Swampland constraints on field ranges and naturalness

Irene Valenzuela

Utrecht University

Universiteit Utrecht

Ibanez,Martin-Lozano,IV [arXiv:1706.05392 [hep-th]]

Ibanez,Martin-Lozano,IV [arXiv: 1707.05811 [hep-th]]

Grimm,Palti,IV [arXiv:1802.08264 [hep-th]]

50 years of the Veneziano model

Galileo Galilei Institute, Firenze, May 2018

50 years of the Veneziano Model:
From Dual Models to Strings, M-theory and Beyond

50 years of the Veneziano Model:
From Dual Models to Strings, M-theory and Beyond

What can String Theory tell us about our universe?

50 years of the Veneziano Model:
From Dual Models to Strings, M-theory and Beyond

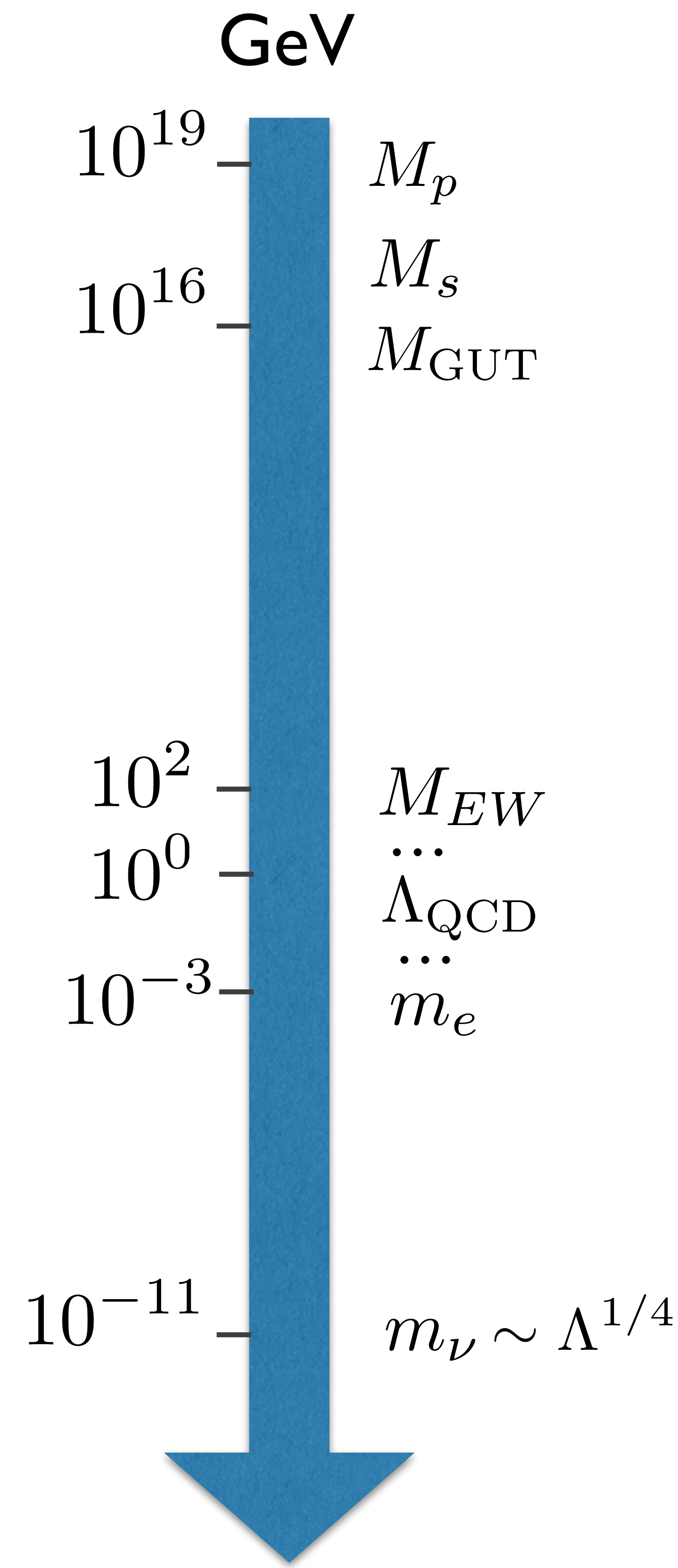
What can String Theory tell us about our universe?

What questions would we like to answer?

Successful Standard Model of Particle Physics and Cosmology!

Expectation of 'separation of scales':

IR effective theory not very sensitive to UV physics



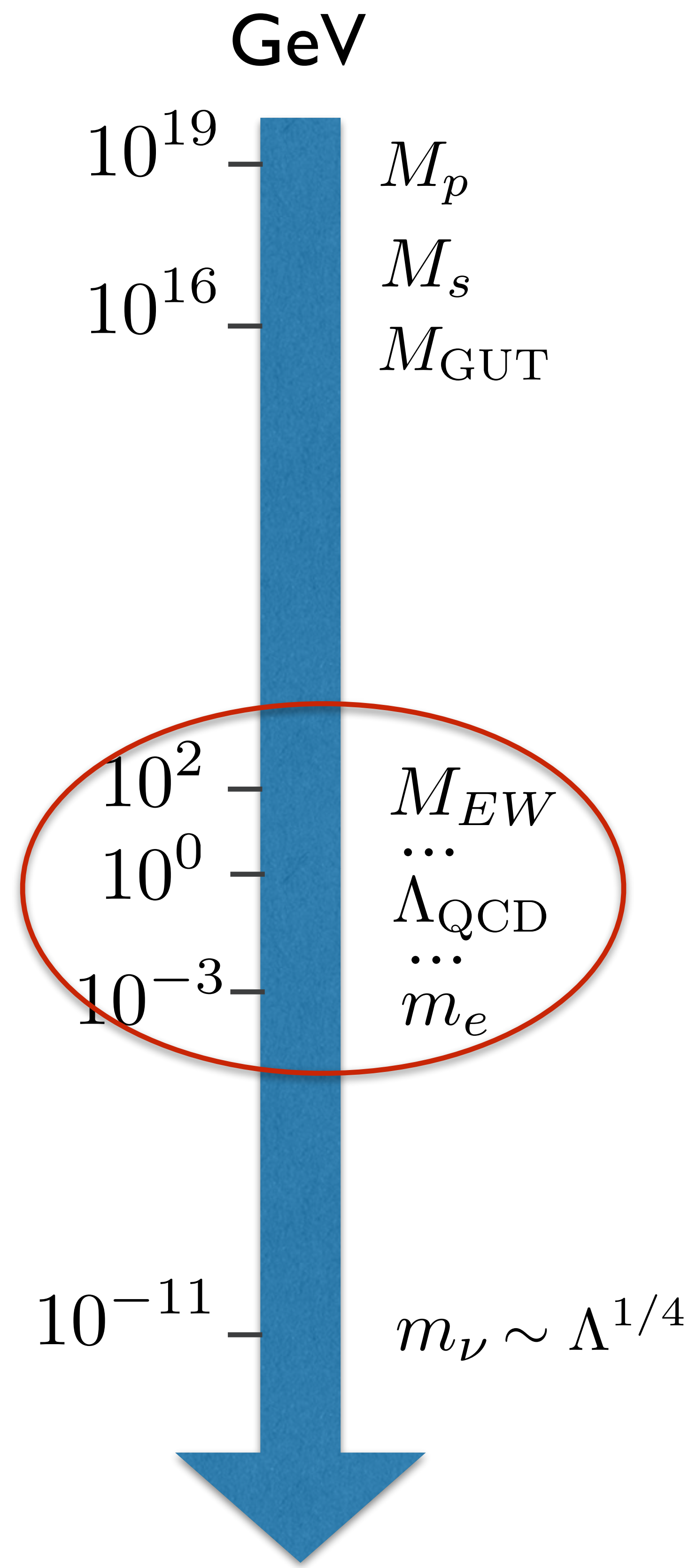
Successful Standard Model of
Particle Physics and Cosmology!

Expectation of ‘separation of scales’:

IR effective theory not very
sensitive to UV physics

Shall we care about Quantum Gravity?

Any implication for low energy physics?

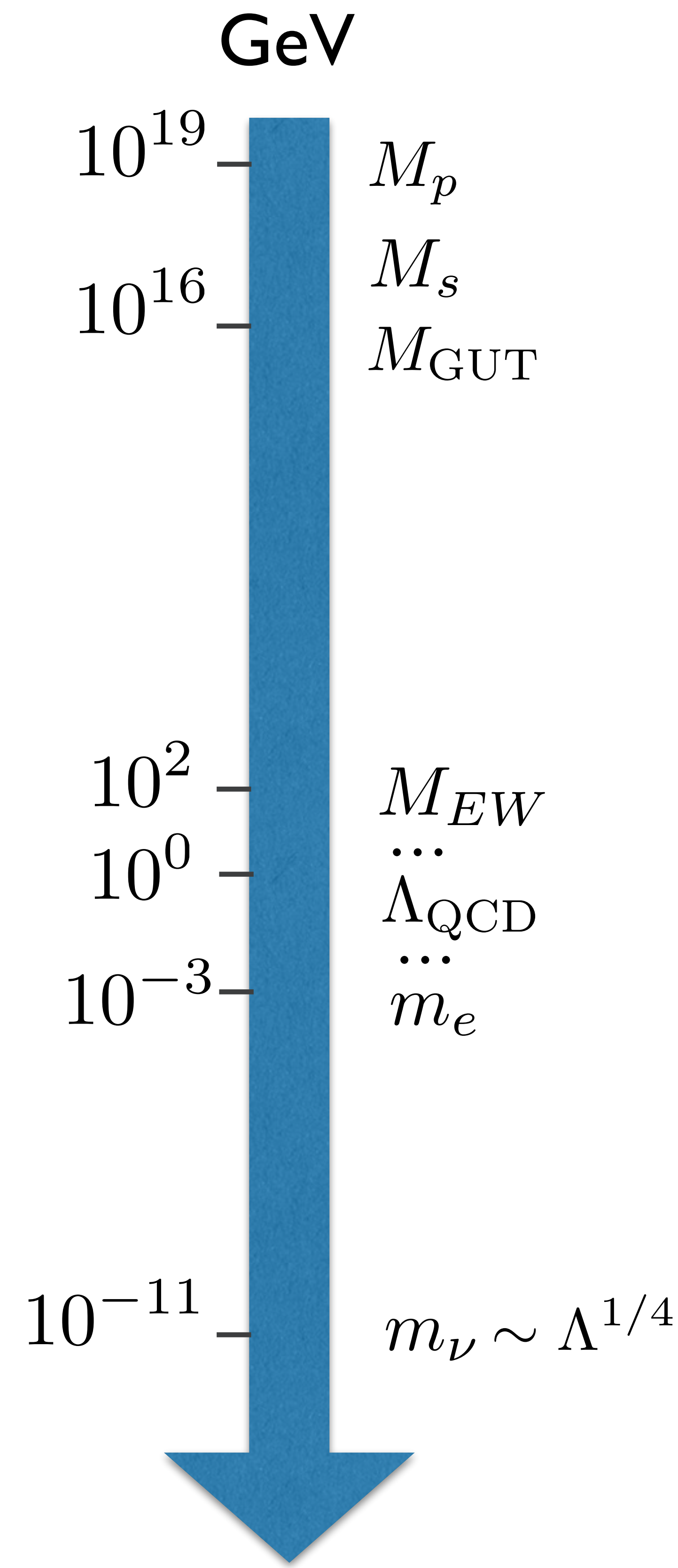


Successful Standard Model of Particle Physics and Cosmology!

Expectation of 'separation of scales':

IR effective theory not very sensitive to UV physics

This picture fails!



Successful Standard Model of
Particle Physics and Cosmology!

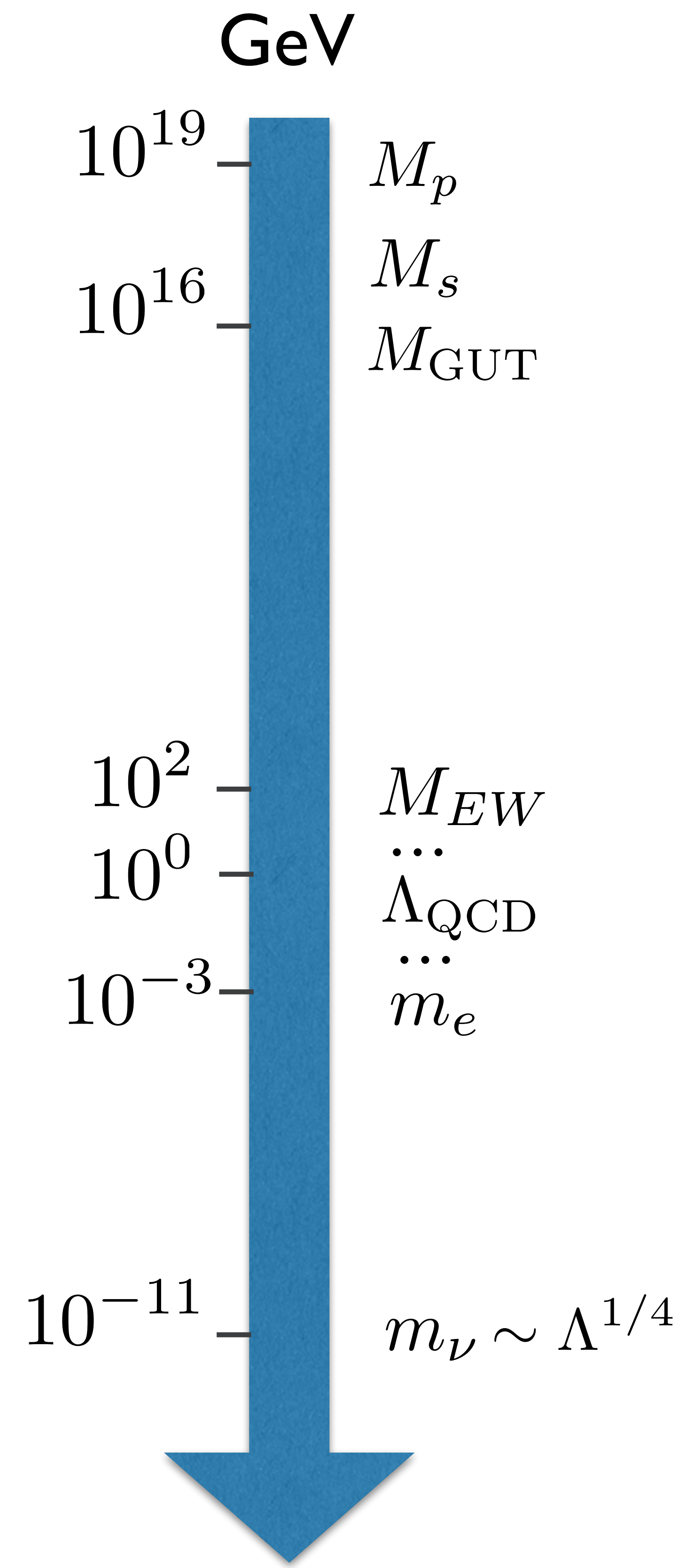
Expectation of 'separation of scales':

IR effective theory not very
sensitive to UV physics

This picture fails!

