On the Origin of the Elliptic Flow and its Dependence on the Equation of State in Heavy Ion Reactions at Intermediate Energies

by A. Le Fèvre\textsuperscript{1}, Y. Leifels\textsuperscript{1}, C. Hartnack\textsuperscript{2} and J. Aichelin\textsuperscript{2,3}

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Introduction

The Quantum Molecular Dynamics approach

Elliptic flow at mid-rapidity: the strongest sensitivity to the Nuclear Equation of State

Survey of the reaction

Collisions versus mean field

Incident energy dependance

Summary
Introduction

![Graph showing $v_2$ vs. beam energy (GeV/nucleon)](image)

- Out-of-plane
- In-plane

Key Experiments:
- FOPI
- EOS
- E895
- E877
- CERES
- NA49
- STAR
- Phenix
- Phobos
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At ultra-relativistic energies: measured $v_2$ and centrality dependence $\Leftrightarrow$ expansion of initially highly compressed almond shaped fireball $\Rightarrow v_2 > 0$ as predicted by hydrodynamics


![Elliptic flow graph](image-url)
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At ultra-relativistic energies, $v_2$ and centrality depend on the expansion of initially highly compressed almond shaped fireball.

Flows at high density in heavy-ion collisions:

$$\frac{dN}{d(\phi - \phi_R)}(y, p_t) = \frac{N_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos n(\phi - \phi_R) \right)$$

- $y$ = rapidity
- $p_t$ = transverse momentum
- $\Phi_R$ = reaction plane azimuthal angle

$V_1$ = 'side/directed flow', $\cos(\Phi-\Phi_R)$ mode

$V_2(y, p_t) = \left( \begin{array}{c} p_x^2 - p_y^2 \\ p_t^2 \end{array} \right)$

'Elliptic flow': $\cos(2(\Phi-\Phi_R))$ mode, competition between 'in-plane' ($V_2>0$) and 'out-of-plane' ejection ($V_2<0$).
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⇒ a negative $v_2$ coefficient up to $E_{inc} \approx 6$ AGeV

⇒ with a minimum at around 0.4-0.6 AGeV


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\[ V_2 \quad \text{(GeV/nucleon)} \]

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At higher incident energies: the expansion takes place after the spectator matter has passed the compressed zone $\Rightarrow v_2$ is determined by the shape of the overlap region only $\Rightarrow v_2 > 0$. 

![Graph showing elliptic flow $v_2$ vs. beam energy](image-url)
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At even lower incident energies: $v^2$ becomes positive again: attractive NN forces outweigh the repulsive NN collisions.
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Here, we quote only how this approach allows for an exploration of the nuclear EoS

Nucleons are represented as Gaussian wave functions -> single-particle Wigner density:

\[ f_i(\mathbf{r}, \mathbf{p}, t) = \frac{1}{\pi^3 \hbar^3} e^{-\frac{2}{L^2} (\mathbf{r} - \mathbf{r}_i(t))^2} e^{-\frac{L}{2\hbar^2} (\mathbf{p} - \mathbf{p}_i(t))^2} \]

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\[ V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_i, \mathbf{p}_j) = G + V_{\text{Coul}} \]
\[ = V_{\text{Skyrme}} + V_{\text{Yuk}} + V_{\text{mdi}} + V_{\text{sym}} + V_{\text{Coul}} \]
\[ = t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + t_2 \delta(\mathbf{r}_i - \mathbf{r}_j)\rho^{-1}(\mathbf{r}_i) + \]
\[ t_3 \exp\left\{ -\frac{||\mathbf{r}_i - \mathbf{r}_j||}{\mu} \right\} + \]
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The Quantum Molecular Dynamics approach

Details of the Quantum Molecular Dynamics (QMD) approach have been published in

Comparisons to experimental benchmark data measured in the incident energy region under consideration are published in
- W. Reisdorf et al. [FOPI Collaboration], Nucl. Phys. A 876 (2012) 1

Here, we quote only how this approach allows for an exploration of the nuclear EoS

