

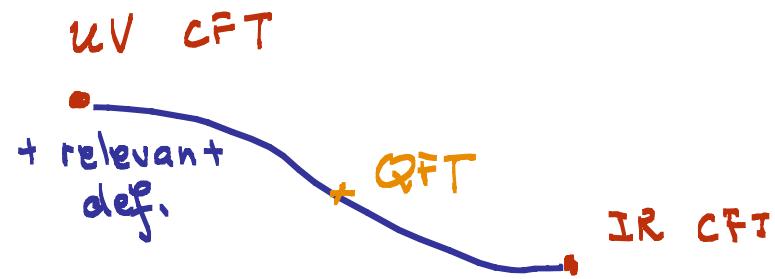
J \bar{T} - deformed CFTs and their
holographic interpretation

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Motivation

- usual , local QFT framework



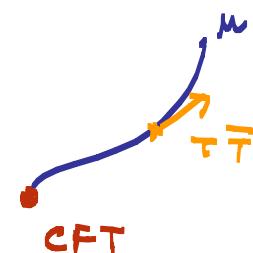
- examples of non-local , UV complete QFTs ?
 - quantum gravity
 - holography (not asymptotically AdS)

$T\bar{T}$ -deformed CFT₂

Smirnov, Zamolodchikov '16
 Caraglià, Negro, Szecsenyi, Tateo '16
 Dubovski, Flauger, Gorbenko '12-'17
 + others

- universal deformation of 2d QFTs

$$S_\mu = S_{\text{CFT}} + \int d\mu \int d^2z (T\bar{T} - \Theta^z)_\mu$$



- deformation irrelevant but integrable
 - finite-size spectrum $E(\mu, R)$, thermodyn, KdV charges (universal!)
- theory non-local (scale μ), but well-defined UV

S-matrix : $S_\mu = e^{\frac{i s \mu}{4}} S_0$

Interesting applications

- $CFT_2 = 24$ free bosons \rightarrow worldsheet bosonic string ($\mu = l_s^2$)
(2d quantum gravity)
Dubovsky et al., Tateo et.al.
- $CFT_2 = \text{large } N, \text{ large gap}, \mu < 0 \rightarrow$ holographically dual to
 AdS_3 gravity w/ finite bulk cutoff $r_c = 1/\sqrt{-\mu}$
Mc Gough, Mezei, Verlinde '16
- $CFT = \text{Sym} (CFT_n)^N$, $\sigma_{T\bar{T}} = \sum_{i=1}^N T_i \bar{T}_i \rightarrow$ holographic dual to
an asymptotically linear dilaton backgnd
Giveon, Itzhaki, Kutasov '17

This talk

- another universal deformation of 2d CFTs w/ a $U(1)$ current

$$S_\mu = S_{\text{CFT}} + \mu \int d^2z (\bar{J} \bar{\top} - \bar{\bar{J}} \bar{\Theta})$$

(1,2)

- breaks Lorentz invariance
- preserves $SL(2, \mathbb{R})_L \times U(1)_R \leftarrow$ non-local
- deformation irrelevant but integrable

Why interesting?

- expected UV complete \rightarrow would like to understand this new type of UV behaviour (non-local)
- expected local and conformal on the left \rightarrow handle on correlation functions ?
- $SL(2, \mathbb{R})_L \times U(1)_R$ \mapsto interesting symmetry enhancement ?
- may help understand holographic dual to extremal black holes (non-AdS holography)

Plan

- field-theoretical analysis of the $J\bar{T}$ deformation
 - definition, spectrum, thermodynamics, etc.
- holographic analysis -//-
- relevance to extremal black holes & future directions

Field-theoretical analysis

The $J\bar{T}$ operator

- assume deformed CFT can be treated as a quasi-local QFT below some scale
- translation & $U(1)$ invariance \rightarrow local conserved currents

$$\bar{\partial} \underbrace{T_{zz}}_{\bar{T}} + \partial \underbrace{T_{\bar{z}\bar{z}}}_{\Theta} = 0$$

by $SL(2, \mathbb{R})_L$

$$\bar{\partial} \underbrace{T_{z\bar{z}}}_{\Theta} + \partial \underbrace{T_{\bar{z}z}}_{\bar{T}} = 0$$

$$\bar{\partial} J + \partial \bar{J} = 0$$

$$O_{J\bar{T}}(z) = \lim_{z' \rightarrow z} J(z') \bar{T}(z) - \bar{J}(z') \Theta(z) + \text{derivatives}$$