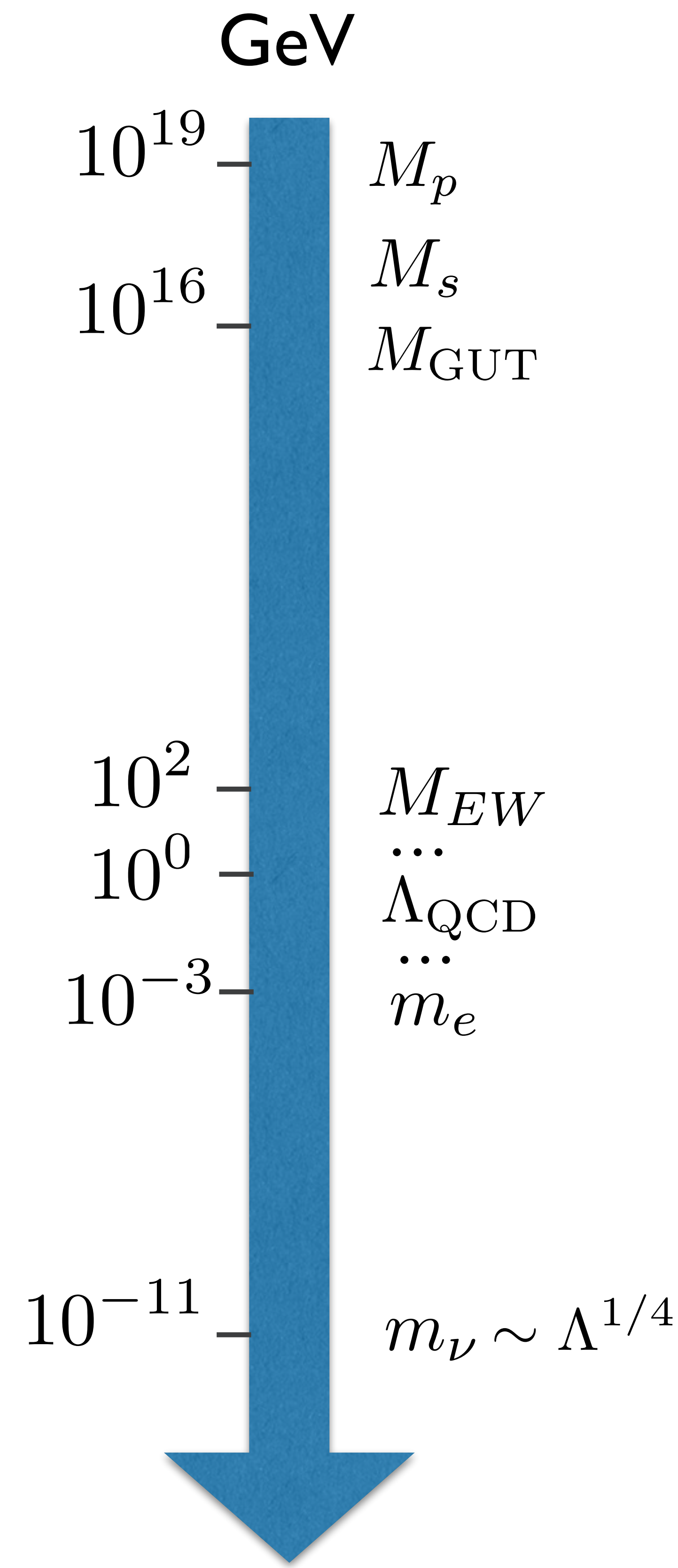
Naturalness problems:

- Cosmological constant
- EW hierarchy problem



Absence of new physics is also a hint!

Naturalness is not a good guiding principle
to progress in high energy physics…
new ideas?

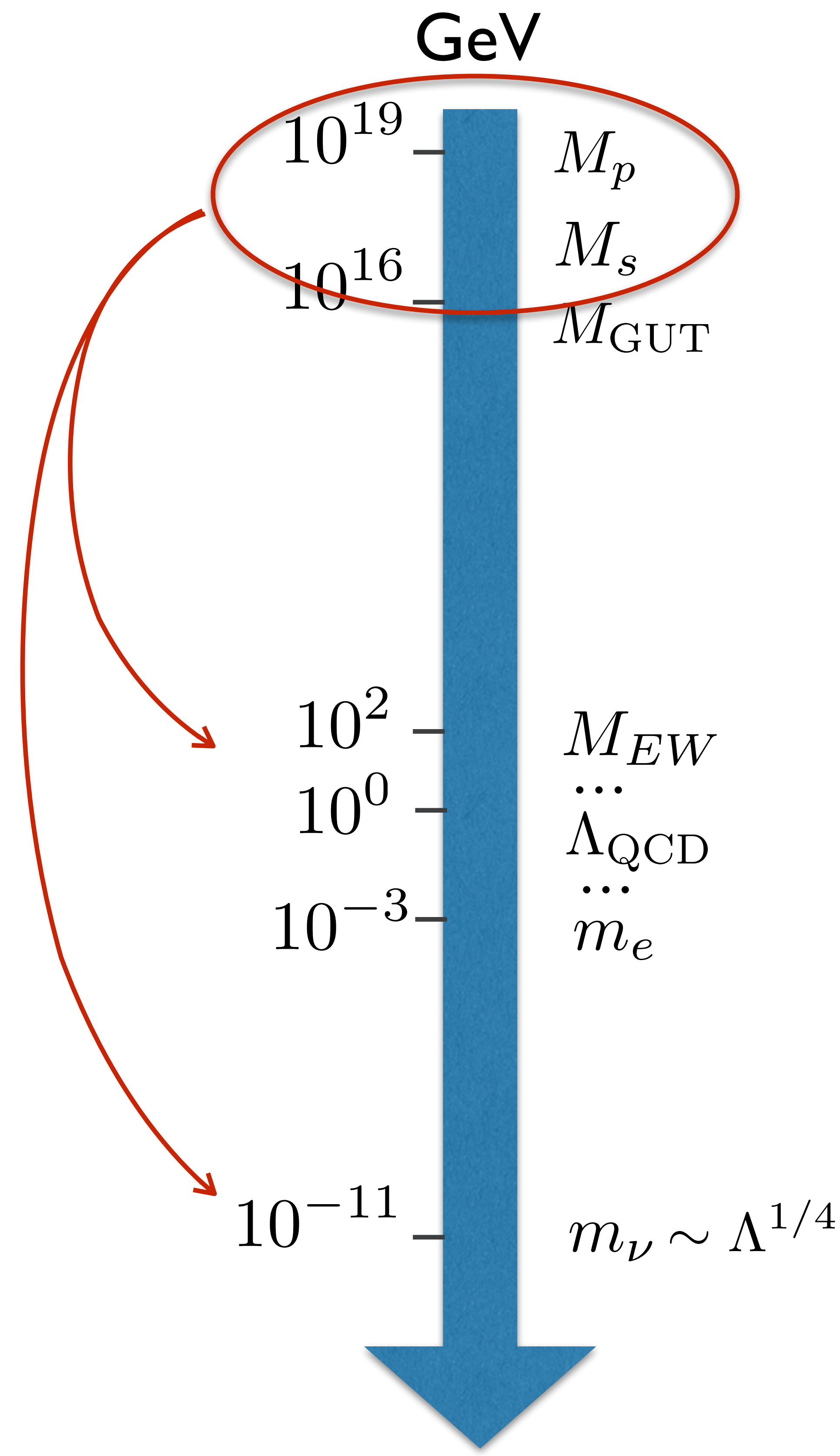


Absence of new physics is also a hint!

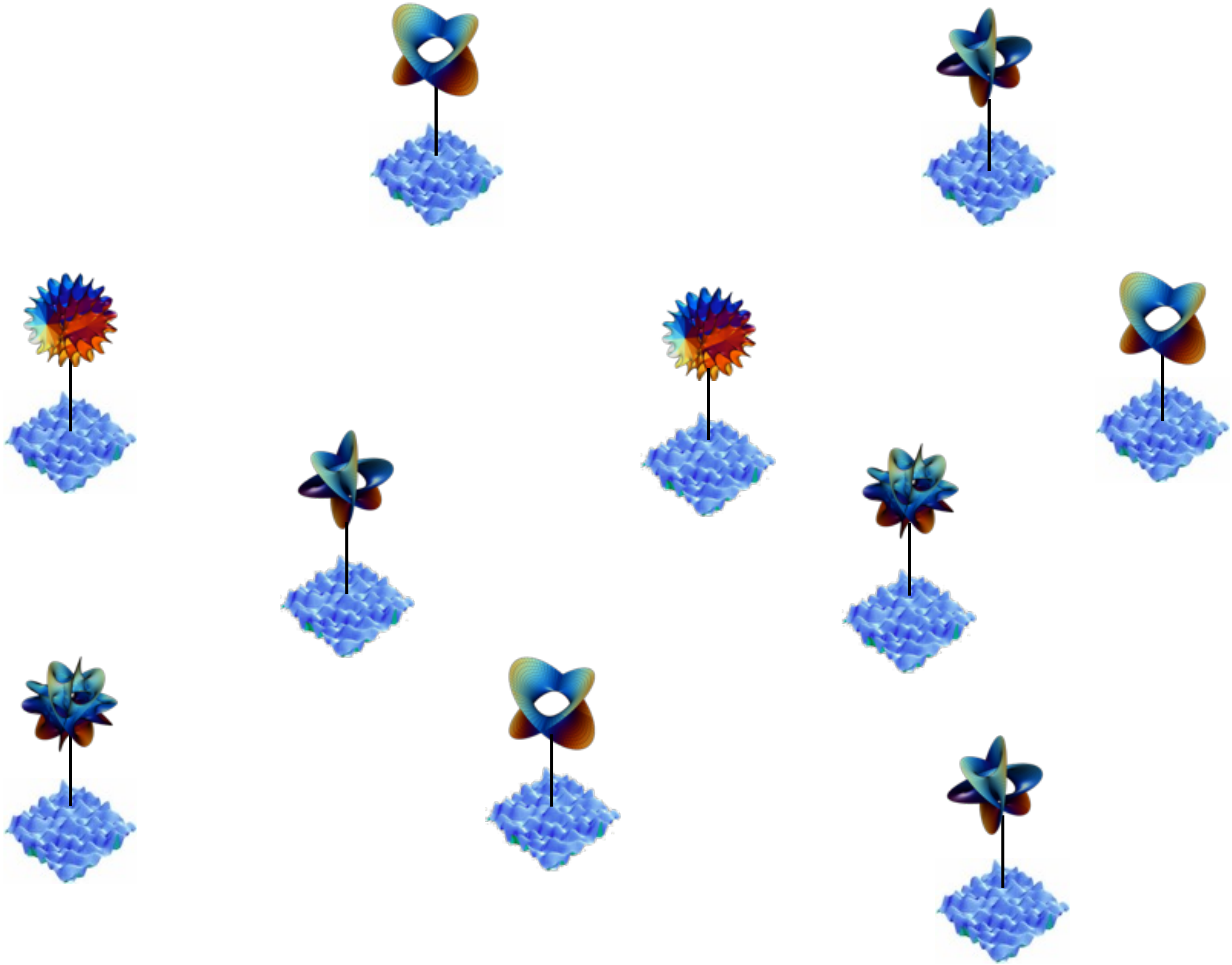
Naturalness is not a good guiding principle
to progress in high energy physics…
new ideas?

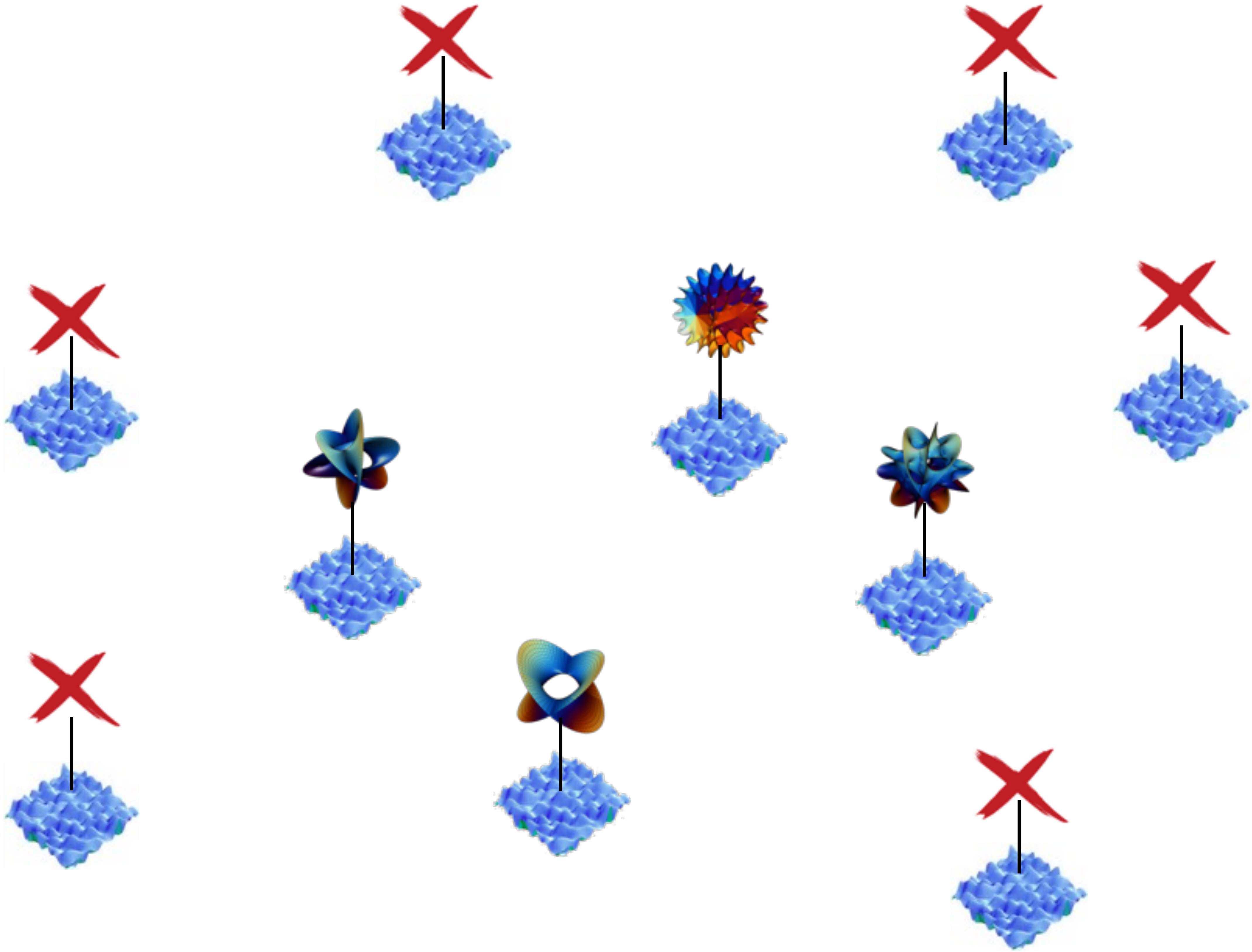
UV/IR mixing induced by gravity?

