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Spontaneous symmetry breaking in particle physics: a case of cross fertilization

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QUARK MATTER ITALIA, 22-24 aprile 2009

Spontaneous (dynamical) symmetry breaking

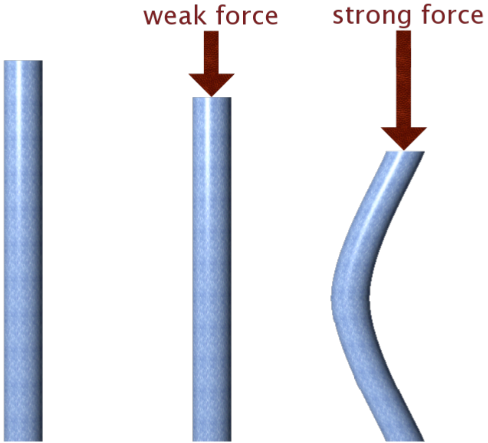


Figure: Elastic rod compressed by a force of increasing strength

Other examples

physical system	broken symmetry
ferromagnets	rotational invariance
crystals	translational invariance
superconductors	local gauge invariance
superfluid ^4He	global gauge invariance

When spontaneous symmetry breaking takes place, the ground state of the system is degenerate

Quasi-particles in superconductivity

Electrons near the Fermi surface are described by the following equation

$$\begin{aligned} E\psi_{p,+} &= \epsilon_p \psi_{p,+} + \phi \psi^{\dagger}_{-p,-} \\ E\psi^{\dagger}_{-p,-} &= -\epsilon_p \psi^{\dagger}_{-p,-} + \phi \psi_{p,+} \end{aligned}$$

with eigenvalues

$$E = \pm\sqrt{\epsilon_p^2 + \phi^2}$$

Here, $\psi_{p,+}$ and $\psi^{\dagger}_{-p,-}$ are the wavefunctions for an electron and a hole of momentum p and spin $+$

Analogy with the Dirac equation

In the Weyl representation, the Dirac equations reads

$$\begin{aligned} E\psi_1 &= \boldsymbol{\sigma}\cdot\boldsymbol{p}\psi_1 + m\psi_2 \\ E\psi_2 &= -\boldsymbol{\sigma}\cdot\boldsymbol{p}\psi_2 + m\psi_1 \end{aligned}$$

with eigenvalues

$$E = \pm\sqrt{p^2 + m^2}$$

Here, ψ_1 and ψ_2 are the eigenstates of the chirality operator γ_5

Nambu-Goldstone boson in superconductivity

Y. Nambu, Phys. Rev. **117**, 648 (1960)

Approximate expressions for the charge density and the current associated to a quasi-particle in a BCS superconductor

$$\begin{aligned}\rho(x,t) &\simeq \rho_0 + \frac{1}{\alpha^2}\partial_t f \\ \boldsymbol{j}(x,t) &\simeq \boldsymbol{j}_0 - \boldsymbol{\nabla} f\end{aligned}$$

where $\rho_0 = e\Psi^\dagger \sigma_3 Z \Psi$ and $\boldsymbol{j}_0 = e\Psi^\dagger(\boldsymbol{p}/m)Y\Psi$ with Y, Z and α constants and f satisfies the wave equation

$$\left(\nabla^2 - \frac{1}{\alpha^2}{\partial_t}^2\right) f \simeq -2e\Psi^\dagger\sigma_2\phi\Psi$$

Here, $\Psi^\dagger = (\psi_1^\dagger, \psi_2)$

Plasmons

The Fourier transform of the wave equation for f gives

$$\tilde{f} \propto \frac{1}{q_0^2 - \alpha^2 q^2}$$

The pole at $q_0^2 = \alpha^2 q^2$ describes the excitation spectrum of the Nambu-Goldstone boson.

A better approximation reveals that, due to the Coulomb force, this spectrum is shifted to the plasma frequency $e^2 n$, where n is the number of electrons per unit volume. In this way the field f acquires a mass.

The Goldstone theorem

J. Goldstone, Nuovo Cimento **19**, 154 (1961)

Whenever the original Lagrangian has a continuous symmetry group, which does not leave the ground state invariant, massless bosons appear in the spectrum of the theory.

physical system	broken symmetry	massless bosons
ferromagnets	rotational invariance	spin waves
crystals	translational invariance	phonons

The axial vector current

Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960)

Electromagnetic current		Axial current
$\bar{\psi}\gamma_\mu\psi$	$\Longleftrightarrow$	$\bar{\psi}\gamma_5\gamma_\mu\psi$

The axial current is the analog of the electromagnetic current in BCS theory. In the hypothesis of exact conservation, the matrix elements of the axial current between nucleon states of four-momentum p and p' have the form

$$\mathit{\Gamma}_\mu^A(p',p) = \left(i\gamma_5\gamma_\mu - 2m\gamma_5 q_\mu/q^2\right) F(q^2) \qquad q = p' - p$$

Conservation is compatible with a finite nucleon mass m provided there exists a massless pseudoscalar particle, the Nambu-Goldstone boson.

