Dualities, Dualities, Dualities…

GGI 11th May 2018

Veneziano's Model Fest(a young 50)

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J'ai souhaité être parmi
et avec ceux d'aujourd'hui
à offrir à Garnier un hommage
qui resterait chez lui
Marc Chagall

What is Duality?
Look in Mathematics
For a precision

- Alexander duality
- Alvis–Curtis duality
- Araki duality
- Beta-dual space
- Coherent duality
- De Groot dual
- Dual abelian variety
- Dual basis in a field extension
- Dual bundle
- Dual curve
- Dual (category theory)
- Dual graph
- Dual group
- Dual object
- Dual pair
- Dual polygon
- Dual polyhedron
- Dual problem
- Dual representation
- Dual q-Hahn polynomials
- Dual q-Krawtchouk polynomials
- Dual space
- Dual topology
- Dual wavelet
- Duality (optimization)
- Duality (order theory)
- Duality of stereotype spaces
- Duality (projective geometry)
- Duality theory for distributive lattices
- Dualizing complex
- Dualizing sheaf
- Eckmann–Hilton duality
- Esakia duality
- Fenchel's duality theorem
- Haag duality
- Hodge dual
- Jónsson–Tarski duality
- Lagrange duality
- Langlands dual
- Lefschetz duality
- Local Tate duality
- Opposite category
- Poincaré duality
  - Twisted Poincaré duality
- Poitou–Tate duality
- Pontryagin duality

S-duality (homotopy theor

Schur–Weyl duality

Serre duality

Spanier–Whitehead dualit

Stone's duality

Tannaka–Krein duality

Verdier duality

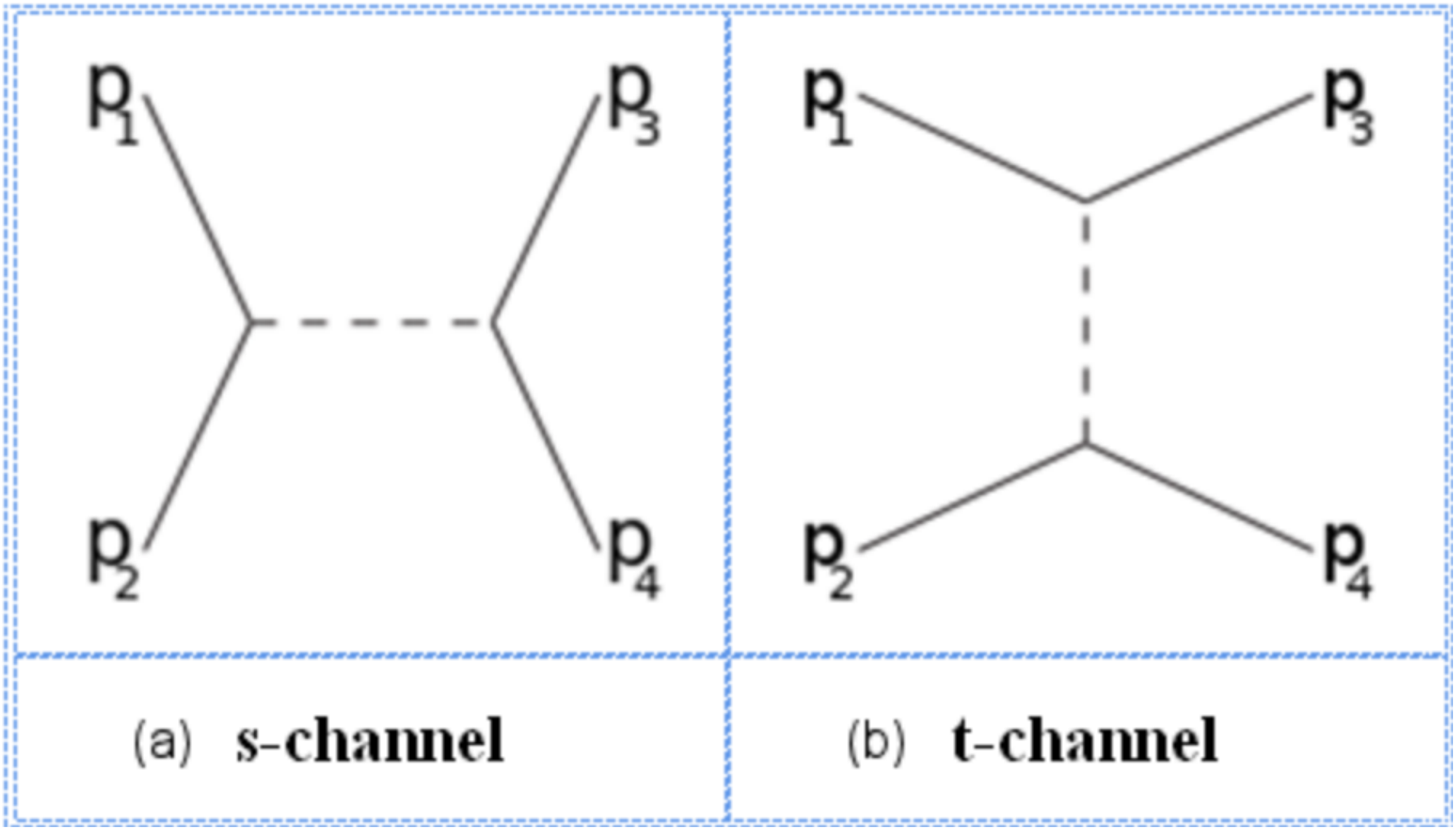
Duality is Multifaceted in Physics as well.

Particle/ Wave Duality- concept changing

Kramers- Wannier Duality- Narrative

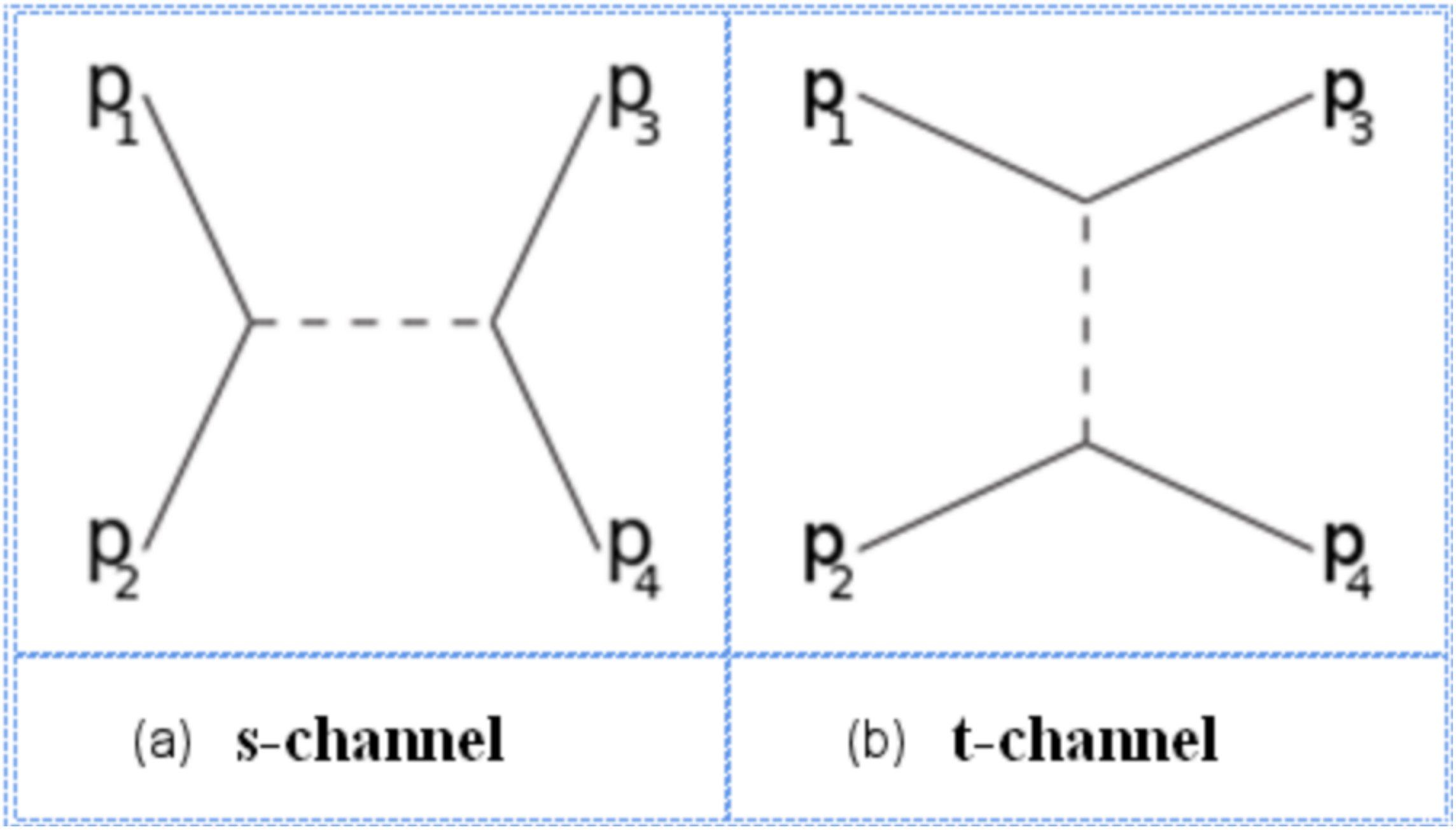
and we will encounter more

QFT- Electron Positron scattering



Add all such diagrams

BUT PION AND NUCLEON SAY - NOT FOR US



Either Or like particle/wave

NATURE CRIES OUT - WE WANT

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B\big(-\alpha(s),-\alpha(t)\big) = -\sum_{n=0}^{\infty} \frac{(\alpha(s)+1)\cdots(\alpha(s)+n)}{n!}\,\frac{1}{\alpha(t)-n}$$

Nature cries out - we want a dual model!