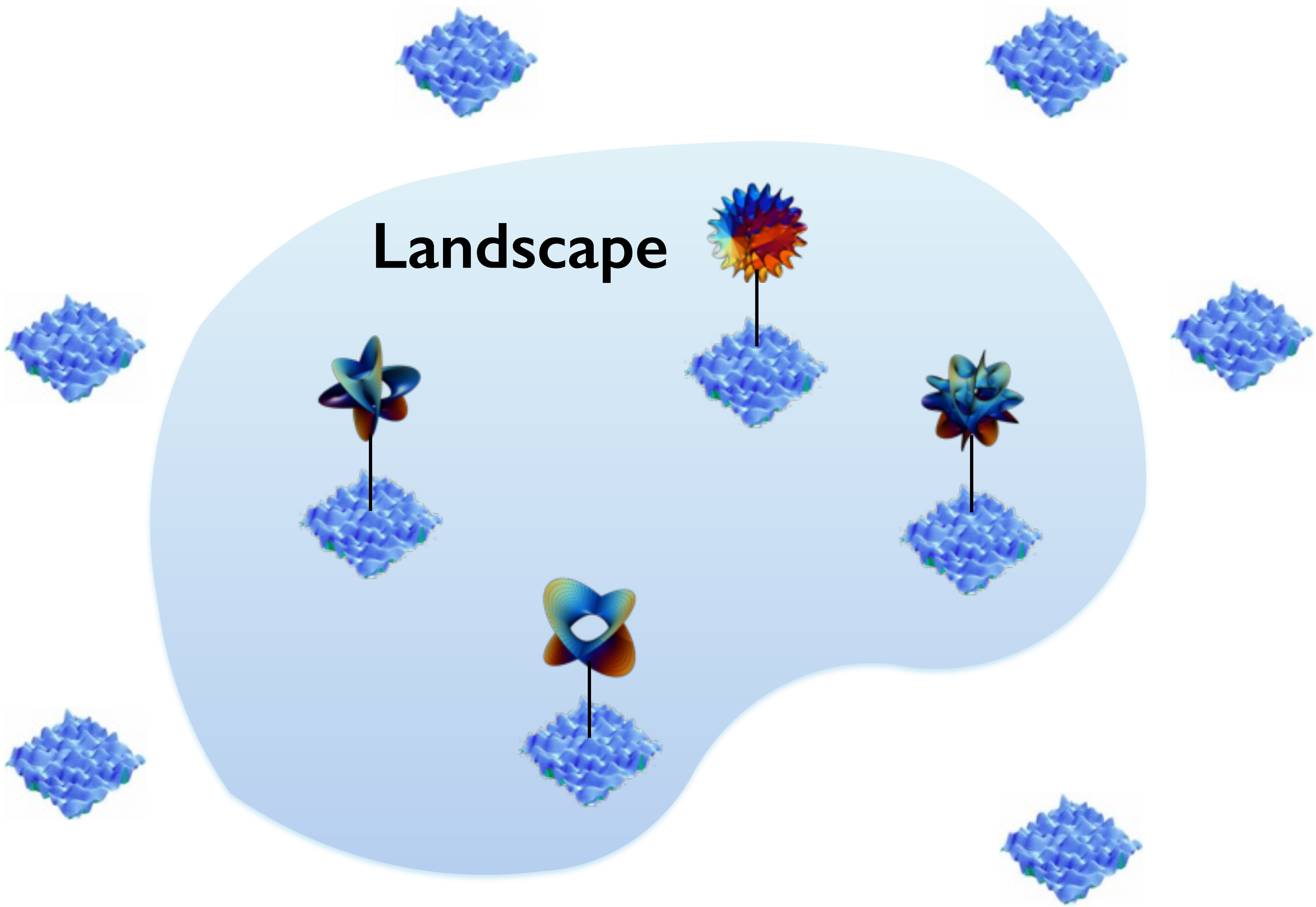
Quantum gravity constraints?

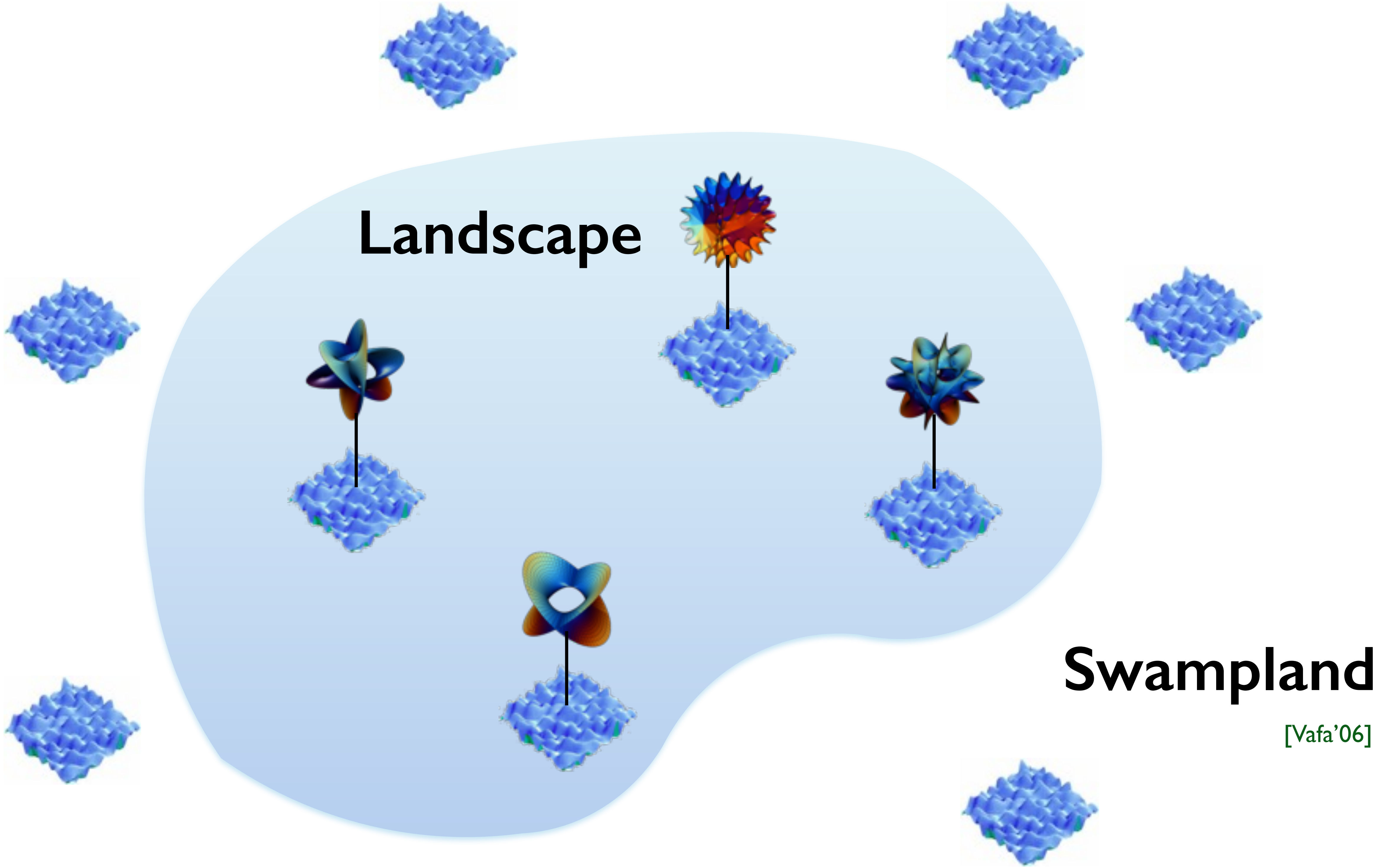


String Landscape



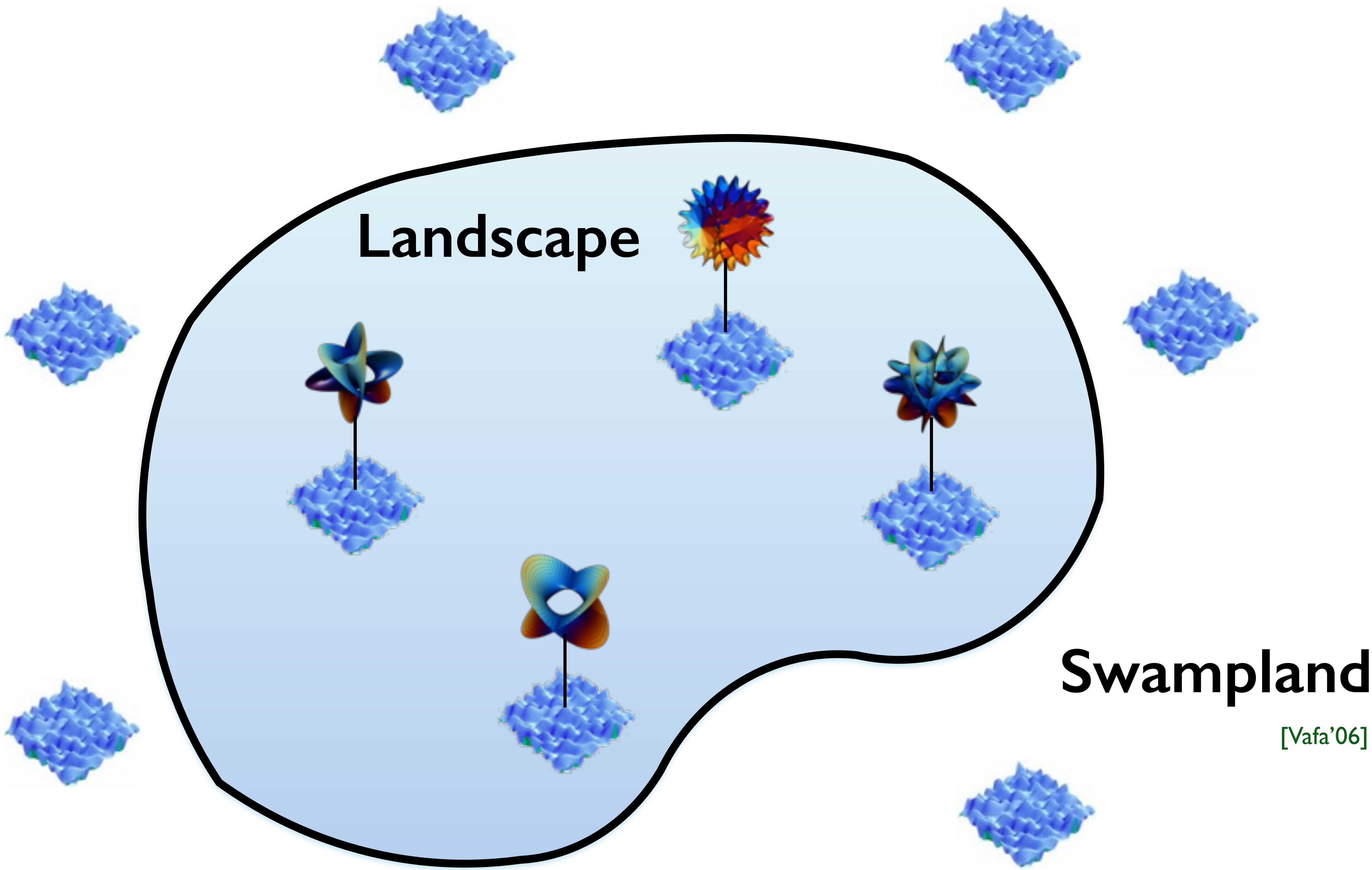



Landscape


Landscape
Swampland
[Vafa'06]

Not everything is consistent with quantum gravity!

What distinguishes the landscape from the swampland?



What are the constraints that an effective theory must satisfy to be consistent with quantum gravity?

First guess: Anomalies

QFT of scalars
and fermions

First guess: Anomalies

QFT of scalars and fermions → QFT of scalars and fermions + gauge fields

Anomaly constraints

example. QFT of one fermion with SU(2) global symmetry

There is a Witten anomaly when coupling the theory to a gauge field!

First guess: Anomalies

QFT of scalars and fermions → QFT of scalars and fermions + gauge fields → QFT of scalars and fermions + gauge fields + gravity

Anomaly constraints

First guess: Anomalies

QFT of scalars
and fermions

Anomaly
constraints

QFT of scalars
and fermions
+ gauge fields

QFT of scalars
and fermions
+ gauge fields
+ gravity

Gravitational anomalies
are not enough

First guess: Anomalies

QFT of scalars and fermions → QFT of scalars and fermions + gauge fields → QFT of scalars and fermions + gauge fields + gravity

Anomaly constraints

Gravitational anomalies are not enough

Not every apparently consistent (anomaly-free) effective theory can be UV embedded in quantum gravity

UV imprint = Quantum Gravity/String Theory predictions!

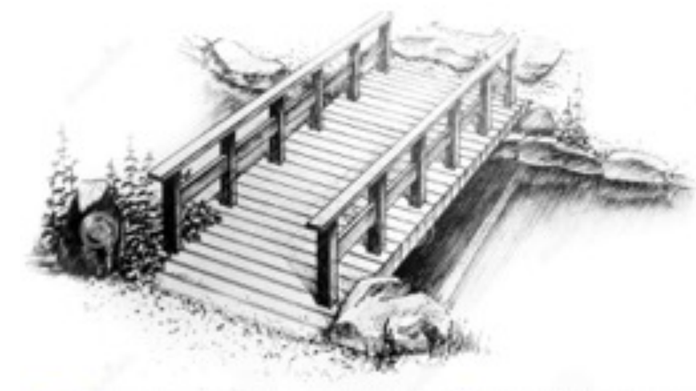
Quantum Gravity Conjectures

Motivated by observing recurrent features of the string landscape and "model building failures", as well as black hole physics

- Absence of global symmetries [Banks-Dixon'88] [Abbott,Wise,Coleman,Lee…] [Horowitz,Strominger,Seiberg…]
- Weak Gravity Conjecture [Arkani-Hamed et al.'06]
- Swampland Distance Conjecture [Ooguri-Vafa'06]

…

Formal ST



StringPheno

They can have significant implications in low energy physics!

1) Weak Gravity Conjecture

Weak Gravity Conjecture

[Arkani-Hamed et al.'06]

Given an abelian gauge field, there must exist an electrically charged particle with

$$m \leq Q$$

(mass) $\leq$ (charge)

so gravity acts weaker than the gauge force.

Original motivation:

in order to allow extremal black holes to decay.

(see also [Aalsma,van de Schaar'18])

Weak Gravity Conjecture

[Arkani-Hamed et al.'06]

Given an abelian gauge field, there must exist an electrically charged particle with

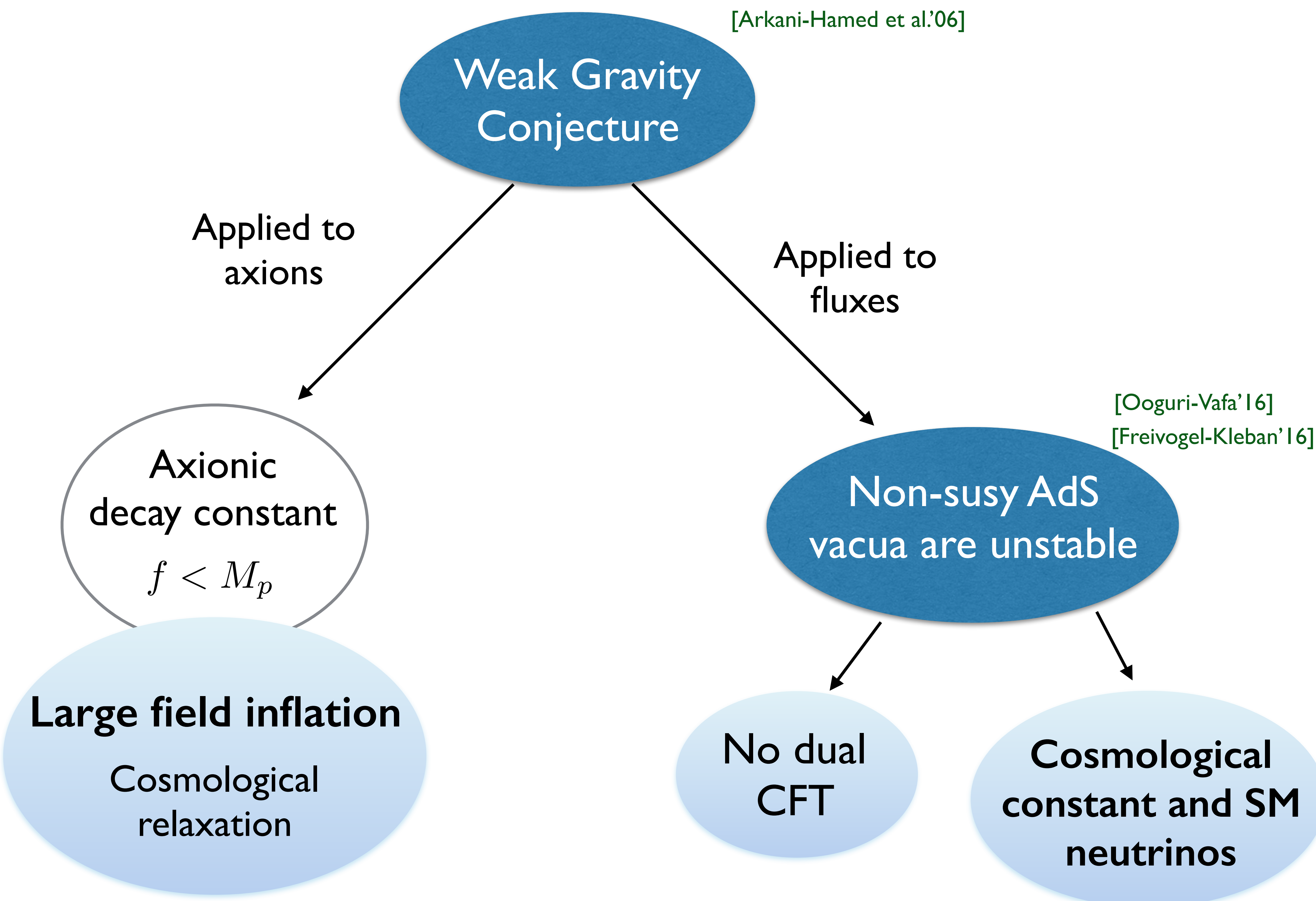
$$m \leq Q$$

(mass) $\leq$ (charge)

so gravity acts weaker than the gauge force.