Nucleons are represented as Gaussian wave functions

The potential consists of several terms:

\[ V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_i, \mathbf{p}_j) = G + V_{\text{Coul}} \]
\[ = V_{\text{Skyrme}} + V_{\text{Yuk}} + V_{\text{mdi}} + +V_{\text{sym}} + V_{\text{Coul}} \]
\[ = t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + t_2 \delta(\mathbf{r}_i - \mathbf{r}_j)\rho^{\gamma - 1}(\mathbf{r}_i) + \]
\[ t_3 \exp\left\{-\frac{|\mathbf{r}_i - \mathbf{r}_j|}{\mu}\right\} + t_4 \ln^2(1 + t_5 (\mathbf{p}_i - \mathbf{p}_j)^2)\delta(\mathbf{r}_i - \mathbf{r}_j) + \]
\[ t_6 \frac{1}{\mu^2} T_i T_j \delta(\mathbf{r}_i - \mathbf{r}_j) \]

Convolution of the distribution functions \( f_i \) and \( f_j \) → single-particle potential (« mean-field ») = \( V_{\text{Skyrme}} + V_{\text{mdi}} \) (local interactions + momentum dependence)

\[ U_i(\mathbf{r}_i, t) = \alpha \left( \frac{\rho_{\text{int}}}{\rho_0} \right) + \beta \left( \frac{\rho_{\text{int}}}{\rho_0} \right) + \delta \ln^2 \left( \varepsilon (\Delta \mathbf{p})^2 + 1 \right) \left( \frac{\rho_{\text{int}}}{\rho_0} \right) \]

In nuclear matter \( t_1, t_2, t_4, t_6 \) uniquely related \( \alpha, \beta, \delta, \) and \( \varepsilon \) : given by fits to the optical potential extracted from elastic scattering data in pA collisions.

\( \alpha, \beta, \gamma : 2 \) are constrained by volume energy has a minimum of \( E/A(\rho_0) = -16 \text{ MeV} \) at \( \rho_0 \).

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Arnaud Le Fèvre - IWM-EC – May 2018 – INFN, Catania, Sicily, Italy
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Here, we quote only how this approach allows for an exploration of the nuclear EoS.

Nucleons are represented as Gaussian wave functions:

\[ f_i(\mathbf{r}, \mathbf{p}, t) = \frac{1}{\pi^3 \hbar^3} e^{-\frac{2}{L^2} (\mathbf{r} - \mathbf{r}_i(t))^2} e^{-\frac{L}{2\hbar^2} (\mathbf{p} - \mathbf{p}_i(t))^2} \]

The total one-body Wigner density is the sum of the Wigner densities of all nucleons:

\[ \sum_i f_i(\mathbf{r}, \mathbf{p}, t) \]

The potential consists of several terms:

\[ \kappa + V_{\text{mdi}} + V_{\text{sym}} + V_{\text{Coul}} + t_2 \delta(\mathbf{r}_i - \mathbf{r}_j) \rho^{-1}(\mathbf{r}_i) + \frac{\mathbf{r}_i}{\mu} + \frac{\mathbf{r}_j}{\mu} + (\mathbf{r}_i - \mathbf{r}_j)^2 \delta(\mathbf{r}_i - \mathbf{r}_j) + Z_i Z_j e^2 \mathbf{r}_i \mathbf{r}_j |r_i - r_j| \]

In nuclear matter, \( t_1, t_2, t_4, t_5 \) uniquely related \( \alpha, \beta, \delta, \) and \( \varepsilon \) and \( \delta \) are given by fits to the optical potential extracted from elastic scattering data in pA collisions.

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Elliptic flow at mid-rapidity: the strongest sensitivity to the Nuclear Equation of State

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the strongest sensitivity to the Nuclear Equation of State

Complete shape of $v_2(y_0)$: a new observable:

$$v_{2n} = |v_{20}| + |v_{22}|,$$

from fit

$$v_2(y_0) = v_{20} + v_{22} \cdot y_0^2$$

Elliptic flow at mid-rapidity: the strongest sensitivity to the Nuclear Equation of State

$v_{2n}(E_{\text{beam}})$ varies by a factor $\approx 1.6$, $>>$ measured uncertainty ($\approx 1.1$)

$\Rightarrow$ clearly favors a ‘soft’ EOS.

