Open quantum system approach to heavy quarks

Davide De Boni

"Secondo incontro sulla fisica con ioni pesanti a LHC", Torino, 9-10 October 2017

Swansea University
Prifysgol Abertawe

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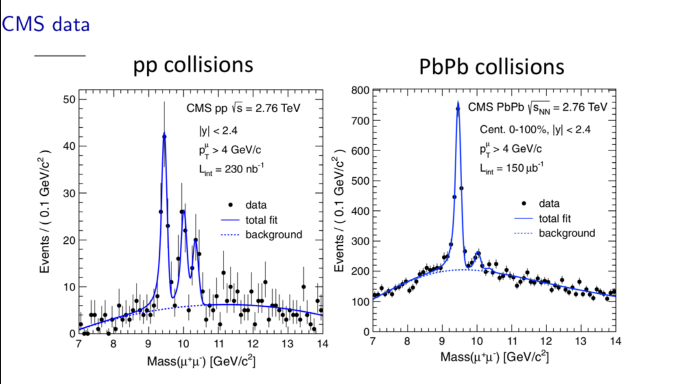
Bottomonium at the LHC

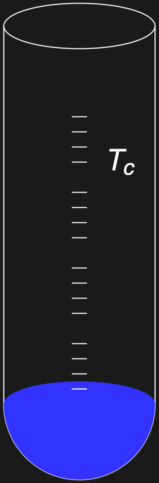
Do we understand what is going on?

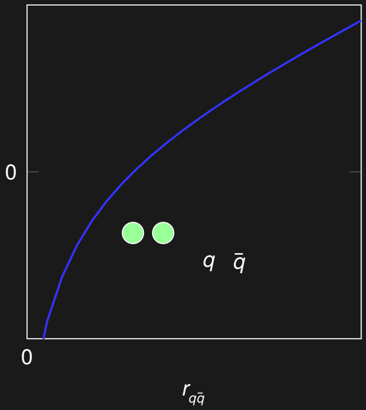


- Do bound states melt?
- Complications with feed-down mechanism
- Are there in-medium modifications?
  - Peaks very similar to the vacuum ones

Quarkonium dissociation

J/Ψ suppression, [Matsui-Satz (86)]





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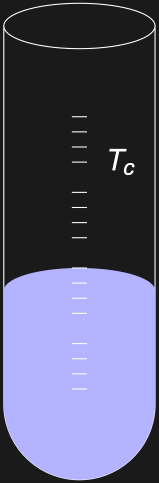
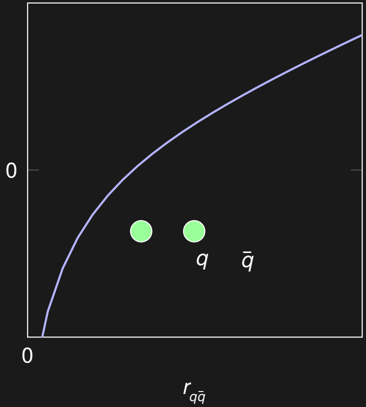


Interquark potential



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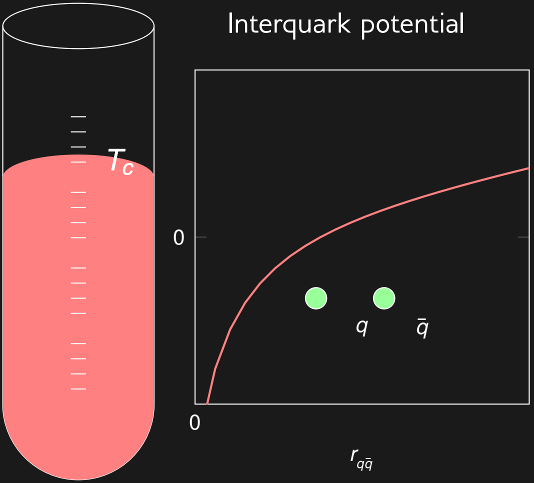
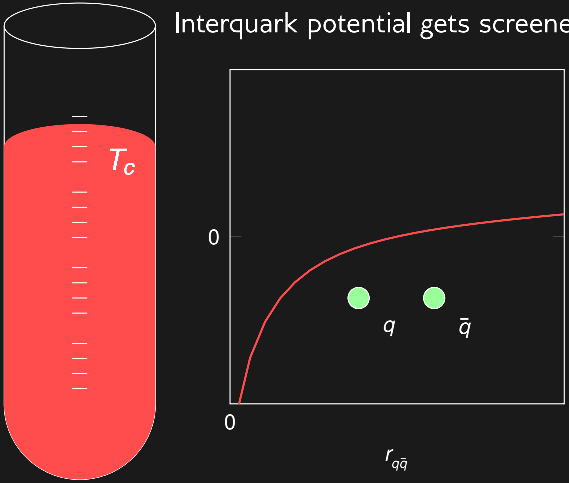


