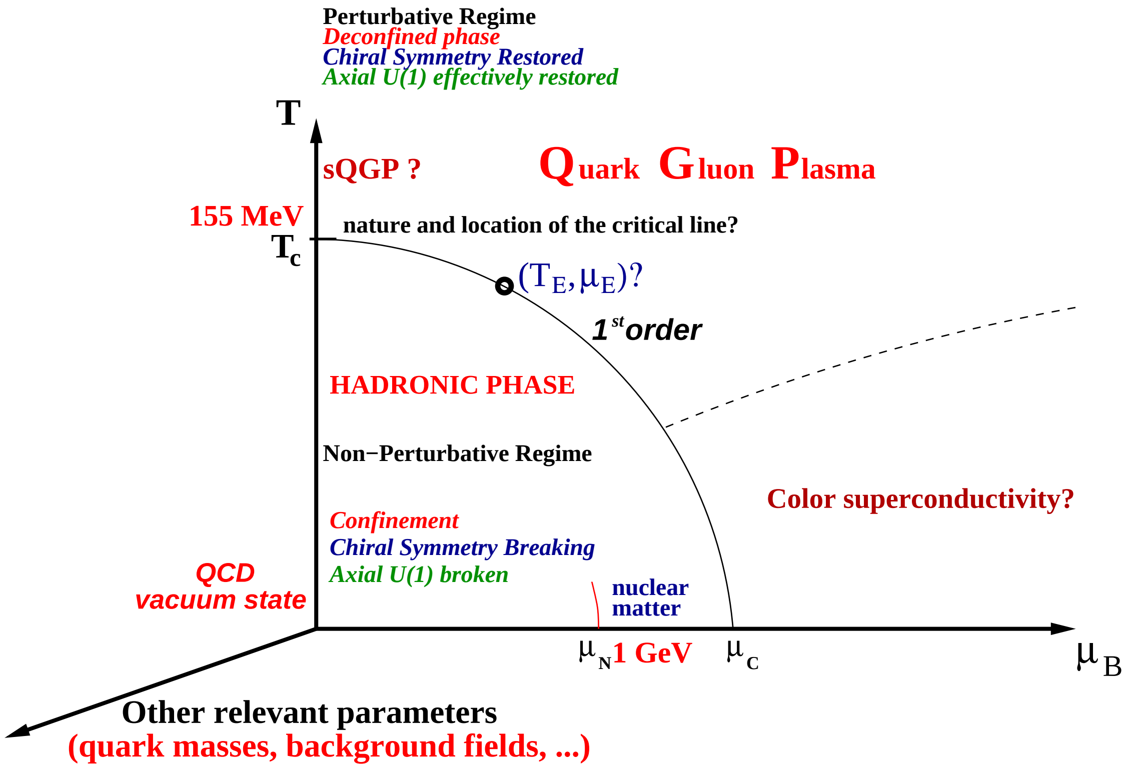
# Perspectives on Lattice QCD inputs for Heavy Ion Physics


**Massimo D'Elia**

University of Pisa & INFN


Secondo incontro sulla fisica con ioni pesanti a LHC, Torino, 10/10/2017

# Lattice QCD is our first principle tool to investigate the QCD phase diagram




## What we would like to know:

- **Location and nature of deconfinement/chiral symmetry restoration as a function of other external parameters ($\mu_B$, external fields, ...)**
- **Critical endpoint at finite $\mu_B$?**
- **Properties of the various phases of strongly interacting matter**

# Problems in lattice QCD at $\mu_B \neq 0$

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

$\det M[\mu_B]$ **complex** $\Longrightarrow$ **Monte Carlo simulations are not feasibile.**

**By now, we can rely on a few approximate methods, viable only for small $\mu_B/T$, like**

- **Taylor expansion of physical quantities around $\mu = 0$**

  **Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003**

- **Reweighting (complex phase moved from the measure to observables)**

  **Barbour et al. 1998; Z. Fodor and S, Katz, 2002**

- **Simulations at imaginary chemical potentials (plus analytic continuation)**

  **Alford, Kapustin, Wilczek, 1999; Lombardo, 2000; de Forcrand, Philipsen, 2002; MD, Lombardo 2003.**

**Others are being developed but still not fully operative (Langevin simulations, density of states, Lefschetz thimble, rewriting the partition function in terms of dual variables, ...)**

**Continuation to real $\mu$ or Taylor expansion is conceivable for quantities with an expected analytic behavior around $\mu = 0$**

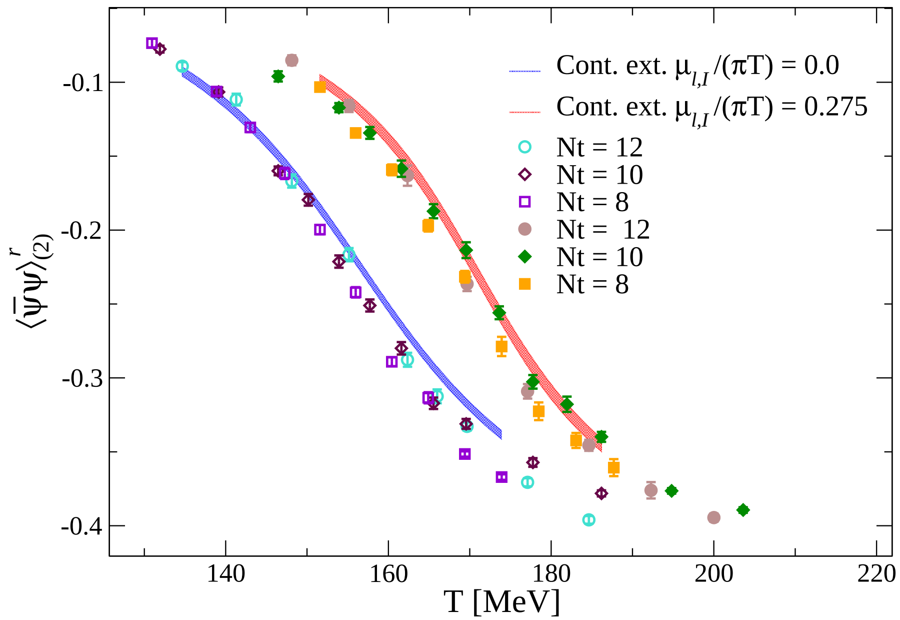
**An example is the dependence of $T_c$ on $\mu_B$:**

$$\frac{T(\mu_B)}{T_c} \simeq 1 - \kappa \left(\frac{\mu_B}{T(\mu_B)}\right)^2 = 1 - 9\kappa \left(\frac{\mu}{T(\mu)}\right)^2$$

