GRAVITATIONAL WAVES FROM COALESCING NEUTRON STARS: THE ROLE OF THE EQUATION OF STATE

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The IHES/INFN effective-one-body (EOB) code: https://eob.ihes.fr
JOINT LIGO-VIRGO DETECTION

Table I. Source parameters for GW170814: median values with 90% credible intervals. We quote source-frame masses; to convert to the detector frame, multiply by (1 + z) [126,127]. The redshift assumes a flat cosmology with Hubble parameter $H_0 = 67.9$ km s$^{-1}$ Mpc$^{-1}$ and matter density parameter $\Omega_m = 0.3065$ [128].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary black hole mass $m_1$</td>
<td>$30.5^{+5.7}<em>{-3.0}M</em>\odot$</td>
</tr>
<tr>
<td>Secondary black hole mass $m_2$</td>
<td>$25.3^{+3.9}<em>{-1.2}M</em>\odot$</td>
</tr>
<tr>
<td>Chirp mass $\mathcal{M}$</td>
<td>$24.1^{+3.4}<em>{-1.1}M</em>\odot$</td>
</tr>
<tr>
<td>Total mass $M$</td>
<td>$55.9^{+3.4}<em>{-2.7}M</em>\odot$</td>
</tr>
<tr>
<td>Final black hole mass $M_f$</td>
<td>$53.2^{+3.2}<em>{-2.5}M</em>\odot$</td>
</tr>
<tr>
<td>Radiated energy $E_{\text{rad}}$</td>
<td>$2.7^{+0.4}<em>{-0.3}M</em>\odot,c^2$</td>
</tr>
<tr>
<td>Peak luminosity $\ell_{\text{peak}}$</td>
<td>$3.7^{+0.5}_{-0.3} \times 10^{56}$ erg s$^{-1}$</td>
</tr>
<tr>
<td>Effective inspiral spin parameter $\chi_{\text{eff}}$</td>
<td>$0.06^{+0.12}_{-0.12}$</td>
</tr>
<tr>
<td>Final black hole spin $a_f$</td>
<td>$0.70^{+0.07}_{-0.08}$</td>
</tr>
<tr>
<td>Luminosity distance $D_L$</td>
<td>$540^{+130}_{-210}$ Mpc</td>
</tr>
<tr>
<td>Source redshift $z$</td>
<td>$0.11^{+0.03}_{-0.01}$</td>
</tr>
</tbody>
</table>
THE THEORY...

Is needed to compute waveform templates for characterizing the source (GWs were detected...but WHAT was detected?)

Theory is needed to study the 2-body problem in General Relativity (dynamics & gravitational wave emission)

Theory: SYNERGY between Analytical and Numerical General Relativity (AR/NR)

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]
HOW TO MEASURE: MATCHED FILTERING!

To extract/do parameter estimation of the GW signal from detector’s output (lost in broadband noise $S_n(f)$)

\[ \langle \text{output} | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h_{\text{template}}^* (f) \]

Detector's output

Template of expected GW signal

Need waveform templates!
Templates for GWS from BBH Coalescence

Brady, Craighton & Thorne, 1998

**Inspiral** (PN methods) **Ringdown** (perturbation theory)

**Merger:** Numerical Relativity

**Effective-One-Body** (Buonanno & Damour (2000))
**PN-resummation** (Damour, Iyer, Sathyaprakash (1998))

Numerical Relativity: >= 2005 (F. Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code: M. Scheel et al., 2008 (SXS collaboration)

Spectral code
Extrapolation (radius & resolution)

Phase error:
< 0.02 rad (inspiral)
< 0.1 rad (ringdown)
EFFECTIVE ONE BODY (EOB): 2000

Numerical Relativity was not working (yet...)
EOB formalism was predictive, qualitatively and semi-quantitatively correct (10%)

- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

\[ \nu = \frac{m_1 m_2}{(m_1 + m_2)^2} = \frac{\mu}{M} \]

