\odot}]$$



Current Density | B Field Strength

Subtractive Currents $\xi < 0$

Additive Currents $\xi > 0$

Bucciantini et al. 2015

Induced deformation

$\bar{e} \simeq \mathcal{F}\left([1 + a_\xi \xi]\frac{H}{W}\right)$ $\bar{e} = [1 + d_\xi\,\xi]\,d_{\rm B}B_{17}^2$

where $a_\xi = -2.8 \times 10^{-3}$, $d_\xi = 4.1 \times 10^{-3}$

difficult to parametrise the effects of concentrated current distributions

❖ Mathematical Framework

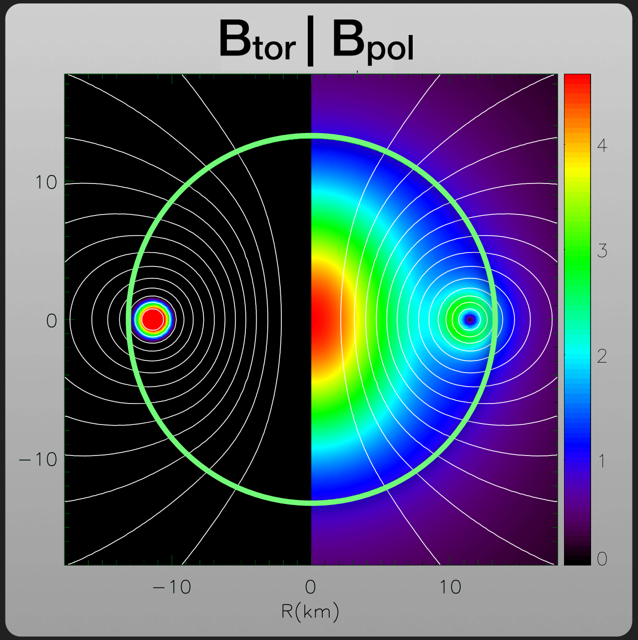
Mixed Field- TT configurations

Stability criterion in stratified star $0.2 \leq \frac{H_{\rm tor}}{H} \leq 0.8$

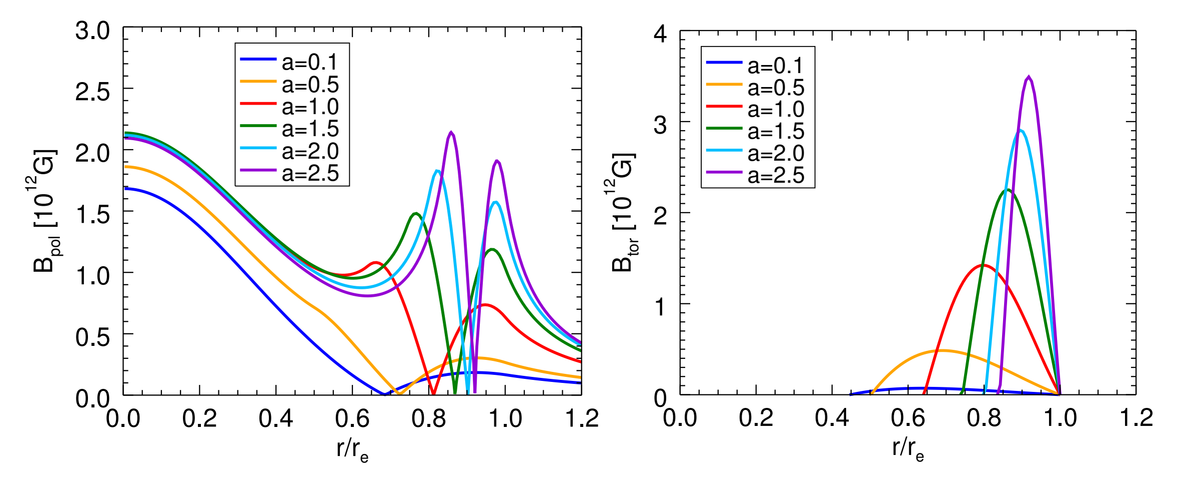
$$\mathcal{M}(\Psi) = k_{\rm pol}\Psi\left(1 + \frac{\xi}{\nu+1}\Psi^{\nu}\right) \qquad \mathcal{I} = \frac{a}{\zeta+1}\Theta[\Psi - \Psi_{\rm sur}](\Psi - \Psi_{\rm sur})^{\zeta+1}$$

a - TT magnetisation

ζ - magnetisation index

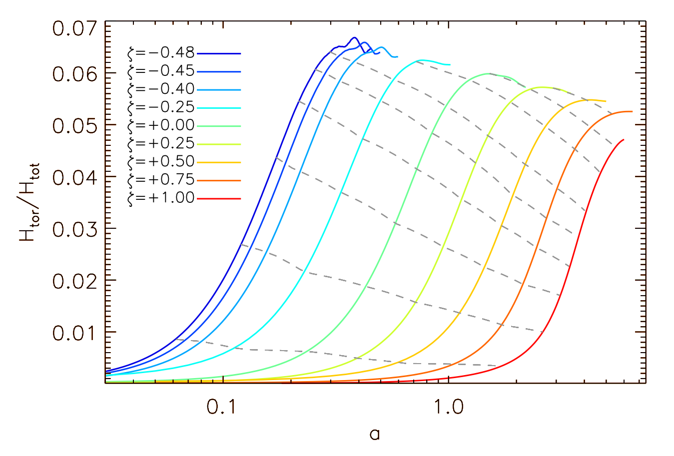

B_{tor} | B_{pol}

Trends of B_{pol} and B_{tor} with a



Higher B-field ⇒smaller toroidal loop

Magnetic energy ratio



⇒saturation of H_{tor}/H ~0.1

⇒oblate deformation

Oppositely flowing currents might allow for toroidal dominated configurations [Ciolfi et al. 2013, Fujisawa et al. 2013,2015]

- hard to achieve current inversion
- boundary conditions @ surface?
- stellar stratification?