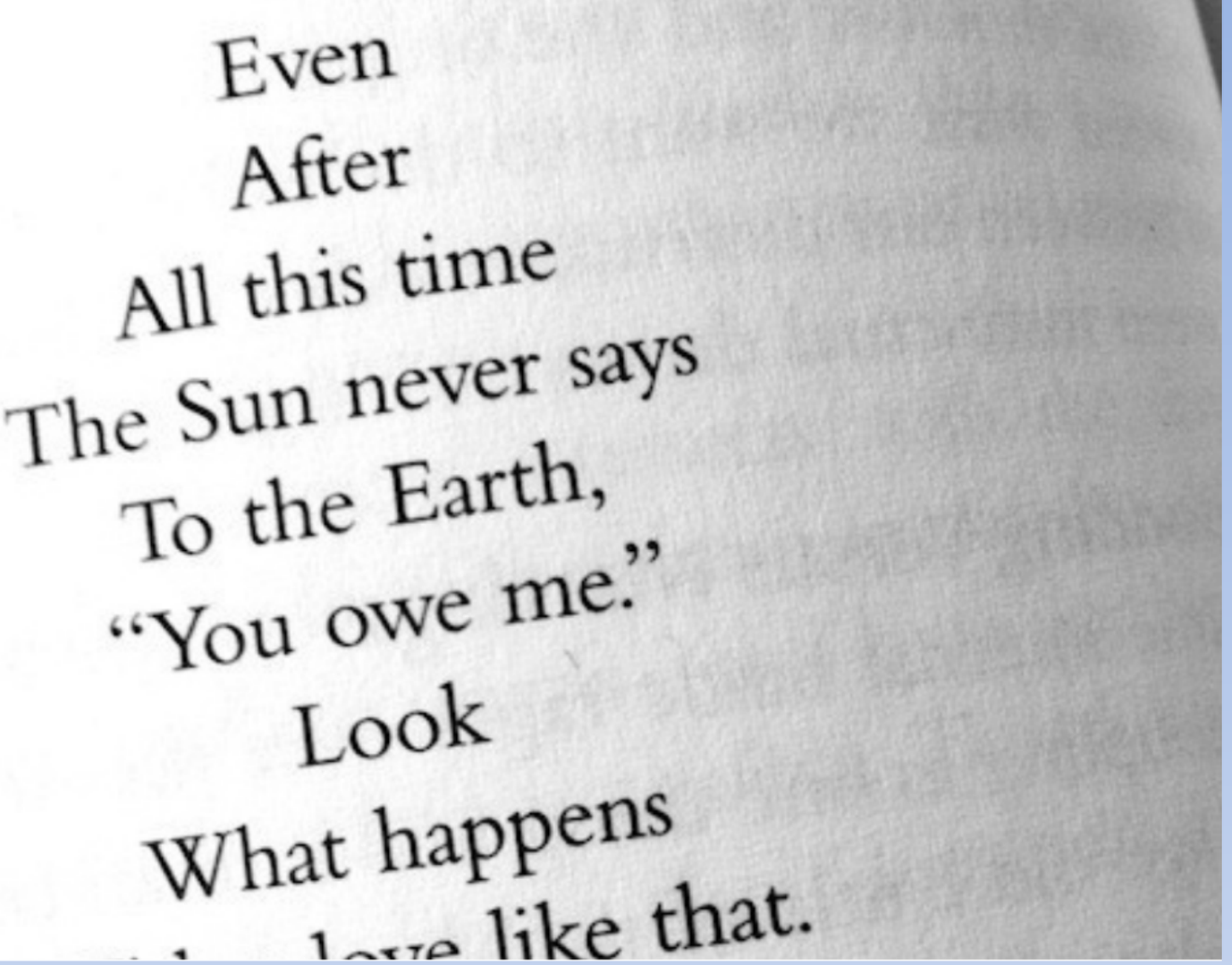
Find a framework

String Theory

What do we want?

Numbers as data, Numbers to measure?

25496 3187	36712 4589	36728 4591	37512 4689
37528 4691	38152 4769	41896 5237	42968 5371
46312 5789	46328 5791	46712 5839	47136 5892
47328 5916	47368 5921	51832 6479	53928 6741
54312 6789	54328 6791	54712 6839	56984 7123
58496 7312	58912 7364	59328 7416	59368 7421
63152 7894	63528 7941	65392 8174	65432 8179
67152 8394	67352 8419	67512 8439	71456 8932
71536 8942	71624 8953	71632 8954	73248 9156
73264 9158	73456 9182	74528 9316	74568 9321
74816 9352	75328 9416	75368 9421	76184 9523
76248 9531	76328 9541		


Even
After
All this time
The Sun never says
To the Earth,
"You owe me."
Look
What happens
like that.

A Story /Narrative

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+ i\bar{\Psi}\not{D}\psi$$
$$+ D_\mu\Phi^\dagger D^\mu\Phi - V(\Phi)$$
$$+ \bar{\Psi}_L\hat{Y}\Phi\Psi_R + h.c.$$

D=5,6 ?

This brings Dualities

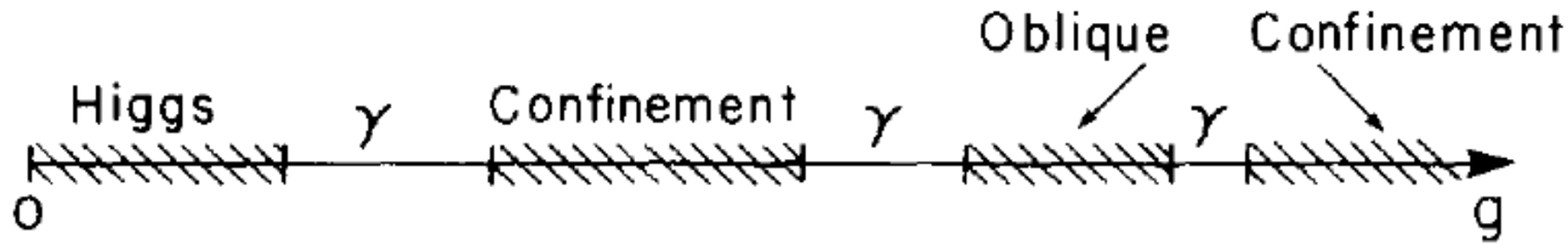
Dualities in QFT

Z(N) Lattice Theories with Global d=2, Local symmetries in d=4

$$Z(T) = Z(c/T)$$

Narrative : order or disorder parameters serve a simpler effective description

$$Z = \mathrm{Tr} \exp\left[-\frac{1}{2g^2} \sum_{\mathrm{P}} \left(\Delta_\mu \phi_\nu - \Delta_\nu \phi_\mu - 2\pi S_{\mu\nu} \right)^2 + ip \sum_{\mathrm{L}} n_\mu \phi_\mu \right]$$



Typical phase diagram. The number of oblique confinement phases depends on θ, and the Coulomb phases (γ) are absent for small p.

Dualities in QFT

Electric- Magnetic Type Dualities

Add a θ parameter and get

on the lattice with Z(N) and

N=4 SUSY d=4 an

SL(2,Z) symmetry/duality

on θ +i/g

How to decide if theories are really dual to each other?

Mostly compared partition functions.

What is needed to establish DUALITY

1 point functions, 2 point functions, 3 point functions

IN CFT, Other theories

Local operators, Non local operators, lines …

boundary effects, change the world volume, the

geometry , the topology.

How does this effect the dualities.

In string theory sum over all genera

- **The Magic of String Theory**

T Duality

Large R or 1/R

Local Momentum Modes

Non Local Winding Modes

$$S = \frac{1}{4\pi\alpha'} \int_0^{2\pi} d\sigma \int d\tau \, [\sqrt{g} g^{\alpha\beta} \hat{G}_{ij}(X) \partial_\alpha X^i \partial_\beta X^j + \varepsilon^{\alpha\beta} \hat{B}_{ij}(X) \partial_\alpha X^i \partial_\beta X^j - \tfrac{1}{2}\alpha' \sqrt{g} \hat{\Phi}(X) R^{(2)}]$$

$$E = G + B.$$

$$M(E) = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}.$$

$$E' \equiv g(E) = (aE + b)(cE + d)^{-1}$$

$$\mathcal{G} = O(d, d, \mathbb{R})$$

It is a gauge symmetry

Can the symmetry “break”?

Are there anomalies?

Back to QFT

$$\langle (O_{\pm})^n \rangle_{\text{exact}} = \frac{(\eta_{\pm})^2}{\eta_{\pm} - 1}$$

Eq. (4.11) now reads

$$\eta_{\text{TS}}^{\pm} = \eta_{\text{ws}}^{\pm}\left(1 - \eta_{\text{ws}}^{\pm}\right). \tag{4.15}$$

Two types of duality relations arise. One can simply interchange the two parameters:

$$\eta_{\text{ws}}^{+} \to \eta_{\text{ws}}^{-}, \qquad \eta_{\text{ws}}^{-} \to \eta_{\text{ws}}^{+}. \tag{4.16}$$

This is a perturbative duality, preserving the nature of the world-sheet theory genus by genus up to a transformation of the type (2.16), which interchanges O_+ and O_-. This transformation has a line of fixed points corresponding to $\eta^+ = \eta^-$. The target space theory is of course also preserved up to a similar transformation. Another duality transformation is suggested by (4.15)

$$\begin{aligned} &\eta_{\text{ws}}^{+} \to 1 - \eta_{\text{ws}}^{+}, \quad \eta_{\text{ws}}^{-} \to \eta_{\text{ws}}^{-}, \\ &\eta_{\text{ws}}^{+} \to \eta_{\text{ws}}^{+}, \qquad \eta_{\text{ws}}^{-} \to 1 - \eta_{\text{ws}}^{-}, \end{aligned} \tag{4.17}$$

or both.

U Duality SL(d,Z)xSL(2,Z)

$$d = 3 : \ (\bar{3}, 2) \text{ of } SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z}),$$

$$d = 4 : \ \bar{10} \text{ of } SL(5, \mathbb{Z}),$$

$$d = 5 : \ 16 \text{ of } SO(5, 5; \mathbb{Z}),$$

$$d = 6 : \ \bar{27} \text{ of } E_{6(6)}(\mathbb{Z}),$$

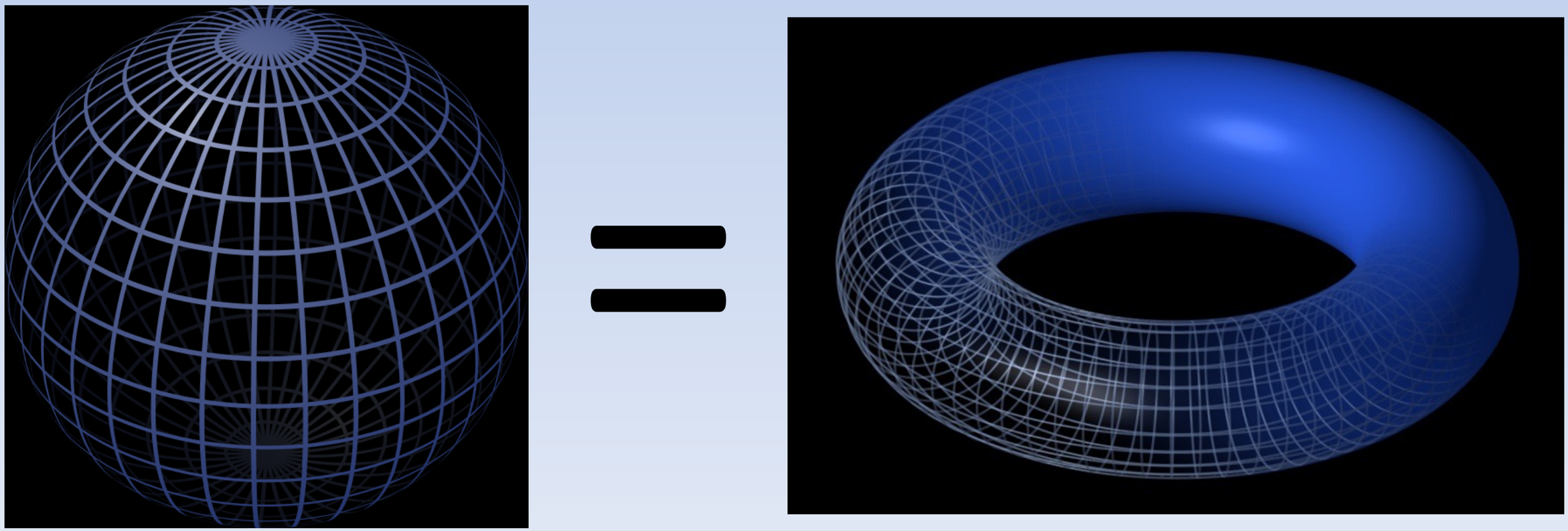
$$d = 7 : \ 56 \text{ of } E_{7(7)}(\mathbb{Z}),$$