Quarkonium dissociation

J/Ψ suppression, [Matsui-Satz (86)]

Interquark potential gets screened



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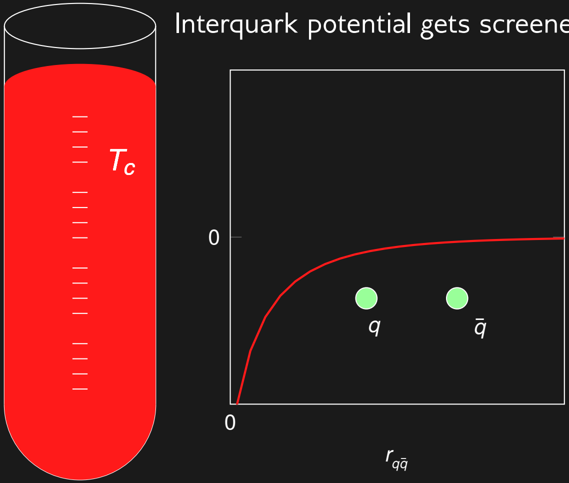
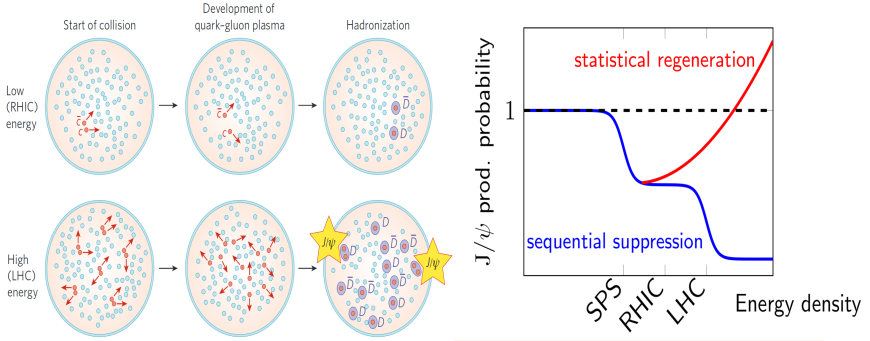


Potential develops an **imaginary part** ⇒ Dissociation

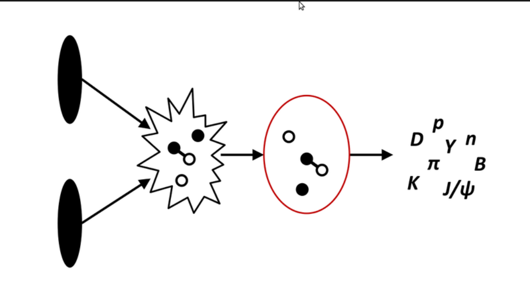
Quarkonium regeneration

[Matsui (87), BraunMunzinger-Stachel (00), Thews-Schroedter-Rafelski (01)]



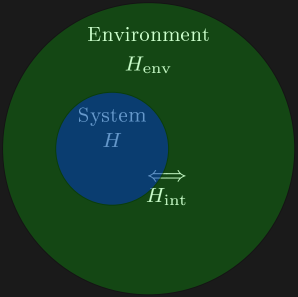
High mobility of heavy quarks in the QGP ⇒ Regeneration