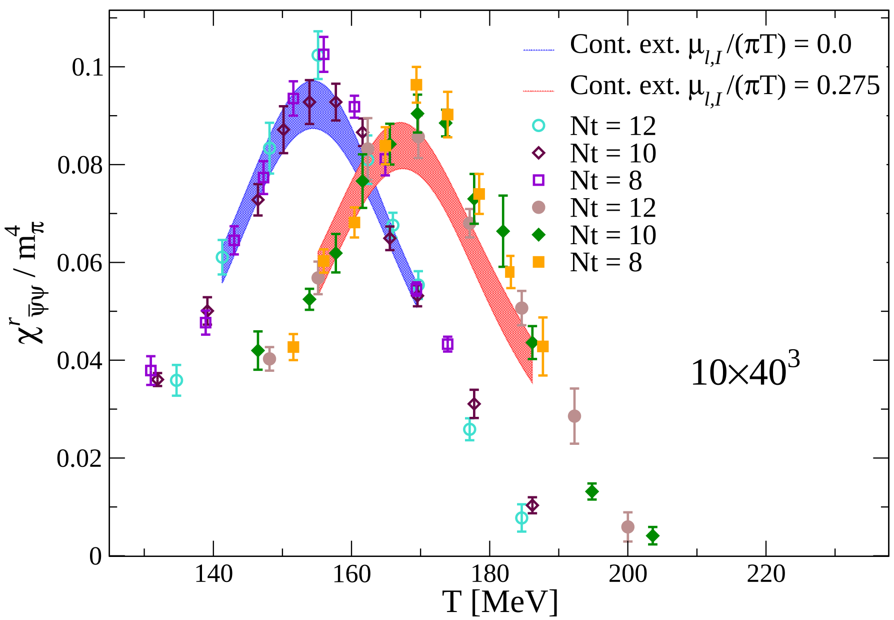
**$\mu$ is the quark chemical potential, $\kappa$ is the curvature of the pseudo-critical line at $\mu_B = 0$ and can be obtained either by Taylor expansion technique or by numerical simulations at imaginary $\mu_B$, assuming analyticity around $\mu_B = 0$:**

$$\frac{T(\mu_I)}{T_c} \simeq 1 + 9\kappa \left(\frac{\mu_I}{T(\mu_I)}\right)^2$$

**In the imaginary chemical potential approach, $T_c$ is computed as a function of $\mu_I$ from various quantities**




**chiral condensate**



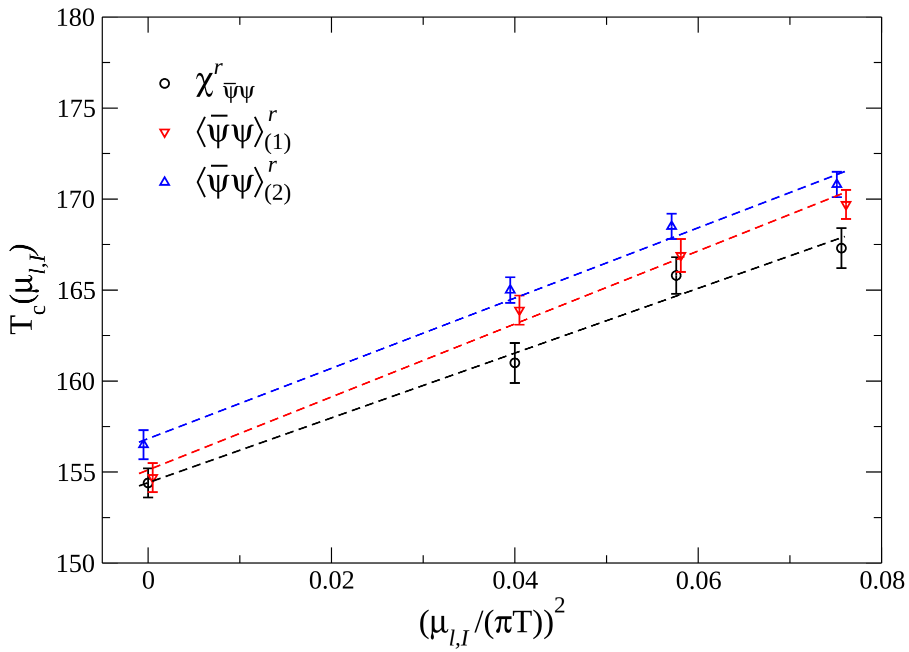

**chiral susceptibility**

**Localizing the pseudocritical temperature for various imaginary chemical potentials from various observables (continuum extrapolation)**

**(results from Bonati et al., arXiv:1507.03571)**

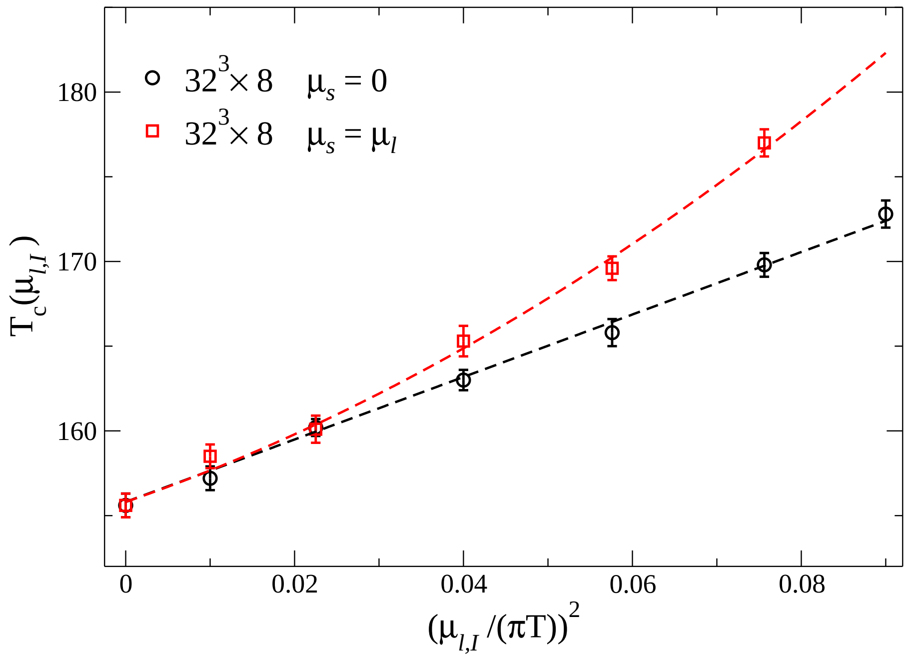
then, assuming analyticity, $\kappa$ is extracted by fitting a linear dependence in $\mu_I^2$ for small $\mu_I$.