Comparing deformations

deformations induced by purely poloidal and purely toroidal B
cannot be easily combined in TT

- integrability conditions → link between currents
- H_{tor}/H_{tot}<10% → 40% reduction of oblateness
- dependency on the current distribution

❖ Mathematical Framework

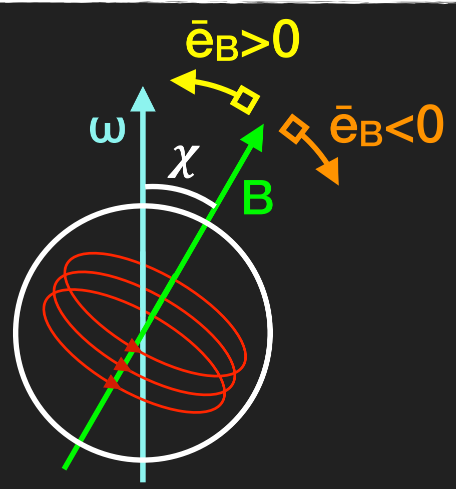
Toy Gravitational Waves

Spin-flip

Evolution of the wobble angle due to dissipative processes

$\bar{e}_B>0$ → parallel rotator

$\bar{e}_B<0$ → orthogonal rotator (maximize GW emission)



$$E_{\rm rot} = \frac{J^2}{2I_0}\left(1 - \epsilon_\Omega - \epsilon_B \cos^2\chi\right)$$

- Spin-down

$$\dot\omega = -K_d\omega^3 - K_{GW}\omega^5$$

- Evolution of χ

$$\dot\chi = (\tan\chi\,\tau_{diss})^{-1} \qquad \dot\chi = -\tan\chi(\tau_{diss})^{-1}$$