Different stages of the heavy-ion collision



- Quarkonium production in heavy-ion collisions
- Early-time dynamics
- Dynamics in the quark gluon plasma $\Leftarrow$ Our focus
- Hadronization
- Vacuum feed-down

Open quantum systems



Schrödinger equation for closed quantum system (heavy particles + plasma)

$$i\hbar \frac{d\rho_{\text{tot}}}{dt}(t) = [H_{\text{tot}}, \rho_{\text{tot}}(t)] \qquad H_{\text{tot}} = H \otimes \mathbb{I}_{\text{env}} + \mathbb{I} \otimes H_{\text{env}} + H_{\text{int}}$$

- ρ_{tot} is the density operator of the total (closed) system
- $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ for a pure state

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Lindblad equation

Master equation for open quantum system (heavy particles)

$$
\begin{aligned}
i\hbar \frac{d\rho}{dt}(t) &= \mathrm{Tr}_{\mathrm{env}} \{[H_{\mathrm{tot}}, \rho_{\mathrm{tot}}(t)]\} \\
&= [H, \rho(t)] + \mathrm{Tr}_{\mathrm{env}} \{[\mathbb{I} \otimes H_{\mathrm{env}} + H_{\mathrm{int}}, \rho_{\mathrm{tot}}(t)]\} \\
&\equiv [H, \rho(t)] + i\, \mathcal{D}\rho(t)
\end{aligned}
$$

$$\rho \equiv \mathrm{Tr}_{\mathrm{env}} \rho_{\mathrm{tot}}$$

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\end{aligned}
$$

$$\rho \equiv \mathrm{Tr}_{\rm env}\,\rho_{\rm tot}$$

Most general master equation in the Markovian limit

- $1/m_{\rm D} \sim \tau_{\rm env} \ll \tau_{\rm sys} \sim 1/\Delta E \Rightarrow$ no memory effects
- $\Delta E \ll m_{\rm D}$ like in effective-field-theory models [Brambilla et al. (10,13)]

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \frac{1}{2\hbar}\sum_{\mu}\left([L_\mu \rho, L_\mu^\dagger] + [L_\mu, \rho L_\mu^\dagger]\right)$$

$L_\mu,\ L_\mu^\dagger$ are the Lindblad operators

Open quantum system approaches

- Lindblad equation (QED case, Singlet-Octet effective model in QCD) [DDB (17), Akamatsu (15), Brambilla, Escobedo et al. (16,17)]
  - Comes directly from the gauge theory, contains diffusion, dissipation and decoherence
  - Langevin and Fokker-Planck dynamics in the classical limit
- Stochastic potential model [Akamatsu, Rothkopf (12)]
  - Comes directly from the gauge theory, easier to simulate but it does not contain dissipation (good only for short times)
- Schrödinger-Langevin equation [Katz, Gossiaux (16)]
  - Easy to simulate, it contains dissipation but does not come from the gauge theory

Abelian model

[Blaizot, DDB, Faccioli, Garberoglio (16), DDB (17)]

- Plasma in thermal equilibrium described by ψ, $\bar{\psi}$
- N heavy quarks and antiquarks propagating out of equilibrium in the plasma
  - **non relativistic heavy particles** (neglect magnetic effects)
  - $m \ll T \ll M$

$$
\begin{aligned}
H_{\rm tot} &= \frac{1}{2M}\sum_{i=1}^{N}\left(\mathbf{p}_i^2+\overline{\mathbf{p}}_i^2\right)+\int \mathrm{d}\mathbf{x}\;\overline{\psi}(\mathbf{x})\;(-\mathrm{i}\gamma^i\partial_i+m)\;\psi(\mathbf{x})+\\
&+\underbrace{\frac{1}{2}\iint \mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{y}\;j^0_{\rm tot}(\mathbf{x})\frac{1}{4\,\pi|\mathbf{x}-\mathbf{y}|}j^0_{\rm tot}(\mathbf{y})}_{\text{Coulomb interactions}}
\end{aligned}
$$