$T_c$ location depends on the observable, slope in $\mu_I^2$ is much less sensitive



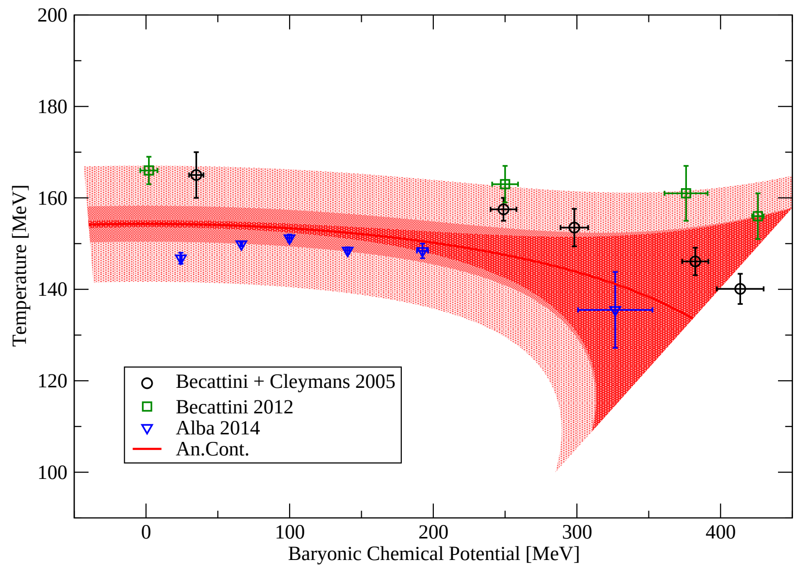

Results obtained for $\mu_u = \mu_d = \mu_l; \;\; \mu_s = 0$

Comparison with $mu_s = \mu_u = \mu_d = \mu_l$ shows deviations, but limited to higher order terms in $\mu^2$, curvature is unaffected




Experimental conditions: $\mu_s \sim 0.25\mu_l$ (to ensure strangeness neutrality)

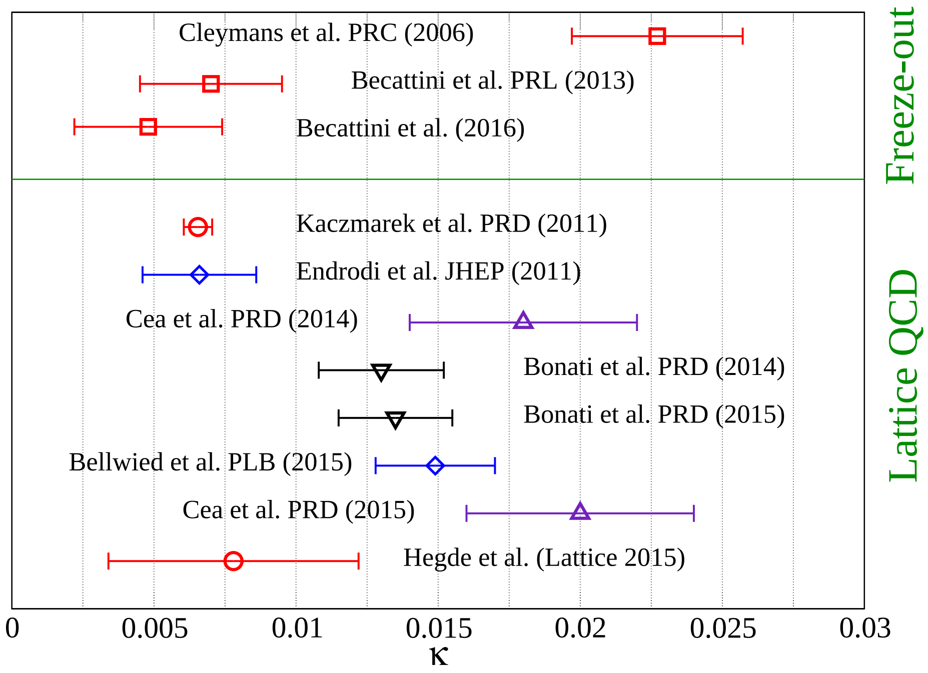
The pseudo-critical line from analytic continuation, compared with determinations of the freeze-out line from heavy ion experiments




The curvature of the pseudo-critical line: various lattice determinations and comparison with freeze-out

Convergence of most recent results indicates good control over possible systematic effects.

Is there a tension between Taylor expansion and analytic continuation?

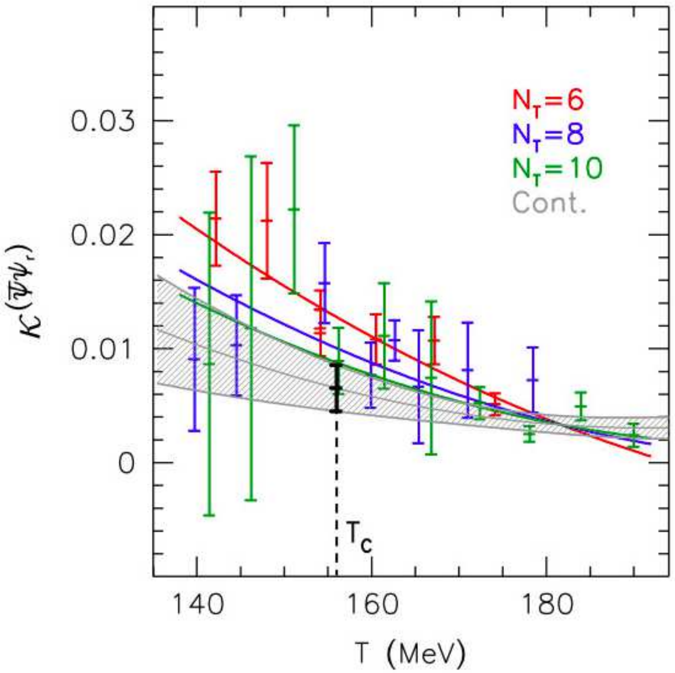


**Some earlier results from Taylor expansion, based on the determination of**

$$\kappa = T_c(0)\frac{\left(\partial\langle\overline{\psi}\psi\rangle/\partial\mu^2\big|_{\mu=0}\right)}{\left(\partial\langle\overline{\psi}\psi\rangle/\partial T\big|_{\mu=0}\right)}$$