$\bar{e}_B<0$ $\bar{e}_B>0$

$$\tau_{diss} = 2E_{pre}/\dot{E}_{diss}$$

$$E_{prec} \propto IP^{-2}\bar{e}_B f(\chi)$$

$$\dot{E}_{diss} \propto T^6P^{-4}R_{NS}^3 f(\chi)$$

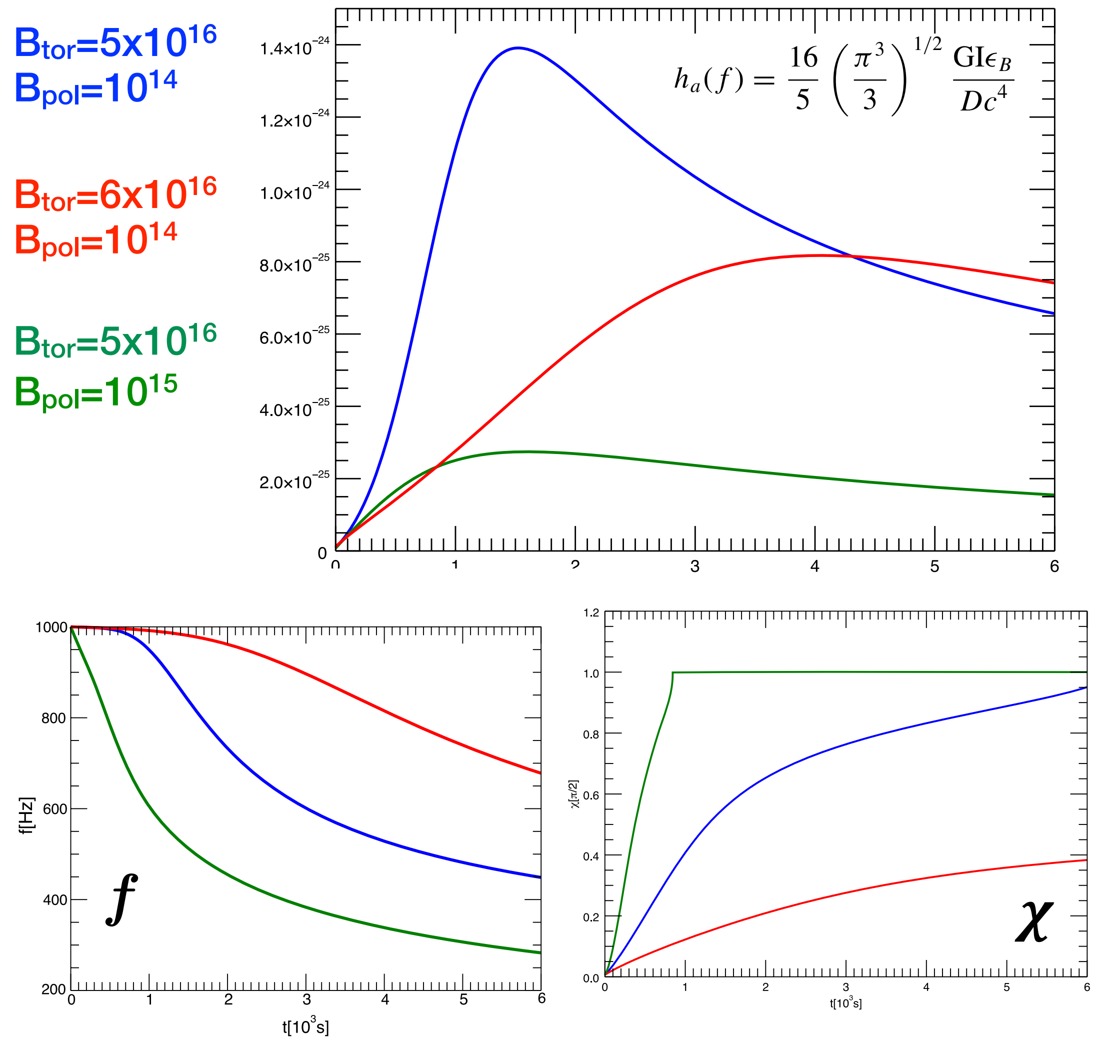
E_{prec} precession energy

$\dot{E}_{diss}$ viscosity dumping rate

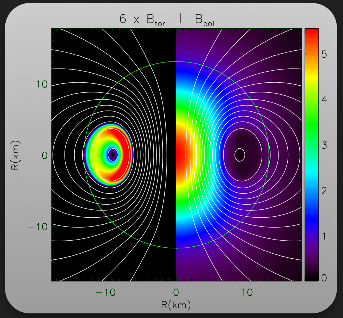
Dall'Osso et al. 2009

$$\bar{e}_B = -d_B^{tor}B_{tor}^2 + d_B^{pol}B_{pol}^2$$

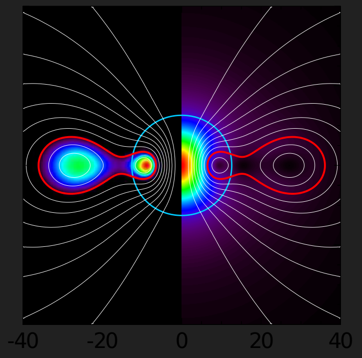
Strain amplitude



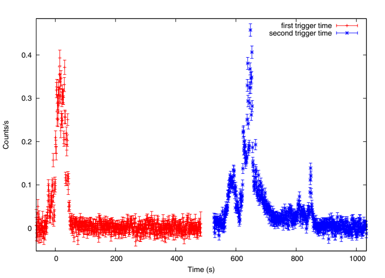
Conclusions & Perspectives



- Test different morphologies of the magnetic field and associated currents distribution
- General parametrisation for the induced deformation



- Study of twisted magnetosphere models
- Limit on the magnetospheric twist: self-regulating mechanism between internal and external currents



- Effects of quark deconfinement (transition from Hadron Star to Quark Star) in the phenomenology of long GRBs

Next

- Evolutionary models
  - X-ECHO code - dynamical Einstein+GRMHD evolution
  - stability of magnetic configuration
  - evolution of the magnetic field in porto-magnetar

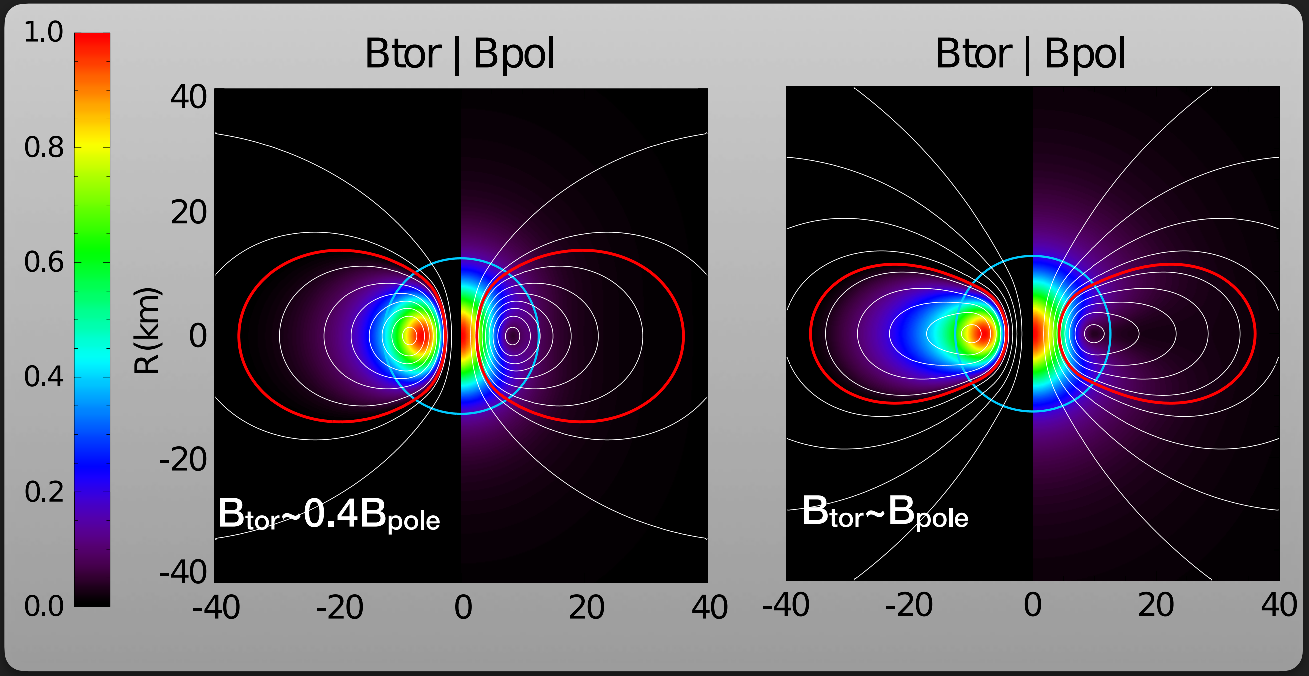
Thank you!