> 2005: Developing EOB & interfacing with NR
  2 groups did (and are doing) it
  - A.Buonanno+ (AEI)
  - T.Damour & AN+ (>2005)

\[ h_+ - i h_\times = \frac{1}{r} \sum_{\ell m} \ell m \cdot -2 Y_{\ell m}(\theta, \phi) \]
(i) Effective-one-body (EOB) models for Inspiral-Merger-Ringdown (IMR):
- AEI: NR-calibrated: SEOBNRv2, SEOBNRv3 (P), SEOBNRv4 (LVC)
- IHES/INFN: NR-informed; TEOBResumS (Matlab & c++; LAL-ification in progress) [UniTo]

(ii) IMRPhenomenological models (UIB & Cardiff)
- IMRPhenomD (LVC)
- IMRPhenomP (Pv2) (LVC)

(iii) Postmerger-ringdown: Damour & AN 2014, Del Pozzo & AN, PRD 95 (2017) 124034 [UniTo&UniPi]

BNSs: Tidal effects
- T-EOB-AEI (including NS oscillations)
- TEOBResumS (no NS oscillations, including GSFs & spin. Matlab & c++; LAL-inprogress)
- TTaylorF2 [UniTo & UniPr]
- NRTides [Unipr, Bernuzzi+2017]
IMPORTANCE OF AN ANALYTICAL FORMALISM

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).

- **Practical**: need hundreds of thousands of accurate GWs templates for detection and data analysis. Need analytical templates: $h(m_1, m_2, \vec{S}_1, \vec{S}_2)$

- **Solution**: synergy between analytical & numerical relativity

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**Perturbation Theory**

Post-Newtonian (PN)

$\frac{v^2}{c^2} \ll 1$

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**Resummed**

PN theory: 

EOB (LAPTOP) + ROM

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**Numerical Relativity**

(SUPERCOMPUTERS)

- Strong-field information
  - **EOBNR models**

- Complementary route: **IMRPhenom** models
  - PN_glue_NR, EOB_glue_NR hybrids (**glued waveforms**) to build **phenomenological templates** [Khan et al., 2015]

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RESULTS: EOBNR/NR WAVEFORMS (NO SPIN)

\[ \Delta \phi_{22}^{\text{EOBNR}} / A_{22}^{\text{NR}} \]

\[ \chi = 0 \]

equal-mass BBH, nonspinning

TEOBR\text{e}\text{s}umS: equal-mass, no spin

Nagar, Damour, Reisswig & Pollney, PRD 93 (2016), 04404
\[ q = 1 \quad M = 2.7M_\odot \]

- Tidal effects
- Love numbers (tial "polarization" constants)
- EOS dependence & "universality"
- EOB/NR for BNS

See:
- Damour\&Nagar, PRD 2009
- Damour\&Nagar, PRD 2010
- Damour, Nagar et al., PRL 2011
- Bini, Damour\&Faye, PRD 2012
- Bini\&Damour, PRD 2014
- Bernuzzi, Nagar, et al, PRL 2014
- Bernuzzi, Nagar, Dietrich, PRL 2015
- Bernuzzi, Nagar, Dietrich \& Damour, PRL, 2015
- Dietrich, Bernuzzi \& Tichy, 2017
ANALYTICALLY: MOTION AND GW IN GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRINKS with time.

Waveform

General Relativity is NONLINEAR!

Post-Newtonian (PN) approximation: expansion in $\frac{v^2}{c^2}$.
\[ H_{\text{real}}(q,p) = H_N(q,p) + H_{1\text{PN}}(q,p) + H_{2\text{PN}}(q,p) + H_{3\text{PN}}(q,p), \quad (4.27) \]

where

\[ H_N(q,p) = \frac{p^2}{2} - \frac{1}{q}, \quad \text{Newton (OPN)} \quad (4.28a) \]

\[ H_{1\text{PN}}(q,p) = \frac{1}{8}(3\nu - 1)(p^2)^2 - \frac{1}{2} [(3 + \nu)p^2 + \nu(n \cdot p)^2] \frac{1}{q} + \frac{1}{2q^2}, \quad (1\text{PN, 1938}) \quad (4.28b) \]