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\begin{aligned}
H_{\text{tot}} &= \frac{1}{2M}\sum_{i=1}^{N}\left(\mathbf{p}_i^2 + \bar{\mathbf{p}}_i^2\right) + \int \mathrm{d}\mathbf{x}\, \bar{\psi}(\mathbf{x})\, (-\mathrm{i}\gamma^i\partial_i + m)\, \psi(\mathbf{x}) + \\
&+ \underbrace{\frac{1}{2}\iint \mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{y}\; j^0_{\text{tot}}(\mathbf{x})\frac{1}{4\,\pi|\mathbf{x}-\mathbf{y}|}j^0_{\text{tot}}(\mathbf{y})}_{\text{Coulomb interactions}}
\end{aligned}
$$

$$
j^0_{\text{tot}}(\mathbf{x}) = \overbrace{g\sum_{i=1}^{N}\left(\delta(\mathbf{x}-\mathbf{q}_i) - \delta(\mathbf{x}-\bar{\mathbf{q}}_i)\right)}^{\text{heavy quarks+antiquarks}} + \overbrace{g\,\bar{\psi}(\mathbf{x})\,\gamma^0\psi(\mathbf{x})}^{\text{plasma fields}}
$$

Lindblad eqn for a $q\bar{q}$ pair (CoM frame)

- $W(\mathbf{r})$ is the imaginary part of the $q\bar{q}$ potential. $V(\mathbf{r})$ is the real screened part.
- W has a correlation length of $1/m_\mathrm{D} \sim 1/(gT)$

$\mathbf{r} = \frac{1}{2}(\mathbf{q}+\mathbf{q}')\,, \qquad \mathbf{y} = \frac{1}{2}(\mathbf{q}-\mathbf{q}')$

$$\frac{\partial \rho(t,\mathbf{r},\mathbf{y})}{\partial t} = \Bigg(\overbrace{\frac{\mathrm{i}\hbar}{M}\frac{\partial}{\partial \mathbf{r}}\cdot\frac{\partial}{\partial \mathbf{y}} - \frac{\mathrm{i}}{\hbar}\left(V(\mathbf{r}+\mathbf{y}/2) - V(\mathbf{r}-\mathbf{y}/2)\right)}^{\text{Schrödinger}}$$

$$\underbrace{-\frac{g^2}{\hbar}\left(2W(\mathbf{y}) - 2W(\mathbf{r}) + W(\mathbf{r}+\mathbf{y}) + W(\mathbf{r}-\mathbf{y}) - 2W(0)\right)}_{\text{diffusion, decoherence}}$$

$$\underbrace{-\frac{g^2\hbar}{2MT}\left(\frac{\partial W(\mathbf{y})}{\partial \mathbf{y}}\cdot\frac{\partial}{\partial \mathbf{y}} - \frac{\partial W(\mathbf{r})}{\partial \mathbf{r}}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{\partial^2 W(\mathbf{r})}{\partial \mathbf{r}^2}\right)}_{\text{dissipation}}\Bigg)\rho(t,\mathbf{r},\mathbf{y})$$

Dissociation, recombination and quantum decoherence

- Probability of having the state $|\psi\rangle$ at time t
  $\rightarrow$ To study $q\bar{q}$ dissociation and recombination

  $$P(\psi, t|\psi_0, t_0) = \int \mathrm{d}q \int \mathrm{d}q' \psi(q') \psi^*(q) \rho(t, q, q')$$

- Linear entropy (proxy of thermal entropy $S = -\mathrm{Tr}\,[\rho \ln \rho]$)

  $$S_L = \mathrm{Tr}\rho - \mathrm{Tr}\rho^2 = 1 - \mathrm{Tr}\rho^2$$

  $\rightarrow$ To study how fast the system becomes classical (quantum decoherence)

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  $\rightarrow$ To study how fast the system becomes classical (quantum decoherence)

