Cold dense matter in compact stars

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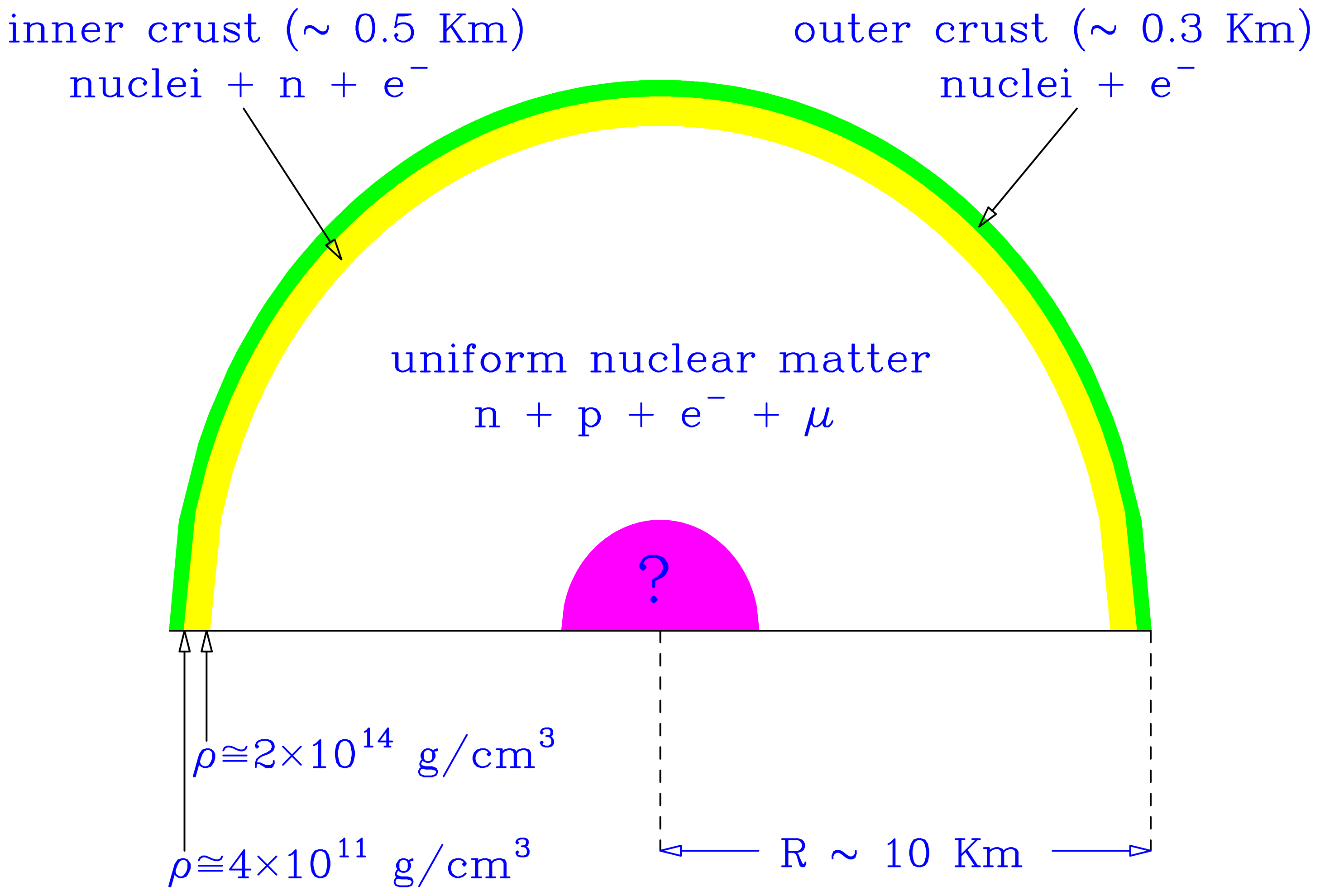
- Understanding the properties of these systems requires the knowledge of the equation of state (EOS) of matter over a huge density range

Overview of Compact Star Structure

- $\rho_0 \approx 0.16\ \mathrm{nucl/fm^3} = 2.67 \times 10^{14}\ \mathrm{g/cm^3}$



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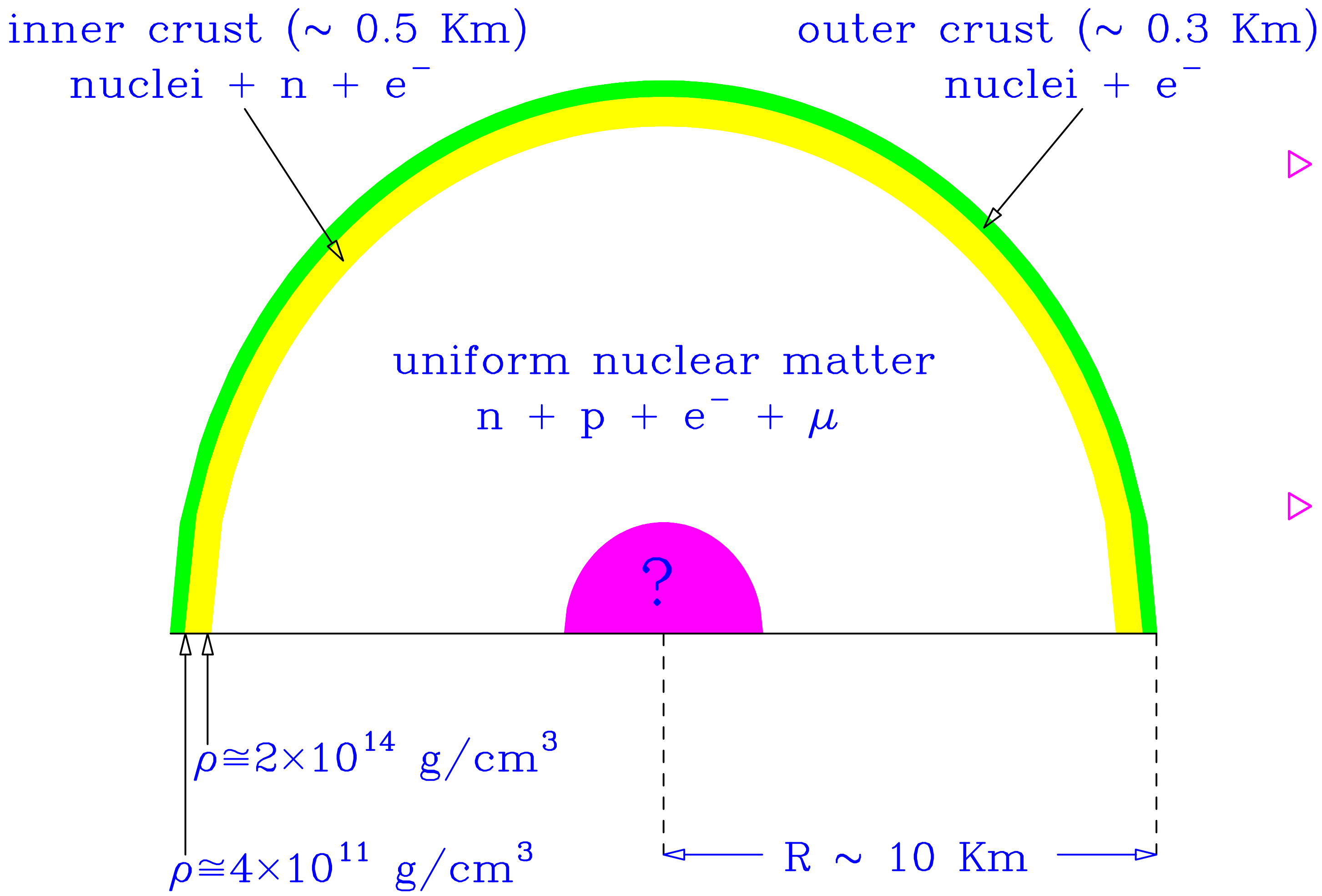
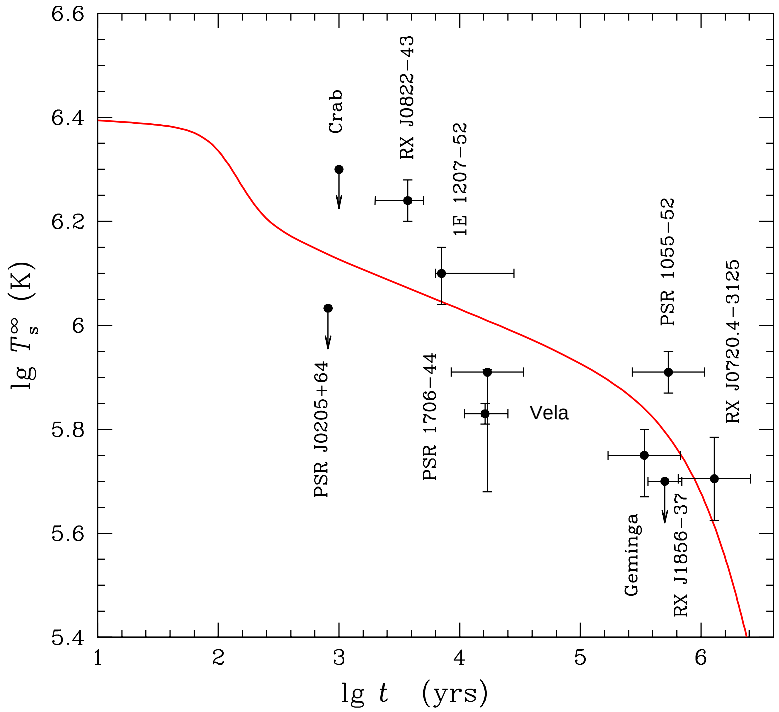