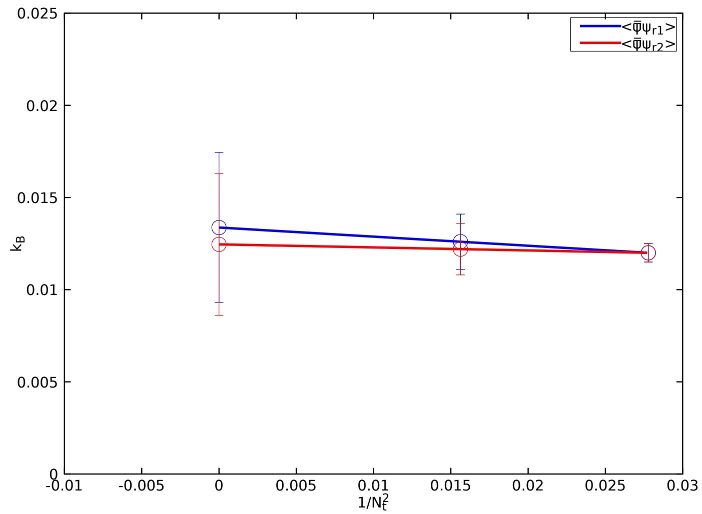
**might be affected by large continuum extrapolation systematics**

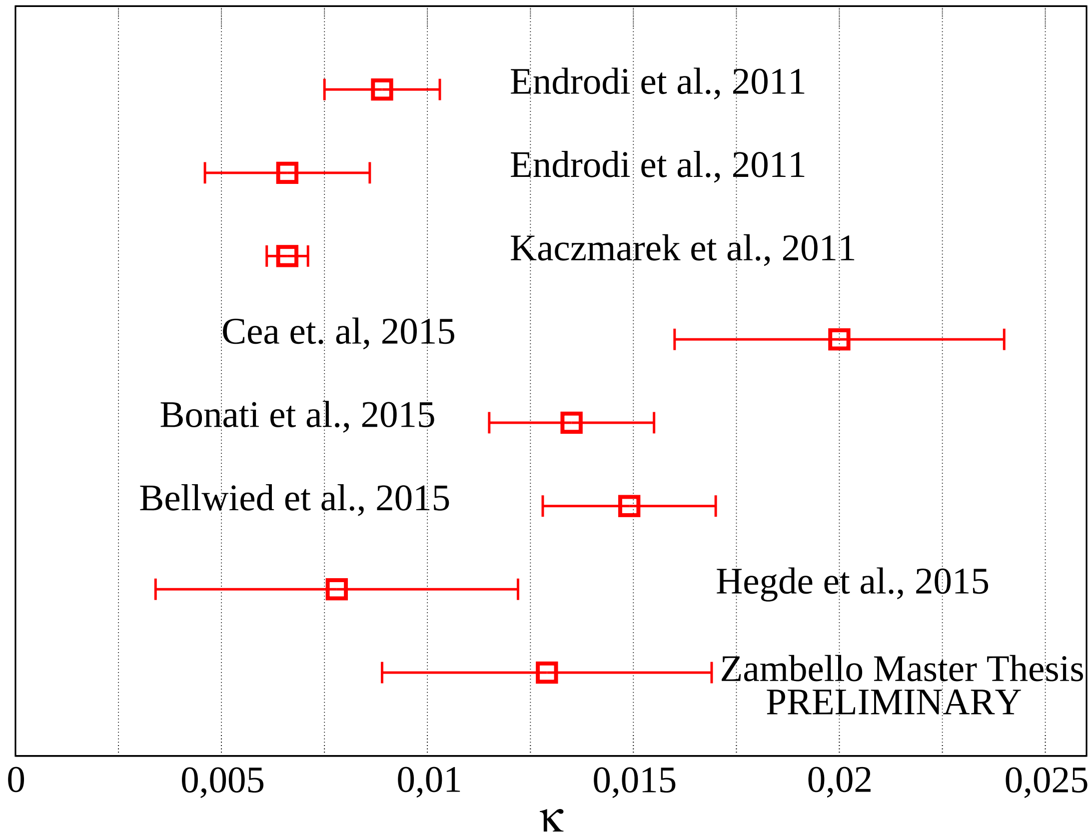
**Endrodi, Fodor, Katz and Szabo, arXiv:1102.1356**




**New results exploiting the same discretization and observables seem to results more in line with analytic continuation**

**Kevin Zambello, Master Thesis, Pisa U., unpublished**






**CONCLUSION: most recent results show a convergence between analytic continuation and Taylor expansion results**

# Perspectives on the pseudo-critical line:

- **Can we obtain consistent information about quartic corrections in the next future?**

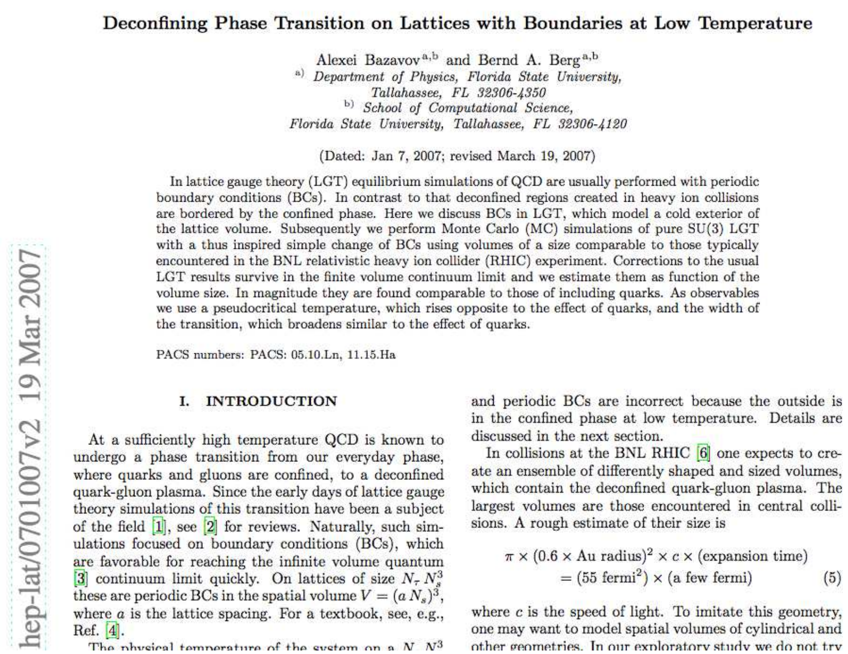
  The task seems not easily achievable by Taylor expansion methods (higher order derivative operators are needed)

  Analytic continuation could be successful however systematic uncertainties due to the choice of the fitting function are more relevant.

- **Are finite size corrections important?**

  The fireball has a finite size, how does that influence the pseudocritical temperature and the curvature?