Zamolodchikov '04

Finite-size spectrum

- cylinder $z = \varphi + i\tau$, $\varphi \sim \varphi + R$
- eigenstates $|n\rangle$ of the energy E_n , momentum P_n , charge Q_n
- want $E_n(\mu, R)$ (P_n, Q_n quantized)

① $\frac{\partial E_n}{\partial \mu} = R \cdot \langle n | O_{J\bar{J}} | n \rangle$

② factorization $\langle O_{J\bar{J}} \rangle_n = \langle J \rangle_n \langle \bar{J} \rangle_n - \langle \bar{J} \rangle_n \langle J \rangle_n$

③ re-express $\langle T_{\tau\bar{\tau}} \rangle_n \sim E_n$, $\langle J_z \rangle_n \sim Q_n$ etc.

universal spectrum (E_n, P_n, Q_n) : J chiral ($J_{\bar{z}} = 0$)

Deformed spectrum

- introduce $E_{L,R} = \frac{1}{2} (E_n \pm p_n)$
- solution to eqn: $E_R(\mu, R) = E_R(R - \frac{\mu Q_n}{2})$
- deformed finite-size spectrum

$$E_R = \frac{h_R - c/24}{R - \mu Q/2}$$

$$E_L = \frac{h_L - c/24}{R} + \frac{\mu Q}{2} \frac{h_R - c/24}{R(R - \frac{\mu Q}{2})}$$

- breaks down for $R < \frac{\mu Q}{2}$ for both signs of μ .

Thermodynamics

- in the original CFT

$$S = 2\pi \sqrt{\frac{c}{6} \left(h_L - \frac{c}{24} - \frac{Q^2}{4K} \right)} + 2\pi \sqrt{\frac{c}{6} \left(h_R - \frac{c}{24} \right)}$$

- $+ \mu \rightarrow$ energy levels are continuously deformed $\Rightarrow S(h_{L,R}; Q) = \text{const.}$
- $S(E, P, Q)$ only through dependence of $h_{L,R}(E, P, Q)$
- thermodynamic quantities all diverge as $R \rightarrow \mu \frac{Q}{2}$

(e.g. $T = \left(\frac{\partial S}{\partial E} \right)_{R,Q}^{-1}$)

Superluminal propagation

- $J\bar{T} = CFT + \text{field-dependent diffeomorphism}$
- under a diffeomorphism $\delta S = - \int T^{\lambda}{}_{\alpha} \partial_{\lambda} \xi^{\alpha} = \mu \int J\bar{T}$

$$\boxed{z \rightarrow z' = z \quad \bar{z} \rightarrow \bar{z}' = \bar{z} - \frac{\mu}{2} \int_z^{\bar{z}} J(w) dw}$$

- CFT on $ds^2 = dz' dz' = dz(d\bar{z} - \frac{\mu}{2} J dz) = d\varphi^2 - dt^2 - \frac{\mu J}{2} (d\varphi + dt)^2$
- superluminal propagation for $\mu \langle J \rangle > 0$ (ok, b/c no Lorentz invariance)
- CTCs for $R < \mu Q/2$ ($J = \frac{Q}{R}$)

Holographic analysis

Double-trace deformations in AdS/CFT

- $J\tilde{T}$ is a double-trace operator
- double trace operators \leftrightarrow mixed bnd. cond. for dual bulk field
- variational principle

$$\delta S_{\text{on-shell}} = \int \langle \phi \rangle \delta J_{\text{source}} - \delta \int \frac{\mu}{2} \langle \phi \rangle^2 = \int \langle \phi \rangle \underbrace{\delta(J - \mu \langle \phi \rangle)}_{\tilde{J}} \quad \begin{matrix} \nearrow \text{(new) vev} \\ \text{new source} \end{matrix}$$