$$d = 8 : \ 240(\subset 248) \text{ of } E_{8(8)}(\mathbb{Z}).$$

Magic of String Theory

With extended objects
Surface of a Sphere=Torus in some topological models

Topology

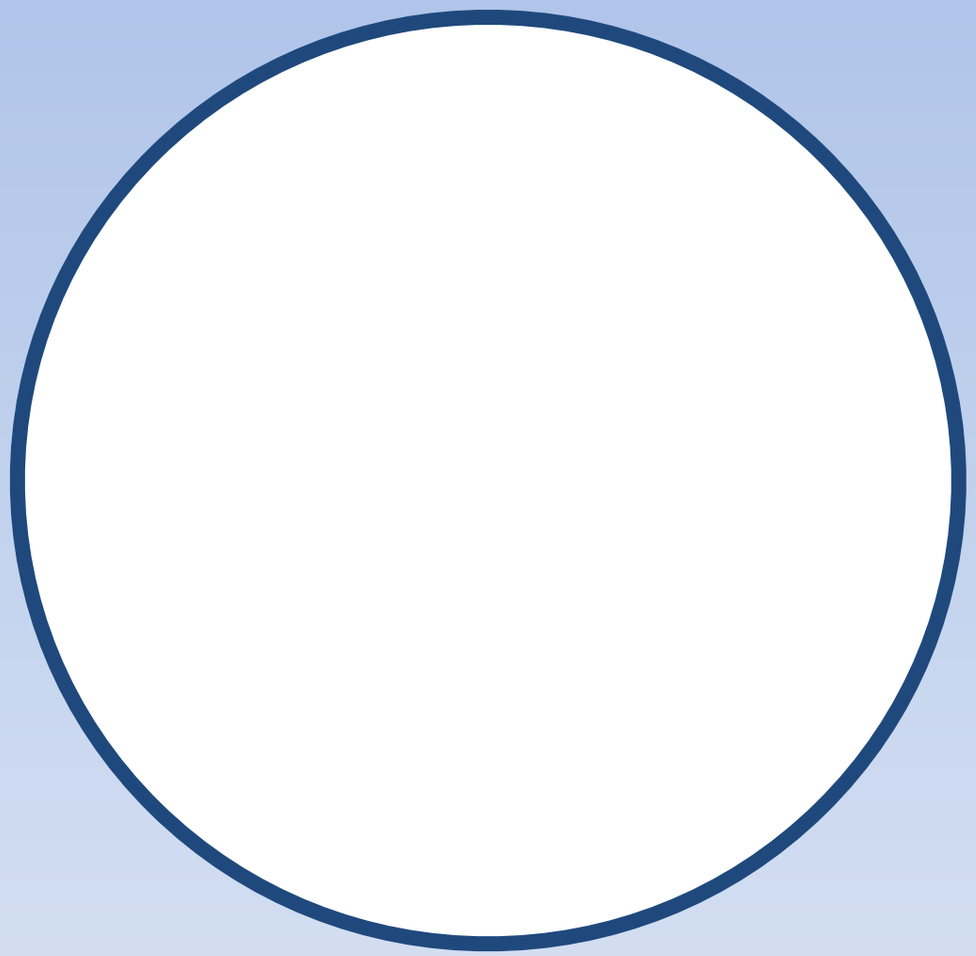


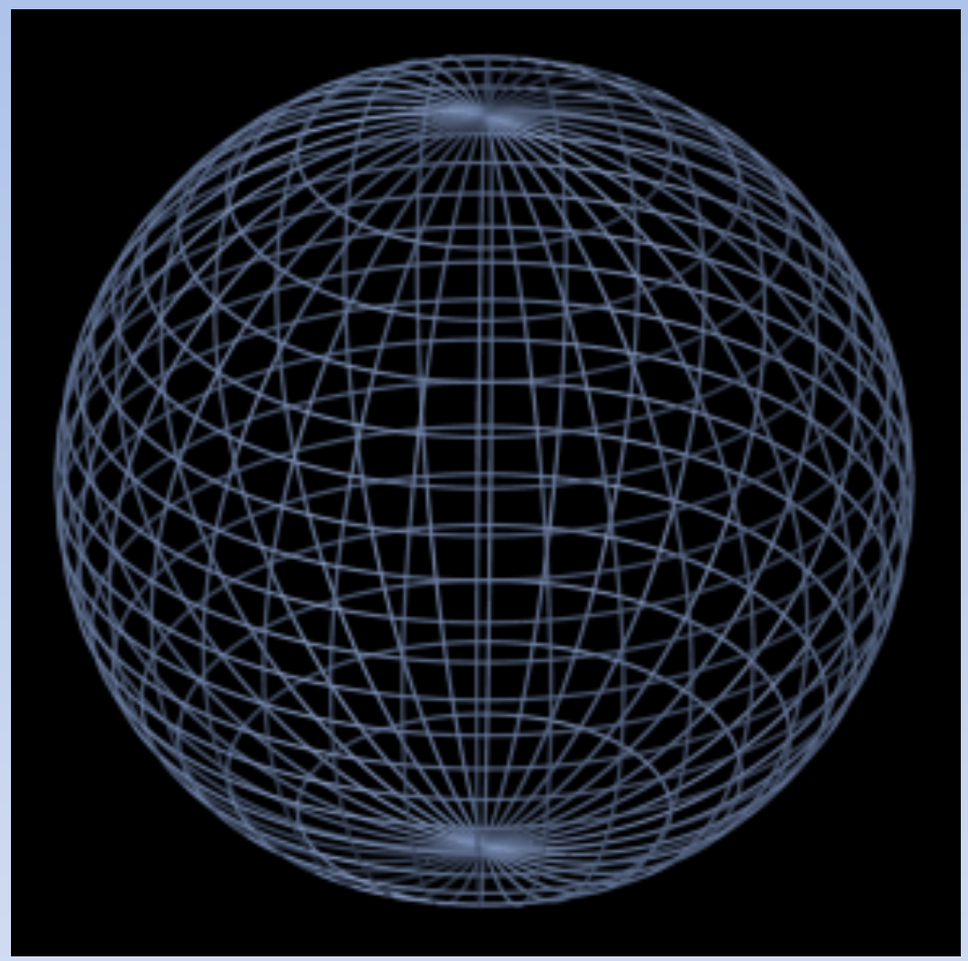
Magic of String Theory

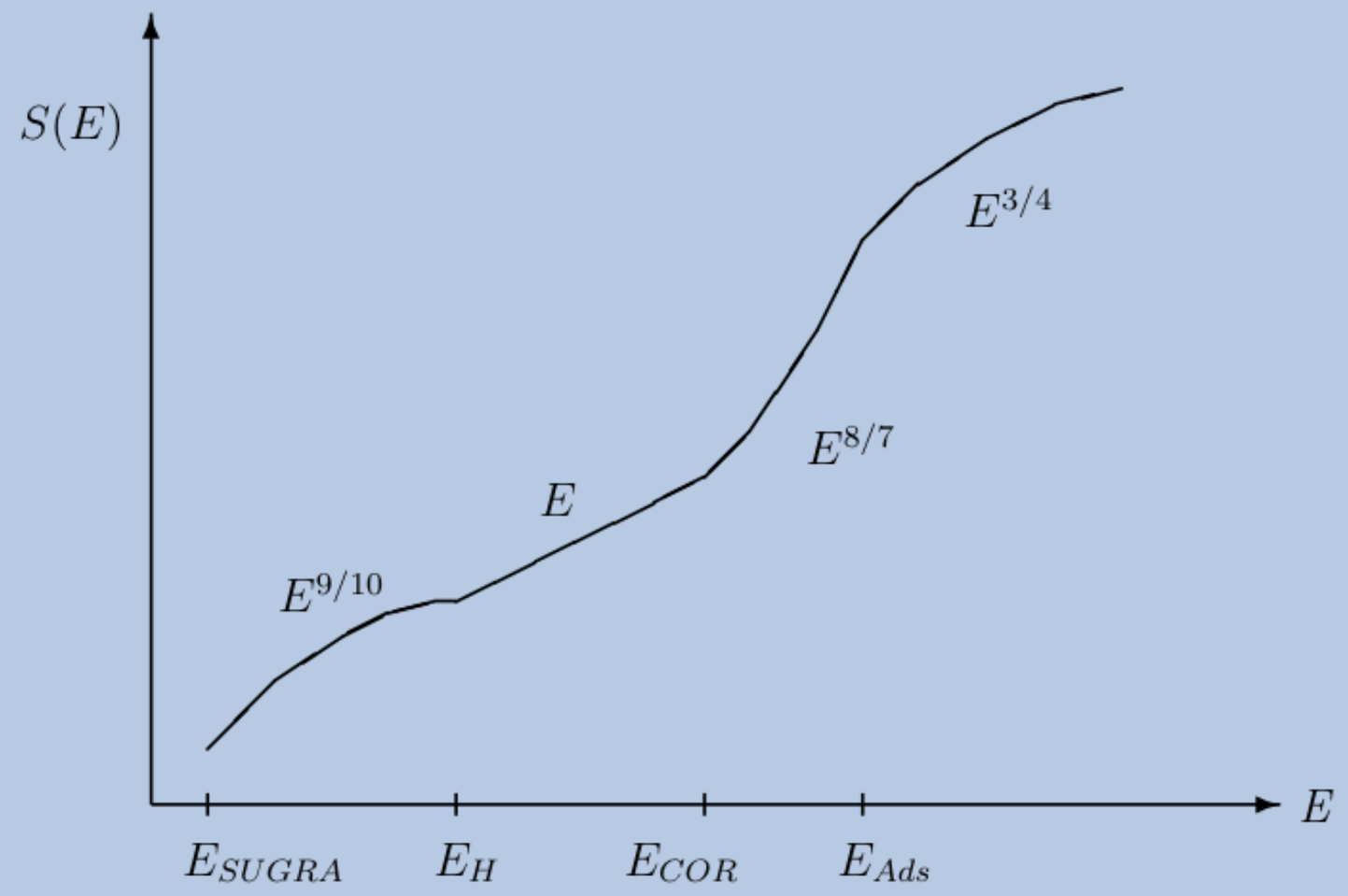
Dimension

With extended objects
1=3 , 4=10

SU(2) dim=3, rank=1
dim G=rank G (k=1)