Evidence:

- Plethora of examples in string theory (not known counter-example)
- Relation to modular invariance of the 2d CFT [Heidenreich et al'16] [Montero et al'16]
- Relation to entropy bounds [Cottrell et al'16] [Fisher et al'17] [Cheung et al'18]
- Relation to cosmic censorship [Crisford et al'17]


[Arkani-Hamed et al.'06]
Weak Gravity Conjecture
Applied to axions
Applied to fluxes
Axionic decay constant
$f < M_p$
Large field inflation
Cosmological relaxation
[Ooguri-Vafa'16]
[Freivogel-Kleban'16]
Non-susy AdS vacua are unstable
No dual CFT
Cosmological constant and SM neutrinos

Weak Gravity Conjecture for fluxes

Given a p-form gauge field, there must exist an electrically charged state with

$$T \leq Q$$

T : tension (mass)

Q : charge

[Arkani-Hamed et al.'06]

Sharpened WGC: Bound is saturated only for a BPS state in a SUSY theory

[Ooguri-Vafa'17]

Weak Gravity Conjecture for fluxes

Given a p-form gauge field, there must exist an electrically charged state with

$$T \leq Q$$

T : tension (mass)

Q : charge

[Arkani-Hamed et al.'06]

Sharpened WGC: Bound is saturated only for a BPS state in a SUSY theory

[Ooguri-Vafa'17]

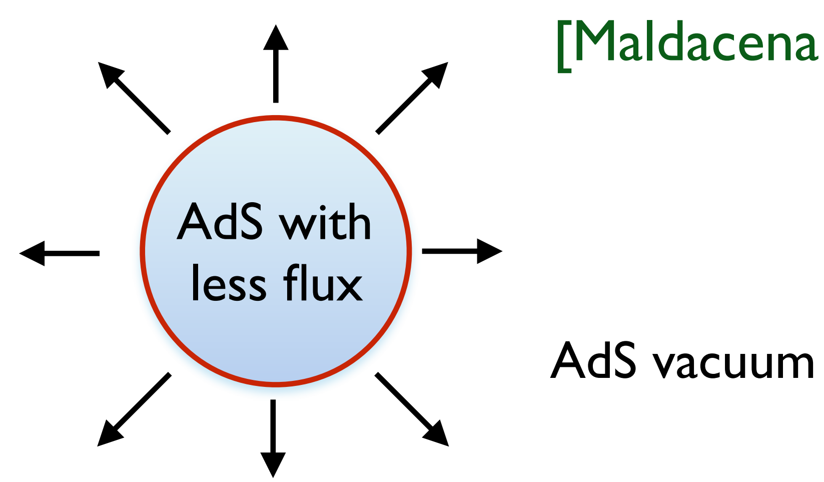
Non-susy vacuum supported by internal fluxes

$$f_0 \sim \int_{\Sigma_p} F_p$$

→ Brane (domain wall) with $T < Q$

Instability of the vacuum!

[Maldacena et al.'99]



Weak Gravity Conjecture for fluxes

Given a p-form gauge field, there must exist an electrically charged state with

$$T \leq Q$$

T : tension (mass)

Q : charge

[Arkani-Hamed et al.'06]

Sharpened WGC: Bound is saturated only for a BPS state in a SUSY theory

[Ooguri-Vafa'17]

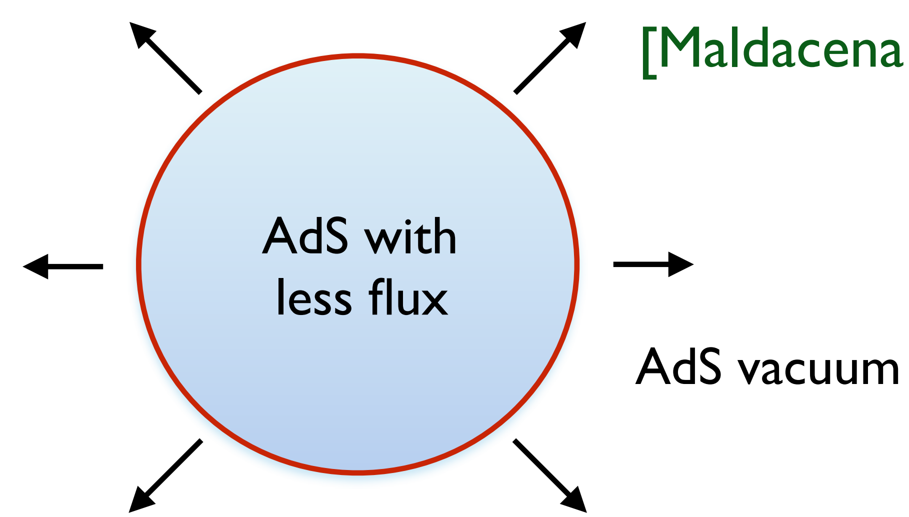
Non-susy vacuum supported by internal fluxes

$$f_0 \sim \int_{\Sigma_p} F_p$$

Brane (domain wall) with $T < Q$

Instability of the vacuum!

[Maldacena et al.'99]



Weak Gravity Conjecture for fluxes

Given a p-form gauge field, there must exist an electrically charged state with

$$T \leq Q$$

T : tension (mass)

Q : charge

[Arkani-Hamed et al.'06]

Sharpened WGC: Bound is saturated only for a BPS state in a SUSY theory

[Ooguri-Vafa'17]

Non-susy vacuum supported by internal fluxes

$$f_0 \sim \int_{\Sigma_p} F_p$$

→ Brane (domain wall) with $T < Q$

Instability of the vacuum!

[Maldacena et al.'99]

Non-susy AdS vacua (supported by fluxes) are unstable

Non-susy AdS vacua are at best metastable

[Ooguri-Vafa'16]
[Freivogel-Kleban'16]

Non-susy stable AdS vacua are in the Swampland!

Implications:

→ **AdS/CFT:** Unstable AdS vacua have no dual CFT

Non-susy CFT cannot have a gravity dual which is Einstein gravity AdS

→ **Low energy physics?**

Compactification of the SM to 3d

Standard Model + Gravity on S^1: [Arkani-Hamed et al.'07] (also [Arnold-Fornal-Wise'10])

$$V(R) = \frac{2\pi\Lambda_4}{R^2} + \text{Casimir energy}$$

$\frac{2\pi\Lambda_4}{R^2}$ → tree-level

Casimir energy → one-loop corrections → exponentially suppressed for $m \gg 1/R$

Depending on the light mass spectra and the cosmological constant, we can get AdS, Minkowski or dS vacua in 3d

We should not get stable non-susy AdS vacua from compactifying the SM !!!
(background independence)

Compactification of the SM to 3d

Standard Model + Gravity on S^1: [Arkani-Hamed et al.'07] (also [Arnold-Fornal-Wise'10])

$$V(R) = \frac{2\pi\Lambda_4}{R^2}$$ + Casimir energy

↓ tree-level

↓ one-loop corrections → exponentially suppressed for $m \gg 1/R$

[Ibanez,Martin-Lozano,IV'17]

We impose the absence of non-susy stable 3d AdS vacua → Constraints on light spectra of SM

Assumption: 4d instabilities are not transferred to 3d

$R_{\text{buble}} > l_{AdS_3}$ (large bubbles)

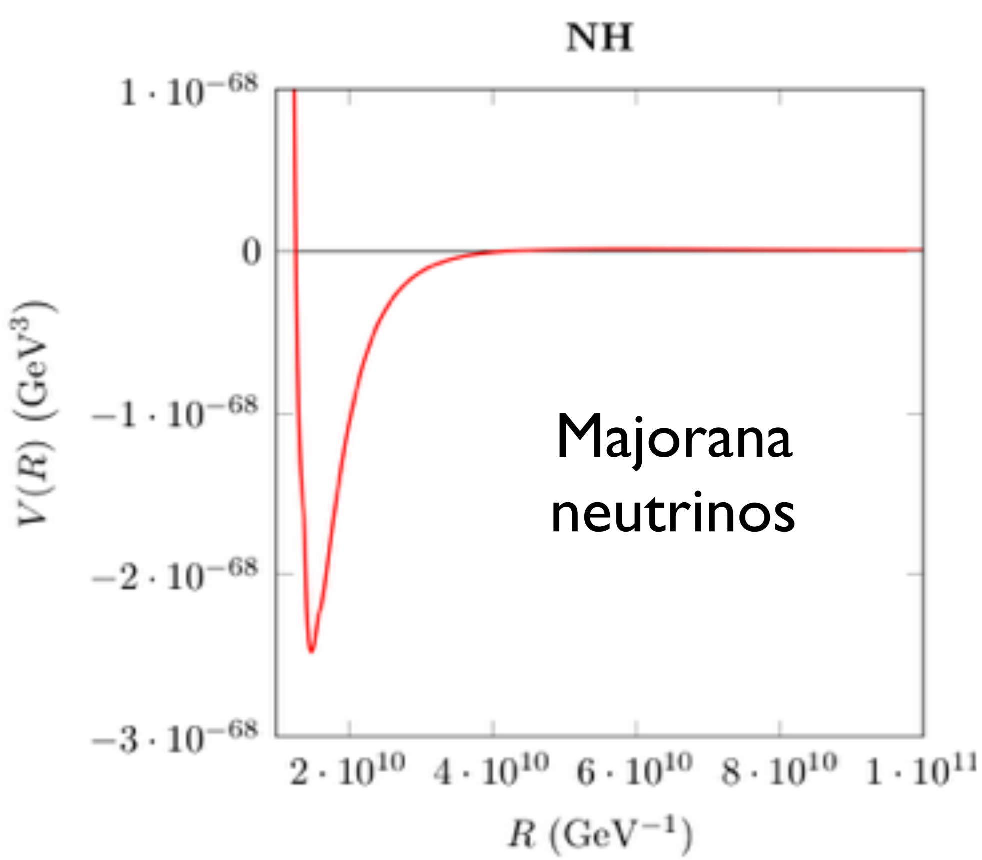
Compactification of the SM to 3d

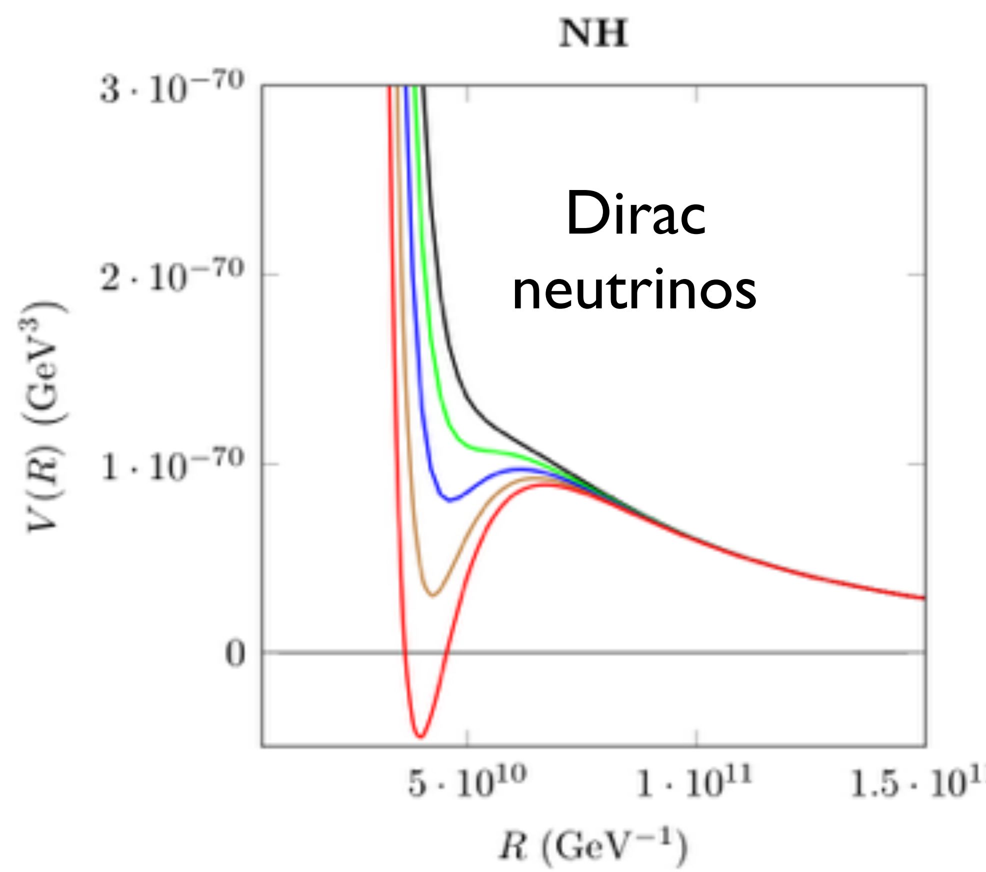
Standard Model + Gravity on S^1:

graviton, photon

massive particles: neutrinos,…

$$V(R) = \frac{2\pi\Lambda_4}{R^2} - \frac{4}{720\pi R^6} + \sum_i \frac{(2\pi R)}{R^3}(-1)^{s_i} n_i \rho_i(R)$$





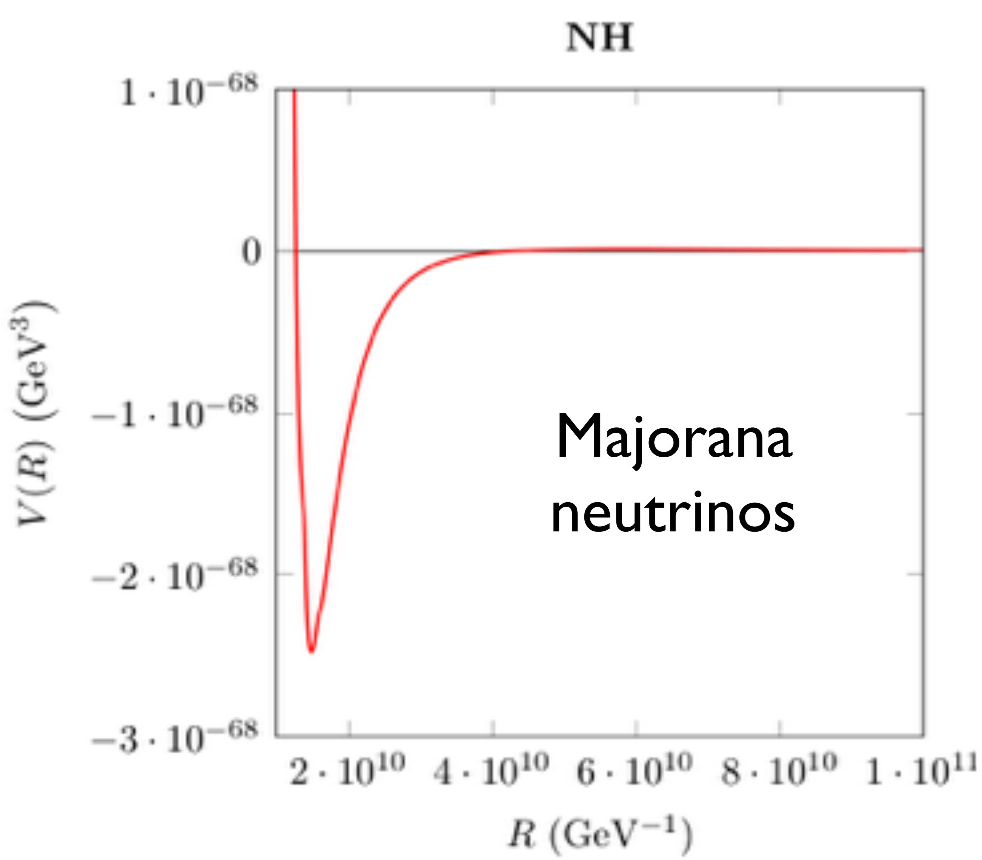
The more massive the neutrinos, the deeper the AdS vacuum