- ? : hyperons, π-condensate, K-condensate, quark matter . . .
- note: most of the neutron star mass is in the region $\rho > \rho_0$

Main assumptions

- cold matter: $T = 0\,^\circ K$



▷ typical neutron star temperature $\sim 10^9\,^\circ K$

▷ to be compared to average kinetic energies in the range

$$10^{11} < \langle T \rangle < 10^{12}\,^\circ K$$

$$@ \;\; \rho_0 < \rho < 4\rho_0$$

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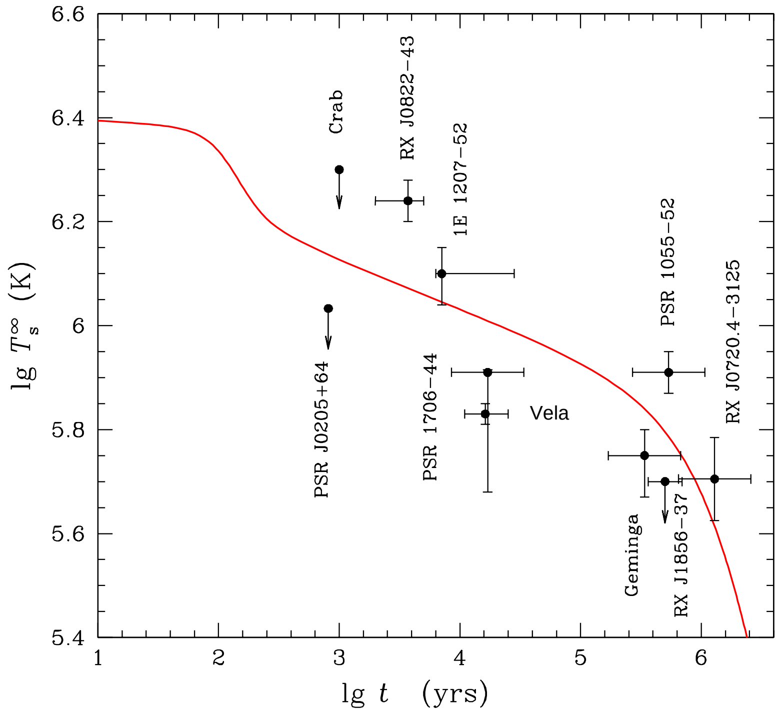

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- transparency to neutrino:

$$\lambda_\nu >> 10 \text{ Km} \quad @ \quad T \sim 10^9\,^\circ K$$

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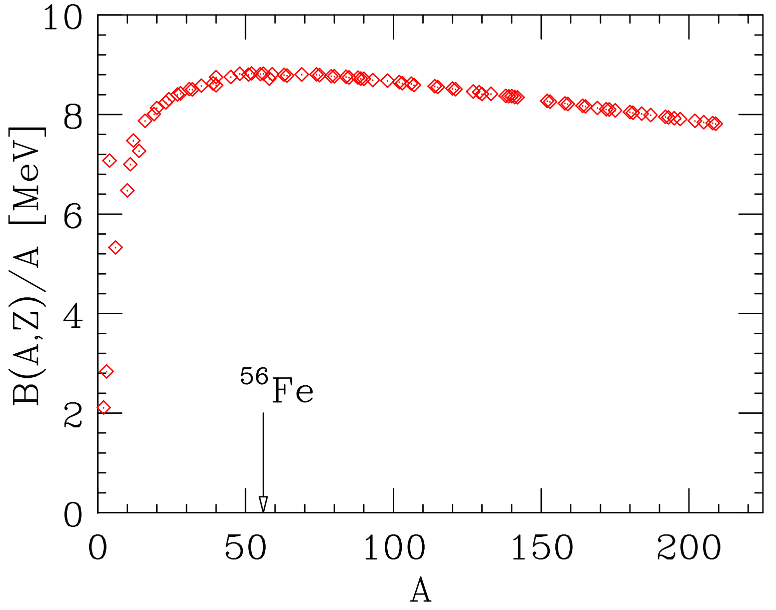
- nuclei are self-bound systems consisting of Z protons and A-Z neutrons
- electrostatic repulsion between protons limits A to $\sim$ 200
- cold means that thermal energies are negligible with respect to proton and neutron Fermi energies
- nuclear systematics constrains $e = e(n, T = 0) = E(n, T = 0)/A$, related to the EOS at $T = 0$ through

$$P(n, T = 0) = -\left(\frac{\partial E}{\partial V}\right)_{T=0} = n^2 \, \frac{de(n, T = 0)}{dn} \, ,$$

at n close to the central density of atomic nuclei

- A-dependence of the (positive) binding energy per nucleon



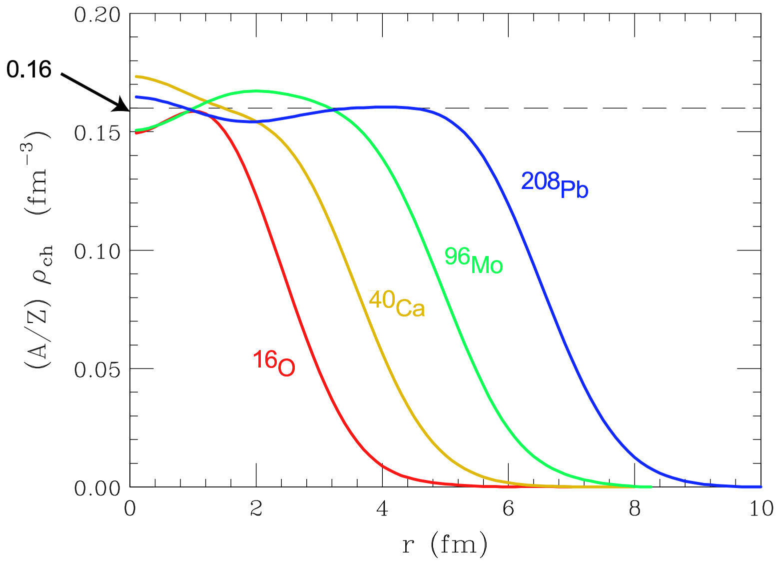
$$\frac{B(\mathrm{Z},\mathrm{A})}{A} = \frac{1}{A}\left[a_\mathrm{V}\mathrm{A} - a_\mathrm{s}\mathrm{A}^{2/3} - a_\mathrm{c}\frac{\mathrm{Z}^2}{\mathrm{A}^{1/3}} - a_\mathrm{A}\frac{(\mathrm{A}-2\mathrm{Z})^2}{4\mathrm{A}} + \lambda\, a_\mathrm{p}\frac{1}{\mathrm{A}^{1/2}}\right]$$

‘

- in the absence of Coulomb repulsion

$$\lim_{A\to\infty}\frac{B(\mathrm{A}/2,\mathrm{A})}{A} = a_\mathrm{V} \sim 16\ \frac{\mathrm{MeV}}{\mathrm{A}}$$

- nuclear densities measured by elastic electron scattering



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- saturation of nuclear densities indicates that the equilibrium density of nuclear matter is

$$n_0 \sim .16 \text{ fm}^{-3} \rightarrow \rho \sim 2.5 \times 10^{14} \text{g/cm}^3$$

- Nuclear systematics provides a single point of the energy-density curve for symmetric nuclear matter, corresponding to equilibrium at $T = 0$

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- Expanding in the vicinity of the equilibrium density yields

$$e(n, T = 0) \approx e_0 + \frac{1}{2} \, \frac{K}{9} \, \frac{(n - n_0)^2}{n_0^2}$$

$$K = 9 \, n_0^2 \left(\frac{\partial^2 e}{\partial n^2} \right)_{n=n_0} = 9 \left(\frac{\partial P}{\partial n} \right)_{n=n_0}$$

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- in principle, the (in)-compressibility module K can be determined from the excitation energies of the nuclear vibrational states

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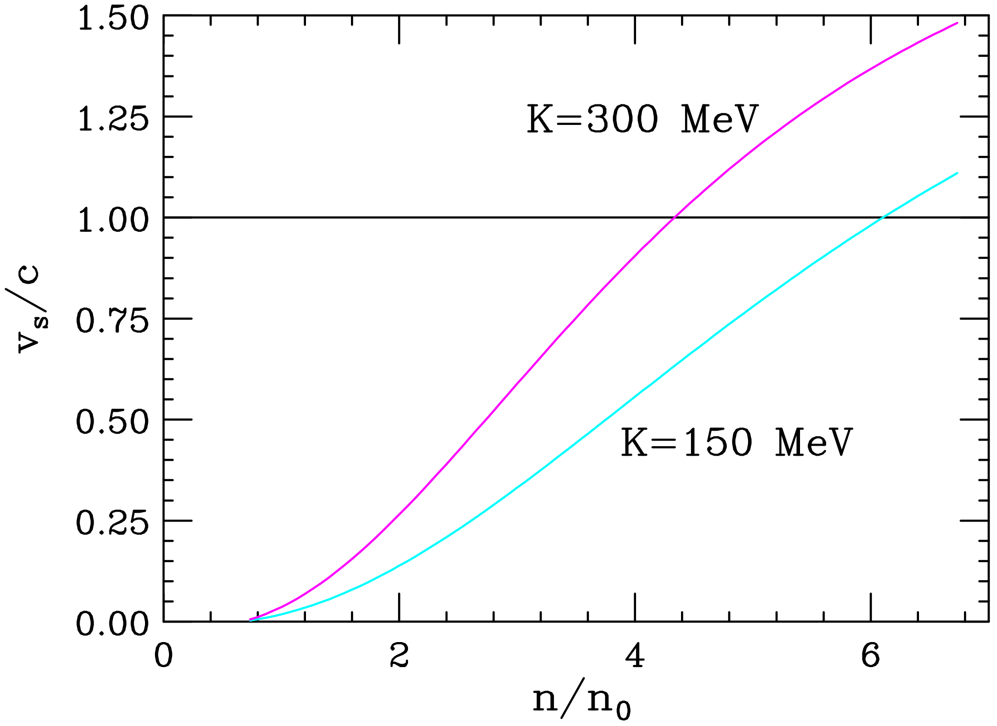
- for any K, a parabolic $e(n, T = 0)$ leads to a speed of sound in matter exceeding the speed of light, i.e. to (m is the nucleon mass)

$$\left(\frac{v_s}{c}\right) = \left(\frac{\partial P}{\partial \epsilon}\right) > 1 \quad , \quad \epsilon = \frac{1}{\Omega}(E + Am) = n(e + m)$$