Last issue has been approached a few years ago in the quenched theory



Deconfining Phase Transition on Lattices with Boundaries at Low Temperature

Alexei Bazavov[a,b] and Bernd A. Berg[a,b]
[a)] *Department of Physics, Florida State University, Tallahassee, FL 32306-4350*
[b)] *School of Computational Science, Florida State University, Tallahassee, FL 32306-4120*

(Dated: Jan 7, 2007; revised March 19, 2007)

In lattice gauge theory (LGT) equilibrium simulations of QCD are usually performed with periodic boundary conditions (BCs). In contrast to that deconfined regions created in heavy ion collisions are bordered by the confined phase. Here we discuss BCs in LGT, which model a cold exterior of the lattice volume. Subsequently we perform Monte Carlo (MC) simulations of pure SU(3) LGT with a thus inspired simple change of BCs using volumes of a size comparable to those typically encountered in the BNL relativistic heavy ion collider (RHIC) experiment. Corrections to the usual LGT results survive in the finite volume continuum limit and we estimate them as function of the volume size. In magnitude they are found comparable to those of including quarks. As observables we use a pseudocritical temperature, which rises opposite to the effect of quarks, and the width of the transition, which broadens similar to the effect of quarks.

PACS numbers: PACS: 05.10.Ln, 11.15.Ha

hep-lat/0701007v2 19 Mar 2007

I. INTRODUCTION

At a sufficiently high temperature QCD is known to undergo a phase transition from our everyday phase, where quarks and gluons are confined, to a deconfined quark-gluon plasma. Since the early days of lattice gauge theory simulations of this transition have been a subject of the field [1], see [2] for reviews. Naturally, such simulations focused on boundary conditions (BCs), which are favorable for reaching the infinite volume quantum [3] continuum limit quickly. On lattices of size $N_\tau N_s^3$ these are periodic BCs in the spatial volume $V = (a\,N_s)^3$, where $a$ is the lattice spacing. For a textbook, see, e.g., Ref. [4].

The physical temperature of the system on a $N_\tau N_s^3$

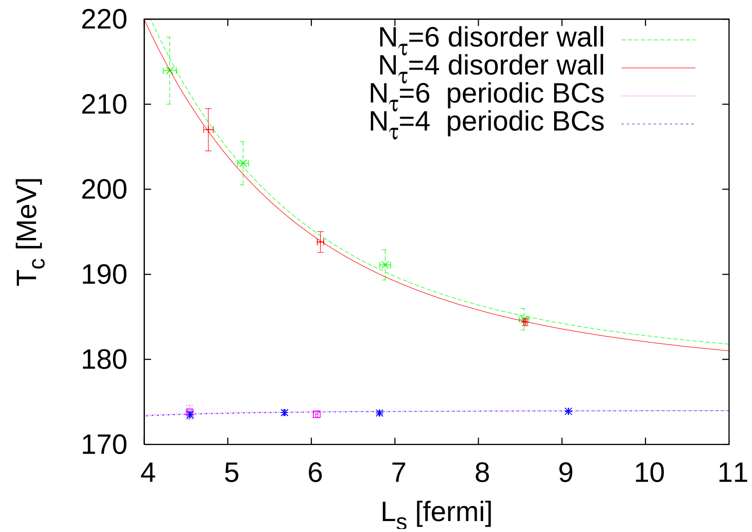
and periodic BCs are incorrect because the outside is in the confined phase at low temperature. Details are discussed in the next section.

In collisions at the BNL RHIC [6] one expects to create an ensemble of differently shaped and sized volumes, which contain the deconfined quark-gluon plasma. The largest volumes are those encountered in central collisions. A rough estimate of their size is

$$\pi \times (0.6 \times \text{Au radius})^2 \times c \times (\text{expansion time}) = (55\ \text{fermi}^2) \times (\text{a few fermi}) \qquad (5)$$

where $c$ is the speed of light. To imitate this geometry, one may want to model spatial volumes of cylindrical and other geometries. In our exploratory study we do not try

Lattice is of course finite by default, but one has to switch from standard periodic to open (or random) spatial boundary conditions to mimic a cold boundary




The quenched result is that $T_c$ increases as an effect of the finite system size

Extension to QCD with dynamical fermions is something which is at hand for the next future

# What about the critical endpoint?

**Until a working solution to the sign problem is found, locating the critical endpoint is a hard task.**

**Recent approaches consider the expansion of the free energy in $\mu_B$ and the so-called generalized susceptibilities, which can be extracted by Taylor expansion or by analytic continuation**

$$\mathcal{F}(T,\mu_B) = \mathcal{F}(T,0) + VT^4 \sum_n \frac{\chi_{2n}^B}{(2n)!} \, (\mu_B/T)^{2n}$$

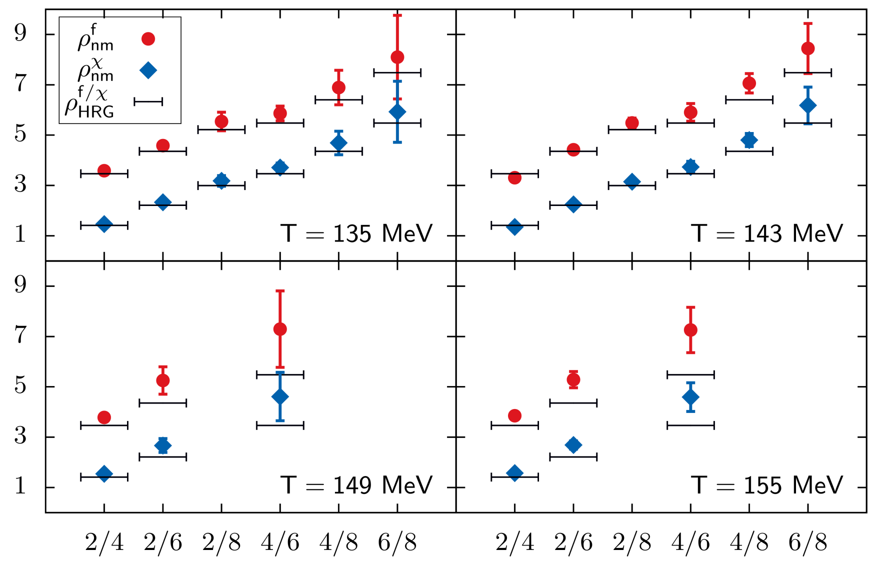
**from them we can reconstruct the free energy dependence and extract estimates of the radius of convergence**

$$\rho_{n,m}^f = \left( \frac{\chi_n^B/n!}{\chi_m^B/m!} \right)^{\frac{1}{(m-n)}} \quad \rho_{n,m}^\chi = \left( \frac{\chi_n^B/(n-2)!}{\chi_m^B/(m-2)!} \right)^{\frac{1}{(m-n)}}$$