Pure states	$\rho = \rho^2 \Rightarrow S_L = 0$
Non-pure states	$\rho \neq \rho^2 \Rightarrow 0 < S_L \leq 1$

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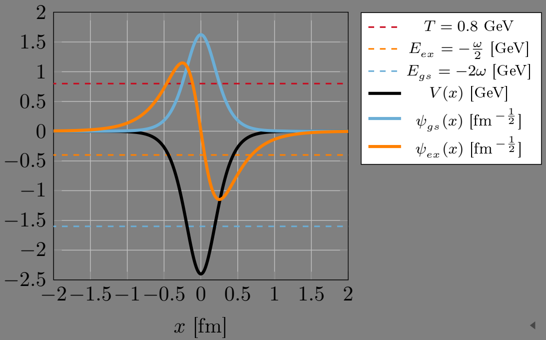
Numerical results for a $q\bar{q}$ pair in 1D

Pöschl-Teller potential:

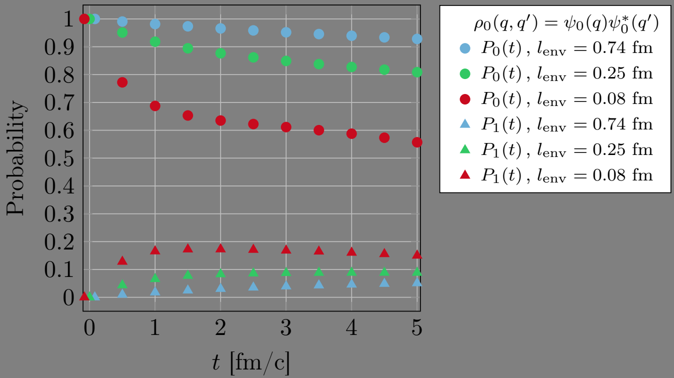
$$V(x) = -\frac{\omega}{2} j(j+1)\mathrm{sech}^2\left[\sqrt{\frac{M\omega}{2\hbar^2}}x\right] \qquad j = 2 \text{ (bound states)}$$

$$W(x) = -\frac{T}{2}\exp\left[-\frac{1}{2}\left(\frac{x}{l_{\mathrm{env}}}\right)^2\right] \qquad l_{\mathrm{env}} \sim \frac{1}{gT}$$



The crucial quantity is the ratio $\lambda_{\mathrm{sys}}/l_{\mathrm{env}}$

$$\lambda_{\mathrm{sys}} = 0.16\ \mathrm{fm}$$



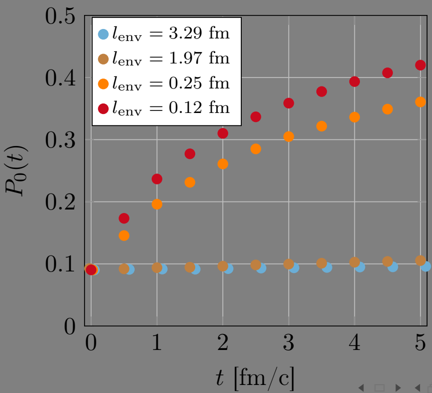
Ground state melts with $P = 1 - P_0 - P_1 \sim 10\%$ after $\Delta t = 5$ fm/c when $l_{\mathrm{env}} = 0.25$ fm

Starting off with a thermal scattering state

$$\psi_{\text{scatt}}(x) \sim e^{-\frac{1}{2}\left(\frac{x}{\delta}\right)^2 + \frac{i}{\hbar} x p} \qquad \delta = \sqrt{2\langle \hat{x}^2 \rangle} = \sqrt{2}\,\lambda_{\text{sys}}$$

$$\lambda_{\text{sys}} = \frac{h}{p_{\text{th}}} = \frac{\sqrt{2}h}{\sqrt{mT}} = 1.77\,\text{fm} \qquad m = 1.2\,\text{GeV}$$

Recombination effects!