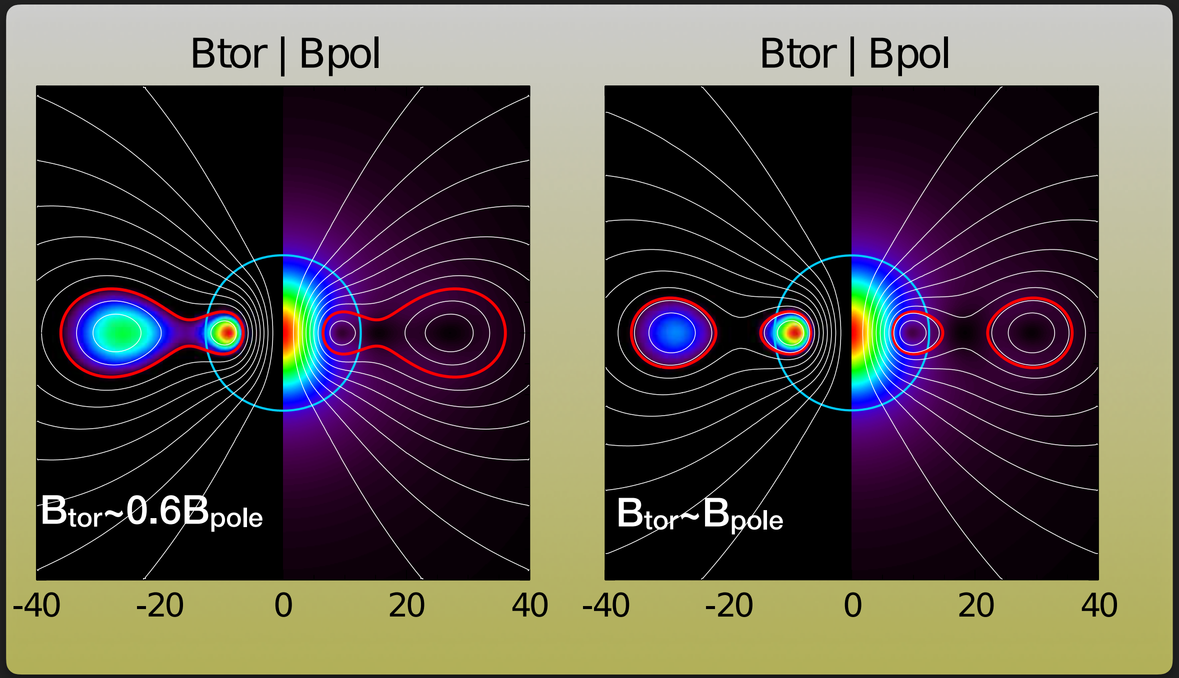
❖ Mathematical Framework

Twisted Magnetosphere

$$\mathcal{M}(\Psi) = k_{\rm pol}\Psi \qquad \mathcal{I}(\Psi) = \frac{a}{\zeta+1}\Theta[\Psi-\Psi_{\rm ext}]\frac{(\Psi-\Psi_{\rm ext})^{\zeta+1}}{(\Psi_{\rm max})^{\zeta+1/2}}$$

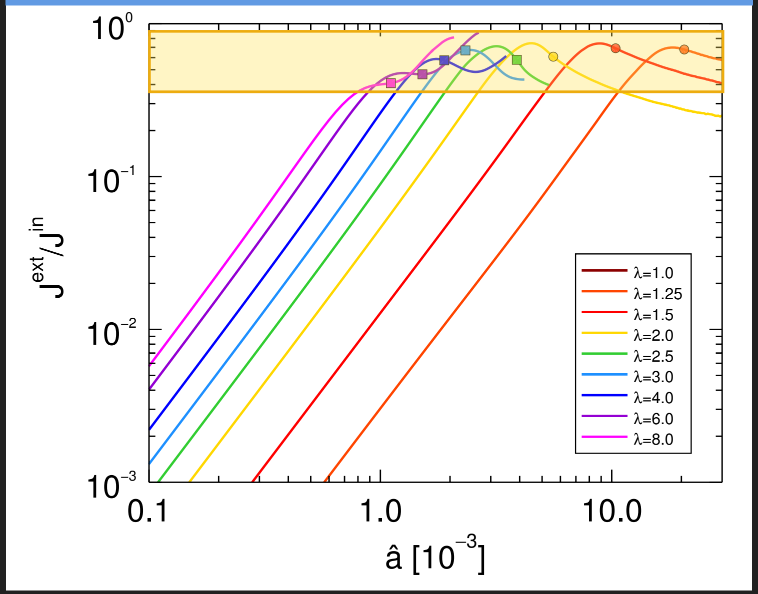
Allow for currents flowing outside the star





- twist angle does not exceed ~2 rad
  (below 3.6 rad found by Parfrey et al. 2013)
- when magnetospheric currents ~ interior currents:
  - progressive change in the topology
  - instable configurations & plasmoids