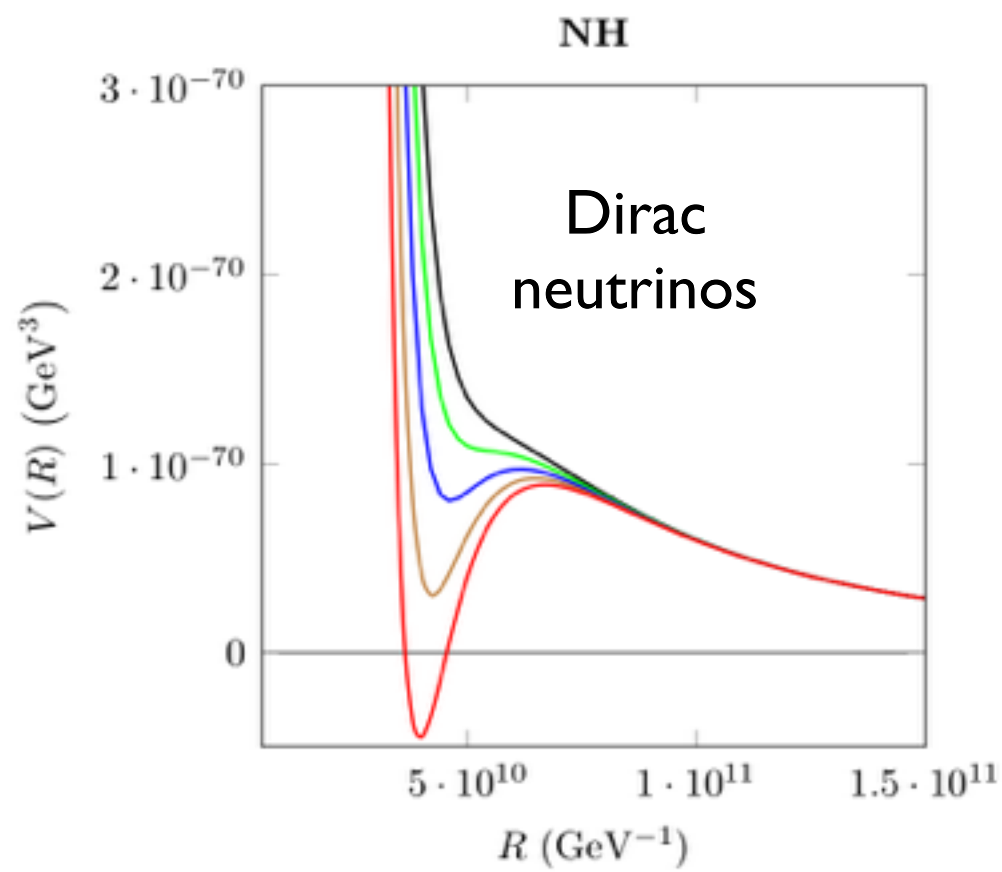
[Ibanez,Martin-Lozano,IV'17] (see also [Hamada-Shiu'17])

Compactification of the SM to 3d

Standard Model + Gravity on S^1:

Absence of AdS vacua implies:





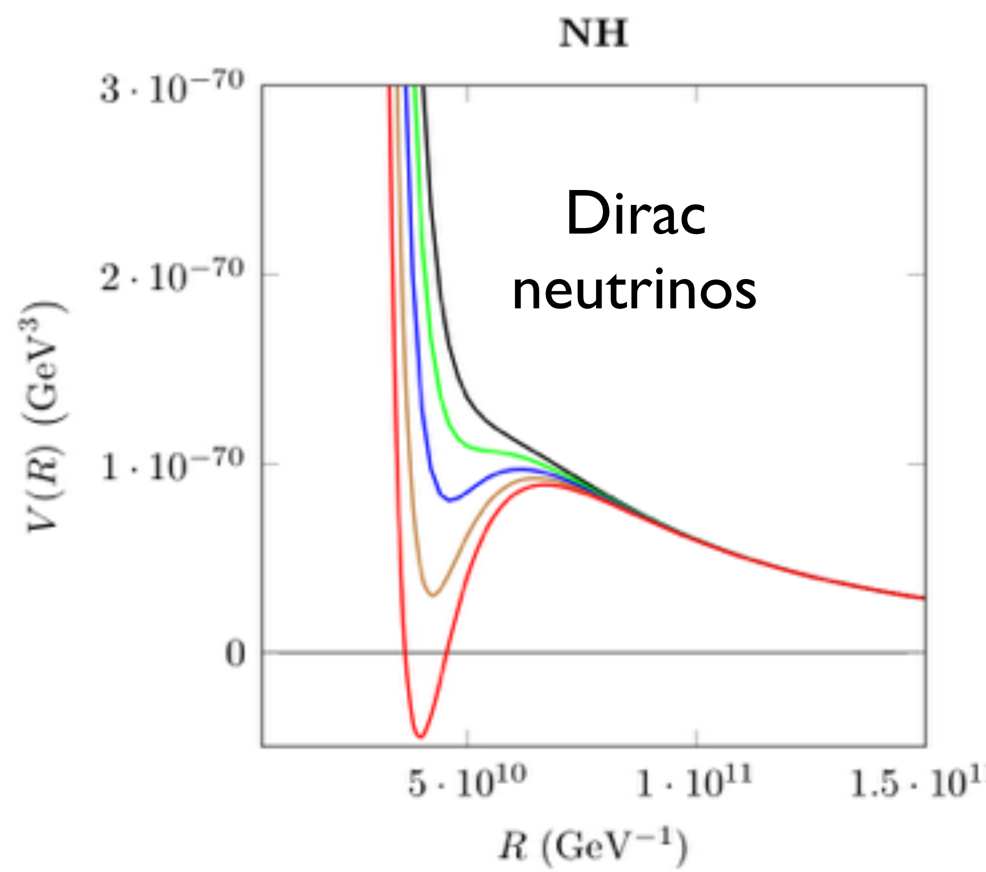
The more massive the neutrinos, the deeper the AdS vacuum

Compactification of the SM to 3d

Standard Model + Gravity on S^1:

Absence of AdS vacua implies:

Majorana neutrinos ruled out!



The more massive the neutrinos, the deeper the AdS vacuum

Compactification of the SM to 3d

Standard Model + Gravity on S^1:

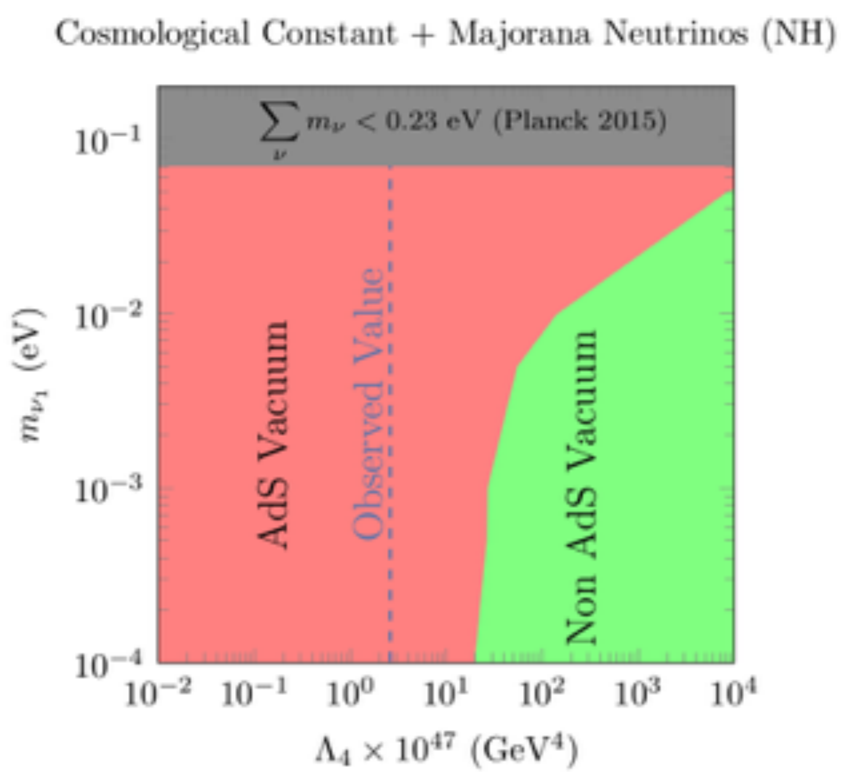
Absence of AdS vacua implies:

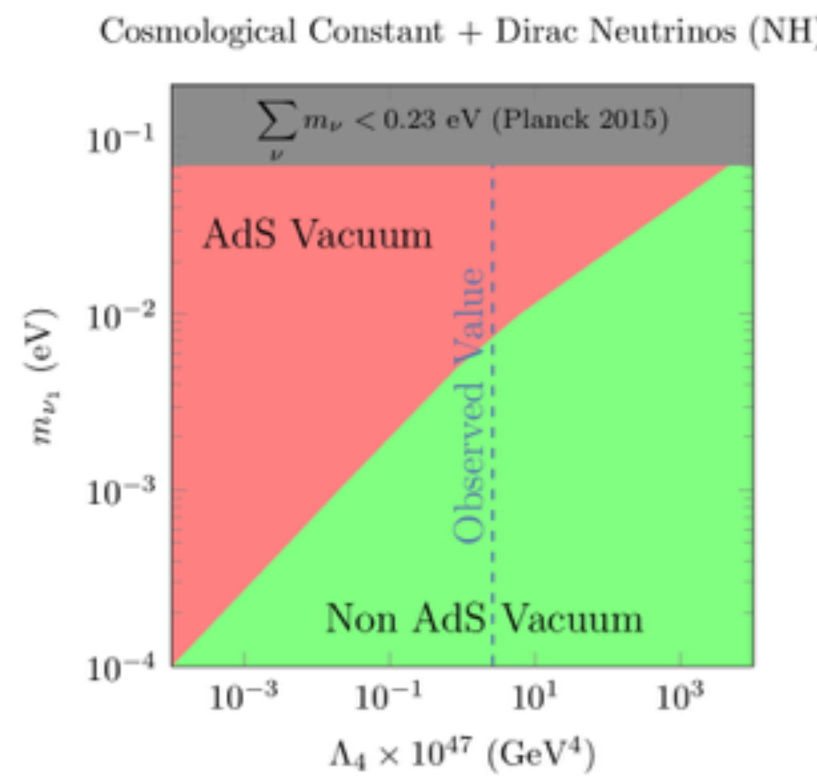
Majorana neutrinos ruled out!

Upper bound for Dirac mass!

$$m_{\nu_1} < 7.7 \text{ meV (NH)}$$
$$m_{\nu_1} < 2.1 \text{ meV (IH)}$$

Lower bound on the cosmological constant





The bound for Λ_4 scales as m_ν^4 (as observed experimentally)

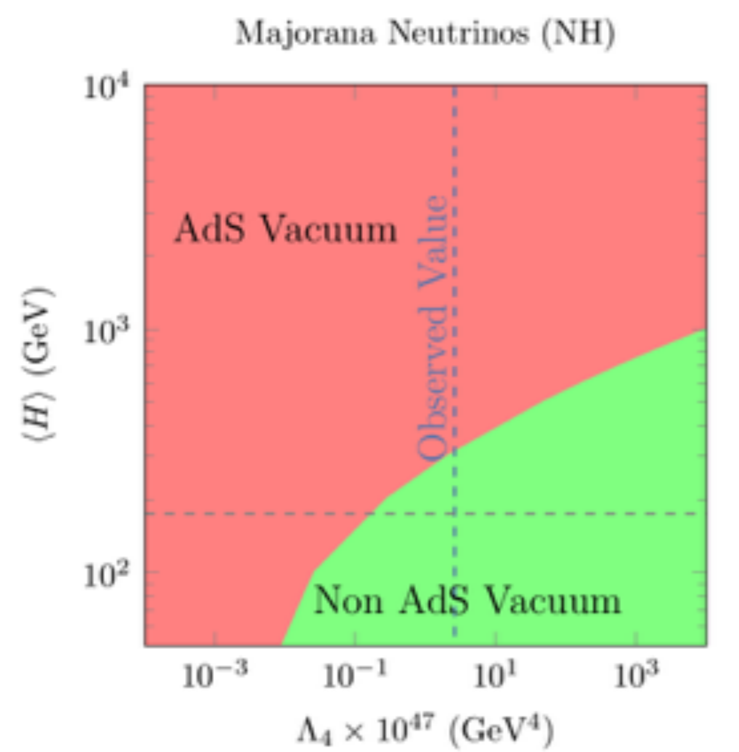
$$\Lambda_4 \geq \frac{a(n_f)30(\Sigma m_i^2)^2 - b(n_f, m_i)\Sigma m_i^4}{384\pi^2}$$

with $a(n_f) = 0.184(0.009)$, $b(n_f, m_i) = 5.72(0.29)$ for Majorana (Dirac)

First argument (not based on cosmology) to have $\Lambda_4 \neq 0$

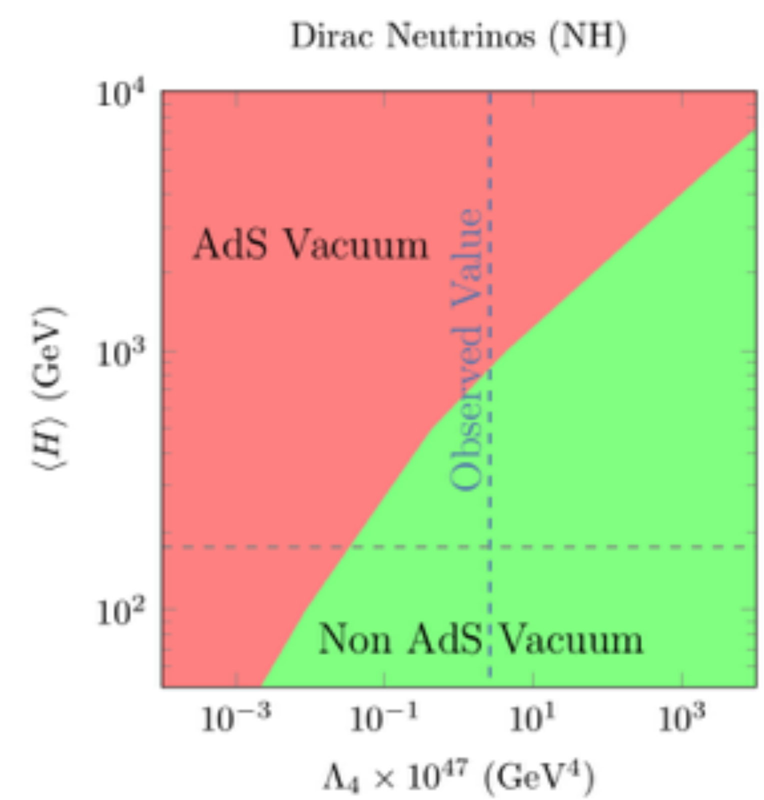
Upper bound on the EW scale

Majorana case: $\langle H \rangle \lesssim \frac{\sqrt{2}}{Y_{\nu_1}} \sqrt{M \Lambda^{1/4}}$