**Results from analytic continuation**

G. Gagliardi, MD, Sanfilippo, 1611.08285, physical point of $N_f = 2+1$ QCD, $N_t = 8$

"Convergence" of the radius of convergence seems slow, and no significant deviations from the HRG are observed

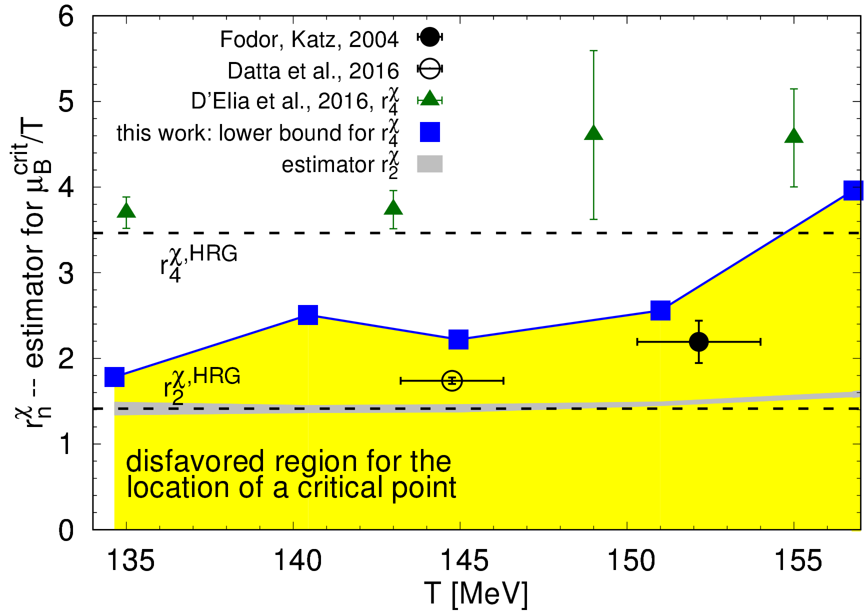



Similar results obtained by the hotQCD collaboration by Taylor expansion up to $O(\mu_B^6)$

A. Bazavov et al., 1701.04325

Lower bounds on the radius of convergence rule out some previous determinations

Note: results mentioned yesterday by Paolo correspond to $\mu_B/T > 10$, so it could just be that we need many more terms in the expansion and lower $T$ (Samanta-Mohanty, 1709.04446, modified HRG, $T_c = 62$ MeV, $\mu_{Bc} = 708$ MeV)



# Transport coefficients: difficulties and prospects

**Lattice QCD is ideally suited for the study of equilibrium properties**

**Nevertheless, the computation of certain Euclidean correlators can give access to spectral functions relevant to linear response theory**

$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\,\rho_E(\omega)\,\frac{\cosh[\omega(\frac{\beta}{2}-\tau)]}{\sinh[\frac{\omega\beta}{2}]}$$

**Difficulties: solve the integral equation with a finite number of determinations of $G_E$. That requires an ansatz on $\rho_E$ which then becomes part of the systematic uncertainty.**

**Increasing high precision and number of points for $G_E$ finally will constrain the systematic uncertainties: extremely fine lattices and extremely precise data are required.**

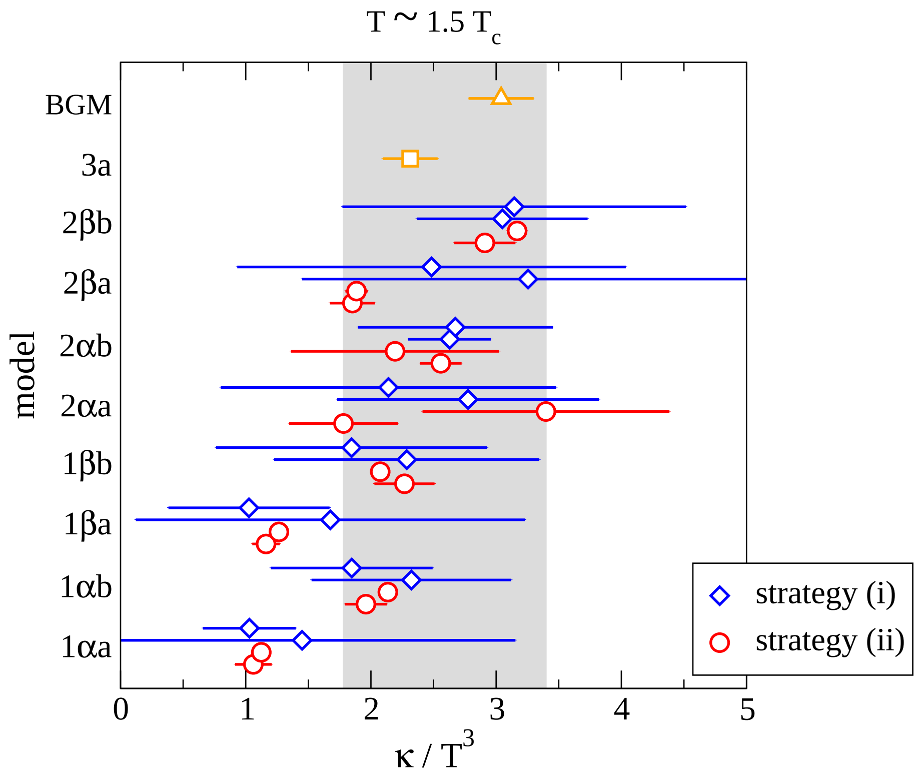
**Successful example in quenched theory for the heavy quark diffusion coefficient in the quenched theory:**

**A. Francis et al., 1508.04053**

- $192^3 \times 48$ **lattice** ($T = 1.5 T_c \to a \simeq 0.01$ **fm)**

- **Luscher-Weisz multilevel update scheme for exponential noise reduction on Polyakov loop correlators**

$$\kappa = \lim_{\omega \to 0} T \rho_E(\omega)/\omega$$




**How difficult is to extend to full QCD?**

- **state of the art** $N_t = 16$

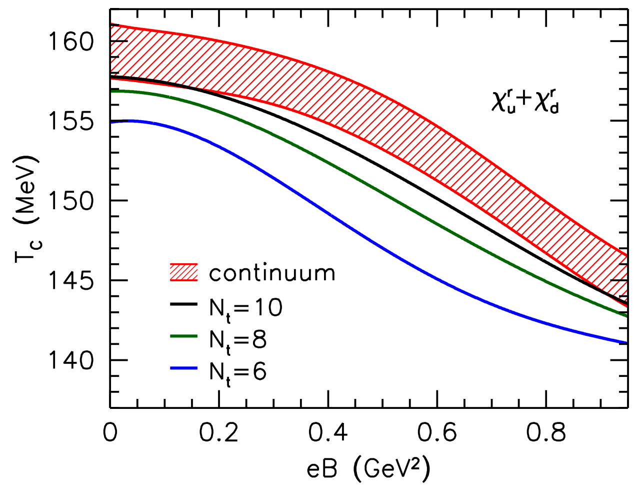
- **multilevel not available yet (domain decomposition algorithms under way)**