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- the stiffest EOS compatible with causality is

$$P = \epsilon \rightarrow \left(\frac{v_s}{c}\right) = 1$$

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    - ▷ meson masses and couplings determined by the equilibrium properties of nuclear matter

- charge neutral matter of neutrons, protons and leptons (e and μ) in β-equilibrium

$$n \leftrightarrow p + \ell$$

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- EOS models can be classified according to their “stiffness”

$$\Gamma = \frac{d \ln P}{d \ln n_B}$$

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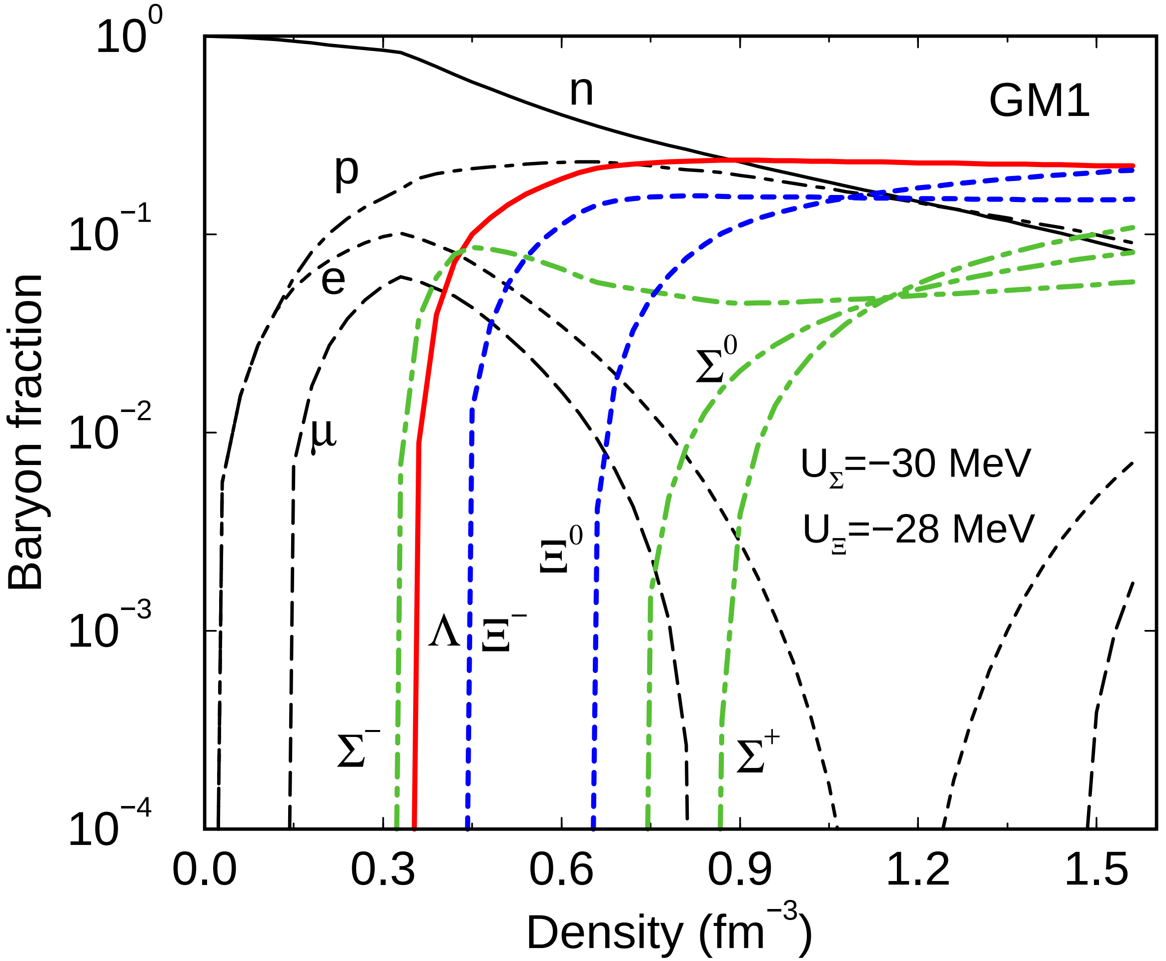
- the appearance of hyperons makes the EOS significanlty softer

Composition of charge neutral and β-stable strange hadronic matter



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- "There may be no neutron stars, only stange stars" (Alcock, Fahri & Olinto, 1986)

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- both P and ϵ are obtained from the thermodynamic potential Ω

$$\Omega = \Omega_{pert} + VB = V \sum_f \sum_n \Omega_f^{(n)} + VB$$

$$P = -\frac{\Omega}{V} = -B - \sum_f \sum_n \Omega_f^{(n)} \quad , \quad \epsilon = -P + \sum_f \mu_f \rho_f$$

Modeling the EOS of quark matter (continued)

- Nambu Jona-Lasinio (NJL) model: deeper dynamical content

$$\mathcal{L} = \bar{q}\,(i\partial\!\!\!/ - \hat{m})\,q + \mathcal{L}_{int} + \mathcal{L}_{det} \quad , \quad \hat{m} = \mathrm{diag}_f(m_u, m_d, m_s)$$

$$\mathcal{L}_{int} = G\,\sum_{a=0}^{8}\left[(\bar{q}\lambda^a q)^2 + (\bar{q}\lambda^a i\gamma_5 q)^2\right]$$

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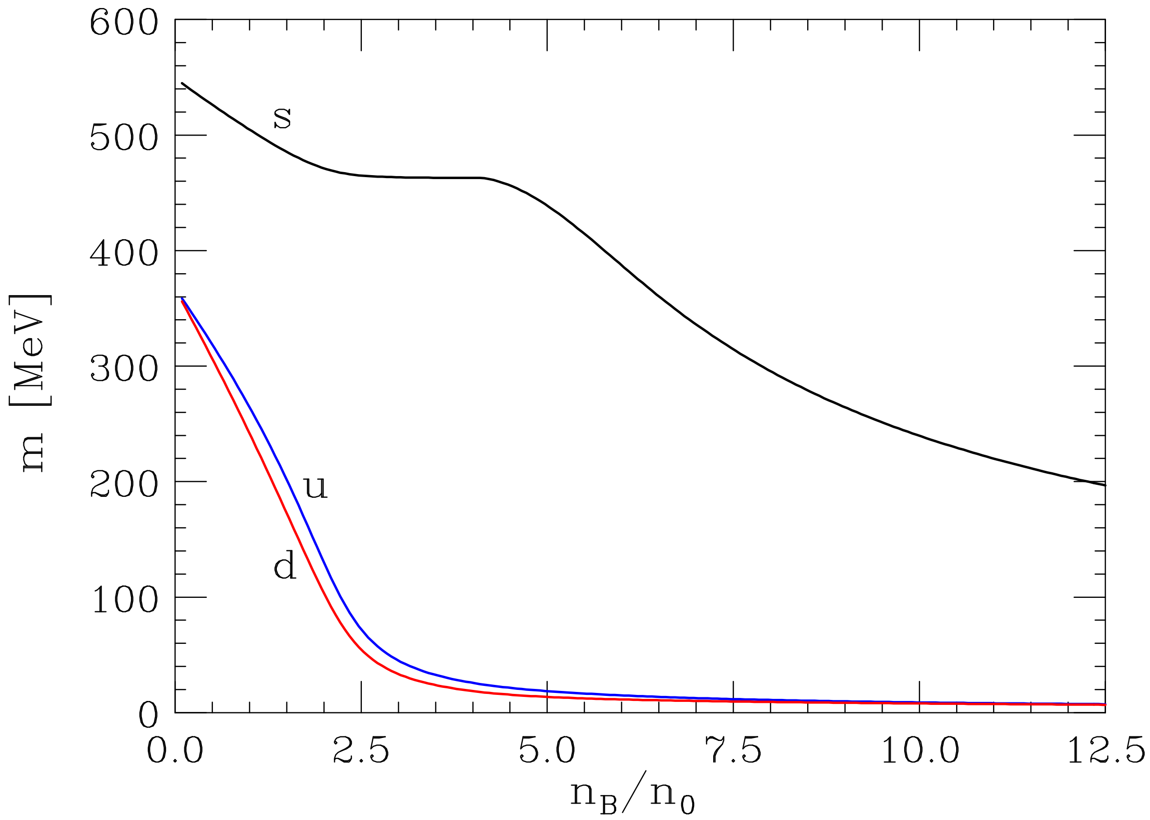
- thermodynamic potential