$M = 10^{10}$ GeV, $Y = 10^{-3}$

Dirac case: $\langle H \rangle \lesssim 1.6 \frac{\Lambda^{1/4}}{Y_{\nu_1}}$



$Y = 10^{-14}$

Parameters leading to a higher EW scale do not yield theories consistent with quantum gravity → No EW hierarchy problem

2) Swampland Distance Conjecture

Swampland Distance Conjecture [Ooguri-Vafa'06]

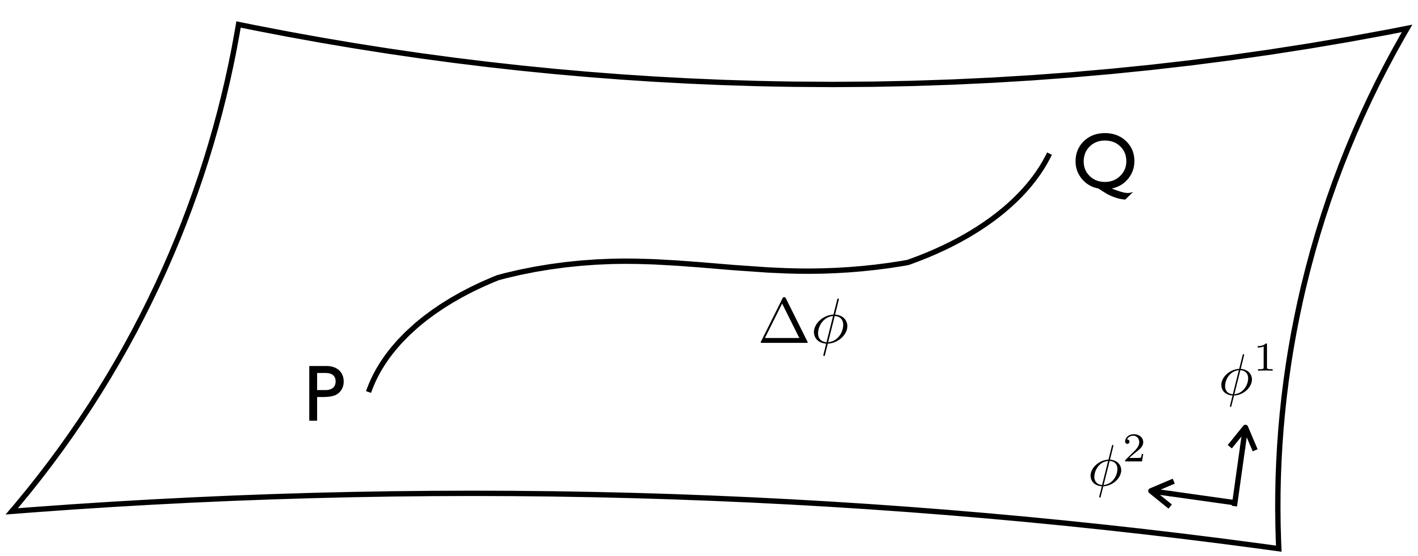
An effective theory is valid only for a finite scalar field variation $\Delta\phi$

because an infinite tower of states become exponentially light

$$m \sim m_0 e^{-\lambda \Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty$$

Consider the moduli space of an effective theory:

$\mathcal{L} = g_{ij}(\phi)\partial\phi^i \partial\phi^j$ → scalar manifold



$\Delta\phi =$ geodesic distance between P and Q

$$m(P) \lesssim m(Q) e^{-\lambda \Delta\phi}$$

Swampland Distance Conjecture [Ooguri-Vafa'06]

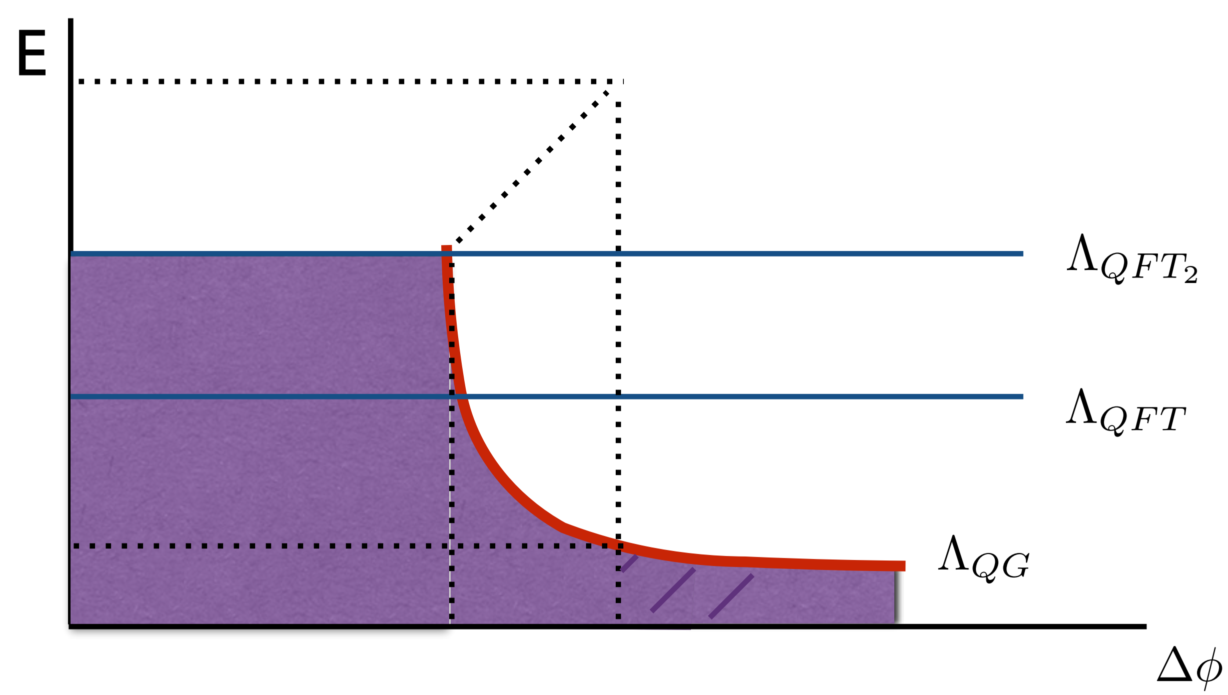
An effective theory is valid only for a finite scalar field variation $\Delta\phi$

because an infinite tower of states become exponentially light

$$m \sim m_0 e^{-\lambda\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty$$

This signals the breakdown of the effective theory:

$$\Lambda_{\text{cut-off}} \sim \Lambda_0 \exp(-\lambda\Delta\phi)$$



Swampland Distance Conjecture [Ooguri-Vafa’06]

Potential implications for inflation!

Large field inflation is at the edge of validity (large field range and high energy)

- Also applies to axions of Type II flux compactifications realising axion monodromy (upon taking into account back-reaction on kinetic term) [Baume,Palti'16] [I.V.,'16]

- Examples compatible with the Refined SDC: [Klaewer,Palti'16]

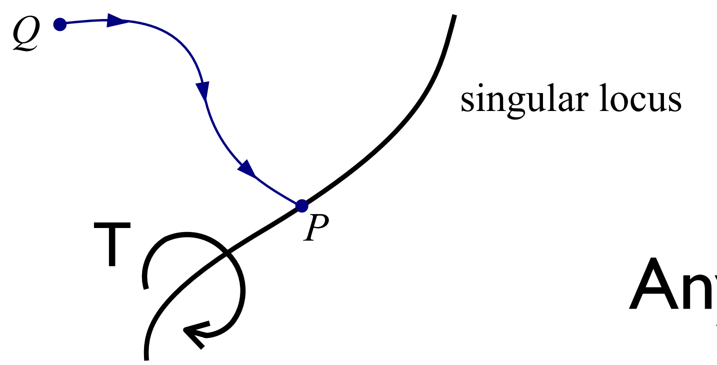
  → exponential drop-off at the Planck scale $\Delta\phi \lesssim M_p$

Evidence: based on particular examples in string theory compactifications

[Ooguri,Vafa’06] [Baume,Palti'16] [I.V.,’16] [Bielleman,Ibanez,Pedro,I.V.,Wieck’16] [Blumenhagen,I.V.,Wolf’17] [Hebecker,Henkenjohann,Witkowski’17] [Cicoli,Ciupke,Mayhrofer,Shukla’18][Blumenhagen et al.’18]

Systematic analysis in the complex structure moduli space of Type IIB Calabi-Yau string compactifications

[Grimm,Palti,IV.'18]



Infinite distance locus:

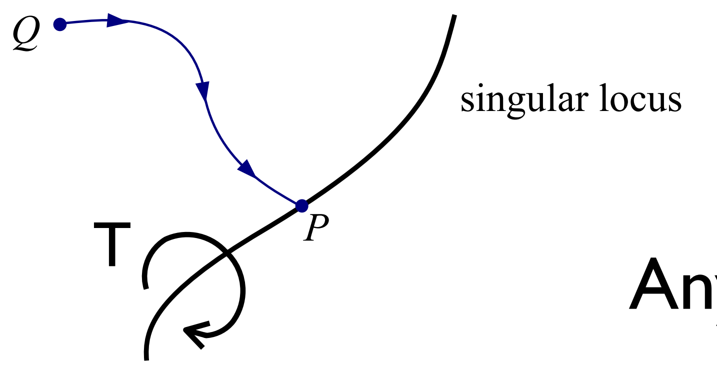
Any trajectory approaching P has infinite length

Infinite tower of states: BPS D3 branes

➡ The mass decreases exponentially fast in the field distance
(due to the universal behaviour of the metric near these points)

Systematic analysis in the complex structure moduli space of Type IIB Calabi-Yau string compactifications

[Grimm,Palti,IV.'18]



Infinite distance locus:

Any trajectory approaching P has infinite length

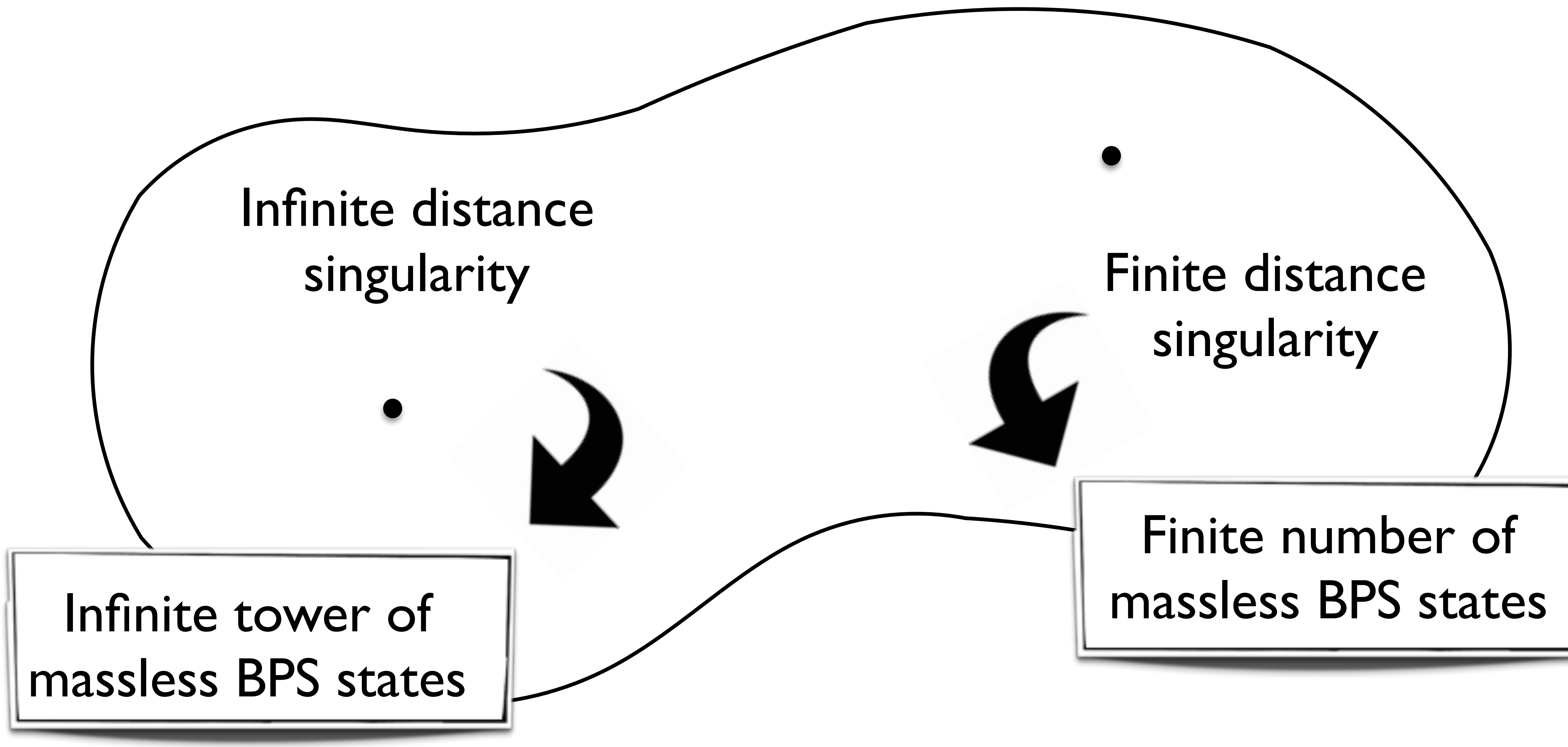
Infinite tower of states: BPS D3 branes

→ The mass decreases exponentially fast in the field distance (due to the universal behaviour of the metric near these points)

- SDC as a quantum gravity obstruction to restore a global axionic symmetry at the singular point
- Infinite field distance is emergent from integrating out the infinite tower of states (see also [Heidenreich,Reece,Rudelius'18])

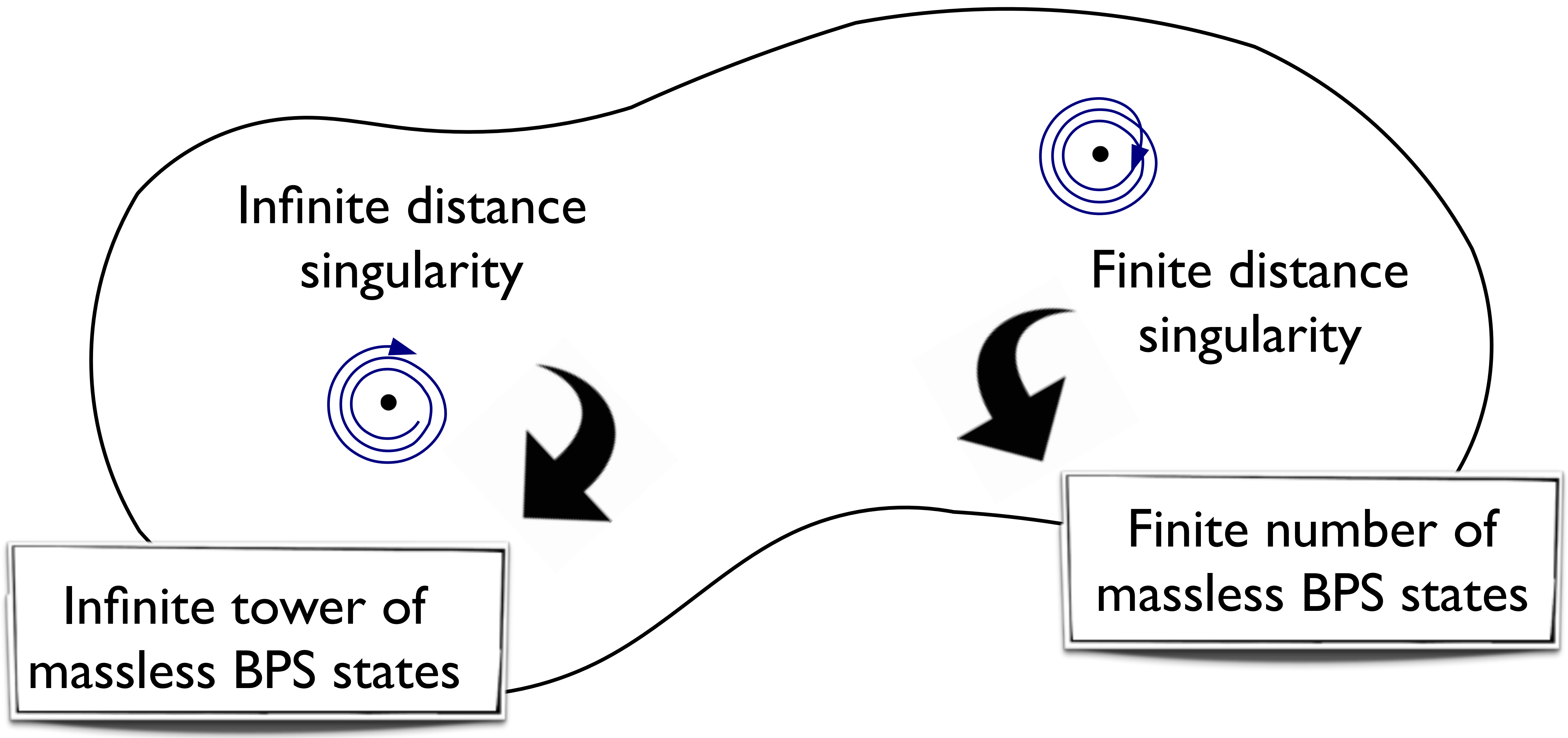
Systematic analysis in the complex structure moduli space of Type IIB Calabi-Yau string compactifications