external J/ internal J



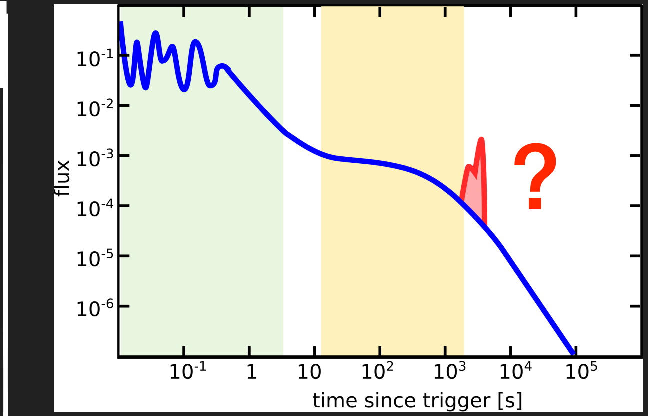
❖ Quark Deconfinement and GRBs

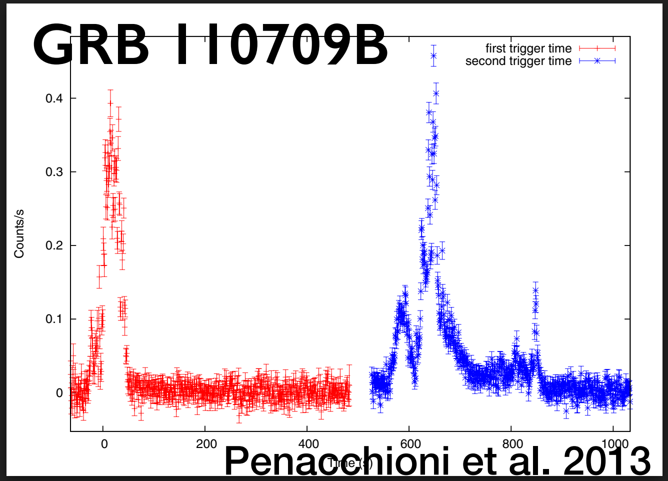
Flares in GRBs & the Two families scenario

Protomagnetar model for GRB

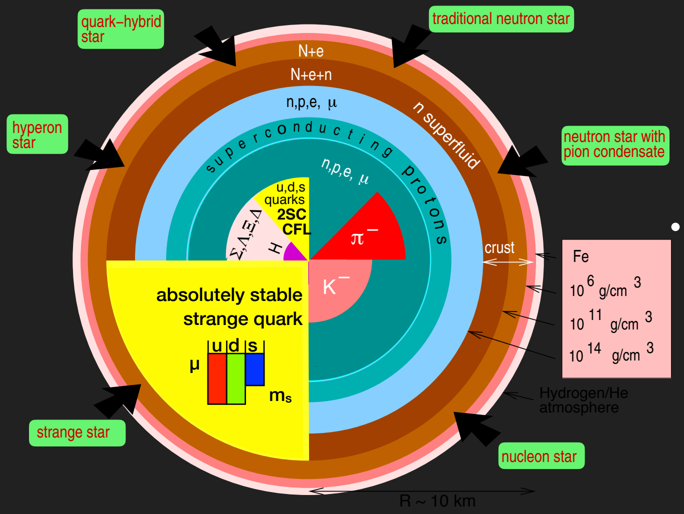
Difficult to explain late flare (~30 sources)

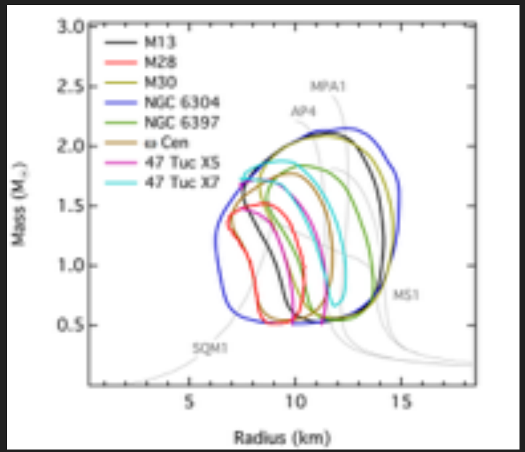
- late mass accretion
- magnetospheric instabilities
- phase transition from NS to Quark Star





Penacchioni et al. 2013





Mass Radius Relation and the EoS

- M~2.0 M$_\odot$ [Demorest 2010, Antoniadis 2016] → stiff EoS
- R~10 km [Guillot 2013, Özel 2016] → soft EoS

difficult to satisfy both with a unique EoS
(including Δs, maximum mass ~ 1.5 M$_\odot$)

The Two Family Scenario: the Main Idea

Coexistence of Hadron Stars (HS) with soft EoS (hyperons and Δs)
Quark Star (QS) with stiff EoS (strange matter)

- conversion at a critical density $\rho_c \sim 10\rho_0$ on a timescale of 10-100 s
- neutrino luminosity ~10^{52}-10^{50} erg/s [Drago et al. 2015-2016]