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Survey of the reaction

Only protons are considered in the following, Au+Au with b=6 fm as illustration

z: beam direction
x: impact parameter direction
y: perpendicular to reaction plane

\( t_{\text{pass}} \) = passing time

IQMD (SM) \( ^{197}\text{Au} + ^{197}\text{Au} \) at 1.5 AGeV, \( b = 6 \text{ fm} \)

yield/event/fm²

reaction plane

transversal plane
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Only protons are considered in the following, Au+Au with $b=6$ fm as illustration:

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Central (participant) matter is highly compressed at max. overlap ($t = 0.5t_{\text{pass}}$).
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Projectile and target remnants stay connected for longer than \( t_{\text{pass}} \) by a ridge with a quite high particle density. This ridge will disintegrate when projectile and target remnants separate further.
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The importance of this ridge can be seen in the zy plane at max. overlap \( \rightarrow \) the highest density at \( z=0 \), in the ridge.
Survey of the reaction

The choice of the EoS influences the reaction scenario predicted by the model \(\Rightarrow\) reflected by the difference (SM-HM) of the proton densities projected onto the \(ij\) plane,

\[
\Delta \rho_{ij} = \rho_{ij}^{SM} - \rho_{ij}^{HM} \text{ (event}^{-1}\text{fm}^{-2})
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The excess in \(x\)-direction has its origin in the in-plane flow of the spectator matter expressed by a finite directed flow \((v1)\):

\(v1\) (hard) >> \(v1\) (soft)
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The excess in x-direction has its origin in the in-plane flow of the spectator matter expressed by a finite directed flow \((v_1)\):
\(v_1\text{ (hard)} \gg v_1\text{ (soft)}\)

In y-direction the surplus in density of the hard EoS is concentrated at around \(z=0\), being less extended but stronger at higher energies. The emission of these particles is caused by a stronger density gradient (and hence a stronger force) in y-direction for a hard (HM) EoS as compared to a soft (SM) one.
Survey of the reaction at $t_{\text{pass}}$

In velocity space we observe a complementary distribution.
Survey of the reaction

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In the xy plane, the shift of protons in x direction is smaller for a soft (SM) than for a hard (HM) EoS
Survey of the reaction at $t_{\text{pass}}$

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This is due to a smaller acceleration yielding a weaker in-plane flow and hence a smaller velocity in $x$-direction.
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The soft EoS leads also to less stopping.
We select now fast moving particles in the transverse direction at mid-rapidity: $|y_0| < 0.2$, $ut_0 > 0.4$ (used by the FOPI collaboration for the $v_2$ investigation) in color. Compared to all (black contours).
We select now fast moving particles in the transverse direction at mid-rapidity:

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At full overlap:

- the innermost participants = a dense almond shaped core, out-of-plane elongated, compression is highest.
We select now fast moving particles in the transverse direction at mid-rapidity:

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Survey of the reaction

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At full overlap:

- the innermost participants = a dense almond shaped core, out-of-plane elongated, compression is highest.
- the outermost participants = more dilute, extending in-plane, aligned with the spectator distribution.

At passing time, the innermost (compressed) participants expand in-plane, but not with enough pressure to produce a positive elliptic flow \( v_2 \) (seen later), in contrast to higher bombarding energies.
Survey of the reaction

Formation of an in-plane ridge between the bulk of the spectators.
Incident energy $\uparrow \Rightarrow$ ridge & initial almond core density $\uparrow$
The elliptic flow time evolution

\[ v_2(t) = \frac{p_x^2(t) - p_y^2(t)}{p_x^2(t) + p_y^2(t)} \]

at mid-rapidity
The elliptic flow time evolution

\[ v_2(t) = \frac{p_x(t) - p_y(t)}{p_x(t) + p_y(t)} \]

\[ v_2 \text{ starts to develop after approximately max. overlap and evolves rapidly.} \]

at mid-rapidity

\[ 0.6 \text{ A GeV} \]

\[ 0.1 \]

\[ 0.05 \]

\[ 0 \]

\[ -0.05 \]

\[ -0.1 \]

\[ 1.5 \text{ A GeV} \]

\[ 0.1 \]

\[ 0.05 \]

\[ 0 \]

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- \( v_2 \) starts to develop after approximately max. overlap and evolves rapidly.
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- Negative for most of the collision times and for both energies.

\[ 0.1 \quad -0.05 \quad -0.05 \quad 0 \quad 0.05 \quad 0.05 \]
\[ 0.6 \text{ A GeV} \]

\[ 0 \quad 20 \quad 40 \quad 60 \]

\[ 1.5 \text{ A GeV} \]
The elliptic flow time evolution

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- But a tendency to be positive in the early stage of the collision.
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SM vs HM: \( v_2 \) at mid-rapidity depends strongly on the EoS; effect enhanced for fastest protons.
The elliptic flow: collisions versus mean field