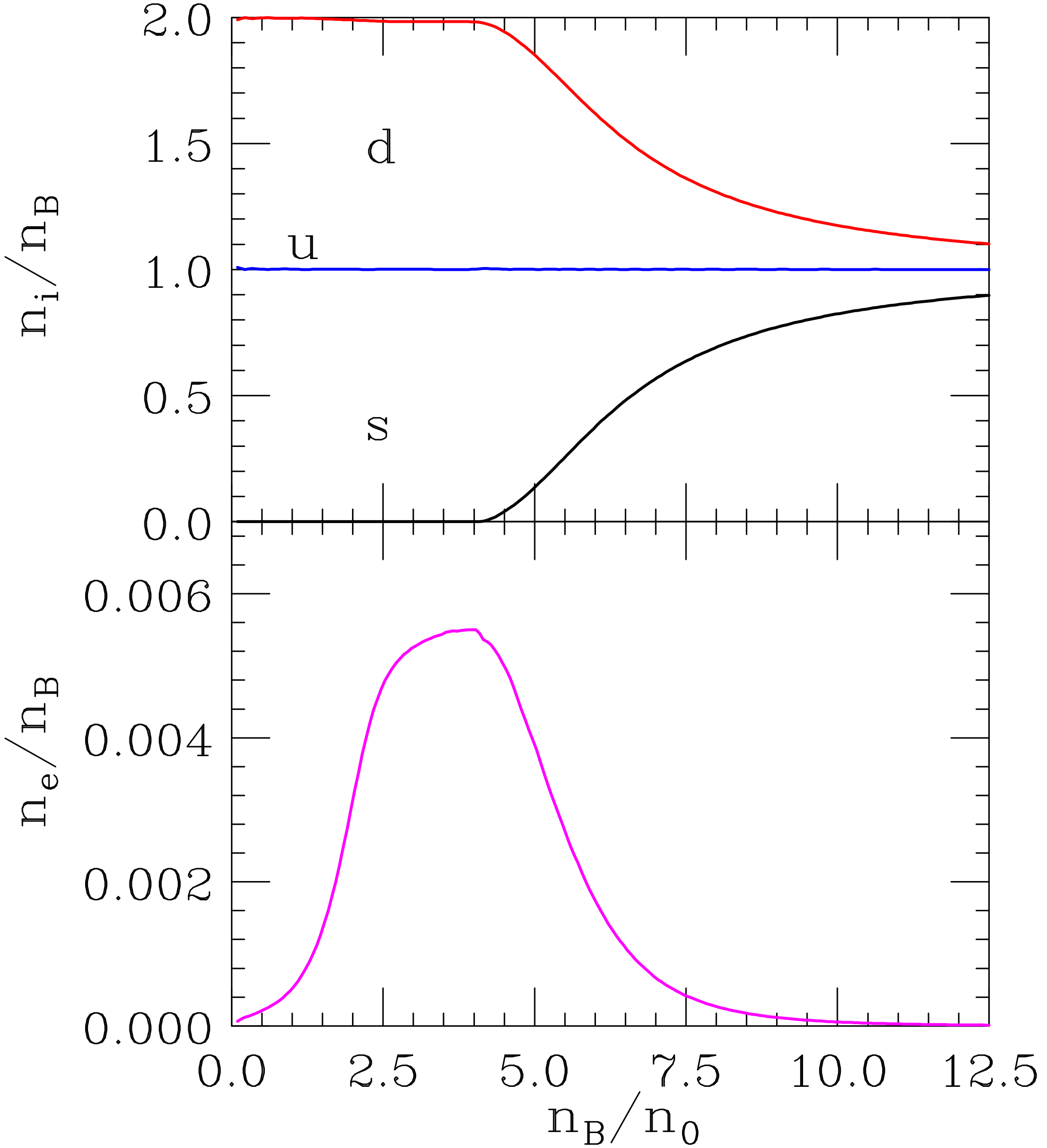
$$\Omega = \sum_f \Omega_f^{(0)} + 2G\,(\phi_u + \phi_d + \phi_s) - 4K\phi_u\phi_d\phi_s$$

NJL model with three flavors



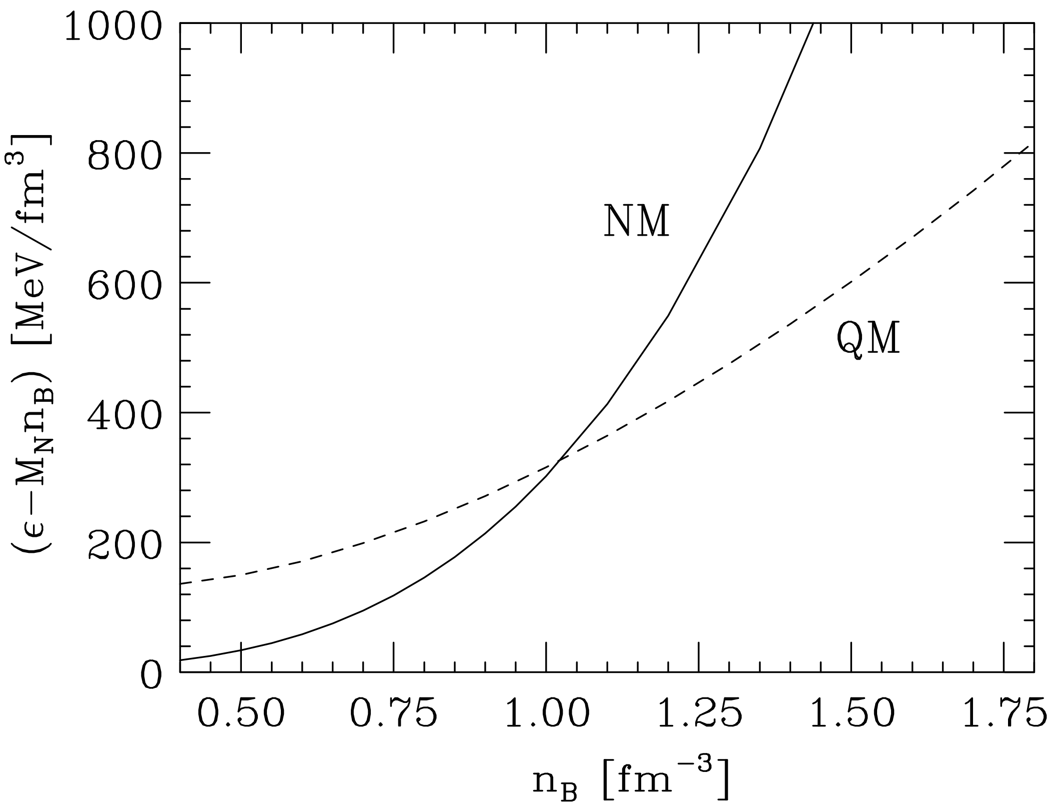
$$m_u = m_d = 5.5\ \mathrm{MeV} \quad , \quad m_s = 140.7\ \mathrm{MeV}$$

Composition of charge neutral β-stable quark matter



From hadronic matter to quark matter

- nucleon matter vs quark matter



▷ as $\rho_B \to \infty$

$$\left(\frac{E}{N_B}\right)_{NM} \propto \rho_B$$

$$\left(\frac{E}{N_B}\right)_{QM} \propto \rho_B^{1/3}$$

- The transition takes place either at constant pressure or with formation of a mixed phase

EOS and properties of nonrotating neutron stars

- given the EOS, mass and radius of a nonrotating star can be obtained from the Tolman-Oppenheimer-Volkov (TOV) equations (hydrostatic equilibrium + Einstein eqs)

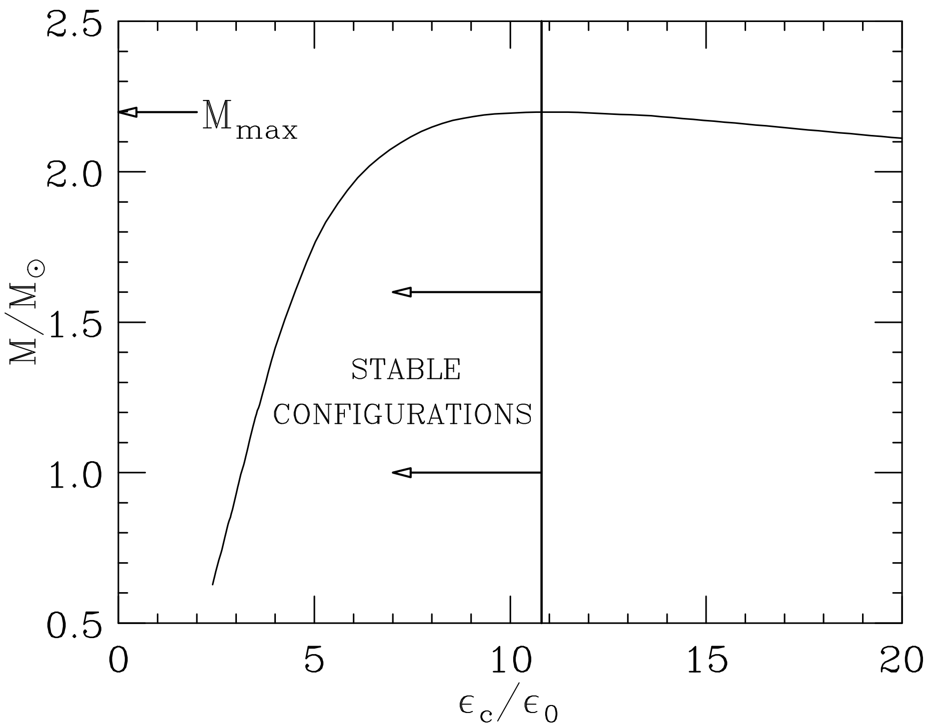
$$\frac{dP(r)}{dr} = -G \frac{[\epsilon(r) + P(r)]\,[M(r) + 4\pi r^3 P(r)]}{r^2\,[1 - 2GM(r)/r]}$$

$$M(r) = 4\pi \int_0^r r'^2 dr' \epsilon(r') \quad , \quad \epsilon(r = 0) = \epsilon_c$$

- solving TOV equations one obtains a set of neutron star confi gurations, characterized by the radius R, defi ned through $P(R) = 0$, and the mass $M = M(R)$