$S(E)$
$E^{3/4}$
$E^{8/7}$
E
$E^{9/10}$
E
E_{SUGRA}
E_H
E_{COR}
E_{Ads}

Magic of String Theory

commutativity $[x,y]=0$

With extended objects

Non commutative

$[s,t] \neq 0$

Magic of String Theory

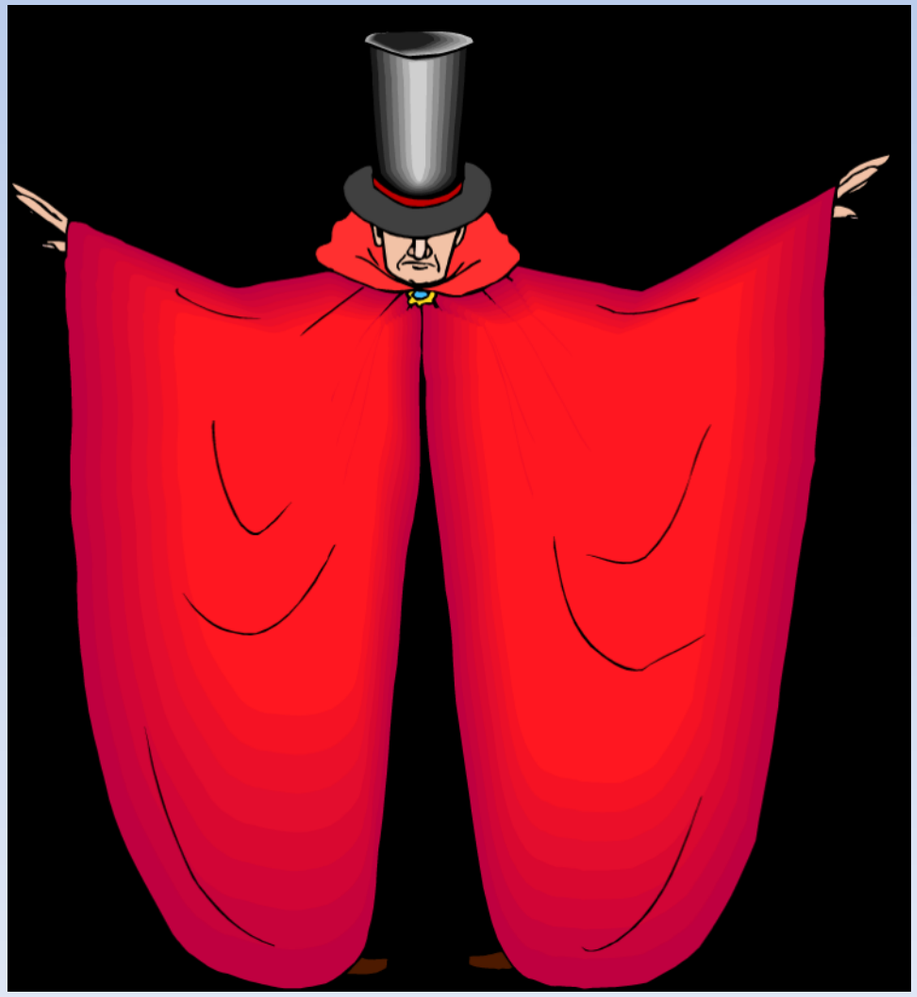
Singularities

With extended objects

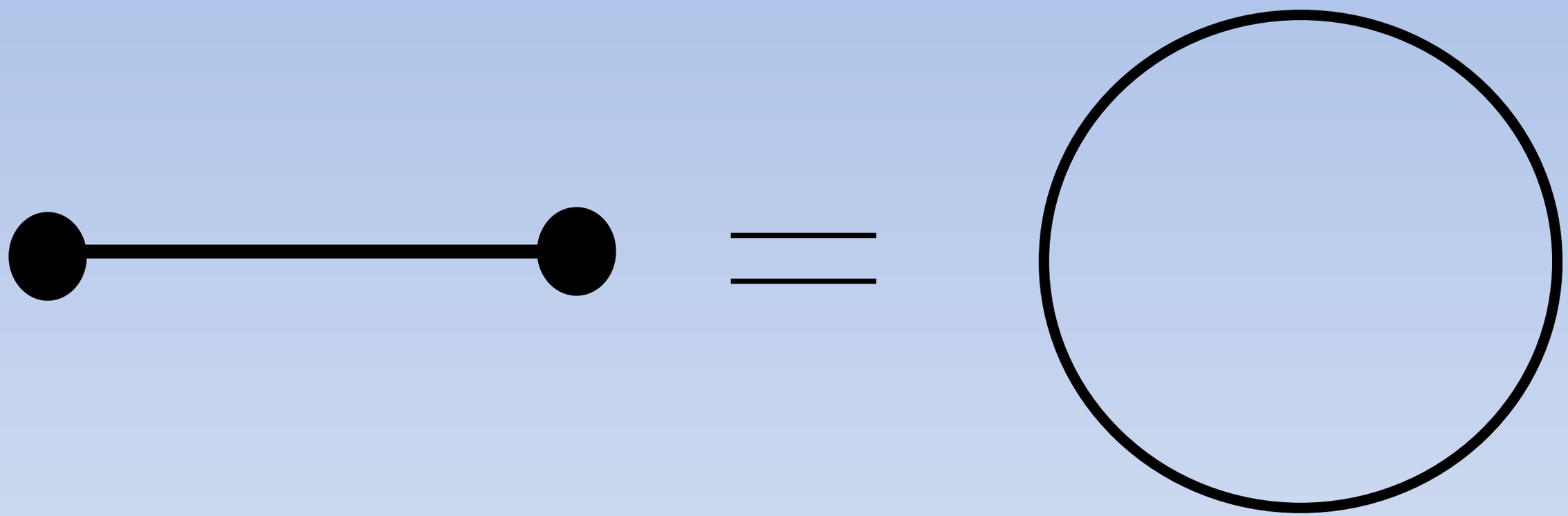
Magic of String Theory

Singularities are a reflection of a breakdown of an approximation.

Gravity has magic cloaks to hide its secrets



HORIZONS



The Big Crunch

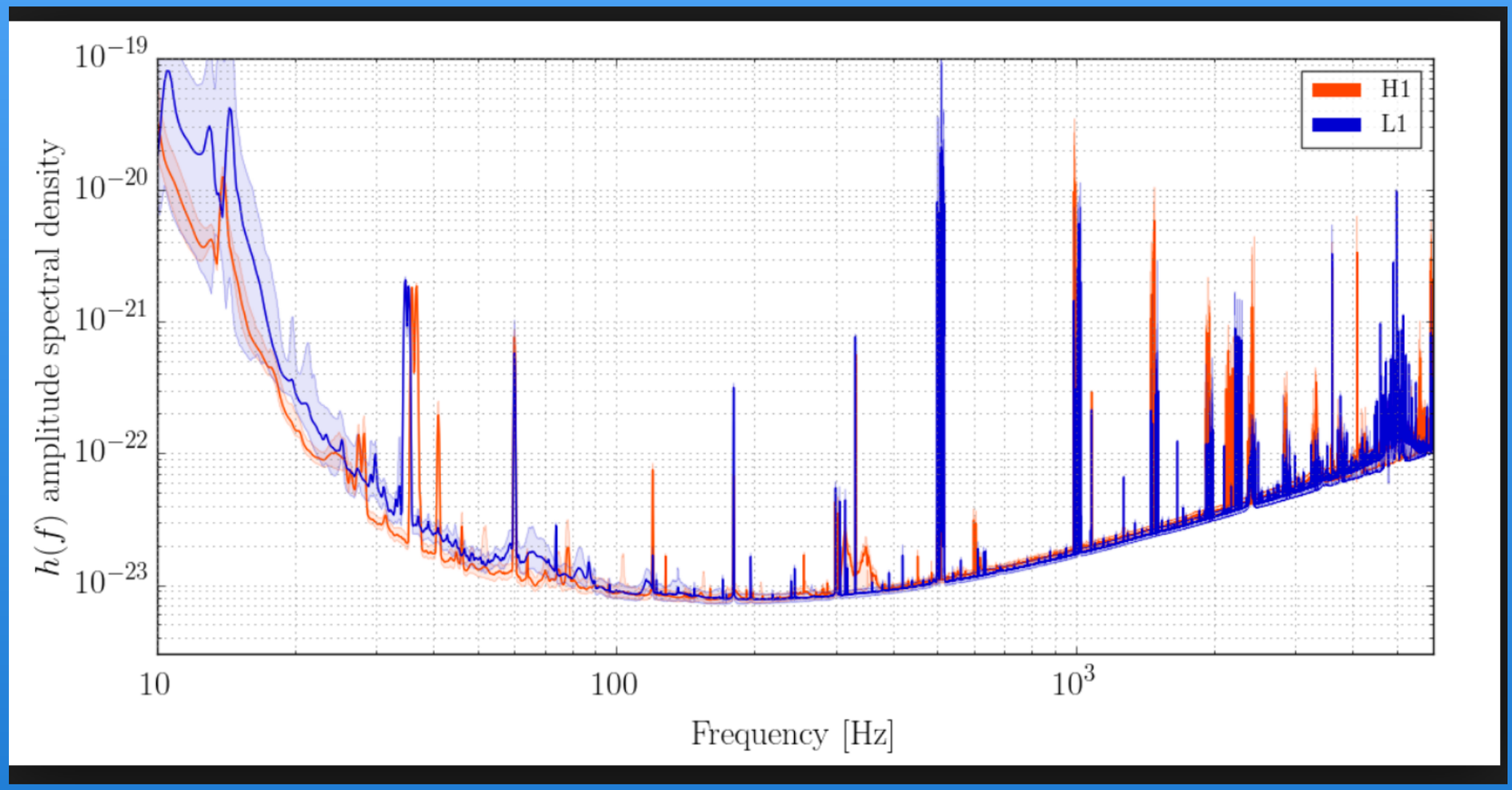
A role for
Non Local Operators?!