Stochastic potential model (Abelian case)

$$\psi_{q\bar{q}}(t+\Delta t) = \exp\left[-\mathrm{i}H(\theta)\Delta t\right]\psi_{q\bar{q}}(t) = U(\Delta t, \theta)\psi_{q\bar{q}}(t)$$

$$H(\theta) = -\frac{\nabla^2}{m} + V + \theta(t, \frac{\mathbf{r}}{2}) + \theta(t, -\frac{\mathbf{r}}{2})$$

Evolution operator is unitary but contains an imaginary part:

$$\theta \sim \frac{1}{(\Delta t)^{1/2}} \Rightarrow U(\Delta t, \theta) = 1 - \mathrm{i}H(\theta)\Delta t + \Delta t \underbrace{\left(W(\mathbf{r}) - W(0)\right)}_{\text{diffusion, no dissipation}}$$

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Non-unitary evolution of averaged wavefunction:

$$\langle\psi_{q\bar{q}}(t+\Delta t)\rangle_\theta = \langle U(\Delta t,\theta)\rangle_\theta\langle\psi_{q\bar{q}}(t)\rangle_\theta$$

Density matrix ($\rho_{q\bar{q}}(t,q,q') = \langle\psi_{q\bar{q}}(t,q)\psi^*_{q\bar{q}}(t,q')\rangle_\theta$):

$$\rho_{q\bar{q}}(t+\Delta t) = \underbrace{\langle U(\Delta t,\theta)U^*(\Delta t,\theta)\rangle_\theta}_{\text{trace preserved}}\,\rho_{q\bar{q}}(t)$$

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Classical level: Langevin and Fokker-Planck dynamics

Fokker-Planck and Langevin equations

Wigner function

$$\rho(t, \mathbf{r}, \mathbf{p}) = \int \mathrm{d}\mathbf{y}\, \rho(t, \mathbf{r}, \mathbf{y}) \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\mathbf{p}\cdot\mathbf{y}}$$

Fokker-Planck equation for one heavy quark (semiclassical limit):

$$\left[\partial_t + \frac{\mathbf{p}}{M} \cdot \partial_{\mathbf{r}} - \partial_{\mathbf{r}} V_{\mathrm{ext}}(\mathbf{r}) \cdot \partial_{\mathbf{p}}\right] \rho(t, \mathbf{r}, \mathbf{p}) = \gamma \left[MT\, \nabla_{\mathbf{p}}^2 + \partial_{\mathbf{p}} \cdot \mathbf{p}\right] \rho(t, \mathbf{r}, \mathbf{p})$$

Fokker-Planck and Langevin equations

Wigner function

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Corresponding Langevin equation for one heavy quark:

$$M\ddot{\mathbf{r}} + M\gamma\dot{\mathbf{r}} + \nabla_{\mathbf{r}} V_{\mathrm{ext}}(\mathbf{r}) = \boldsymbol{\eta}(\mathbf{r}, t) \qquad \gamma \sim W''(\mathbf{r} = 0)$$

(γ is space-depent in the many-quark case)
Noise vector corresponds to a stochastic force

$$\langle \boldsymbol{\eta}(\mathbf{r}, t)\rangle = 0\,, \qquad \langle \eta_i(\mathbf{r}, t)\eta_j(\mathbf{r}, t')\rangle = 2M\gamma T\delta_{ij}\delta(t - t')$$

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Langevin dynamics

Simulation for faintly bound $q\bar{q}$ pairs

- Pros: Cheap simulations, good approximation after a decoherence time
- Cons: Difficult to implement initial quantum conditions

Conclusions and outlook

- The language of open quantum systems is the appropriate one to study real-time dynamics of quarkonium
- Lindblad equation:
  - comes from gauge theory (e.g. EFT, even non-perturbative
  - quite difficult to simulate numerically
- Stochastic potential:
  - easier to simulate numerically
  - comes from gauge theory but no friction (fine for short time dynamics only)
- Langevin/Fokker-Planck dynamics:
  - easy numerical simulations for many heavy quarks
  - comes from gauge theory (and Lindblad) in the classical limit
  - fair description when quantum information is lost (decoherence time can be short)
- Can we input quantum initial conditions in the Langevin approach?

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