Maximum neutron star mass

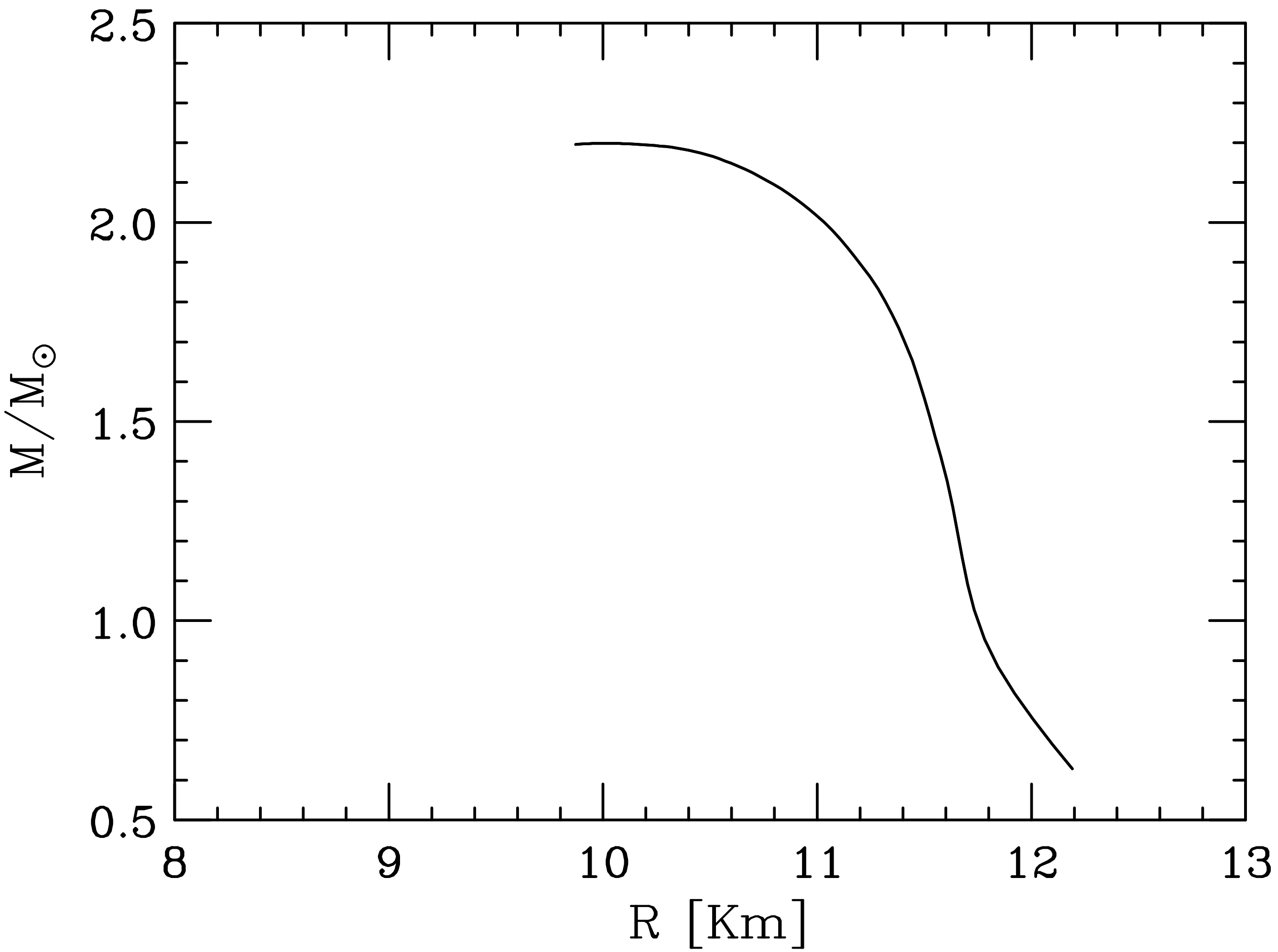
- ▷ typical mass-central energy-density curve



- ▷ maximum mass given by

$$M_{max} = M(\overline{\epsilon}_c) \quad , \quad \left(\frac{dM}{d\epsilon_c}\right)_{\epsilon_c=\overline{\epsilon}_c} = 0$$