Effect of the JT - deformation

- sources : $J^\alpha \leftrightarrow z_\alpha$; $T^\alpha{}_\alpha \leftrightarrow e^\alpha{}_\alpha$
- coupling = covariantize $\mu \int d^2z J\bar{T} \rightarrow \mu_a \int d^2x e T^\alpha{}_\alpha J^\alpha$
- $\mu_a = \mu \delta_a^+$, \mathcal{F} chiral : $\mu_a J^\alpha = 0$
- variational principle :

$$\delta S_{\text{tot}} = \int d^2x \left[e T^\alpha{}_\alpha \delta e^\alpha{}_\alpha + e J^\alpha \delta z_\alpha - \delta \left(e \mu_a T^\alpha{}_\alpha J^\alpha \right) \right]$$

$\underbrace{\qquad\qquad\qquad}_{\text{CFT}}$ $\underbrace{\qquad\qquad\qquad}_{\text{deformation}}$

$$= \int d^2x \tilde{e} \left(\tilde{T}^\alpha{}_\alpha \delta \tilde{e}^\alpha{}_\alpha + \tilde{J}^\alpha \delta \tilde{z}_\alpha \right)$$

new sources/
vers

Holographic dictionary

$$\left\{ \begin{array}{l} \tilde{e}^\alpha_a = e^\alpha_a - \mu_\alpha J^\alpha \quad \tilde{a}_\alpha = a_\alpha - \mu_\alpha T^\alpha_\alpha \quad \text{sources} \\ \tilde{T}^\alpha_\alpha = T^\alpha_\alpha + (e^\alpha_\alpha + \mu_\alpha J^\alpha) \mu_\beta T^\beta_\beta J^\beta \quad \tilde{J}^\alpha = J^\alpha \quad \text{vevs} \end{array} \right.$$

\Rightarrow deformed th. w/ sources $\tilde{e}^\alpha_a, \tilde{a}_\alpha$ (fixed) \leftrightarrow CFT w/ sources

$$e^\alpha_a = \tilde{e}^\alpha_a + \mu_\alpha J^\alpha \quad a_\alpha = \tilde{a}_\alpha + \mu_\alpha T^\alpha_\alpha$$

- use usual AdS_3/CFT_2 dictionary (Einstein grav + $U(1)$ Chern-Simons)

to compute $T^\alpha_\alpha, J^\alpha \Rightarrow \tilde{T}^\alpha_\alpha, \tilde{J}^\alpha$

The asymptotic bulk solution

- fix sources in the deformed theory $\tilde{e}^a{}_a = \delta^a{}_a$, $\tilde{a}_a = 0$
- equivalent to CFT w/ sources $e^a{}_a^{(0)} = \delta^a{}_a + \mu_a J^a(x^+)$, $a_a = \mu_a T^a_{\alpha}$
 $\underbrace{\text{grav+CS.}}$
- Fefferman-Graham expansion $ds^2 = \left(\frac{g_{ii}^{(0)}}{z^2} + g_{ij}^{(1)} + \dots \right) dx^i dx^j + \frac{dz^2}{z^2}$

$$g^{(0)} = \begin{pmatrix} + & - \\ - & \begin{pmatrix} -\frac{\mu}{2} J(x^+) & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \end{pmatrix}$$

$$\cdot \text{Compère, Song, Strominger}$$

$$g^{(1)} = \begin{pmatrix} \mathcal{L}(x^+) + \frac{\mu^2}{4} J^2(x^+) \bar{\mathcal{L}} & -\frac{\mu}{2} J(x^+) \bar{\mathcal{L}} \\ -\frac{\mu}{2} J(x^+) \bar{\mathcal{L}} & \bar{\mathcal{L}}(x^+ - \frac{\mu}{2} \int J(x^+)) \end{pmatrix}$$

- most general asympt. AdS_3 + coordinate transf $x^- \rightarrow x^- - \frac{\mu}{2} \int J(x^+)$
 $(+ \text{gauge transf.})$