# Properties of QCD in a strong magnetic field

**A magnetic background does not pose any technical problem to lattice QCD. The issue of the relevance to peripheral HIC (relaxation time of magnetic field) is open.**

**The magnetic field has strong effects also on QCD thermodynamics and leads to a decrease of the pseudo-critical temperature**

**G. S. Bali et al., arXiv:1111.4956**

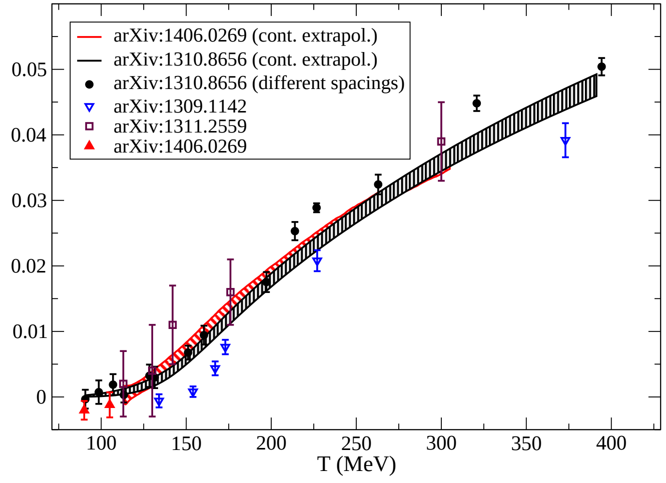



**The thermal QCD medium becomes strongly paramagnetic right above $T_c$. On the left: magnetic susceptibility**

**C. Bonati et al., arXiv:1307.8063, arXiv:1310.8656;**

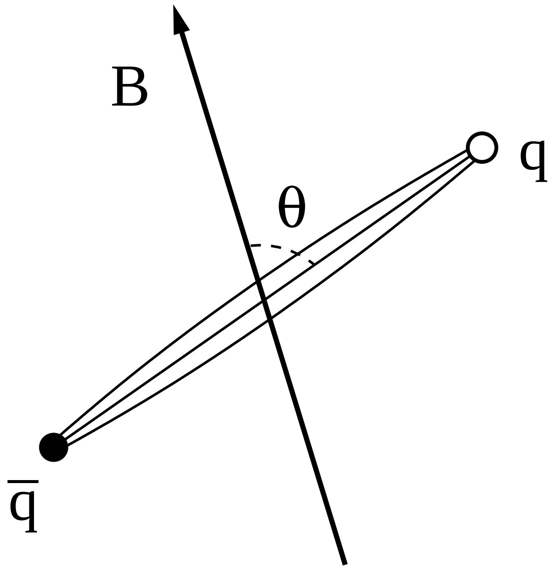
**L. Levkova and C. DeTar, arXiv:1309.1142;**

**G. S. Bali et al., arXiv:1406.0269**

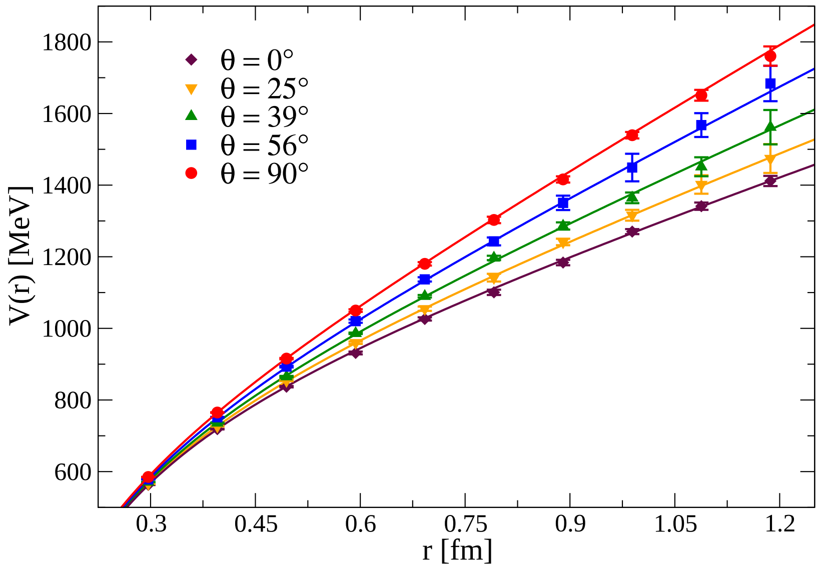


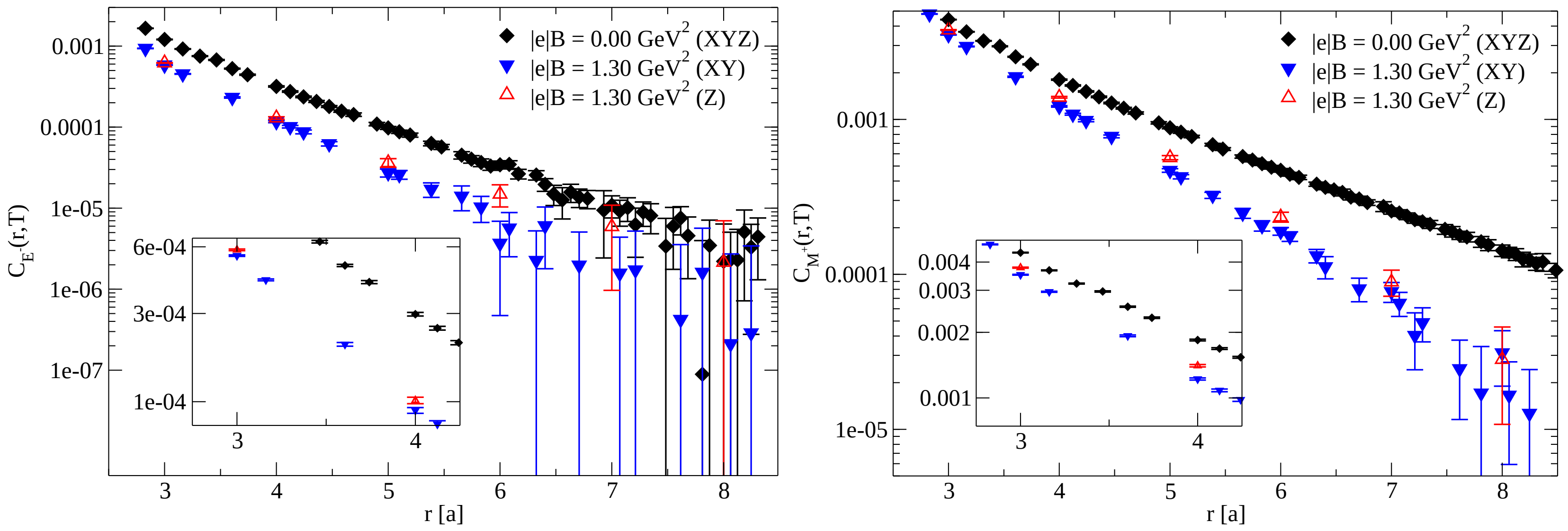
The magnetic field has also shown to strongly influence the interaction between heavy quarks, introducing an anisotropy in the potential.