Dualities in String Theory

- Geometry
- Topology
- Number of dimensions, small and large
- (non-)Commutativity
- Singularity structure
- Associativity

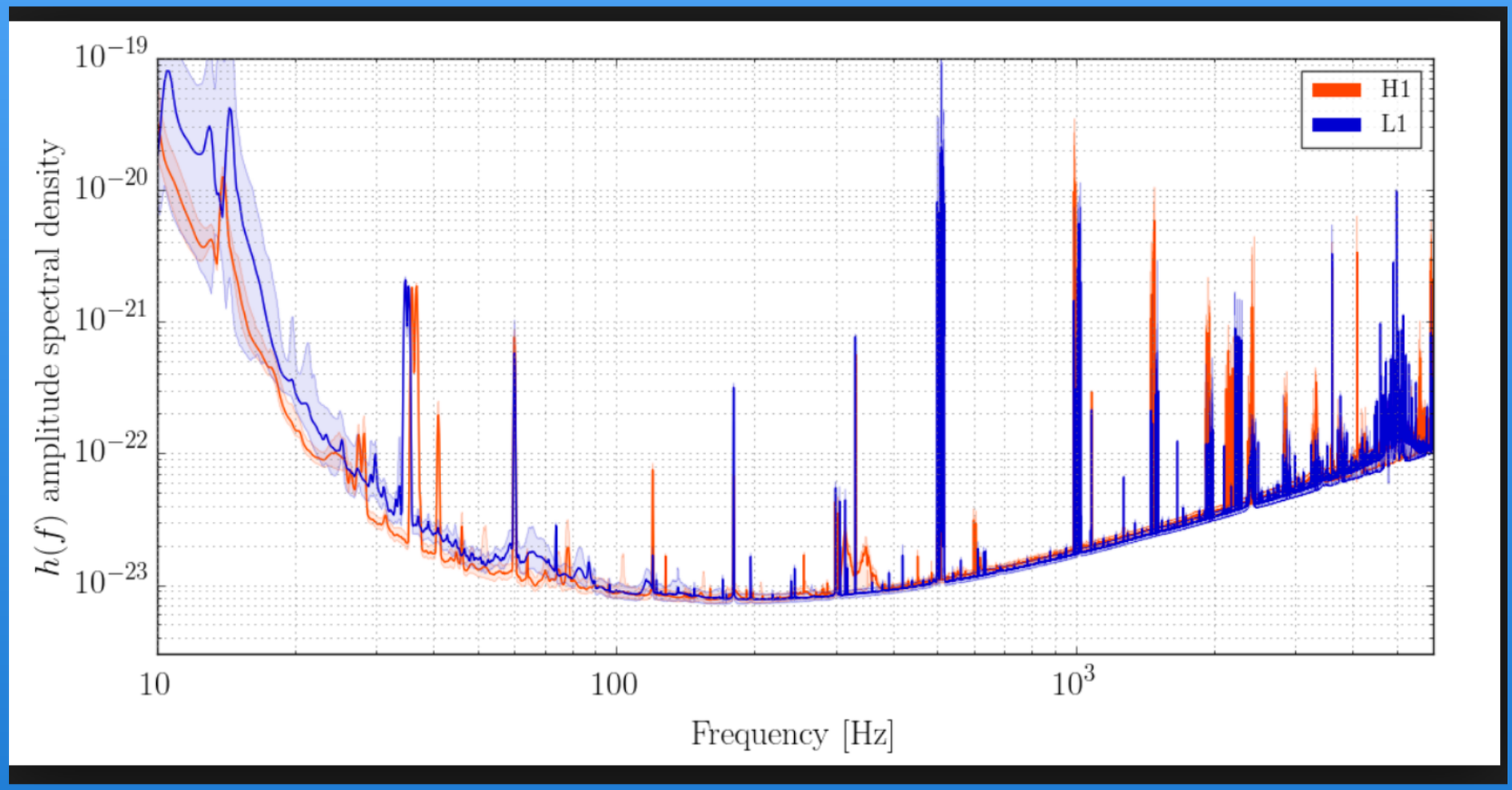
- **ONE UNIFIED ARENA??**
- **A DUALITY INVARIANT FORMULATION?**

NEW LIGO DATA



THE UNIVERSE IS A CIRCLE IN ONE DIRECTION !

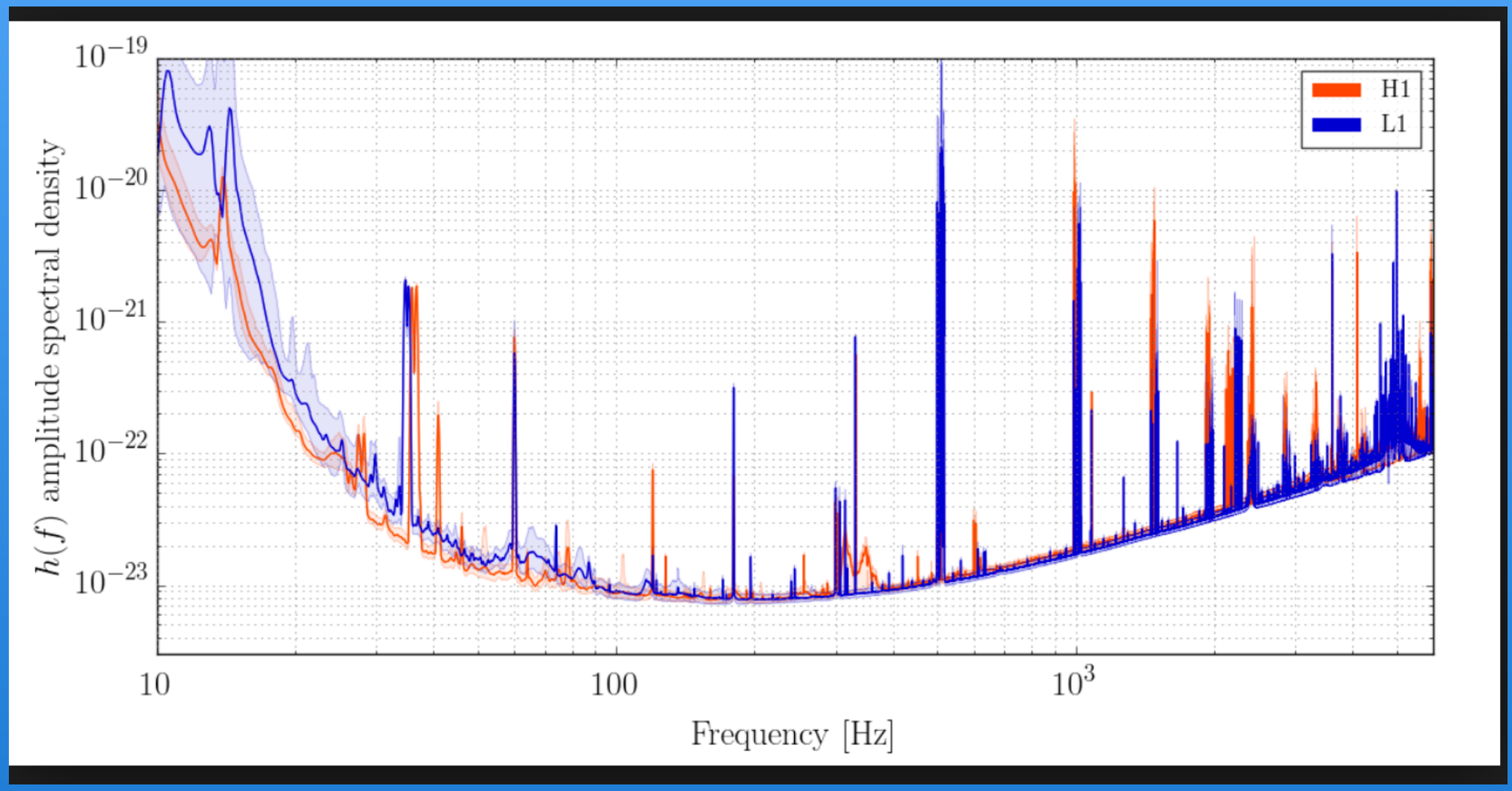
NEW LIGO DATA



THE UNIVERSE IS A CIRCLE IN ONE DIRECTION !

SO IS R LARGE OR SMALL ??

NEW LIGO DATA



SO IS R LARGE OR SMALL??

WE USE LOCAL OPERATORS TO MEASURE !
NON LOCAL/WINDING MORE DIFFICULT

CANT GET OUT OF MY MIND
THE WORLD SHEET AND TARGET SPACE AS ONE UNIT

$$Z_{\text{class}}(\sigma,\tau) = Z_{\text{class}}(\tau,\sigma), \tag{5.6}$$

namely, the classical piece of the mapping from a Riemann surface $\Sigma_g(\tau)$ to a target space $T^{2d}(\sigma)$ is equivalent to the mapping from a Riemann surface $\Sigma_d(\sigma)$ to a target space $T^{2g}(\tau)$.[50]

The result (5.6) can be useful in proving the invariance of genus-g partition functions under generalized target space duality. Notice that there is an isomorphism between $\mathrm{Sp}(2g,\mathbb{Z})$ (the group of modular transformations of the Riemann surface) generated by [235]

N=4 describes also a theory of a string moving in a background a AdS_5 X S^5 And a black hole in AdS_5 X S^5

The AdS/CFT Correspondence

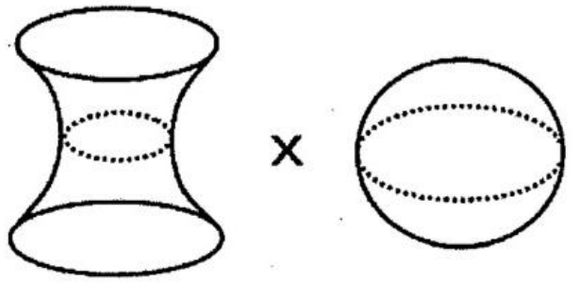
[Maldacena '97]

D=4, N=4 , SUSY Y.M. SU(N)

't Hooft coupling: $\lambda = N g^2_{\rm YM}$ 1/color number: $\frac{1}{N}$ theta angle: $\theta_{\rm YM}$

$\mathcal{N} = 4$ SYM was conjectured to be dual to a string theory:

IIB Superstrings on $AdS_5 \times S^5$



tension: $\frac{R^2}{\alpha'} = \sqrt{\lambda}$ coupling: $g_s = \frac{\lambda}{4\pi N}$ axion: $\langle C \rangle = \theta_{\rm YM}$

ADS/CFT

ALSO BACK TO QFT!