❖ Quark Deconfinement and GRBs

Phenomenology & Modelization

Delayed formation of a QS from ms Magnetar

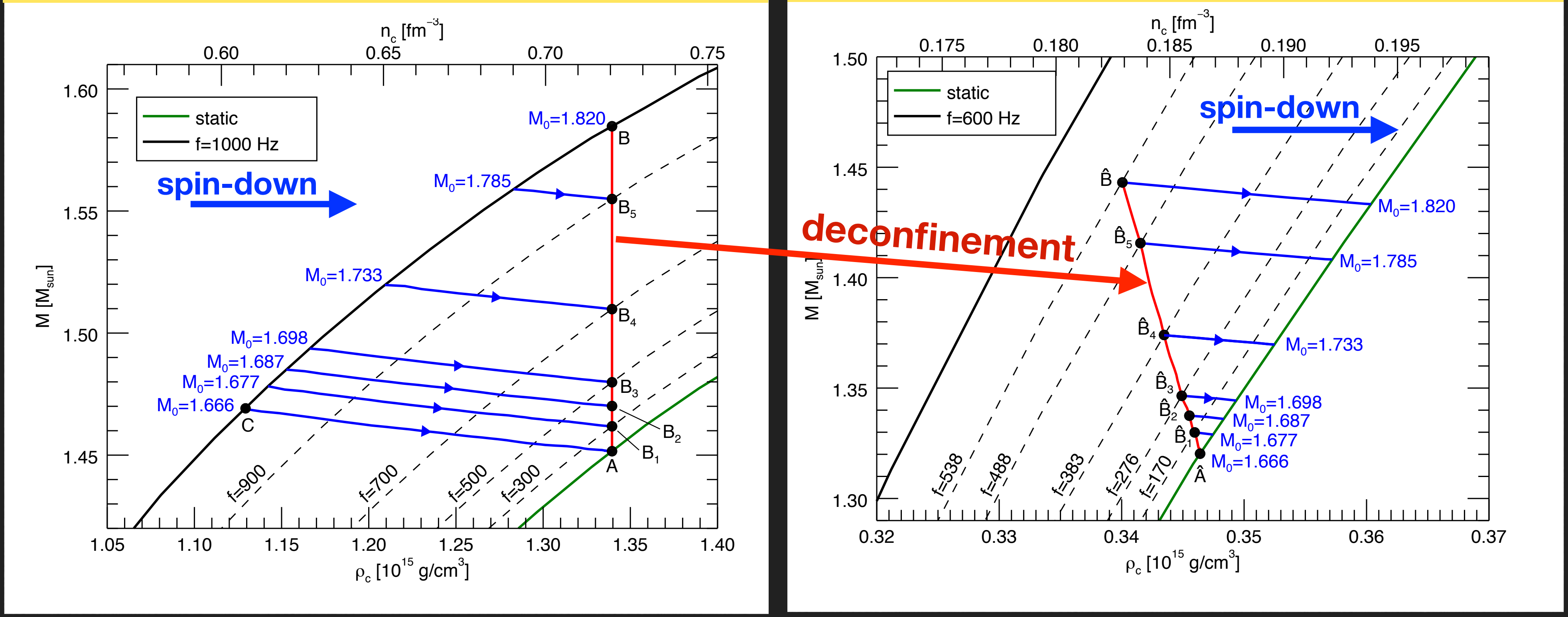
1. First Long GRB - formation of a ms HS magnetar
2. Quiescent phase - magnetic braking spin-down
3. Engine Reactivation - quark deconfinement
4. Second burst - spin down of the QS magnetar

Quasi-stationary spin-down evolution:

- spin down → conservation of baryonic mass M_0
- quark deconfinement → conservation of M_0 and angular momentum

$$\boxed{\frac{\mathrm{d}J}{\mathrm{d}t} = -\frac{B^2R^6\Omega^3}{4}}$$

Hadron Stars **Quark Stars**



- delay~10-10^3 s
- ΔM ~ 10^{53} erg
- ΔK ~ 10^{52} erg
- K(QS) ~ 10^{52} erg

↓

Compatibility with energetics of GRBs

Explain GRB 110709B (two bursts with similar energetic with 300 s delay):

B~10^{15} G P_{ini}~1-1.5 ms M_0~1.7 M_{sun}

❖ Quark Deconfinement and GRBs

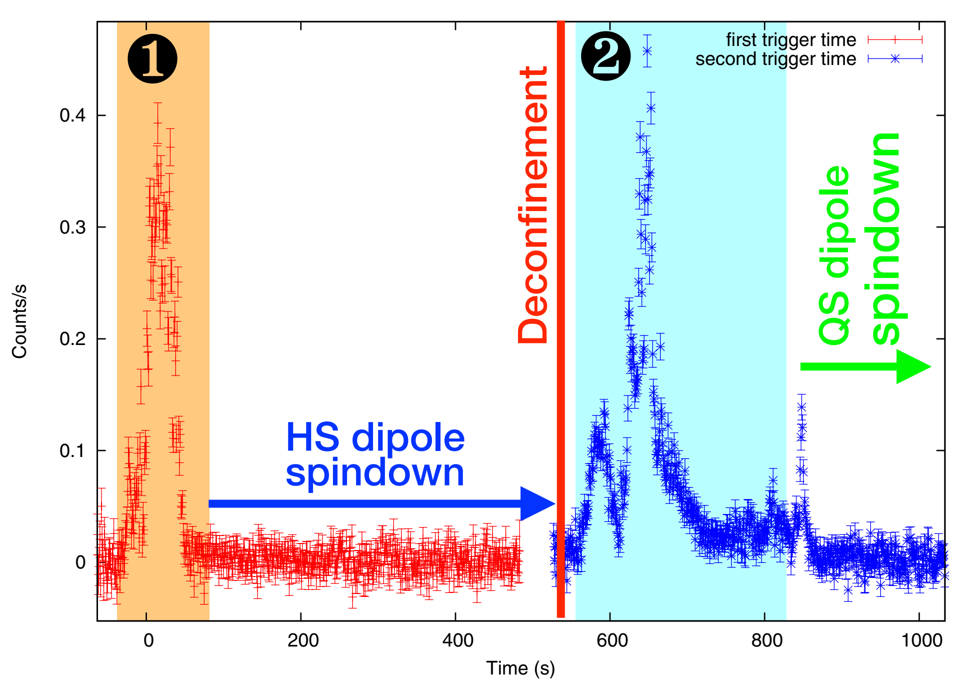
Comparison with GRB 110709B

- cosmological redshift z~1
- beaming correction ~10^{-2}
- delay between bursts ❶ and ❷ ~300 s

❶ $E_{iso}=2.6\times10^{53}$erg $\Delta t^{(1)}$~40 s

❷ $E_{iso}=4.4\times10^{52}$ erg $\Delta t^{(2)}$~150 s

Penacchioni et al. 2013



- initial magnetic field $B=2.0\times10^{15}$G
- during burst: enhanced spin-down (~4)