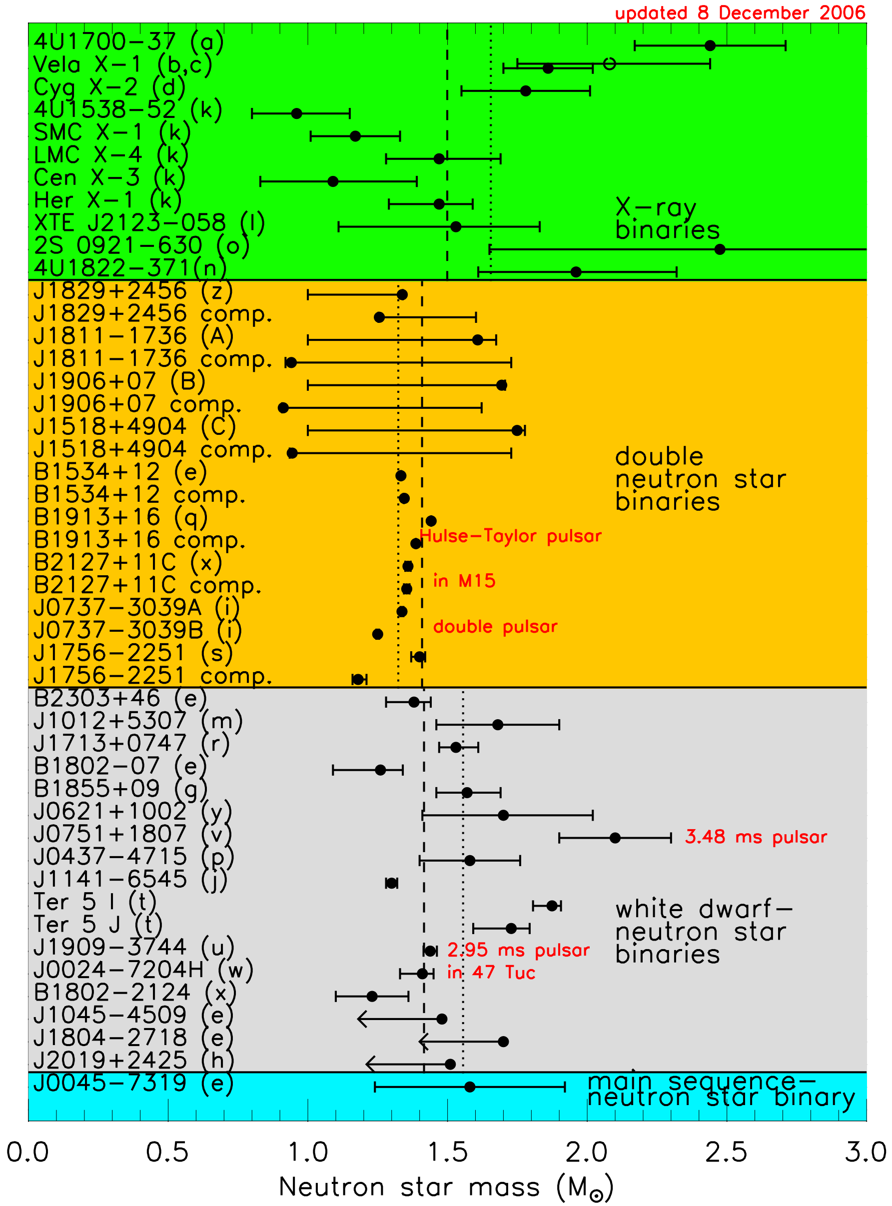
Mass-radius relation

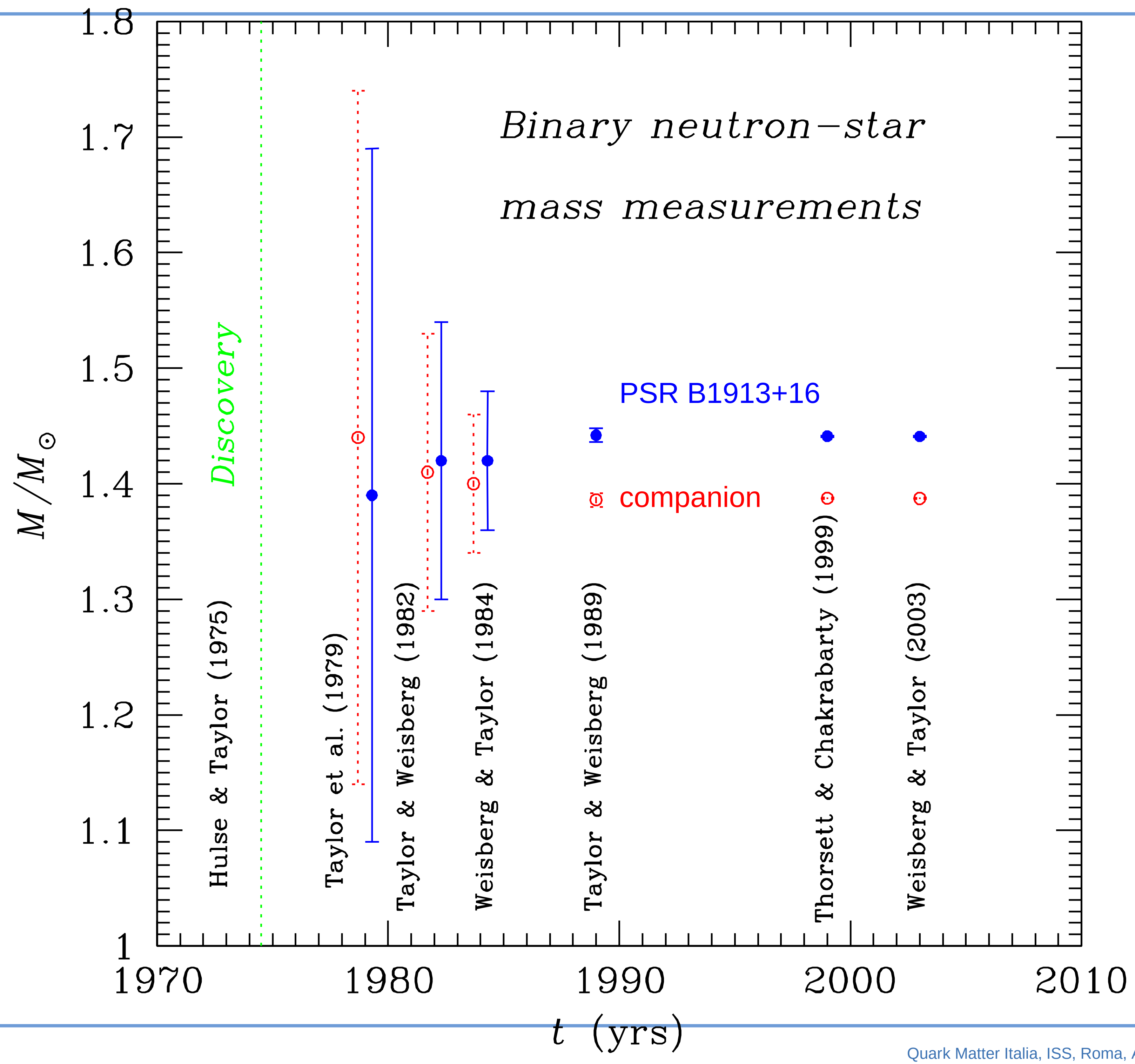


Basic experimental facts

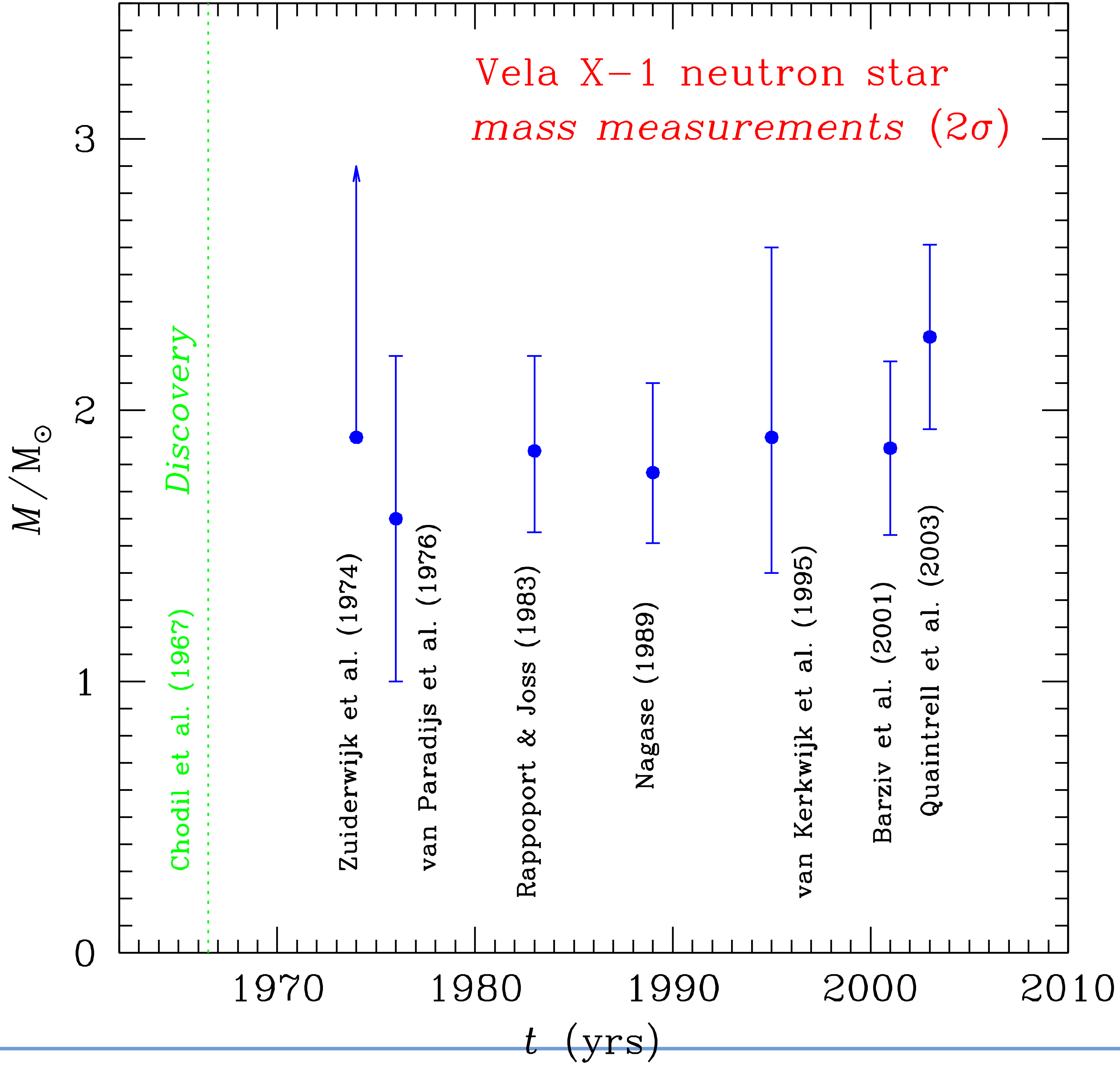
- Mass of the Hulse-Taylor bynary pulsar:
  $M = 1.4411 \pm 0.0007 M_{\odot}$
- $\sim$ 20 accurate measurements from bynary systems yield
  $M = 1.35 \pm 0.1 M_{\odot}$
- recent suggested evidences of heavier neutron stars
  - mass of Vela X-1: $M = 1.87^{+0.23}_{-0.17} M_{\odot}$
  - mass of Cygnus X-2: $M = 1.78 \pm 0.23 M_{\odot}$
  - QPO of galactic X-ray sources seem to indicate that they include neutron stars with $M = \sim 2 M_{\odot}$

Compilation of measured neutron star masses





Binary neutron−star
mass measurements
PSR B1913+16
companion
Discovery
Hulse & Taylor (1975)
Taylor et al. (1979)
Taylor & Weisberg (1982)
Weisberg & Taylor (1984)
Taylor & Weisberg (1989)
Thorsett & Chakrabarty (1999)
Weisberg & Taylor (2003)
M / M⊙
t (yrs)
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1970
1980
1990
2000
2010



Vela X−1 neutron star
mass measurements (2σ)
M / M⊙
t (yrs)
0
1
2
3
1970
1980
1990
2000
2010
Discovery
Chodil et al. (1967)
Zuiderwijk et al. (1974)
van Paradijs et al. (1976)
Rappoport & Joss (1983)
Nagase (1989)
van Kerkwijk et al. (1995)
Barziv et al. (2001)
Quaintrell et al. (2003)

Predicted maximum masses vs data

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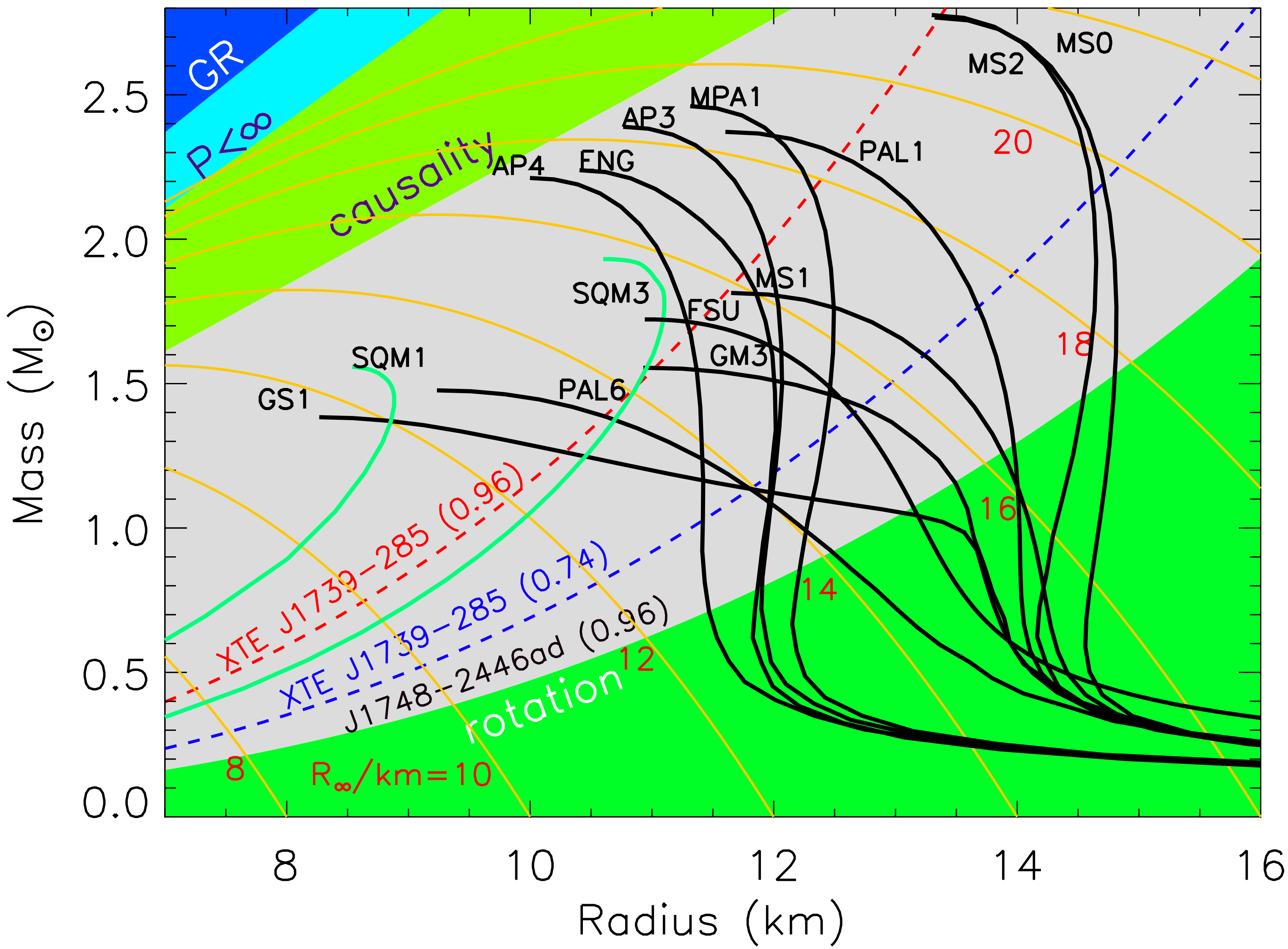
- stable strange stars with $M \lesssim 2 \ \mathrm{M}_\odot$ are also allowed by the MIT bag model, provided perturbative gluon exchange is taken into account
- if confirmed, the measured mass of Vela X-1 will rule out soft EOS, thus leaving little room for the occurrence of hyperonic matter