[Grimm,Palti,IV.'18]



Systematic analysis in the complex structure moduli space of Type IIB Calabi-Yau string compactifications

[Grimm,Palti,IV.'18]



Key ingredient:

Monodromy transformation → Field distance

→ Monodromy orbit of states

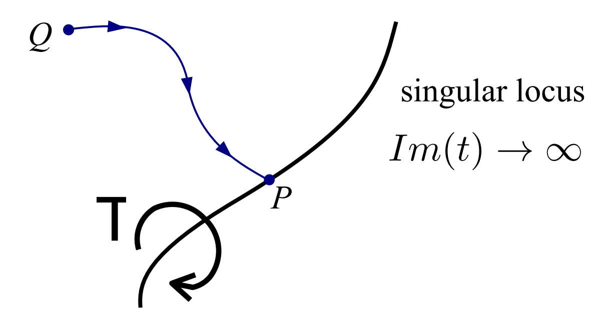
Nilpotent orbit theorem

Distances given by: $d_\gamma(P,Q) = \int_\gamma \sqrt{g_{IJ}\dot{x}^I \dot{x}^J} ds$

$$g_{I\bar{J}} = \partial_{z^I}\partial_{\bar{z}^J} K$$

$$K = -\log\left(-i^D \int_{Y_D} \Omega \wedge \bar{\Omega}\right)$$

Periods of the (D,0)-form: $\Pi^{\mathcal{I}} = \int_{\Gamma_{\mathcal{I}}} \Omega$



transform under monodromy $\Pi(e^{2\pi i z}) = T \cdot \Pi(z)$

(remnant of higher dimensional gauge symmetries)

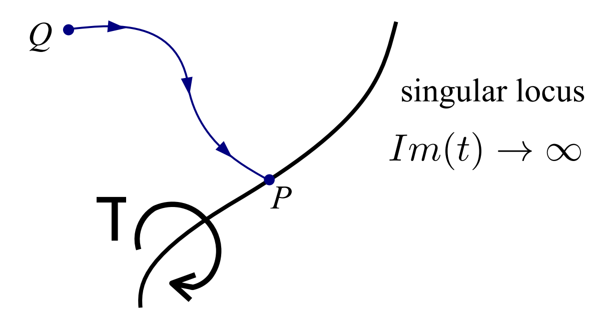
Nilpotent orbit theorem

Distances given by: $d_\gamma(P,Q) = \int_\gamma \sqrt{g_{IJ}\dot{x}^I\dot{x}^J}ds$

$$g_{I\bar{J}} = \partial_{z^I}\partial_{\bar{z}^J}K$$

$$K = -\log\left(-i^D \int_{Y_D} \Omega \wedge \bar{\Omega}\right)$$

Periods of the (D,0)-form: $\Pi^{\mathcal{I}} = \int_{\Gamma_{\mathcal{I}}} \Omega$



transform under monodromy $\Pi(e^{2\pi i z}) = T \cdot \Pi(z)$

(remnant of higher dimensional gauge symmetries)

Nilpotent orbit theorem: [Schmid'73]

$$\Pi(t,\eta) = \exp(tN)\,a_0(\eta) + \mathcal{O}(e^{2\pi i t}, \eta)$$

$$t = \frac{1}{2\pi i}\log z$$

$N = \log T \longrightarrow$ Nilpotent matrix

It gives local expression for the periods near singular locus!

Infinite distances - Infinite towers

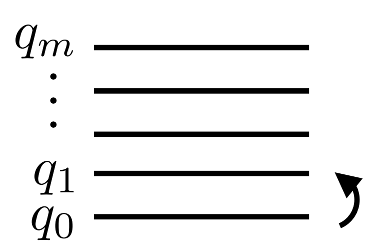
1) Infinite distances only if monodromy is of infinite order

Theorem: P is at infinite distance ⟷ $Na_0 \neq 0$

[Wang'97, Lee'16]

2) Monodromy can be used to populate an infinite orbit of BPS states

Mass given by central charge: $Z = e^K q \cdot \Pi$ $\qquad q = (q_e^I, q_I^m)$



$$q_m = T^m q \qquad m \in \mathbb{Z}$$

3) Universal local form of the metric gives the exponential mass behaviour

$$\frac{M_q(P)}{M_q(Q)} \simeq \exp\left(-\frac{1}{\sqrt{2d}}\, d_\gamma(P,Q)\right)$$

Infinite distances - Infinite towers

Infinite massless monodromy orbit at the singularity → Infinite tower of states becoming exponentially light

Massless: $q^T N^j a_0 = 0\,, \quad j \geq d/2$

Infinite orbit: $Nq \neq 0$

Swampland Distance Conjecture ✓

Tool: mathematical machinery of mixed hodge structure

(finer split of cohomology at the singularity adapted to N)

[Deligne][Schmid][Cattani,Kaplan,Schmid]
[Kerr,Pearlstein,Robles'17]

Emergence from integrating out the states

Famous story: periods near conifold have log-divergence from integrating out a single BPS D3-state

[Strominger'95]

We perform similar analysis at infinite distance singularities:

One-loop corrections from integrating out the tower of BPS states

→ matches geometric result

Emergence from integrating out the states

Famous story: periods near conifold have log-divergence from integrating out a single BPS D3-state

[Strominger'95]

We perform similar analysis at infinite distance singularities:

One-loop corrections from integrating out the tower of BPS states

→ matches geometric result

- Corrections to the field metric:

$$d(\phi_1,\phi_2) \simeq C\int_{\phi_1}^{\phi_2}\sqrt{\sum_{i=1}^{S}(\partial_\phi m_i)^2 d\phi} \simeq C\int_{\phi_1}^{\phi_2}\frac{d}{\sqrt{12c}}\frac{1}{\phi}d\phi = C\frac{d}{\sqrt{12c}}\log\left(\frac{\phi_2}{\phi_1}\right)$$

Emergence from integrating out the states

Famous story: periods near conifold have log-divergence from integrating out a single BPS D3-state

[Strominger'95]

We perform similar analysis at infinite distance singularities:

One-loop corrections from integrating out the tower of BPS states

→ matches geometric result

- Corrections to the field metric:

$$d(\phi_1, \phi_2) \simeq C \int_{\phi_1}^{\phi_2} \sqrt{\sum_{i=1}^{S} (\partial_\phi m_i)^2 d\phi} \simeq C \int_{\phi_1}^{\phi_2} \frac{d}{\sqrt{12c}} \frac{1}{\phi} d\phi = C \frac{d}{\sqrt{12c}} \log\left(\frac{\phi_2}{\phi_1}\right)$$

- Corrections to the gauge kinetic function:

$$\mathrm{Im}\, \mathcal{N}_{IJ}^{IR} \simeq \mathrm{Im}\, \mathcal{N}_{IJ}^{UV} - \sum_{k}^{S} \left(\frac{8\, q_{k,I} q_{k,J}}{3\pi^2} \log \frac{\Lambda_{UV}}{m_k} \right) \quad \rightarrow \quad g_{YM}^2 \sim \phi^{-n} \sim m_0^{2n}$$

(unlike conifold $g_{YM}^2 \sim 1/\log(m_0)$)

Emergence from integrating out the states

Famous story: periods near conifold have log-divergence from integrating out a single BPS D3-state

[Strominger'95]

We perform similar analysis at infinite distance singularities:

One-loop corrections from integrating out the tower of BPS states

→ matches geometric result

- Corrections to the field metric:

$$d(\phi_1,\phi_2) \simeq C\int_{\phi_1}^{\phi_2}\sqrt{\sum_{i=1}^{S}(\partial_\phi m_i)^2 d\phi} \simeq C\int_{\phi_1}^{\phi_2}\frac{d}{\sqrt{12c}}\frac{1}{\phi}d\phi = C\frac{d}{\sqrt{12c}}\log\left(\frac{\phi_2}{\phi_1}\right)$$

- Corrections to the gauge kinetic function:

$$\mathrm{Im}\,\mathcal{N}_{IJ}^{IR} \simeq \mathrm{Im}\,\mathcal{N}_{IJ}^{UV} - \sum_{k}^{S}\left(\frac{8\,q_{k,I}q_{k,J}}{3\pi^2}\log\frac{\Lambda_{UV}}{m_k}\right) \quad \rightarrow \quad g_{YM}^2 \sim \phi^{-n} \sim m_0^{2n}$$

(unlike conifold $g_{YM}^2 \sim 1/\log(m_0)$)

Infinite distance and weak coupling emerge from integrating out an infinite tower of states!

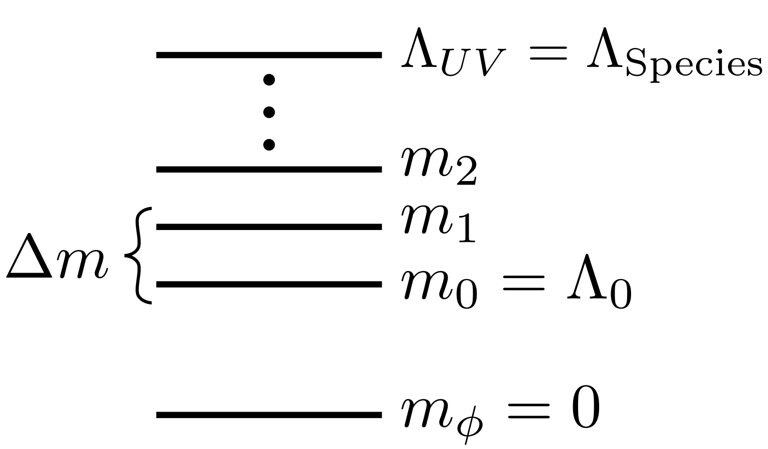
Emergence from integrating out the states

Famous story: periods near conifold have log-divergence from integrating out a single BPS D3-state

[Strominger'95]

We perform similar analysis at infinite distance singularities:

> One-loop corrections from integrating out the tower of BPS states
>
> → matches geometric result



$$\left.\begin{aligned} \Lambda_{UV} &= \frac{M_p}{\sqrt{S}} \\ S &= \frac{\Lambda_{UV}}{\Delta m(\phi)} \end{aligned}\right\} \Rightarrow \Lambda_{UV}(\phi) \sim \Delta m(\phi)^{1/3}$$

Field dependent UV cut-off!

UV cut-off decreases exponentially fast in the proper field distance

Summary

Consistency with quantum gravity implies constraints on low energy physics:

1) AdS Instability Conjecture + stability of 3D SM vacua:

Lower bound on the cosmological const. of order the neutrino masses	⟷	Upper bound on the EW scale in terms of the cosmological const.

New approach to fine-tuning or hierarchy problems?
UV/IR mixing? (see also [Luest-Palti'17])

➡ Generalizations:

- BSM extensions: New light particles, supersymmetry… [Ibanez,Martin-Lozano,IV'17] [Gonzalo,Herraez,Ibanez'18]
- 2d compactifications: Toroidal, orbifolds… [Ibanez,Martin-Lozano,IV'17] [Gonzalo,Herraez,Ibanez'18]

Summary

Consistency with quantum gravity implies constraints on low energy physics:

2) Swampland Distance Conjecture:

Upper bound on the scalar field range: Implications for inflation!

✓ Test in the complex structure moduli space of CY IIB compactifications

- Infinite order monodromy as generator of the infinite tower
- Emergence of infinite field distance

➡ Generalizations:

- Our results are valid for any CY (model-independent)

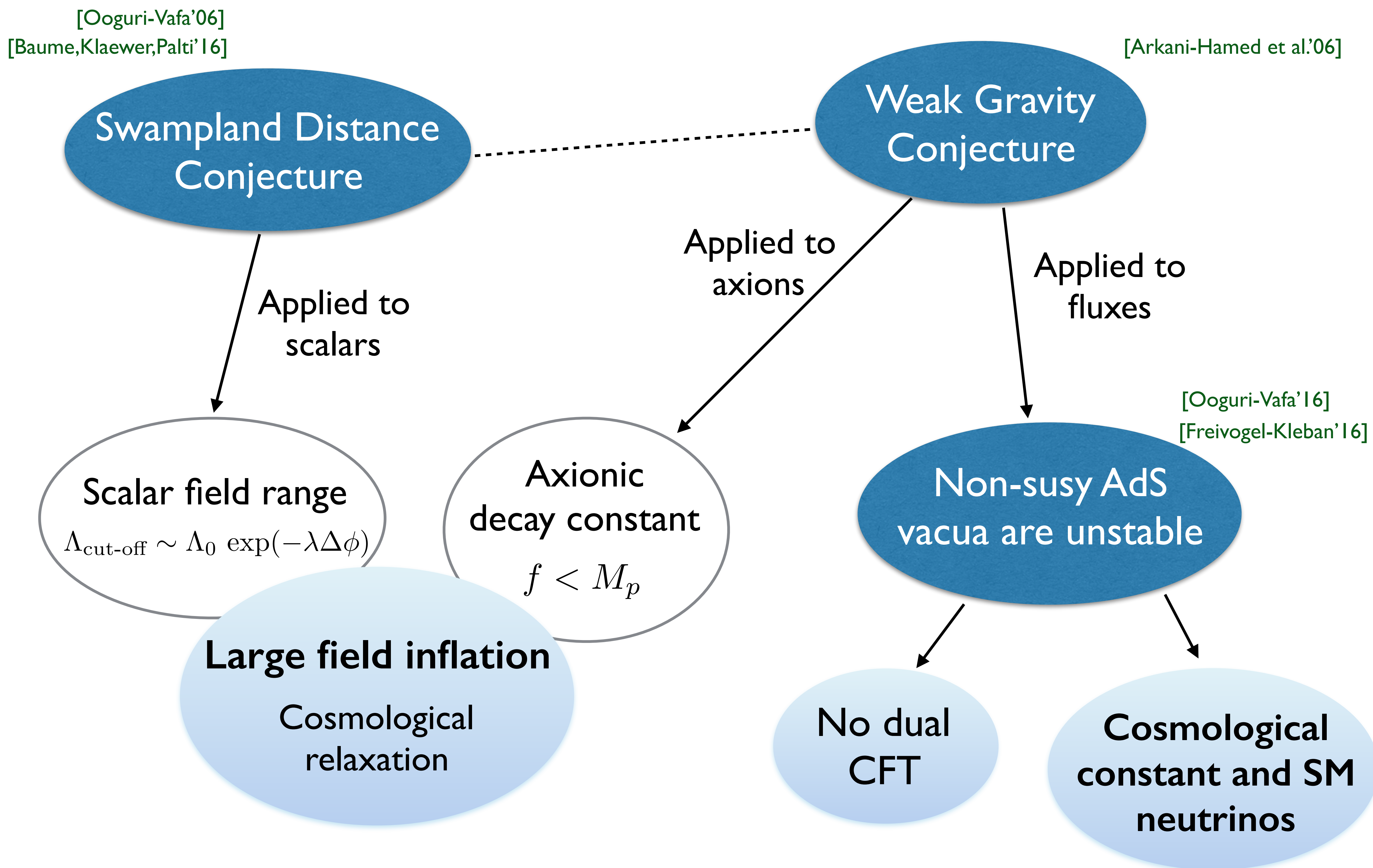
  (but only for infinite distance points that belong to a single singular divisor)

- Other moduli spaces?