\[ H_{2\text{PN}}(q,p) = \frac{1}{16} (1 - 5\nu + 5\nu^2)(p^2)^3 + \frac{1}{8} [(5 - 20\nu - 3\nu^2)(p^2)^2 - 2\nu^2(n \cdot p)^2] \frac{1}{q^2} - \frac{1}{4}(1 + 3\nu)\frac{1}{q^3}, \quad (2\text{PN, 1982/83}) \quad (4.28c) \]

\[ H_{3\text{PN}}(q,p) = \frac{1}{128} (-5 + 35\nu - 70\nu^2 + 35\nu^3)(p^2)^4 \]

\[ + \frac{1}{16} [(-7 + 42\nu - 53\nu^2 - 5\nu^3)(p^2)^3 + (2 - 3\nu)^2(n \cdot p)^2(p^2)^2 + 3(1 - \nu)^2(n \cdot p)^4p^2 - 5\nu^3(n \cdot p)^6] \frac{1}{q} \]

\[ + \left[ \frac{1}{16} (-27 + 136\nu + 109\nu^2)(p^2)^2 + \frac{1}{16} (17 + 30\nu)(n \cdot p)^2p^2 + \frac{1}{12} (5 + 43\nu)(n \cdot p)^4 \right] \frac{1}{q^2} \quad (3\text{PN, 2000}) \]

\[ + \left\{ \left[ -\frac{25}{8} + \left( \frac{16\pi^2}{48} - \frac{335}{48} \right) \nu - \frac{23}{8} \nu^2 \right] p^2 + \left( -\frac{85}{16} - \frac{3}{64} \pi^2 - \frac{7}{4} \nu \right) (n \cdot p)^2 \right\} \frac{1}{q^3} \]

\[ + \left[ \frac{1}{8} + \left( \frac{109}{12} - \frac{21}{32} \pi^2 + \omega_{\text{static}} \right) \nu \right] \frac{1}{q^4}, \quad (4.28d) \]

...and 4PN too, [Damour, Jaranowski & Schaefer 2014/2015] - 4 loop calculation

\[ q = q_1 - q_2 \]

\[ p = p_1 = -p_2 \]
FLUX & WAVEFORM (3.5PN)

\[
\frac{dE}{dt} = -\mathcal{L}
\]

balance equation

**Mechanical loss**  
**GW luminosity**

\[
\mathcal{L} = \frac{32e^5}{5G} \nu^2 x^5 \left(1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(\frac{-44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 + \left(\frac{-8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16x)\right] + \left(\frac{-134543}{7776} + \frac{41}{48}\pi^2\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right)x^3 + \left(\frac{-16285}{504} + \frac{217451728}{3024}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\right) \}
\]

Newtonian quadrupole

\[
h_{22}^{2} = -8\sqrt{\frac{\pi G \nu m}{c^2 R}} e^{-2i\phi} x^4 - x \left(\frac{107}{42} - \frac{55}{42}\nu\right) + x^{3/2} \left[2\pi + 6i \ln\left(\frac{x}{a}\right)\right] - x^2 \left(\frac{2173}{1512} + \frac{1069}{216}\nu - \frac{2047}{1512}\nu^2\right) + x^3 \left[\frac{27027409}{646800} - \frac{856}{105}\gamma_E + \frac{2}{3}\pi^2 - \frac{1712}{105}\ln^2 - \frac{428}{105}\ln x\right] + x^4 \left[\frac{-278185}{33264} - \frac{41}{96}\pi^2\nu - \frac{20261}{2772}\nu^2 + \frac{114635}{99792}\nu^3 + \frac{428i}{105}\pi + 12i\pi \ln\left(\frac{x}{a}\right)\right] + \mathcal{O}(e^{7/2}) \}
\]

\[
C = \gamma_E = 0.5772156649...
\]

\[
x = (M\Omega)^{2/3} \sim \nu^2 / c^2
\]
EFFECTIVE-ONE-BODY (EOB)
approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

1. Replace two-body dynamics \((m_1, m_2)\) by dynamics of a particle 
   \((\mu \equiv \frac{m_1 m_2}{m_1 + m_2})\) in an effective metric \(g_{\mu\nu}^{\text{eff}}(u)\), with
   \[ u \equiv \frac{GM}{c^2 R}, \quad M \equiv m_1 + m_2 \]