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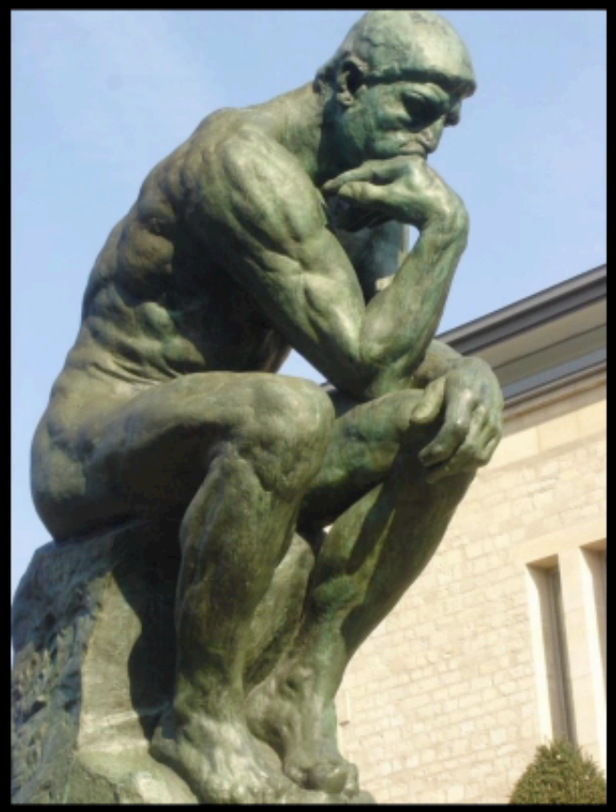
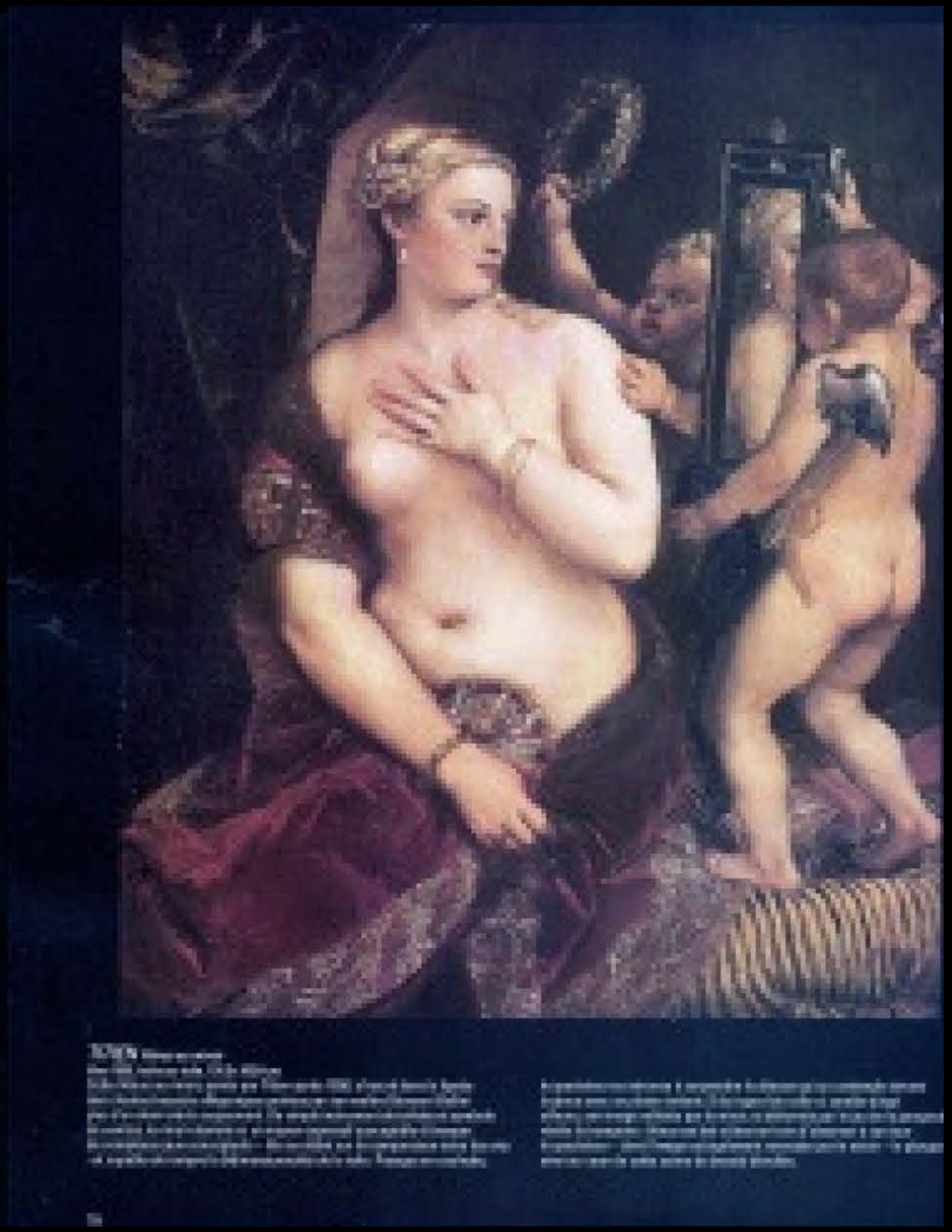
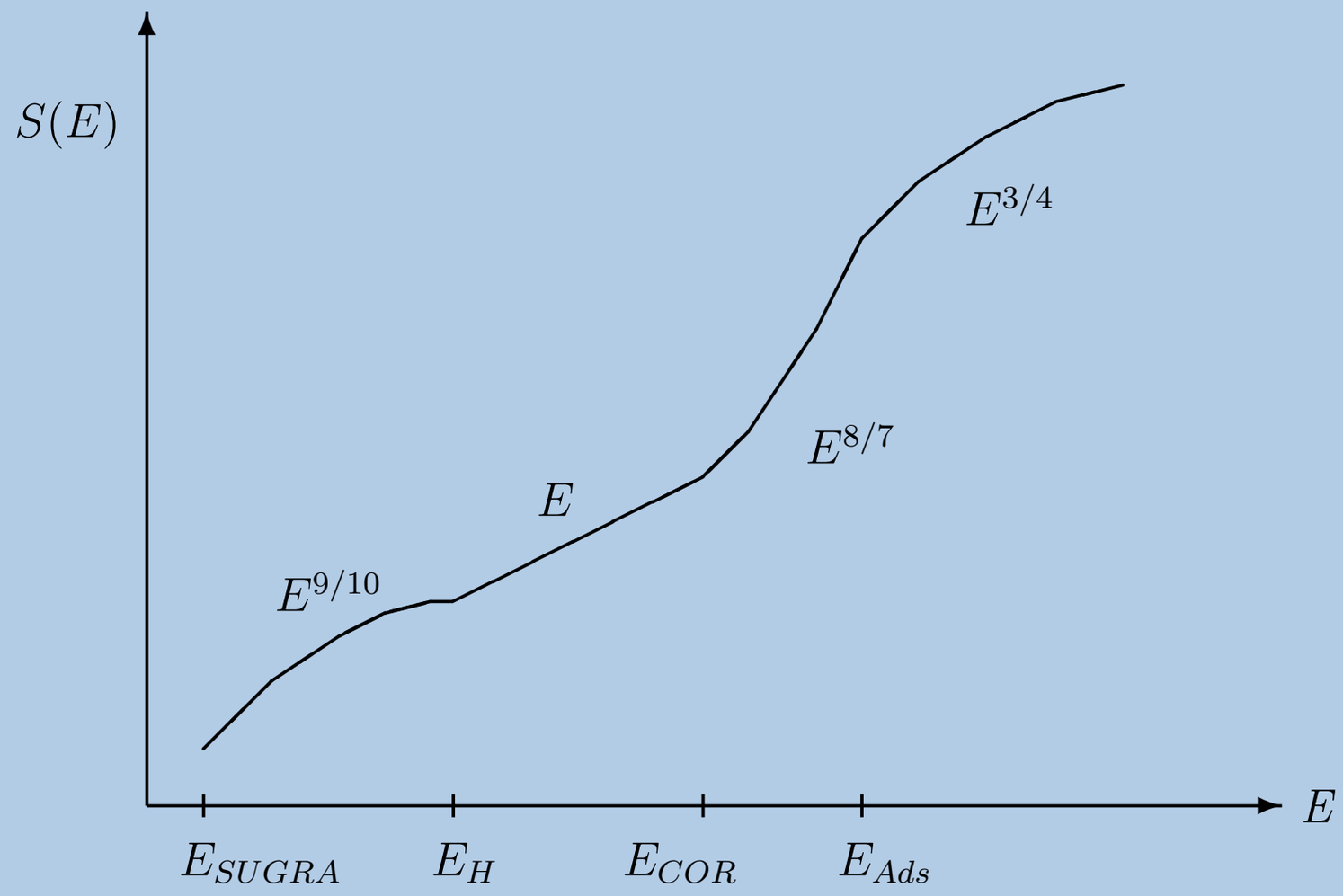
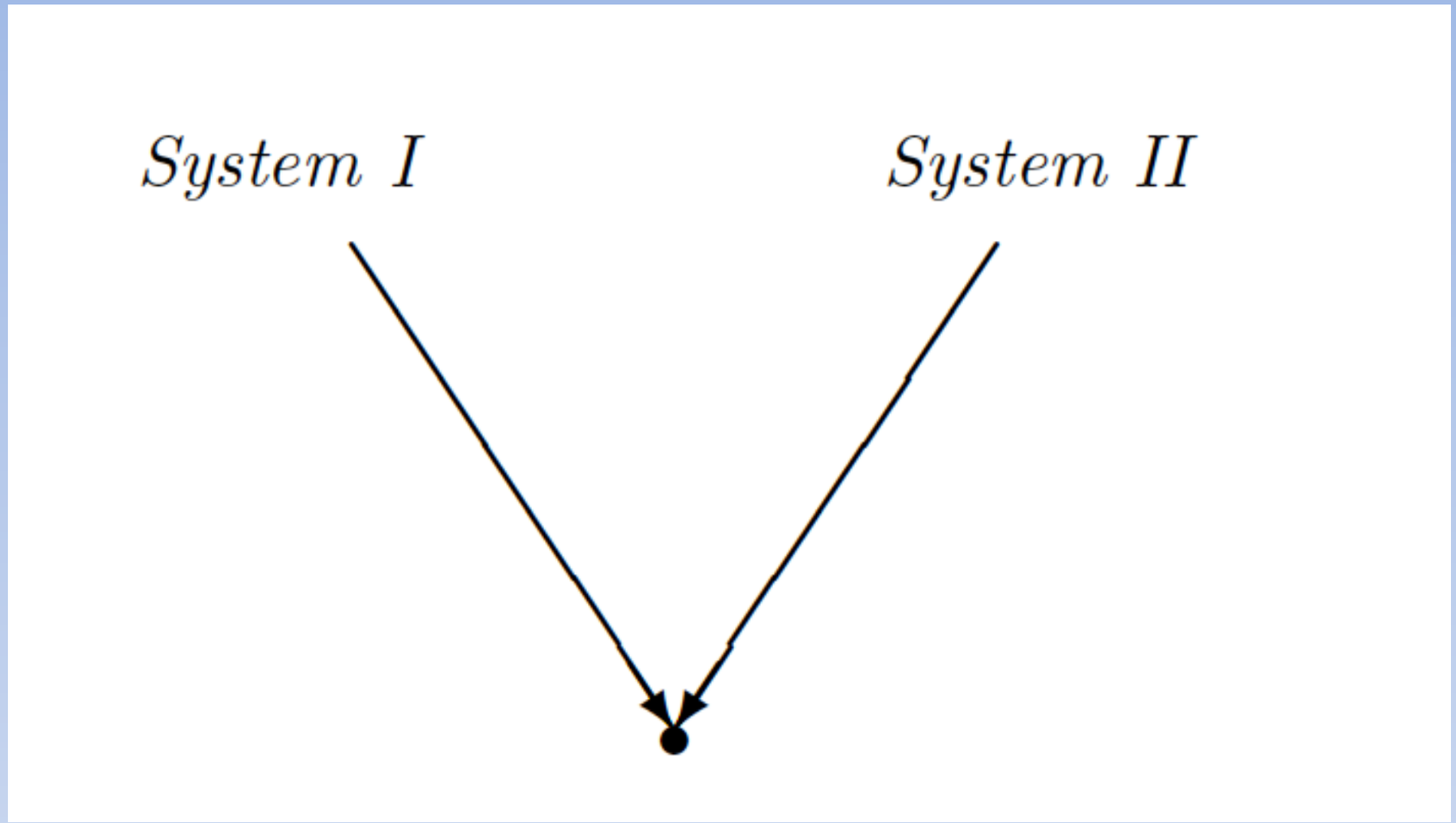