Mass-radius relation



Recent (still controversial) observational developments

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- z is related to the mass-radius ratio through

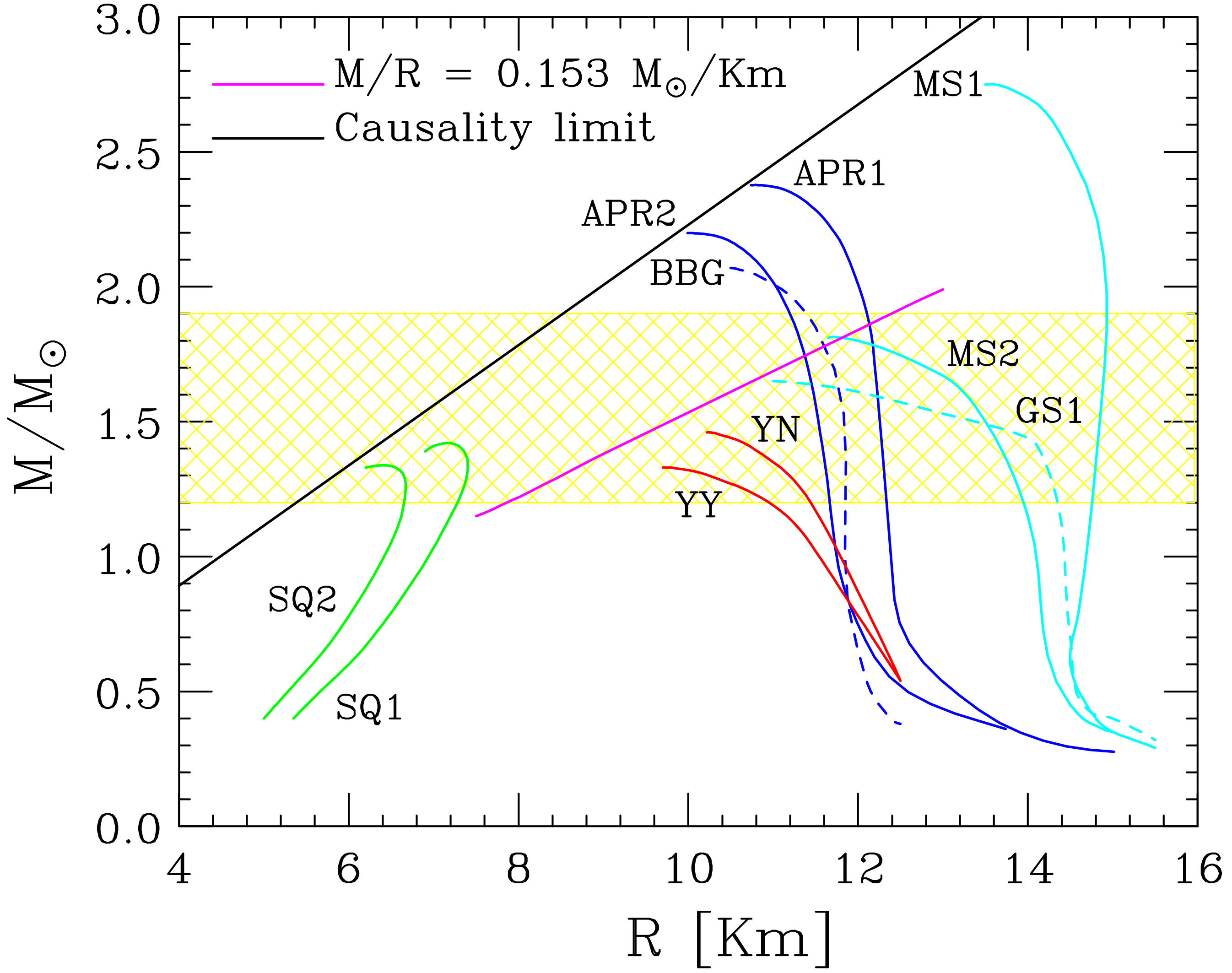
$$R(1+z) = R\left(1 - \frac{2GM}{c^2}\frac{1}{R}\right)^{-1/2}$$

yielding

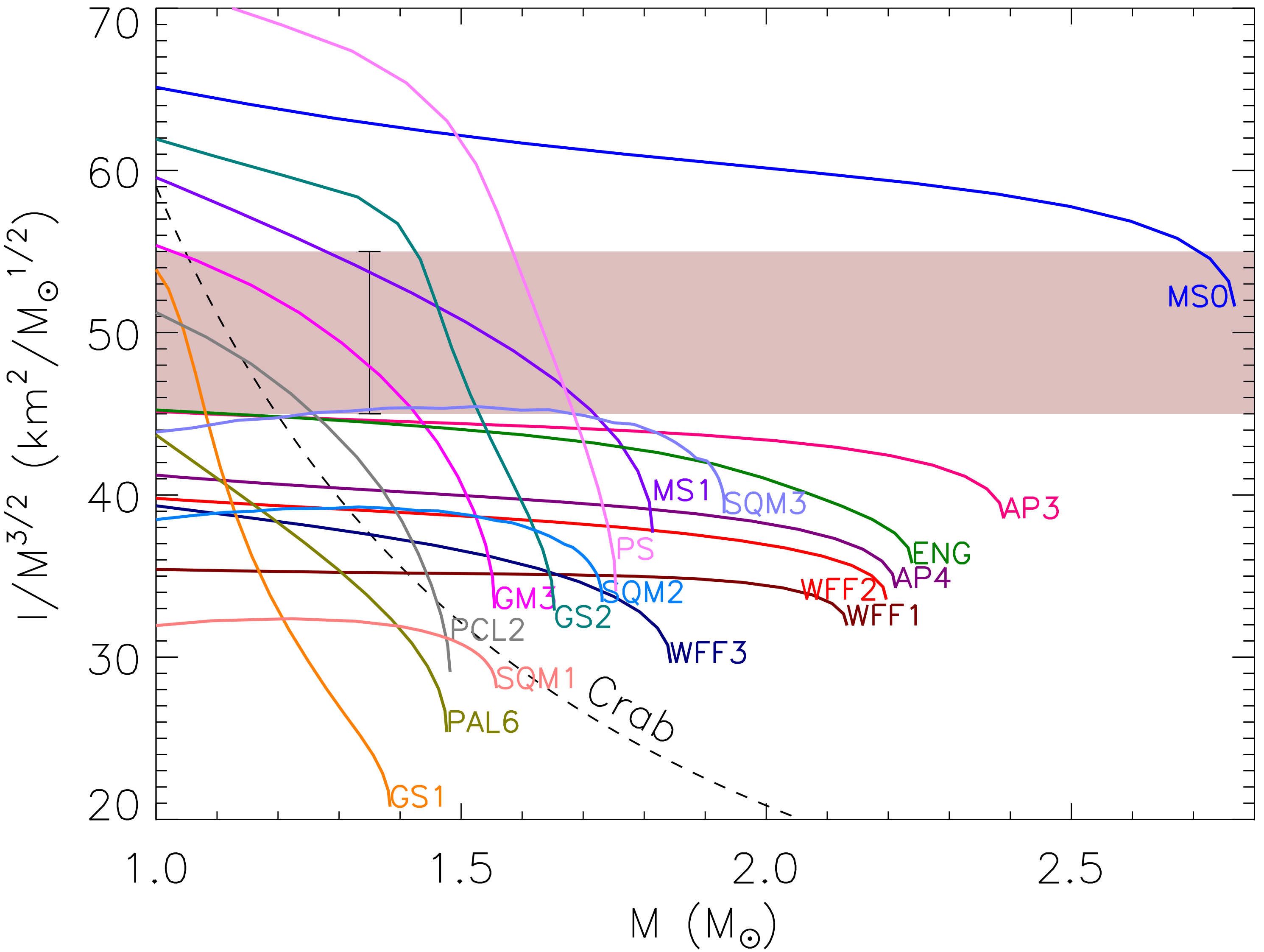
$$\frac{M}{R} = 0.153\,\frac{M_\odot}{\mathrm{Km}}$$

i.e.

$$1.4 \lesssim M/M_\odot \lesssim 1.8 \iff 9 \lesssim R \lesssim 12\ \mathrm{Km}$$