2. Systematically use RESUMMATION of PN expressions (both \(g_{\mu\nu}^{\text{eff}}\) and \(F_{RR}\)) based on various physical requirements

3. Require continuous deformation w.r.t.
   \[ \nu \equiv \mu/M \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} \]
   in the interval \(0 \leq \nu \leq \frac{1}{4}\)
STRUCTURE OF THE EOB FORMALISM

**EOB Hamiltonian**
\[ H_{\text{EOB}} \]

**EOB Rad. Reac. force**
\[ \hat{F}_\varphi \]

**Matching at merger time**
\[ h_{\ell m}^{\text{ringdown}}(t) = \sum_N C_N^+ e^{-\sigma_N^+(t-t_m)} \]

**Resummed (BD99)**

**Resummed (DIS98)**

**Resummed (DN07,DIN08)**

**PN dynamics**
(DDB81,D82,DJS01,IF03,BDIF04)

**PN rad losses**
WW76, BDIWW95, BDEFI05

**PN waveform**
BD89, B95&D05,ABIQ04,

**BH perturbations**
RW57, Z70, Z72

**Factorized waveform**
\[ h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)} \]
\[ \hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_m^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m} \]

**BNS: tides**
(Love numbers)

**EOB waveform**
\[ h_{\ell m}^{\text{EOB}} = \theta(t_m - t) h_{\ell m}^{\text{insplunge}}(t) + \theta(t - t_m) h_{\ell m}^{\text{ringdown}}(t) \]

\[ \frac{dr}{dt} = \left( \frac{A}{B} \right)^{1/2} \partial \hat{H}_{\text{EOB}} \partial p_r, \]
\[ \frac{dp_r}{dt} = - \left( \frac{A}{B} \right)^{1/2} \partial \hat{H}_{\text{EOB}} \partial r, \]
\[ \Omega = \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_\varphi}, \]
\[ \frac{dp_\varphi}{dt} = \hat{F}_\varphi. \]
Explicit Form of the EOB Hamiltonian

**EOB Hamiltonian**

\[
H_{\text{EOB}} = M \sqrt{1 + 2\nu (\hat{H}_{\text{eff}} - 1)}
\]

All functions are a \(\nu\)-dependent deformation of the Schwarzschild ones

\[
A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4
\]

\[
A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3
\]

\[
u = \frac{GM}{c^2 R}
\]

Simple effective Hamiltonian:

\[
\hat{H}_{\text{eff}} \equiv \sqrt{\frac{p_{r_*}^2}{r^2} + A(r) \left(1 + \frac{p_{\varphi}^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}
\]

\[
p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r
\]

Crucial EOB radial potential

Contribution at 3PN
EFFECTIVE POTENTIALS

Newtonian gravity (any mass ratio):
circular orbits are always stable. No plunge.

\[ W_{\text{Newt}}^{\text{eff}} = 1 - \frac{2}{r} + \frac{p_{\varphi}^2}{r^2} \]

Test-body on Schwarzschild black hole:
last stable orbit (LSO) at \( r = 6M \): plunge

\[ W_{\text{Schwarzschild}}^{\text{eff}} = \left( 1 - \frac{2}{r} \right) \left( 1 + \frac{p_{\varphi}^2}{r^2} \right) \]

\[ W_{\text{EOB}}^{\text{eff}} = A(r; \nu) \left( 1 + \frac{p_{\varphi}^2}{r^2} \right) \]

\[ \nu \text{-deformation of the Schwarzschild case!} \]
HAMILTON’S EQUATIONS & RADIATION REACTION

\[
\dot{r} = \left( \frac{A}{B} \right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_{r^*}} \\
\dot{\varphi} = \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_{\varphi}} \equiv \Omega \\
\dot{p}_{r^*} = - \left( \frac{A}{B} \right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial r} + \hat{F}_{r^*} \\
\dot{p}_\varphi = \hat{F}_{\varphi}
\]