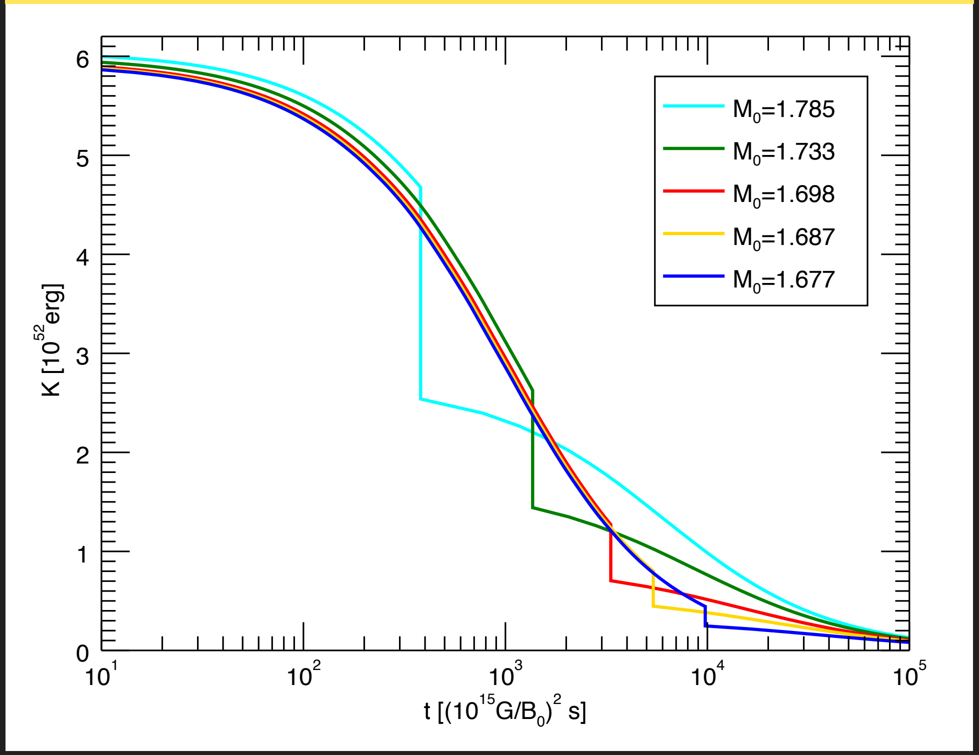
- check energy compatibility with burst ❶
- check time delay for deconfinement
- check energy compatibility with burst ❷

Compatibility with HSs having

M_0=1.72-1.70 $M_\odot$

P= 1-1.6 ms

Rotational energy K(t)

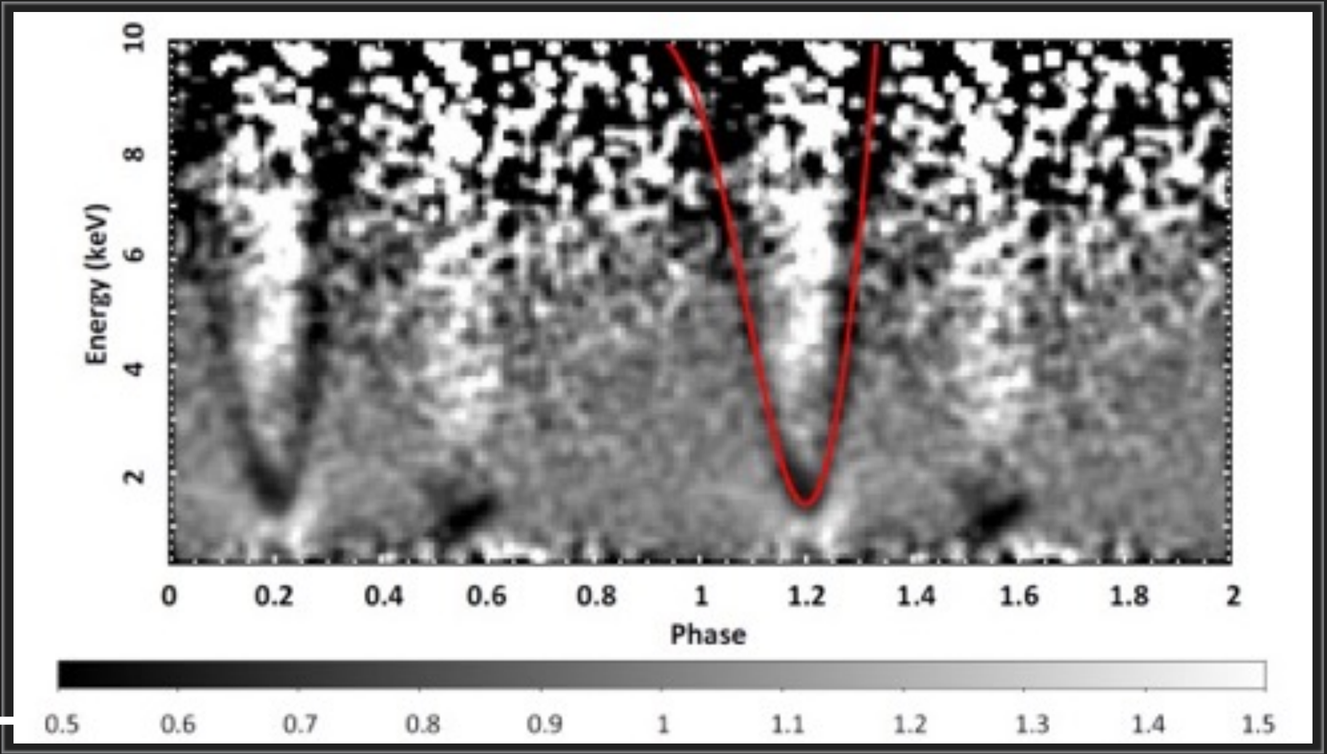


❖ Introduction

Magnetars Phenomenology

SGR 0418+5729

- Low P-dot NS → B_{dipole} ~ 10^{13}G
- intense bursting activity
- observed phase dependent spectral feature