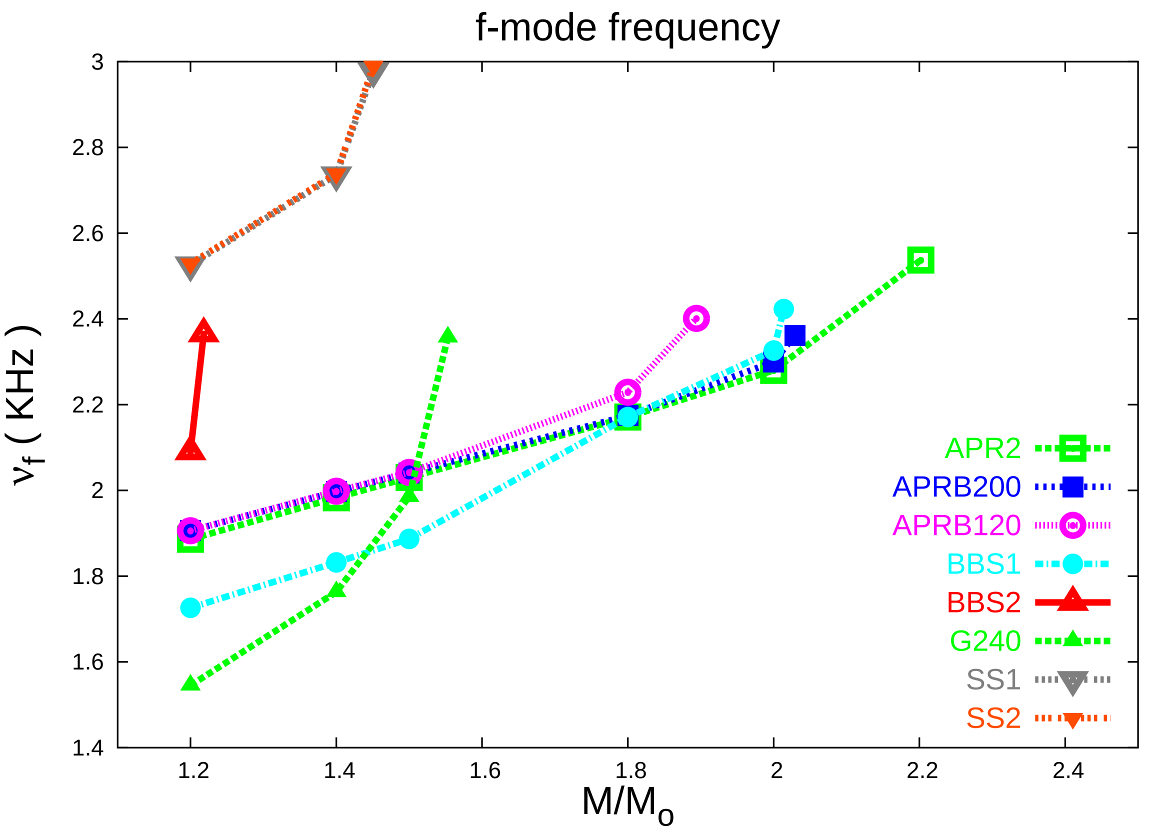

3.0
2.5
2.0
1.5
1.0
0.5
0.0
4
6
8
10
12
14
16
$M/M_{\odot}$
R [Km]
$M/R = 0.153\ M_{\odot}/Km$
Causality limit
MS1
APR1
APR2
BBG
MS2
GS1
YN
YY
SQ2
SQ1

Moment of inertia



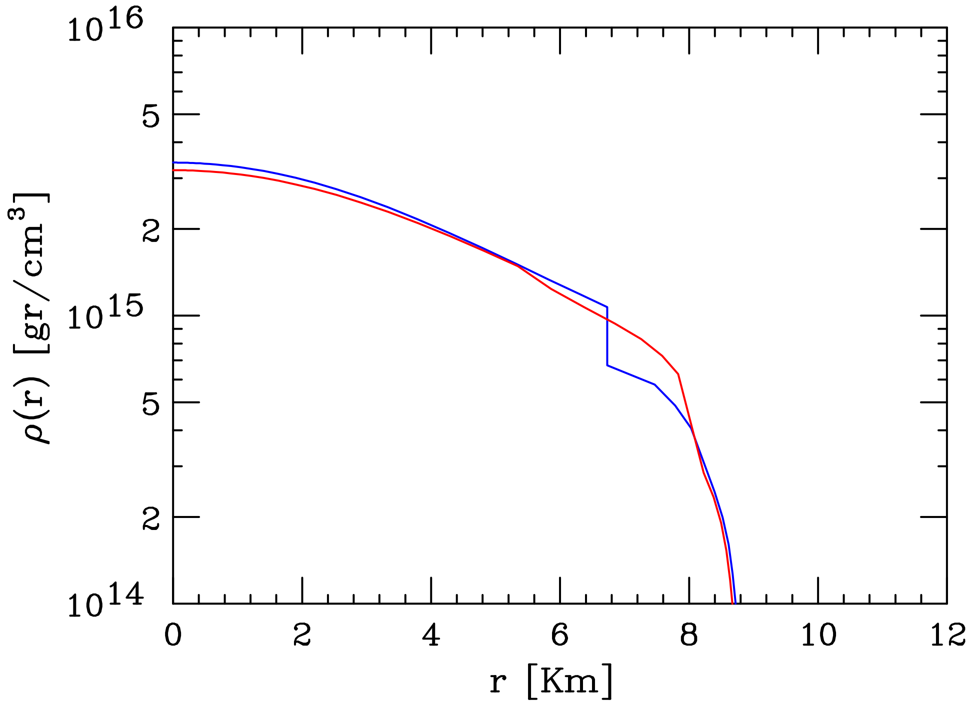
Gravitational wave asteroseismology

- Frequency of the fundamental mode vs mass



Gravitational wave asteroseismology (continued)

- A phase transition at constant pressure leads to the appearance of a discontinuity in the star density profile

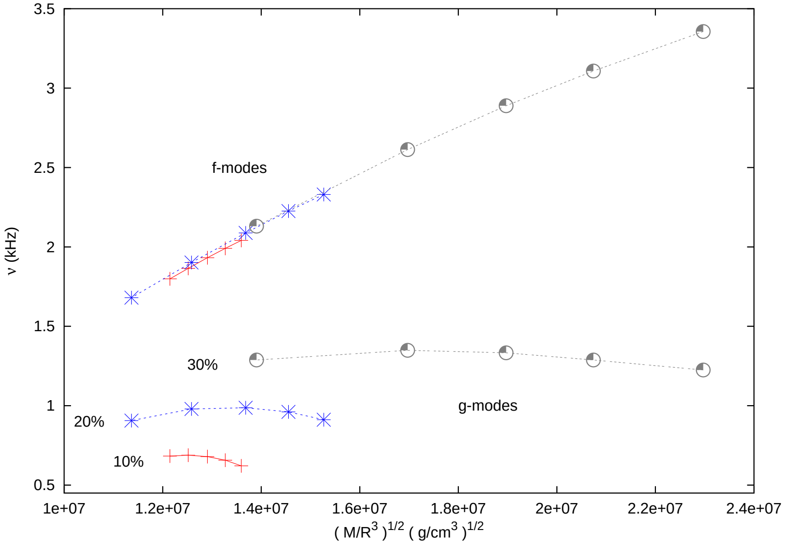


- A class on nonradial oscillations, called g-modes, is associated with the occurrence of a density discontinuity in the interior of the star

- Consider a star described by a polytropic EOS with a density discontinuity Δ_ρ located at $\rho = \rho_D$

$$P(\rho) = \begin{cases} K\left(1+\frac{\Delta_\rho}{\rho_D}\right)^\Gamma \rho^\Gamma & \rho < \rho_D \\ K\rho^\Gamma & \rho > \rho_D + \Delta_\rho \end{cases}$$

- frequency of the f- and g- modes of a star with $M = 1.4\ M_\odot$, as a function of $(M/R^3)^{1/2}$



Backup Slides

Why the EOS ?

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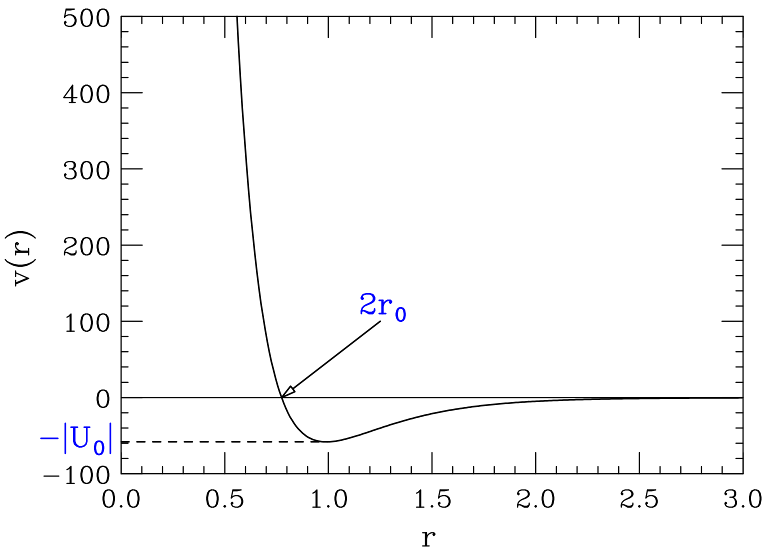
- In the presence of interactions, the EOS carries a wealth of information on the underlying dynamics

Simple example: van der Waals' fluid

- consider a system of particles interacting through the two-body potential



- at $|U_0|/T << 1$ its EOS takes the van der Waals form

$$P = \frac{nT}{1 - nb} - an^2$$

$$b = \frac{16}{3}\pi r_0^3 \qquad\qquad a = \pi \int_{2r_0}^{\infty} |v(r)| r^2 dr$$

NJL model with two flavors