An observable to quantify their respective contribution to it: transverse momentum modification induced projected on the direction of the final momentum:

\[
\langle \Delta P^\perp(t) \rangle = \langle \Delta P(t) \cdot \frac{\mathbf{p}_{final}}{\mathbf{p}_{final}} \rangle
\]
The elliptic flow: collisions versus mean field

An observable to quantify their respective contribution to it: transverse momentum modification induced projected on the direction of the final momentum:

$$\langle \Delta P_t^0(t) \rangle = \langle \Delta P_t(t) \cdot \frac{p_{final}}{|p_{final}|} \rangle$$

From collisions: about an order of magnitude larger than from mean field, set fast in the overlap zone ⇒ this zone of violent collisions expands rapidly keeping its almond shape.
The elliptic flow: collisions versus mean field

An observable to quantify their respective contribution to it: transverse momentum modification induced projected on the direction of the final momentum:

$$\langle \Delta P^o_t(t) \rangle = \langle \Delta P_t(t) \cdot \frac{p_{final}}{|p_{final}|} \rangle$$

From collisions: about an order of magnitude larger than from mean field, set fast in the overlap zone $\Rightarrow$ this zone of violent collisions expands rapidly keeping its almond shape.

From mean field: large out-of plane momentum transfer at the tips of the almond shape because here nucleons are between vacuum and the central densest zone $\Rightarrow$ highest density gradient, largest force $\Rightarrow$ move in y-direction out of the overlap zone.
The elliptic flow: collisions versus mean field

An observable to quantify their respective contribution to it: transverse momentum modification induced projected on the direction of the final momentum:

\[ \langle \Delta P^y_t(t) \rangle = \langle \Delta P_t(t) \rangle \cdot \frac{\rho_{final}}{\rho_{final}} \]

From collisions: about an order of magnitude larger than from mean field, set fast in the overlap zone ⇒ this zone of violent collisions expands rapidly keeping its almond shape.

From mean field: large out-of-plane momentum transfer at the tips of the almond shape because here nucleons are between vacuum and the central densest zone ⇒ highest density gradient, largest force ⇒ move in y-direction out of the overlap zone.

Outer blue areas ⇐ attractive potential of the remnant, deceleration.

Inner blue area: inner density decreases and attraction by the moving spectators ⇒ transverse velocity decreases

0.6 A.GeV, mid-rapidity, \( u_t > 0.4 \)
The elliptic flow: collisions versus mean field

Little difference between 0.6 AGeV and at 1.5 AGeV.

1.5 A.GeV, mid-rapidity, $u_{t0}>0.4$
The elliptic flow: collisions versus mean field

\[ \langle \Delta P_i^0(t) \rangle = \langle \Delta P_i(t) \rangle \cdot \frac{p_{i, \text{final}}}{|p_{i, \text{final}}|} \]

\( v^2 \) directly related to its anisotropy in \( x \) and \( y \).

Collision contribution: always much larger than that of mean field.
The elliptic flow: collisions versus mean field

![Graph showing the comparison between collisions and mean field for different energy levels.](image)
Excess in the y-direction: clearly visible for the mean field AND the collisions. For the collisions: becomes smaller with higher projectile velocity until it vanishes at 1.5 AGeV incident energy.
The elliptic flow: collisions versus mean field

Excess in the y-direction: clearly visible for the mean field AND the collisions. For the collisions: becomes smaller with higher projectile velocity until it vanishes at 1.5 AGeV incident energy.

K_0 has no visible influence on the amplitude of the collisional out-of-plane momentum excess because the number of collisions is almost unchanged by the choice of the EoS.
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Pauli blocking: quenches v_2<0 due to collisions, from the densest phase of the collisions, stronger for SM because larger densities are reached.
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Without Pauli blocking, there would be a collisional contribution to the EoS dependence of $v_2$. 
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Without Pauli blocking, there would be a collisional contribution to the EoS dependence of \( v_2 \).

Mean field contribution to \( v_2<0 \): dependent on incident energy and \( K_0 \): moderate at 0.6 AGeV with the soft EoS, contributing to only 30% of the total \( \Delta P_y^0 - \Delta P_x^0 \), very strong and dominating at 1.5 AGeV with the stiffer EoS.
The elliptic flow: collisions vs mean field

At passing time:
- inner $R_{xy} < 4\ \text{fm}$
- outer $R_{xy} > 4\ \text{fm}$
- id., from m.f.
- id., from coll.