- The system must radiate angular momentum
- How? Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

\[
\hat{F}^{\text{Taylor}}_{\varphi} = - \frac{32}{5} \nu \Omega^5 r^4 \hat{F}^{\text{Taylor}}(v_\varphi)
\]

Plus horizon contribution [AN&Akçay2012]
**THE KNOWLEDGE OF THE CENTRAL A POTENTIAL TODAY**

**4PN analytically complete + 5PN logarithmic term in the A(u) function:**

\[
A_{5PN}^{Taylor} = 1 - 2u + 2\nu u^3 + \left( \frac{94}{3} - \frac{41}{32}\pi^2 \right) \nu u^4 + \nu[a_5^c(\nu) + a_5^{ln} \ln u] u^5 + \nu[a_6^c(\nu) + a_6^{ln} \ln u] u^6
\]

\[
\begin{align*}
1PN & \\
2PN & \\
3PN & \\
4PN & \\
5PN & \\
\end{align*}
\]

\[
\begin{align*}
a_5^{\log} &= \frac{64}{5} \\
a_5^c &= a_5^{c_0} + \nu a_5^{c_1} \\
a_5^{c_0} &= -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\ln(2) + \frac{128}{5}\gamma \\
a_5^{c_1} &= -\frac{221}{6} + \frac{41}{32}\pi^2 \\
a_6^{\log} &= -\frac{7004}{105} - \frac{144}{5}\nu \\
\end{align*}
\]

\{4PN fully known ANALYTICALLY!

**NEED ONE “effective” 5PN parameter from NR waveform data:** \(a_6^c(\nu)\)

**State-of-the-art EOB potential (5PN-resummed):**

\[
A(u; \nu, a_6^c) = P_5^1[A_{5PN}^{Taylor}(u; \nu, a_6^c)]
\]
RESULTS: EOB/NR WAVEFORMS (NO SPIN)

\[ \Delta \phi_{22}^{\text{EOBNR}} \]

\[ \Delta A_{22}^{\text{EOBNR}} / A_{22}^{\text{NR}} \]

\[ \chi = 0 \]

equal-mass BBH, nonspinning

\[ \text{TEOBResumS: equal-mass, no spin} \]

Nagar, Damour, Reisswig & Pollney, PRD 93 (2016), 04404
THE EOB[NR] POTENTIAL

\[ A(r; \nu) = 1 - \frac{2M}{r} \]

From EOB/NR-fitting: \[ a^c_6(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804 \]
\[ q = 1 \quad M = 2.7M_\odot \]

- Tidal effects
- Love numbers (tidal “polarization” constants)
- EOS dependence & “universality”
- EOB/NR for BNS

See:
- Damour\&Nagar, PRD 2009
- Damour\&Nagar, PRD 2010
- Damour, Nagar et al., PRL 2011
- Bini, Damour\&Faye, PRD 2012
- Bini\&Damour, PRD 2014
- Bernuzzi, Nagar, et al, PRL 2014
- Bernuzzi, Nagar, Dietrich, PRL 2015
- Bernuzzi, Nagar, Dietrich \& Damour, PRL, 2015
BNS: ANALYTICAL NEEDS

- Study the response of each neutron star to the tidal field of the companion [theory of relativistic Love numbers (i.e. tidal polarizability coefficients) + tidal corrections to dynamics (beyond Newtonian accuracy)]

- Incorporate the corresponding tidal effects within a theoretical framework able to describe the gravitational wave signal emitted by inspiralling compact binaries (up to merger): EOB-resummed description of dynamics and waveforms

- Compare/check analytical models against NR simulations; possibly, inform, when needed, the EOB model with high-order tidal information extracted from NR

- Assess the measurability of tidal effects within the signal seen by interferometric detectors

- Construct surrogate models (e.g. ROMs) for fast template evaluation [Lackey et al. 2016]
A BIT OF HISTORY...