Match to field theory

- remember deformed spectrum

$$E_R = \frac{h_R - c/24}{R - \mu Q/2} \quad E_L = \frac{h_L - c/24}{R} + \frac{\mu Q}{2} \frac{h_R - c/24}{R(R - \frac{\mu Q}{2})}$$

- reproduce from holography?
- energy eigenstates \rightarrow black holes w/ new asymptotics = BTZ + coord. transf.
- match parameters by requiring that $A_{\text{Pl}}(h_L, h_R)$ be the same
 - reproduces correct deformed thermodynamics
 - holographic vs $\langle \tilde{T}^a_a \rangle_{\text{grav+CS}}$ match E_L, E_R

Symmetry enhancement

- global symmetries deformed CFT $SL(2, \mathbb{R})_L \times U(1)_R \times U(1)_J$
- holographic Ward identities $\tilde{T}_{++}(x^+)$, $\tilde{T}_{--}(x^- - \frac{\mu}{2} \int J(x^+))$, $J(x^+)$
 \Rightarrow infinite family of conserved charges, e.g. $\int d\varphi \chi_L(x^+) \tilde{T}_{++}(x^+)$
- the asymptotic symmetry algebra of the associated Fourier modes

$Virasoro_L \times Virasoro_R \times U(1)$ Kac-Moody

same
 c, k

L.M.

non-local deformation of the original $Vir_R(x^-)$

"state - dependent" "symmetry"

Comments

Correlation functions?

- holography: CFT + field-dependent coord transf ($x^- \rightarrow x^- - \frac{\mu}{2} \int_{-\infty}^{x^+}$)
- more precisely? also in $T\bar{T}$ (Dubovsky et. al.)
- prescription for all correlation functions

$$\left\langle \prod_i O_i(x_i^+, x_i^-) \right\rangle_\mu = \underbrace{\left\langle T_{ii} O_i(x_i^+, x_i^- - \frac{\mu}{2} Q_{\text{rel}}) \right\rangle}_{\text{CFT}}_{Q_{x^+ < x_i^+} - Q_{x^+ > x_i^+}}$$

• e.g. 2pf.

$$\begin{array}{c} x^- - \frac{\mu}{2} q \quad x^- - \frac{\mu}{2} q \\ \hline q \quad -q \end{array}$$

unchanged (no anomalous dim)

3pf

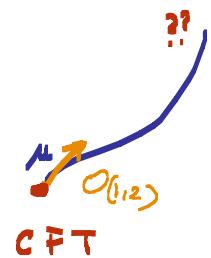
$$\begin{array}{ccc} x^- - \frac{\mu}{2} q & x^- - \mu q & x^- - \frac{\mu}{2} q \\ \hline q & 0 & -q \end{array}$$

~ dipole star product

- compare w/ conformal perturbation theory

Relation to extremal black holes?

- $\mathcal{J}\bar{\mathcal{T}}$ -deformed CFTs share many of the properties of holographic duals to **extremal black holes**
- warped AdS_3
 $S = S_{CFT_2} + \mu \int O_{(1,2)} + \dots$ multitrace ... + stringy
- $SL(2, \mathbb{R})_L \times U(1)_R$, non-local R , CTCs on spacelike circle, expected UV complete
(string.th.)



$\mathcal{J}\bar{\mathcal{T}}$?
= simplest example of "dipole CFT" \leftarrow integrable flow "up the RG"

- new perspective on Cardy entropy, Virasoro symmetry of extremal black holes?

Summary

- new non-local Lorentz-breaking 2d QFTs \rightarrow tractable flow "up the RG"
- properties for J chiral : spectrum, thermodyn, superluminal propagation
- holographically dual to AdS_3 w/ mixed bnd. cond. (perfect match)
- holography suggests : CFT + field-dependent coord. transf.
 - right-moving Virasoro is not broken,
but rather non-locally deformed

Future directions

- are $J\bar{T}$ -deformed CFTs UV-complete?
- understand lightlike non-locality
- meaning of "non-local Virasoro" symmetry? (organisation of the spectrum?)
- first tractable example of "flow up the RG" for a $(1,2)$ operator
 - more (non-trivial) examples?
 - lessons for Kerr/CFT?

Thank you !