$V_2$ vs time ($\text{fm}/c$)

- 0.6 A GeV
- 1.5 A GeV

SM  mid-rapidity  HM
The elliptic flow: collisions vs mean field

Outermost nucleons ($R_{xy} > 4$ fm) = the main source of the overall negative $v_2$:
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* From collisions: the early in-plane screening by the spectators ($\rightarrow v_2 < 0$) affects only the outermost nucleons, whereas the collisions of the inner nucleons create a nearly azimuthally isotropic distribution ($v_2 \approx 0$).
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* Asymptotically, the mean field = the main origin of the overall out-of-plane $v_2$, apart from reactions at energies below 1 AGeV where the collisions contribute equally when the nuclear matter EoS is soft, i.e. the number of collisions is large.
The elliptic flow: incident energy dependance

![Graph showing the elliptic flow as a function of beam energy for Au+Au collisions with b = 4 fm. The graph compares FOPI data with IQMD model predictions with and without u_{t0} cuts.](image-url)
The elliptic flow: incident energy dependance

- Strong beam energy dependence for $E_{\text{inc}} > 0.4$ AGeV

![Graph showing elliptic flow vs beam energy with FOPI data and IQMD predictions.](image-url)
The elliptic flow: incident energy dependance

- Strong beam energy dependence for $E_{\text{inc}} > 0.4$ AGeV
- Maximum of amplitude at around 0.6 AGeV.

\[ u_{10} > 0.8 \]

\[ V^2 \]

![Au+Au, b = 4 fm](image)

- FOPI data
- IQMD (SM) - dashed: no $u_{10}$ cut
- IQMD (HM) - idem

beam energy (GeV/nucleon)
The elliptic flow: incident energy dependence

- Strong beam energy dependence for $E_{\text{inc}} > 0.4$ AGeV
- Maximum of amplitude at around 0.6 AGeV.
- Strength enhanced with protons with a large transverse velocity.
The elliptic flow: incident energy dependance

- Strong beam energy dependence for $E_{\text{inc}} > 0.4$ AGeV
- Maximum of amplitude at around 0.6 AGeV.
- Strength enhanced with protons with a large transverse velocity.
- Comparison with FOPI observations (protons with $u_{t0} > 0.8$, same impact parameter) $\Rightarrow$ good agreement (amplitude and evolution) using the soft (SM) EoS.
Summary
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- Arnaud Le Fèvre - IWM-EC – May 2018 – INFN, Catania, Sicily, Italy
Summary:

❖ The elliptic flow observed in the reactions around $E_{\text{kin}} \approx 1$ AGeV for protons at mid-rapidity ($|y_0| < 0.2$) has two origins:
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❖ The collisional component of $v_2$ is almost independent of the EoS (due to Pauli blocking),
Summary:

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❖ The collisional component of $v_2$ is almost independent of the EoS (due to Pauli blocking),
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❖ The calculations with a soft EoS (SM) are in better agreement with the experimental data than that with a hard equation of state (HM).
Thank you for your attention!
Alternative method: in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.

- limited to $E_{\text{beam}} < 10$ A.GeV ⇔ some kind of a clock is available (sound velocity versus participant-spectator interaction).
- KaoS (1990’s), C+C, Au+Au, K$^+$ yields → 'soft' EOS. But:
  - kaons rare at $E_{\text{beam}} = 0.8$ A.GeV (max. sensitivity to the EOS).
  - all 'bulk' observables (multiplicities, clusterisation, stopping, flow) under control in the transport model?
- EoS (1996), Au+Au @ 0.25 to 1.15 A.GeV, radial & sideward flow, squeeze-out versus QMD → no strong sensitivity on the nuclear incompressibility $K_0$.
- FOPI (2005), Au+Au @ 0.09-1.5 A.GeV, Z=1 elliptic flow, versus 4 different transport codes → 'no strong constraint on the EOS can be derived at this stage'.
- BEVALAC & AGS accelerators, proton flows versus transport theories → $K_0 = 167$-200 MeV (soft) from $V_1$, $K_0 = 300$ MeV (semi-stiff) from $V_2$ → contradictions.
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- Au, K+ yields \( \rightarrow \) 'soft' EOS. But:
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The elliptic flow

- Arnaud Le Fèvre - IWM-EC – May 2018 – INFN, Catania, Sicily, Italy