1983-Damour. Relativistic generalization of Love numbers
1989-1991-Damour-Soffel-Xu: magnetic&electric tidal polarization constants (tidal polarizability)
2007-Hinderer. First Love number computation in GR
2008-Hinderer& Flanagan. Polytropes. Inspiral only - not promising
2009-TD&AN - Binnington&Poisson - multipolar Love numbers
2009- Hinderer. Love numbers for realistic EOS. Measurability inspiral only - not promising
2009- TD&AN - Tidal Effective One Body model (TEOB). Up to merger (contact). 1PN dynamical effects
2010- Vines, Hinderer & Flanagan. 1PN effects in waveforms
2010 - EOB/NR comparisons [Baiotti, Damour, Nagar+]. TaylorT4 ruled out. Strong tides close to merger?
2011 - 2PN tidal terms [Bini, Damour & Faye]
2012 - Damour, Nagar & Villain. EOB. Strong indications for measurability of Love numbers up to merger.
2012 - Bernuzzi, Nagar et al.: high order reconstruction - E(j) correct. Disentangle tides & noise. Strong tides?

2014 - Bini& Damour: GSF-informed tidal potential
2015 - Bernuzzi, Nagar, Dietrich&Damour - EOB-GSF/NR compatibility up to merger. Several EOS.
2015 - Hotokezaka et al. Similar analysis confirming BNDD. Negligible eccentricity.
2015 - Hinderer +: inclusion of f-mode oscillations in EOB model. BHNS relevancy. H4 EOS.
2016 - Hotokezaka+: EOB-NR hybrids & new assessment of LN measurability; several EOS; PN finally ruled out
2016 - Lackey, Bernuzzi+: ROM of BNDD model
2017 - Dietrich-Hinderer: comparisons between TEOB: high level of consistency
2017 - Bernuzzi-Dietrich-Tichy: NR-based, closed form tidal approximants
RESULT (Damour, Nagar, Villain 2012)

Tidal polarizability parameters can actually be measured by adv LIGO with a reasonable SNR=16

Use EOB controlled, accurate, description of the phasing up to BNS merger!

Confirmed by Bayesian analysis:
Del Pozzo+ 2013 Agathos+2015
TIDAL EFFECTS IN EOB FORMALISM

Tidal extension of EOB formalism: **nonminimal worldline couplings**

\[
\Delta S_{\text{nonminimal}} = \sum_{A} \frac{1}{4} \mu_2^A \int d\Sigma_A \left( u^\mu u^\nu R_{\mu\alpha\nu\beta} \right)^2 + \ldots
\]

Damour&Esposito-Farèse96, Goldberger&Rothstein06, TD&AN09

**Modifications of the EOB effective metric...**

\[
A(r) = A_r^0 + A^{\text{tidal}}(r)
\]

\[
A^{\text{tidal}}(r) = -\kappa_T^2 u^6 \left( 1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \ldots \right) + \ldots
\]

And tidal modifications of GW waveform & radiation reaction

Need analytical theory for computing \( \mu_2, \kappa_T^2, \bar{\alpha}_1 \ldots \)

(?) Need accurate NR simulations to check/inform the higher-order PN tidal contributions, that may be quite important during the late inspiral up to merger
THEORY OF RELATIVISTIC TIDAL POLARIZATION

The star acquires induced gravito-electric and gravito-magnetic multipole moments

\[ M^{(A)}_L = \mu^A_L G^{(A)}_L \]
\[ S^{(A)}_L = \sigma^A_L H^{(A)}_L \]

2\( k_\ell \equiv (2\ell - 1)!! \frac{G\mu_\ell}{R^{2\ell+1}} \)
\[ j_\ell \equiv (2\ell - 1)!! \frac{4(\ell + 2)}{\ell - 1} \frac{G\sigma_\ell}{R^{2\ell+1}} \]

Dimensionless relativistic “second” Love numbers

Actual calculation based on star perturbation theory: Love numbers are obtained as boundary conditions (matching interior to exterior perturbations)
RELATIVISTIC LOVE NUMBERS (POLYTROPIC EOS)

"rest-mass polytrope" (solid lines)
\[ p = K \mu^\gamma \]
\[ e = \mu + \frac{p}{\gamma - 1} \]

"energy polytrope" (dashed lines)
\[ p = K e^\gamma \]