Thank you!

back-up slides

Quantum Gravity Conjectures



Casimir energy

Potential energy in 3d:

$$V(R) = \frac{2\pi r^3 \Lambda_4}{R^2} + \sum_i (2\pi R) \frac{r^3}{R^3} (-1)^{s_i} n_i \rho_i(R)$$

Casimir energy density:

$$\rho(R) = \mp \sum_{n=1}^{\infty} \frac{2m^4}{(2\pi)^2} \frac{K_2(2\pi R m n)}{(2\pi R m n)^2}$$

For small mR:

$$\rho(R) = \mp \left[\frac{\pi^2}{90(2\pi R)^4} - \frac{\pi^2}{6(2\pi R)^4} (mR)^2 + \frac{\pi^2}{48(2\pi R)^4} (mR)^4 + \mathcal{O}(mR)^6 \right]$$

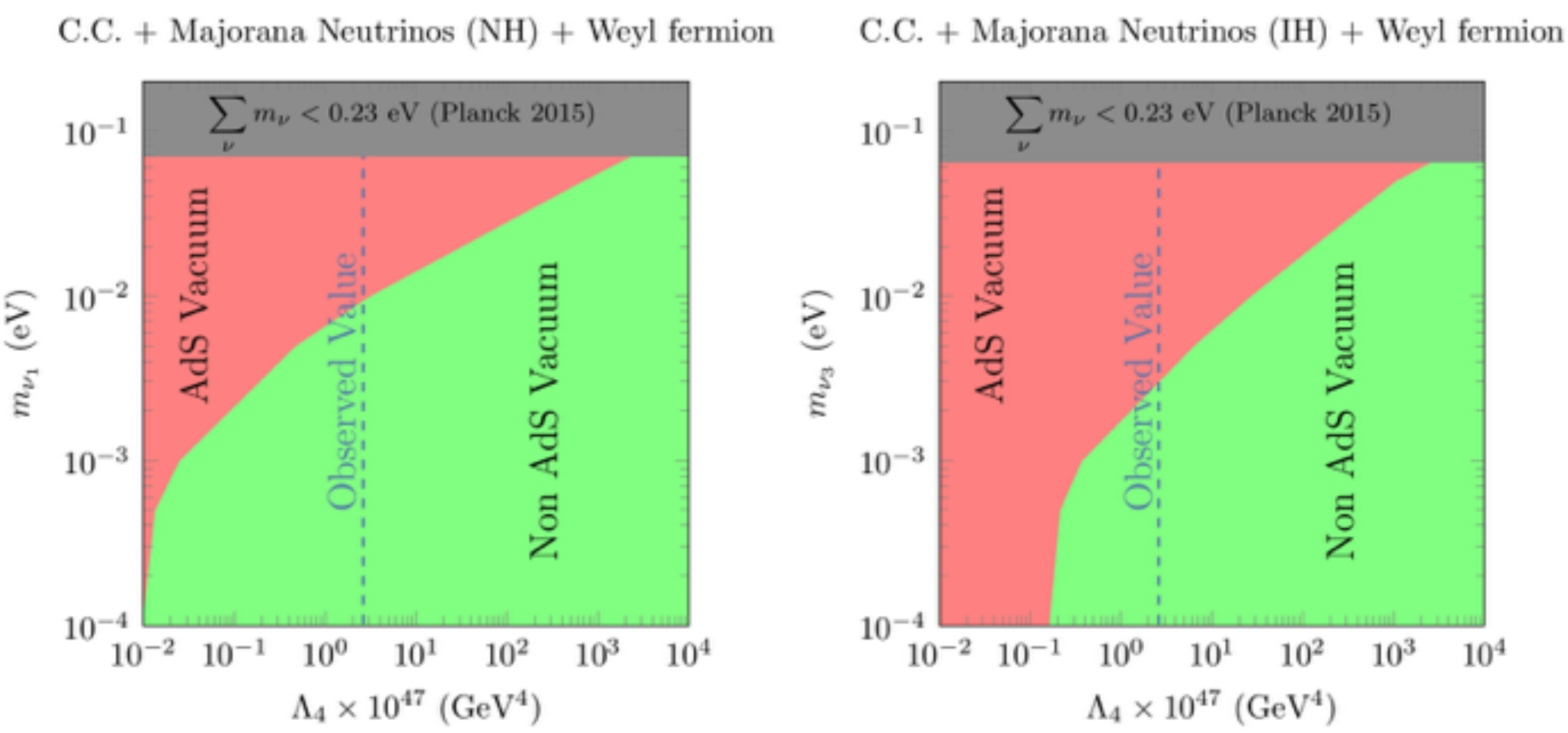
Adding BSM physics

- **Light fermions**

Positive Casimir contribution ⟶ helps to avoid AdS vacuum

Majorana neutrinos are consistent if adding $m_\chi \lesssim 2\ \text{meV}$

example. For $m_\chi = 0.1\ \text{meV}$:

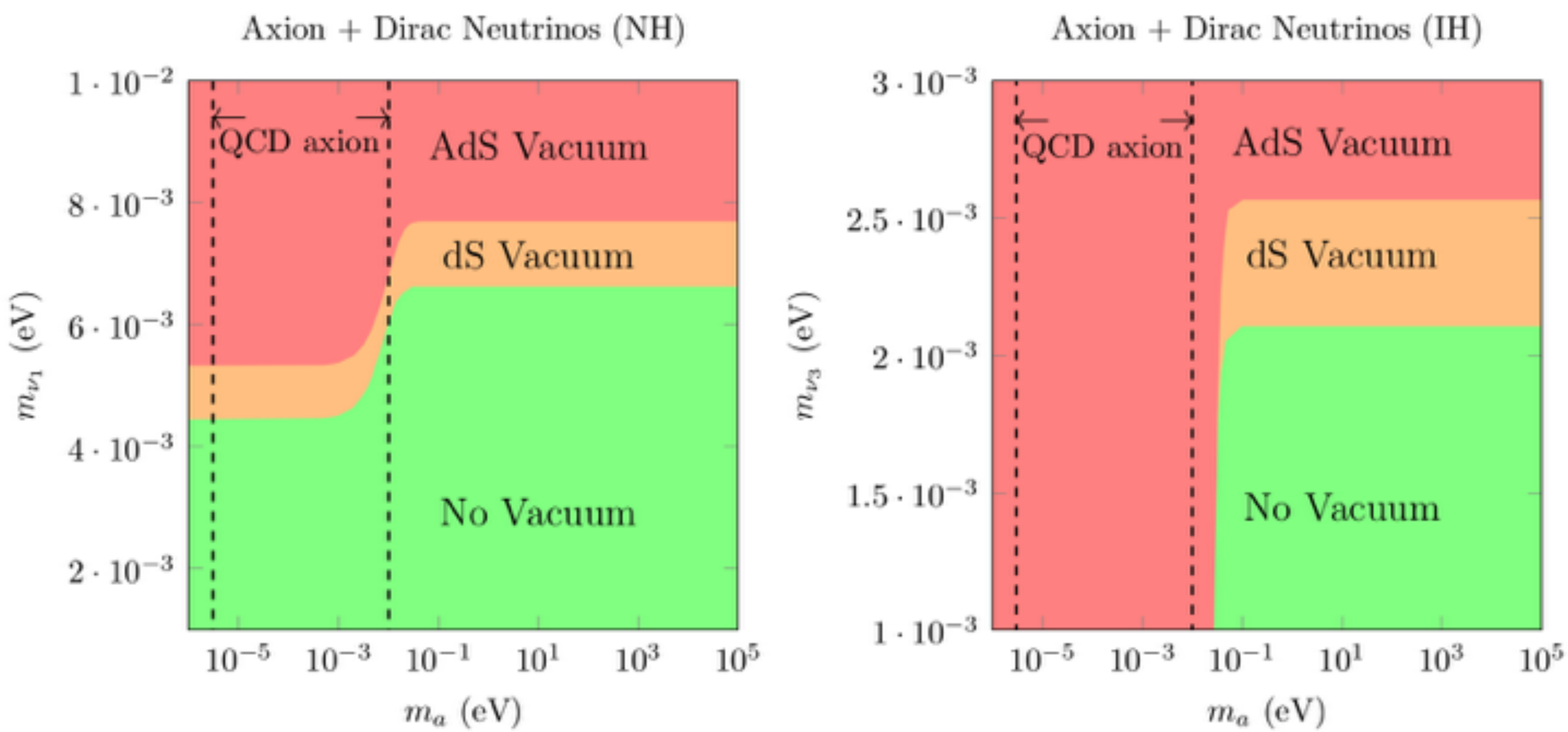


Adding BSM physics

- **Axions**

  1 axion: negative contribution ⟶ bounds get stronger

  Multiple axions: can destabilise AdS vacuum



Bounds on the SM + light BSM physics

Model	Majorana (NI)	Majorana (IH)	Dirac (NH)	Dirac (IH)
SM (3D)	no	no	$m_{\nu_1} \leq 7.7 \times 10^{-3}$	$m_{\nu_3} \leq 2.56 \times 10^{-3}$
SM(2D)	no	no	$m_{\nu_1} \leq 4.12 \times 10^{-3}$	$m_{\nu_3} \leq 1.0 \times 10^{-3}$
SM+Weyl(3D)	$m_{\nu_1} \leq 0.9 \times 10^{-2}$ $m_f \leq 1.2 \times 10^{-2}$	$m_{\nu_3} \leq 3 \times 10^{-3}$ $m_f \leq 4 \times 10^{-3}$	$m_{\nu_1} \leq 1.5 \times 10^{-2}$	$m_{\nu_3} \leq 1.2 \times 10^{-2}$
SM+Weyl(2D)	$m_{\nu_1} \leq 0.5 \times 10^{-2}$ $m_f \leq 0.4 \times 10^{-2}$	$m_{\nu_3} \leq 1 \times 10^{-3}$ $m_f \leq 2 \times 10^{-3}$	$m_{\nu_1} \leq 0.9 \times 10^{-2}$	$m_{\nu_3} \leq 0.7 \times 10^{-2}$
SM+Dirac(3D)	$m_f \leq 2 \times 10^{-2}$	$m_f \leq 1 \times 10^{-2}$	yes	yes
SM+Dirac(2D)	$m_f \leq 0.9 \times 10^{-2}$	$m_f \leq 0.9 \times 10^{-2}$	yes	yes
SM+1 axion(3D)	no	no	$m_{\nu_1} \leq 7.7 \times 10^{-3}$	$m_{\nu_3} \leq 2.5 \times 10^{-3}$ $m_a \geq 5 \times 10^{-2}$
SM+1 axion(2D)	no	no	$m_{\nu_1} \leq 4.0 \times 10^{-3}$	$m_{\nu_3} \leq 1 \times 10^{-3}$ $m_a \geq 2 \times 10^{-2}$
$\geq 2(10)$ axions	yes	yes	yes	yes

Compactifications of SM on T_2 $\longrightarrow$ qualitatively similar, but a bit stronger

(see also [Hamada-Shiu'17])

BPS states and stability

Does a BPS state cross a wall of marginal stability upon circling the monodromy locus?

Consider:

$$\mathbf{q}_C = \mathbf{q}_B + \mathbf{q}_{\bar{A}} \quad \Rightarrow \quad M_{\mathbf{q}_C} \leq M_{\mathbf{q}_B} + M_{\mathbf{q}_{\bar{A}}}$$

Wall of marginal stability: $\varphi(B) - \varphi(A) = 1$ with $\varphi(A) = \frac{1}{\pi}\mathrm{Im}\log Z_{\mathbf{q}_A}$

Upon circling the monodromy locus:

$$\varphi_{\mathrm{I}} \to \varphi_{\mathrm{I}} + \mathcal{O}\left(\frac{1}{Im\, t}\right) \,,\quad \varphi_{\mathrm{II}} \to \varphi_{\mathrm{II}} + 2 + \mathcal{O}\left(\frac{1}{Im\, t}\right)$$

Type I state can only decay to I-II or II-II states!

Stable massless states: $\mathcal{M}_Q = \mathcal{M}/\mathcal{M}_{\mathrm{II}}$

Under n monodromy transformations:

$$\varphi_{\mathrm{I}} \to \varphi_{\mathrm{I}} - \frac{n}{\pi \mathrm{Im}\, t} \quad \Rightarrow$$

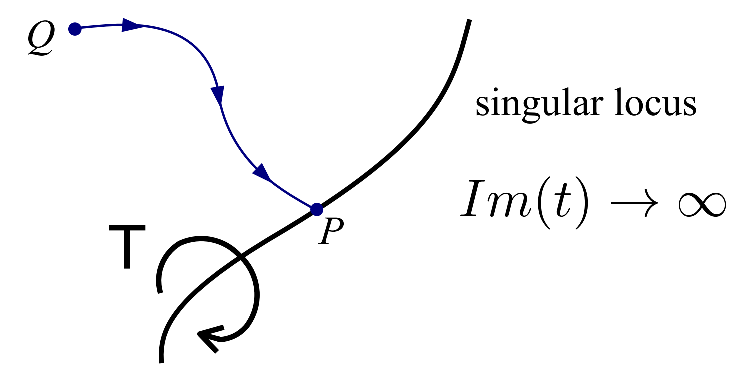
Number of BPS states

$$n \sim Im\,(t)$$

Infinite distances

Nilpotent orbit theorem: [Schmid'73]

$$\Pi(t,\eta) = \exp(tN)\, a_0(\eta) \; + \; \mathcal{O}(e^{2\pi i t}, \eta)$$



$$t = \frac{1}{2\pi i} \log z$$

Nilpotent matrix $N = \log T$ (T of infinite order)

Local form of the metric: $g_{t\bar{t}} = \frac{d}{\mathrm{Im}(t)^2} + \dots$

where *d* is an integer s.t. $N^d a_0 \neq 0 \quad , \quad N^{d+1} a_0 = 0$

Theorem: [Wang'97, Lee'16]

P is at infinite distance ⟷ $N a_0 \neq 0$ (i.e. $d > 0$)

Infinite tower of states

Candidates: BPS wrapping D3-branes

Mass given by central charge: $Z = e^K q \cdot \Pi$ $\qquad q = (q_e^I, q_I^m)$

Massless condition: $q^T N^j a_0 = 0\,, \quad j \geq d/2$ $\longrightarrow$ subtleties regarding stability and counting of BPS states

Monodromy orbit of states:

$$q_m = T^m q \qquad m \in \mathbb{Z}$$

If T is of infinite order $Nq \neq 0$ → Starting wth one state, we generate infinitely many!

Exponential mass behaviour:

$$Z \simeq \frac{\sum_j \frac{1}{j!}(Im\, t)^j q^T N^j a_0}{(2^d/d!)^{1/2}(Im\, t)^{d/2}}$$

$$d_\gamma(P,Q) = \int_Q^P \sqrt{g_{t\bar{t}}}|dt| \sim \frac{\sqrt{2}}{2}\log(\mathrm{Im}\, t)|_Q^P$$

→

$$\frac{M_q(P)}{M_q(Q)} \simeq \exp\left(-\frac{1}{\sqrt{2d}}\, d_\gamma(P,Q)\right)$$