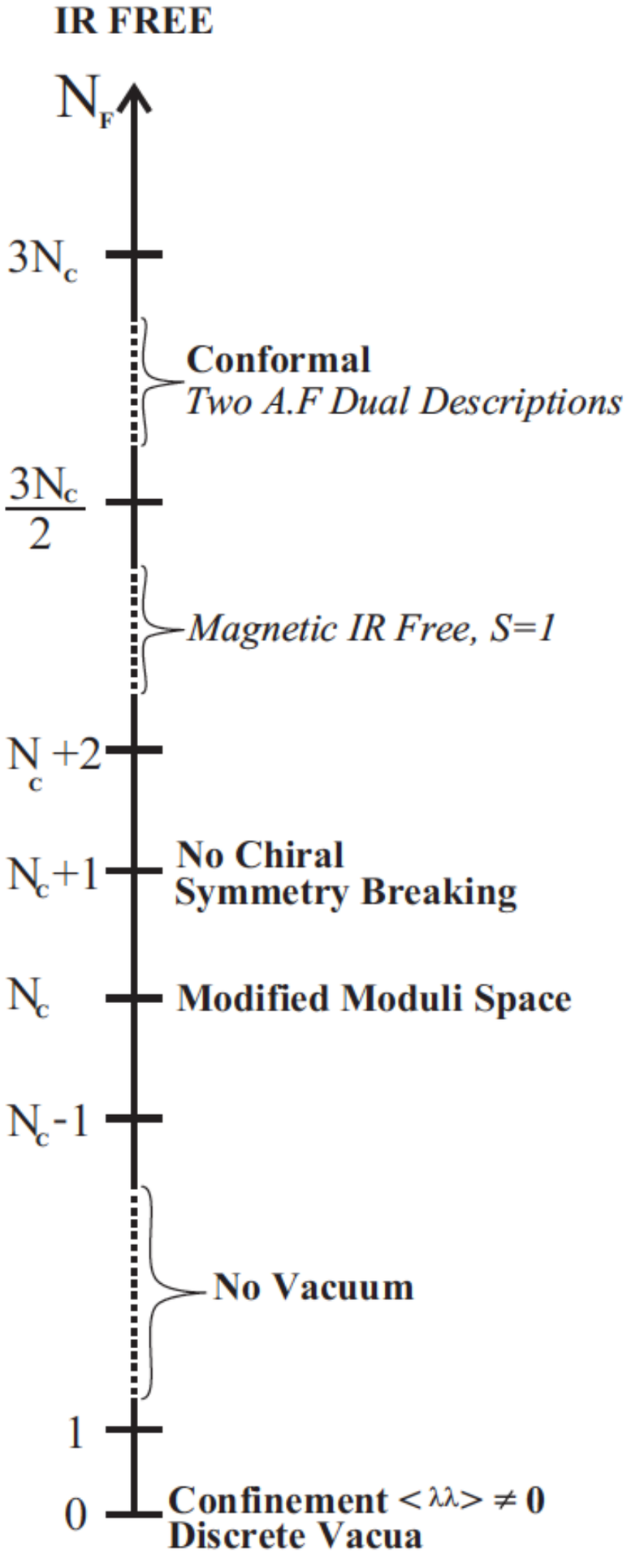



$S(E)$
$E^{3/4}$
$E^{8/7}$
E
$E^{9/10}$
E
E_{SUGRA}
E_H
E_{COR}
E_{Ads}

IR DUALITY



QCD and a Chiral Lagrangian of Pions



N=1 SUSY more structure

Emerging U(1) gauge symmetry in a conformal window

Branes in motion

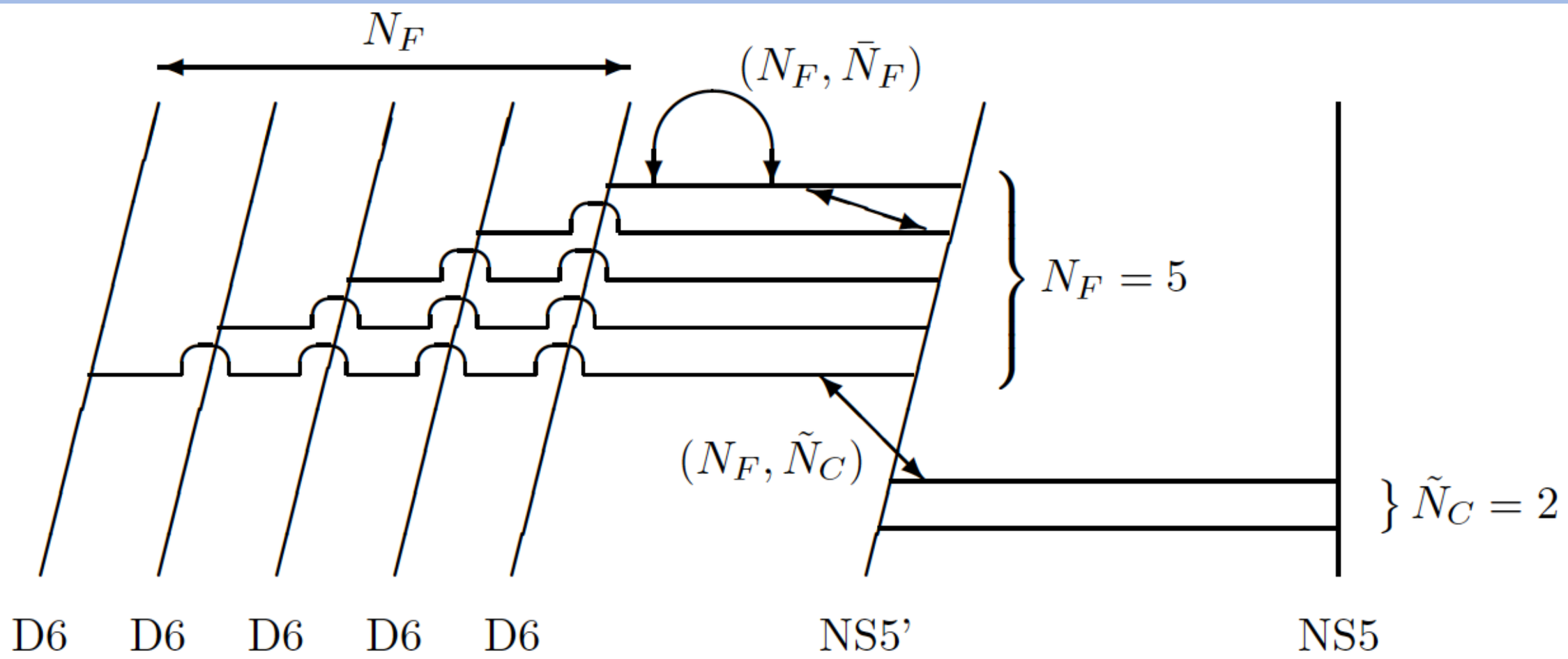
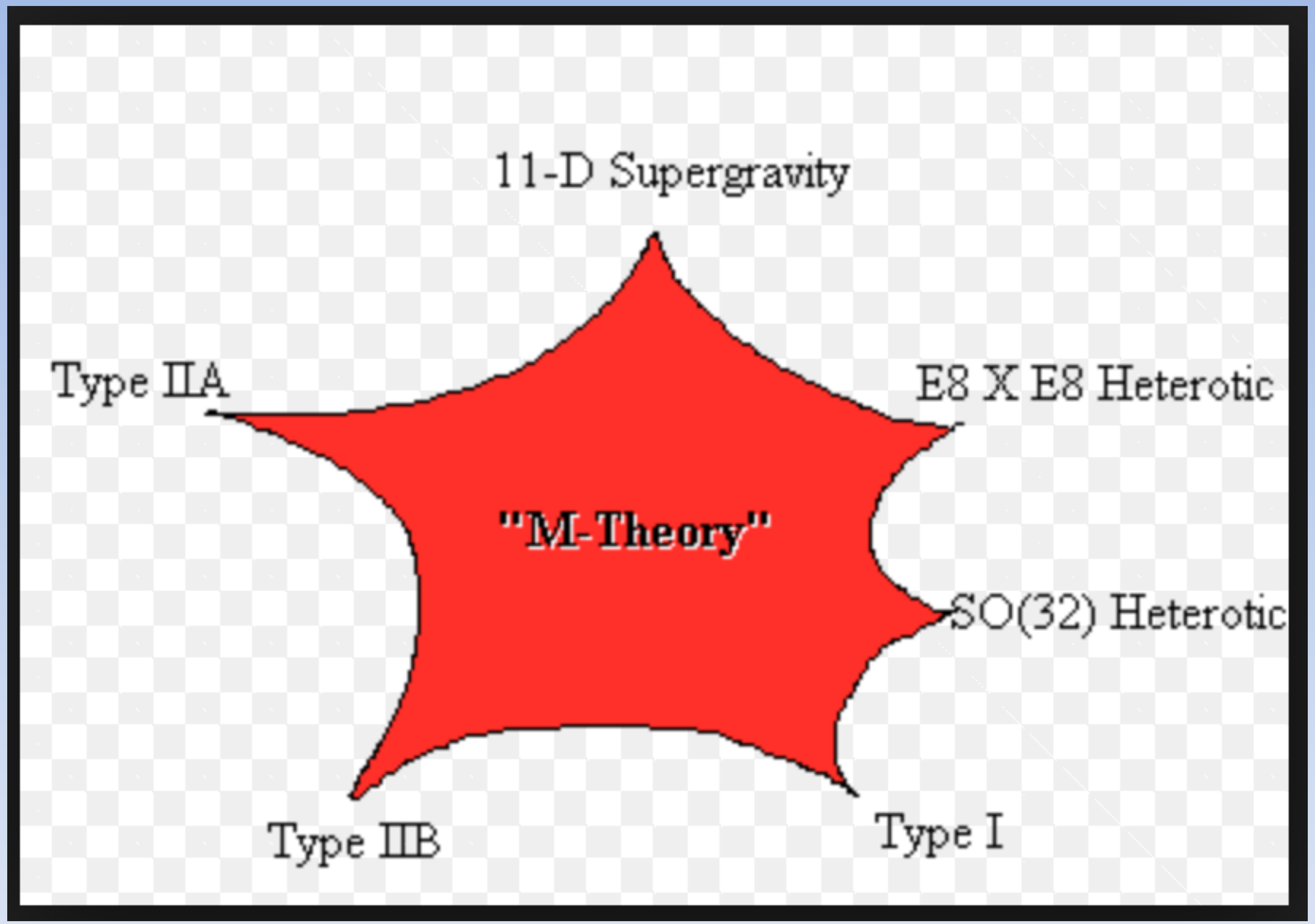