C. Bonati et al., arXiv:1403.6094, arXiv:1607.08160




At fixed $r$, the potential is an increasing function of the angle and reaches a maximum for orthogonal directions





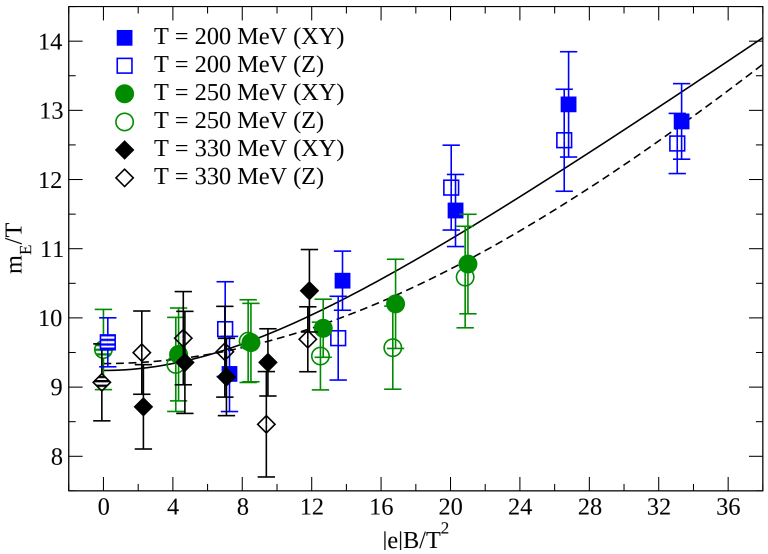

**In the deconfined phase, Polyakov loop correlators give access to electric and magnetic screening masses, which also show a sizable dependence on the magnetic background**

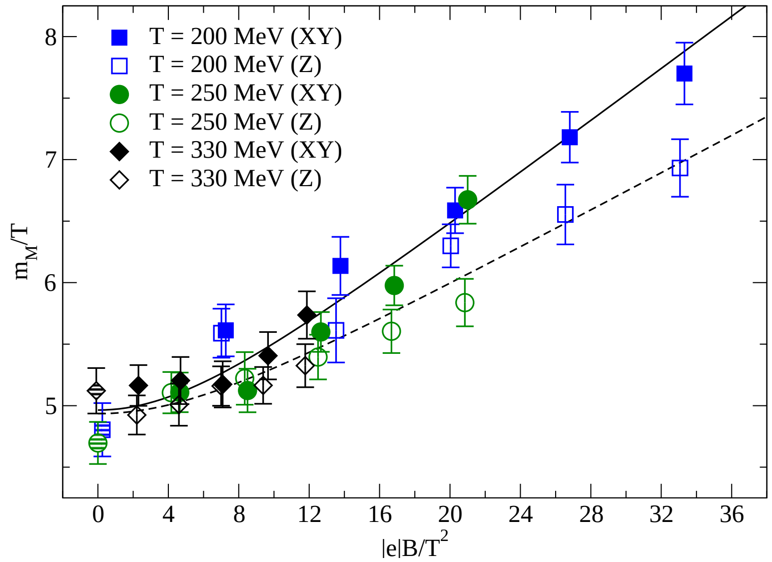
$$C_{M^+} = +\frac{1}{2}\mathrm{Re}\left[C_{LL} + C_{LL^\dagger}\right] - |\langle \mathrm{Tr} L\rangle|^2$$

$$C_{E^-} = -\frac{1}{2}\mathrm{Re}\left[C_{LL} - C_{LL^\dagger}\right] .$$

$$C_{E^-}(\mathbf{r},T)\big|_{r\to\infty} \simeq \frac{e^{-m_E(T) r}}{r}$$

$$C_{M^+}(\mathbf{r},T)\big|_{r\to\infty} \simeq \frac{e^{-m_M(T) r}}{r}$$




**Such masses show a clear (increasing) dependence on $B$: the magnetic background field enhances the color screening properties of the QGP**

$$\frac{m^d_{E/M}}{T} = a^d_{E/M}\left[1 + c^d_{1;E/M}\frac{|e|B}{T^2}\,\mathrm{atan}\left(\frac{c^d_{2;E/M}}{c^d_{1;E/M}}\frac{|e|B}{T^2}\right)\right],$$

**Does $B$ have any influence on heavy quarkonia suppression in the QGP? Not clear, one should also know how the quarkonia wave function is modified by B**

**A direct determination of quarkonia spectral functions in the presence of B would be the most direct way to check, which should be pursued in future lattice simulations.**

# CONCLUSIONS

- **Many issues can be sistematically clarified in the next few years:**
  - quartic corrections in $\mu_B$ to the pseudocritical line
  - effects of the finite fireball size on $T_c$ and on the curvature
  - effects of magnetic background on quarkonia in the QGP
- **In some directions, the perspective still quenches in a dense fog:**

  Information about the critical endpoint would be (almost) immediate after solution of the sign problem. By now, we can just put all our efforts on a refinement of the free energy expansion in terms of $\mu_B$ and then look for the radius of convergence. Luckily enough, this is interesting by itself (conserved charge fluctuations)
- **Transport coefficients:**

  we will likely have a systematic refinement of quenched studies in the next future. That will be the basis for full QCD determinations, which however are order of magnitudes more demanding.