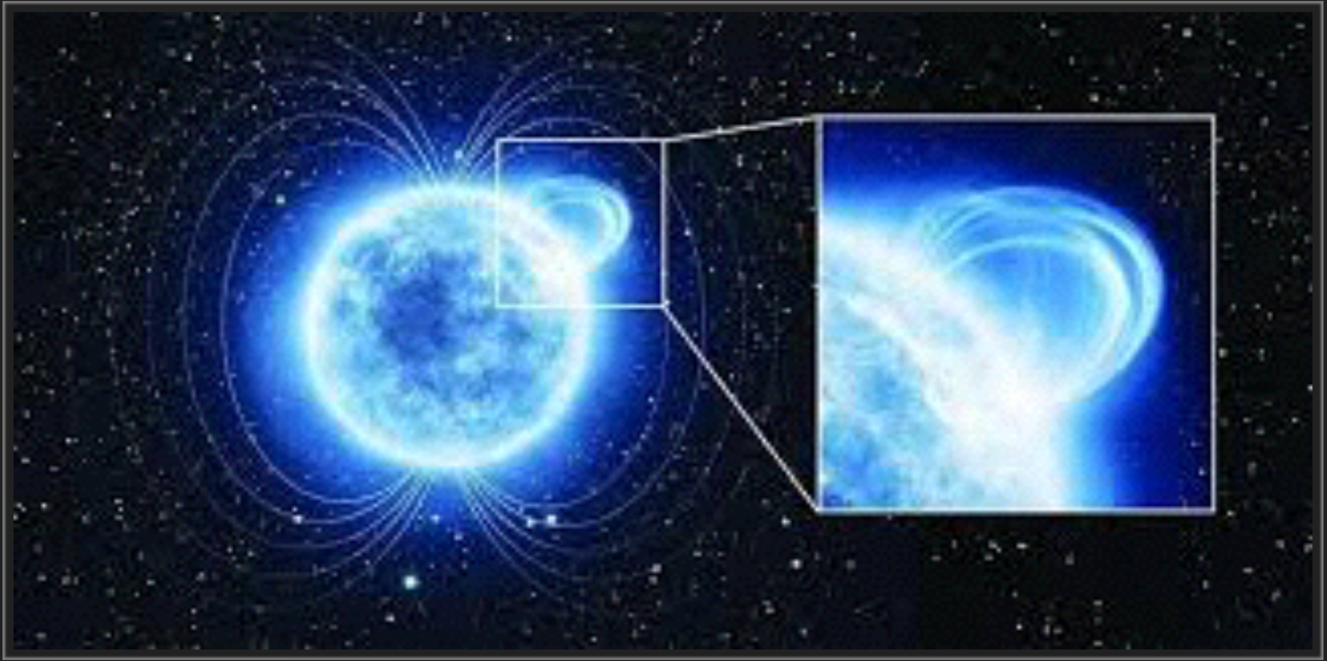
[Tiengo et al 2013]

Resonant Cyclotron Scattering

interaction of magnetospheric current (protons) with thermal photons

$$E_B \sim \frac{11.6}{1+z}\left(\frac{m_e}{m}\right)\left(\frac{B}{10^{12}}\right)\mathrm{keV}$$

require a localised magnetic field strength > 10^{14} G



❖ Mathematical Framework

Einstein equations

CFC equations

$$\Delta\psi = -\left[2\pi E + \frac{1}{8} f_{ik} f_{jl} \tilde{A}^{ij} \tilde{A}^{kl}\right] \psi^5$$

$$\Delta(\alpha\psi) = \left[2\pi(E + 2S) + \frac{7}{8} f_{ik} f_{jl} \tilde{A}^{ij} \tilde{A}^{kl}\right] \alpha\psi^5$$

$$\Delta_L \beta^i := 16\pi\alpha\psi^4 S^i + 2\psi^6 \tilde{A}^{ij} \nabla_j \left(\frac{\alpha}{\psi^6}\right)$$

$\gamma_{ij} = \psi^4 f_{ij}$ Conformally Flat Condition

$K = 0$ Maximal Slicing ⇒ $K_{ij} = \frac{1}{\psi^4} \tilde{A}^{ij}$

- Used in models of Core Collapse, Neutron Stars
- Deviation from full GR negligible (Shibata et al. 2004)
- But uniqueness problems! (Cordero-Carrion et al 2009)

$$K^{ij} = \frac{1}{\psi^{10}} \hat{A}^{ij} \quad \text{with} \quad \hat{A}^{ij} = \boxed{\hat{A}^{ij}_{TT}} + (LW)^{ij}$$

Smaller than the non conformal part
(Cordero-Carrion2009)

$$\hat{A}^{ij} = \nabla^i W^j + \nabla^j W^i - \frac{2}{3}\left(\nabla_k W^k\right) f^{ij}$$

- full decoupling & numerically stable form
- consistency with full GR (10^{-4})

XCFC equations

$$\Delta_L W^i = 8\pi f^{ij} \hat{S}_j$$

$$\Delta\psi = -2\pi \hat{E} \psi^{-1} + \frac{1}{8} f_{ik} f_{jl} \hat{A}^{ij} \hat{A}^{kl} \psi^{-7}$$

$$\Delta(\alpha\psi) = [2\pi(\hat{E} + 2\hat{S})\psi^{-2} + \frac{7}{8} f_{ik} f_{jl} \hat{A}^{ij} \hat{A}^{kl} \psi^{-8}]\alpha\psi$$

$$\Delta_L \beta^i = 16\pi\alpha\psi^{-6} f^{ij} \hat{S}_j + 2\hat{A}^{ij} \nabla_j (\alpha\psi^{-6})$$

where $\hat{S}_j := \psi^6 S_j, \;\; \hat{E} := \psi^6 E, \;\; \hat{S} := \psi^6 S$