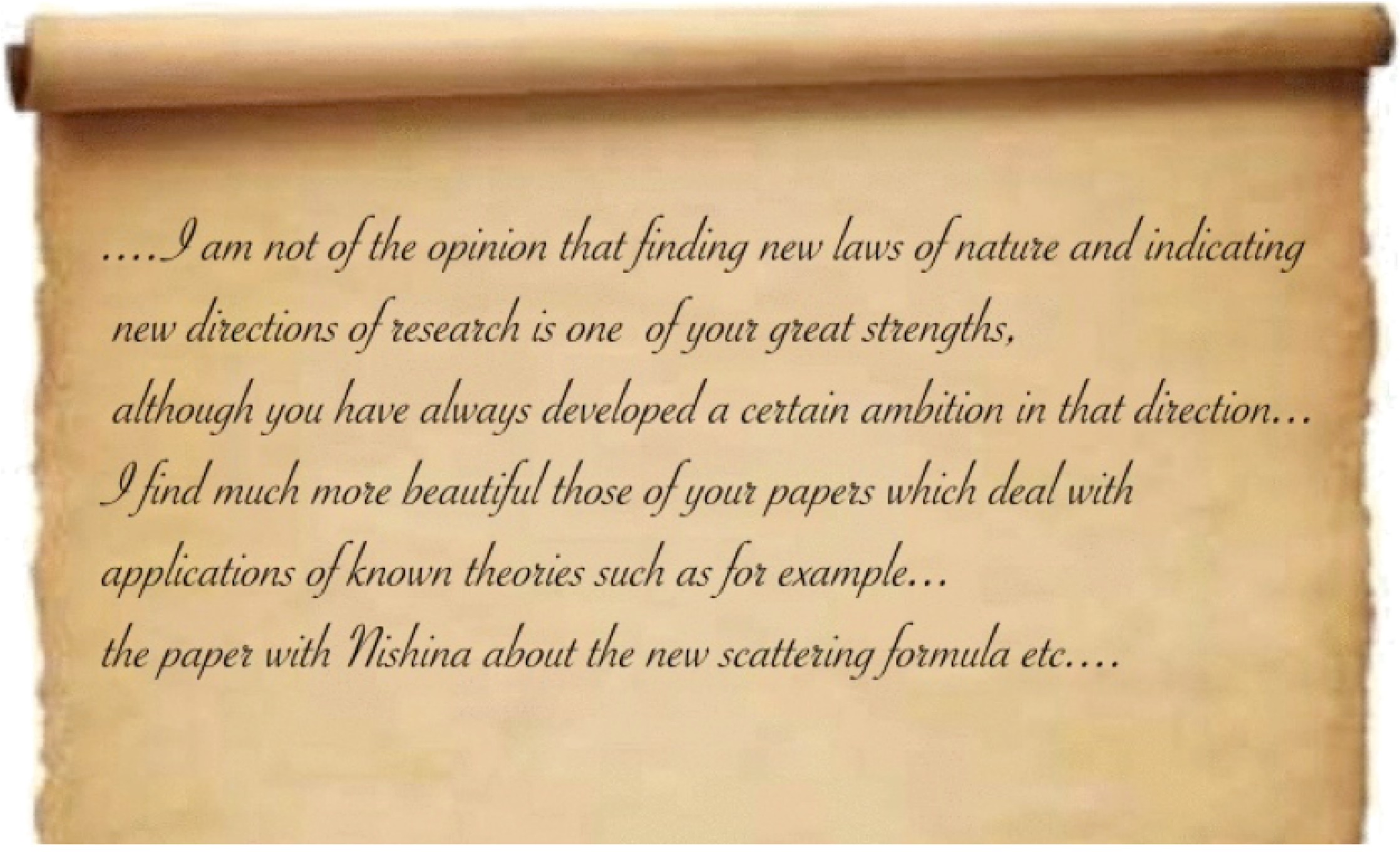
Figure 33: This is the dual configuration to that of a N=1 SUSY gauge theory with N_C=3 and N_F=5 (Fig. 39). The configuration shown has N_C=5-3=2, N_F=5 and, in addition, N_F^2 color singlet massless particles.

LOTS AND LOTS OF DUALITIES - A WEB

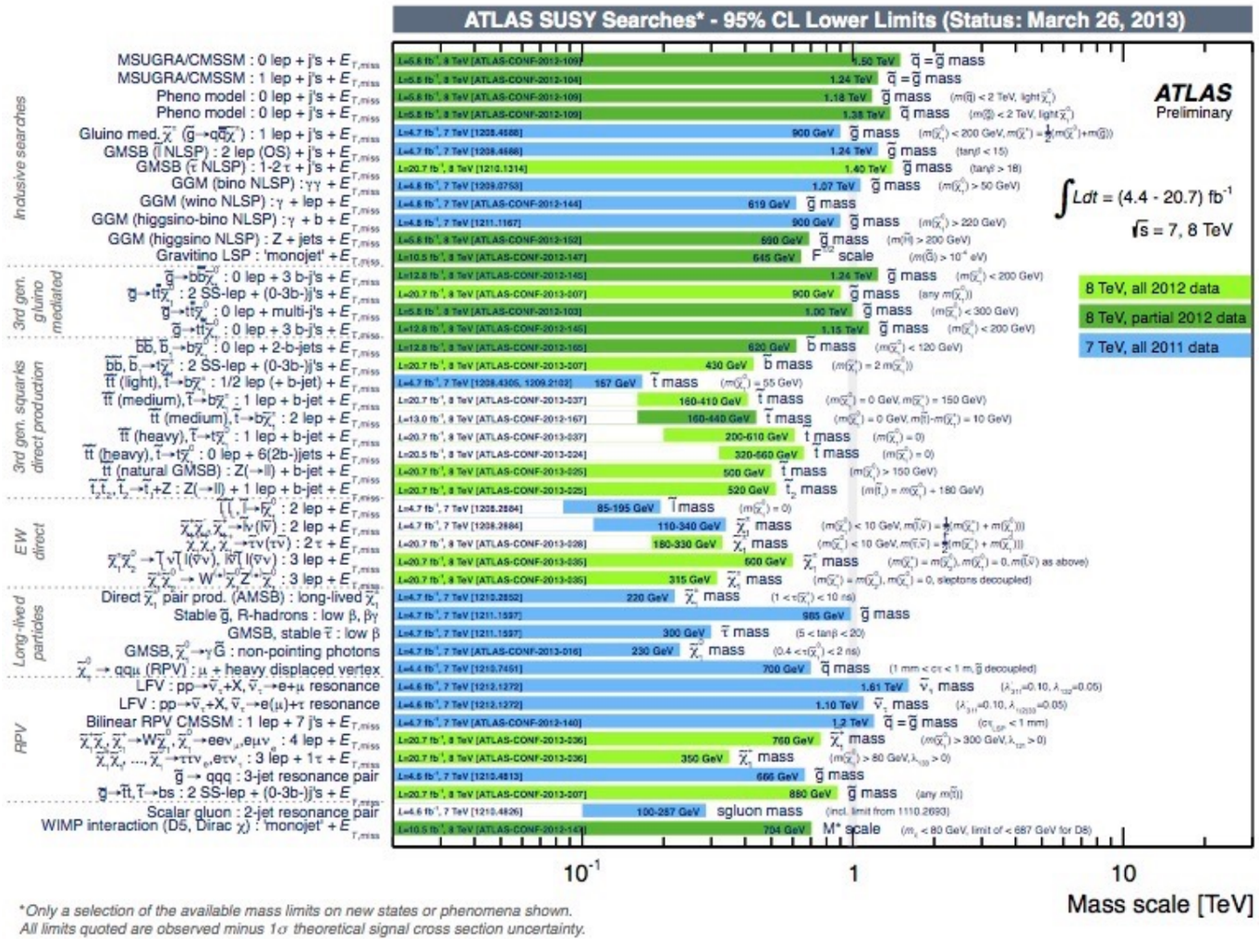


Many many more

Shortly Before arriving in Stockholm, Klein received a letter from his good friend Pauli which reads in part:



From Nature to the LHC, SUSY? STRINGS?



*Only a selection of the available mass limits on new states or phenomena shown.
All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.

Happy Birthday dear Formula

מזל טוב

Fifty years..for advice , we need it .