Tidal polarization parameters
\[ M_L^{(A)} = \mu_\ell^A G_L^{(A)} \]
\[ 2k_\ell \equiv (2\ell - 1)!! \frac{G\mu_\ell}{R^{2\ell+1}} \]
RELATIVISTIC LOVE NUMBERS: (REALISTIC EOS)

DN 2009 (SLy & FPS)
Hinderer et al. 2010 (several tabulated EOS)

$$p = K \rho^{1+1/n}$$

Polytropic EOS

$$p = K \epsilon^{1+1/n}$$
TIDAL INTERACTION POTENTIAL

Central tidal “coupling constant”:

$$\kappa_{\ell}^T \equiv 2 \left[ \frac{1}{q} \left( \frac{X_A}{C_A} \right)^{2\ell+1} k_{\ell}^A + q \left( \frac{X_B}{C_B} \right)^{2\ell+1} k_{\ell}^B \right]$$

$$X_{A,B} \equiv M_{A,B}/M$$

Function of: masses, compactnesses and relativistic Love numbers

In the dynamics:

$$A(u) = A^0(u) + A^{\text{tidal}}$$

$$A^{\text{tidal}} = \sum_{\ell \geq 2} -\kappa_{\ell}^T u^{2\ell+2} \hat{A}_\ell^{\text{tidal}}(u) + \text{PN corrections (NLO, NNLO, ...)}$$

NLO & NNLO tidal PN corrections known analytically

[Binis, Damour & Faye 2011]

$$\hat{A}_2^{\text{tidal}} = 1 + \frac{5}{4} u + \frac{85}{14} u^2$$

$$\kappa_2^T = \frac{1}{8} \frac{k_2}{C^5}$$

“Newtonian” (LO) part

$$\kappa_2^T \sim 100$$

Central tidal “coupling constant”:

$$\kappa_2^T = \frac{1}{8} \frac{k_2}{C^5}$$
THREE RESULTS

1. **Numerical-relativity** matches effective-one-body (EOB) analytical-relativity waveforms and dynamics essentially up to merger. Method to compute GW templates for LIGO/Virgo to measure EOS out of tidal effects

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

“Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to Merger”
[checked by Hotokezaka et al., PRD 91 (2015) 6, 064060, notably with reduced eccentricity]

2. **Quasi-universality** in BNS merger (binding energy, angular momentum, GW frequency vs tidal coupling constant): explained using EOB theory

S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich & M. Ujevic, PRL 112 (2014), 201101

“Quasiuniversal properties of neutron star mergers”

3. **Quasi-universality** of post-merger $M f_2$ frequency vs tidal coupling constant

S. Bernuzzi, A. Nagar & T. Dietrich, arXiv:1504.01764

“Towards a description of the complete gravitational wave spectrum of neutron star mergers”

Unifying description of inspiral, merger and post-merger phases
RESUMMED TIDAL INTERACTION

Bini&Damour (2015) resummed expression for $\hat{A}_\ell^{\text{tidal}}$

Presence of a pole: potential strongly attractive @ mrg

$$A_T^{(+)}(u;\nu) \equiv -\sum_{\ell=2}^{4} \left[ \kappa^{(\ell)} A \nu^{2\ell+2} \hat{A}_A^{(\ell^+)} + (A \leftrightarrow B) \right].$$

$$\hat{A}_A^{(2^+)}(u) = 1 + \frac{3u^2}{1-r_{LR}u} + \frac{X_A \tilde{A}_1^{(2^+)*1SF}}{(1-r_{LR}u)^{7/2}} + \frac{X_A^2 \tilde{A}_2^{(2^+)*2SF}}{(1-r_{LR}u)^p}$$

$p = 4$

Because of analytic considerations (BD 2015)
EOB (orbital) interaction potential

\[ A(u) = A^0(u) + A^{\text{tidal}} \]

\[ A^{\text{tidal}} = \sum_{\ell \geq 2} -\kappa^T_\ell u^{2\ell+2} \hat{A}^{\text{tidal}}_\ell(u) \]

"Newtonian" (LO) part \times PN corrections (NLO, NNLO, ...)

---

\[ A_0 = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\right) \nu u^4 + \nu [a^c_5(\nu) + a^\ln_5 \ln u] u^5 + \nu [a^c_6(\nu) + a^\ln_6 u] u^6 \]

1PN 2PN 3PN 4PN 5PN term "informed" by NR

\[ a^c_6(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804 \]

---

\[ A^{\text{resum}}(u; \nu, a^c_6) = P_5^1[A_0(u; \nu, a^c_6)] \]

\[ A^{(+)}_T(u; \nu) \equiv -\sum_{\ell=2}^{4} \left[ \kappa^{(+)}_\ell u^{2\ell+2} \hat{A}^{(+)}_\ell(u) + (A \leftrightarrow B) \right] \]

\[ \kappa^{(+)}_\ell = 2 \frac{X_B}{X_A} \left(\frac{X_A}{C_A}\right)^{2\ell+1} \kappa^A_\ell \]

Love numbers

\[ X_{A,B} \equiv m_{A,B}/M \]

\[ \nu = 0.25 \]

---

Graph showing the interaction potential \( A(R) \) for different cases.

\[ \kappa^T_2 \approx 73.55 \]

Italian text at the bottom:

lunedì 9 ottobre 17
FIG. 3: Phasing and amplitude comparison (versus NR retarded time) between TEOB_{Resum}, NR and the phasing of TT4 for three representative models. Waves are aligned on a time window (vertical dot-dashed lines) corresponding to $I_{\omega} \approx (0.04, 0.06)$. The markers in the bottom panels indicate: the crossing of the TEOB_{Resum} LSO radius; NR (also with a dashed vertical line) and EOB merger moments.

<table>
<thead>
<tr>
<th>Name</th>
<th>EOS</th>
<th>$k_2^T$</th>
<th>$r_{LR}$</th>
<th>$C_{A, B}$</th>
<th>$M_{A, B} [M_\odot]$</th>
<th>$M_{ADM}^0 [M_\odot]$</th>
<th>$J_{ADM}^0 [M_\odot]^3$</th>
<th>$\Delta \phi_{NRmrg}^{TT4}$</th>
<th>$\Delta \phi_{NRmrg}^{TEOB_{Resum}}$</th>
<th>$\Delta \phi_{NRmrg}^{TEOB_{NLO}}$</th>
<th>$\Delta \phi_{NRmrg}^{TEOB_{Resum}}$</th>
<th>$\delta \phi_{NRmrg}$</th>
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<td>2B135</td>
<td>2B</td>
<td>23.9121</td>
<td>3.253</td>
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<td>-0.19</td>
<td>+0.57^a</td>
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<tr>
<td>SLy135</td>
<td>SLy</td>
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<td>3.701</td>
<td>0.17381</td>
<td>1.35000</td>
<td>2.67760</td>
<td>7.65780</td>
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<td>-1.79</td>
<td>-0.75</td>
<td>±0.40</td>
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<td>$\Gamma = 2$</td>
<td>2.0751</td>
<td>3.728</td>
<td>0.15999</td>
<td>1.64388</td>
<td>3.25902</td>
<td>11.11313</td>
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<tr>
<td>$\Gamma_{151}$</td>
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<td>1.51484</td>
<td>3.00497</td>
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<tr>
<td>MS1b135</td>
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</tr>
</tbody>
</table>

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

lunedì 9 ottobre 17
Implementation: ROM of TEOBResumS

Surrogate model for TEOB_ihes: Lackey, Bernuzzi, Galley, Meidan & Van Den Broeck 2016

Maximum error: 0.043 radians in phase

Implemented in LAL TEOBResum_ROM

T(S)EOB_ihes models are available as stand-alone matlab codes and as c++ codes (LAL-ification in progress with Nikhef group)
SUMMARY


2. TEOBResumS: spin-aligned waveform model for BBH and NS. It can be used for measuring the EOS through the measure of the tidal polarizability constants.

3. Compatibility (within NR errors) between such EOBNR model and state-of-the art NR data over mass ratio and spin

4. BNS waveforms are very mature, and different avatars exists that can be used for DA purposes to measure the EOS