In Nature, the axial current is only approximately conserved. Nambu's hypothesis was that the small violation of axial current conservation gives a mass to the N-G boson, which is then identified with the π meson. Under this hypothesis, one can write

$$\Gamma^A_\mu(p', p) \simeq \left(i\gamma_5\gamma_\mu - \frac{2m\gamma_5 q_\mu}{q^2 + m_\pi^2} \right) F(q^2) \qquad q = p' - p$$

This expression implies a relationship between the pion nucleon coupling constant G_π, the pion decay coupling g_π and the axial current β-decay constant g_A

$$2mg_A \simeq \sqrt{2} G_\pi g_\pi$$

This is the Goldberger–Treiman relation

An encouraging calculation

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961), Appendix

It was experimentally known that the ratio between the axial vector and vector β-decay constants $R = g_A/g_V$ was slightly greater than 1 and about 1.25. The following two hypotheses were then natural:

1. under strict axial current conservation there is no renormalization of g_A;
2. the violation of the conservation gives rise to the finite pion mass as well as to the ratio $R > 1$ so that there is some relation between these quantities.

Under these assumptions a perturbative calculation gave a value of R close to the experimental one. More important, the renormalization effect due to a positive pion mass went in the right direction.

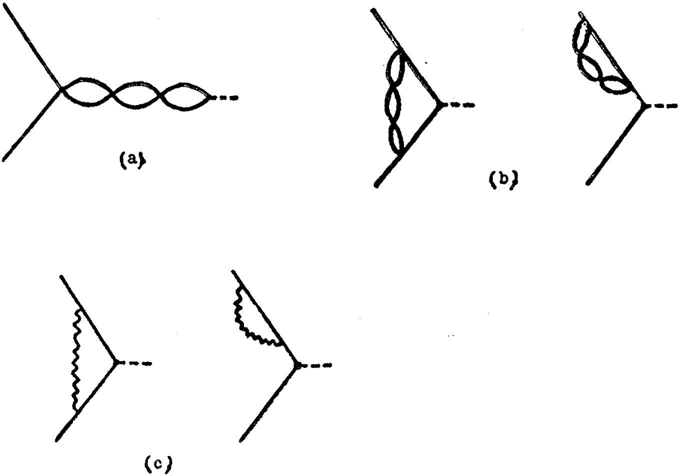


FIG. 2. Typical graphs considered in the evaluation of the axial vector vertex.

The NJL model: an informal presentation

1960 Midwest Conference in Theoretical Physics, Purdue University

<u>A 'SUPERCONDUCTOR' MODEL OF ELEMENTARY PARTICLES</u>
<u>AND ITS CONSEQUENCES</u> by <u>Y. Nambu</u> (University of Chicago)†

(In absence of the author the paper was presented by G. Jona-Lasinio.)

1

In recent years it has become fashionable to apply field-theoretical techniques to the many-body problems one encounters in solid state physics and nuclear physics. This is not surprising because in a quantized field theory there is always the possibility of pair creation (real or virtual), which is essentially a many-body problem. We are familiar with a number of close analogies between ideas and problems in elementary particle theory and the corresponding ones in solid state physics. For example, the Fermi sea of electrons in a metal is analogous to the Dirac sea of electrons in the vacuum, and we speak about electrons and holes in both cases. Some people must have thought of the meson field as something like the shielded Coulomb field. Of course, in elementary particles we have more symmetries and invariance properties than in the other, and blind analogies are often dangerous.

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois
(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.

The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

The Nambu–Jona-Lasinio (NJL) model

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

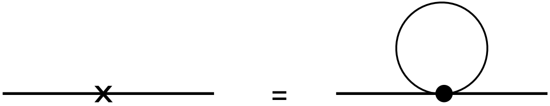
The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi + g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

It is invariant under ordinary and γ_5 gauge transformations

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$
$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

Mean field approximation



$$m = -\frac{g_0 m i}{2\pi^4} \int \frac{d^4 p}{p^2 - m^2 - i\varepsilon} \; F(p, \Lambda)$$

The spectrum of the NJL model

Mass equation

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{m^2}{\Lambda^2} \ln\left(1 + \frac{\Lambda^2}{m^2}\right)$$

where Λ is the invariant cut-off

Spectrum of bound states

nucleon number	mass μ	spin-parity	spectroscopic notation
0	0	0^-	${}^1S_0$
0	$2m$	0^+	${}^3P_0$
0	$\mu^2 > \frac{8}{3}m^2$	1^-	${}^3P_1$
± 2	$\mu^2 > 2m^2$	0^+	${}^1S_0$

Other examples of BCS type SSB

- ^3He superfluidity
- Nuleon pairing in nuclei
- Fermion mass generation in the electro-weak sector of the standard model

Nambu calls the last entry

> *my biased opinion, there being other interpretations as to the nature of the Higgs field*

Broken symmetry and the mass of gauge vector mesons

P. W. Anderson, Phys. Rev. **130**, 439 (1963)
F. Englert, R. Brout, Phys. Rev. Lett. **13**, 321 (1964)
P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964)

A simple example (Englert, Brout). Consider a complex scalar field $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$ interacting with an abelian gauge field A_μ

$$H_{\text{int}} = ieA_\mu \varphi^\dagger \overleftrightarrow{\partial_\mu} \varphi - e^2 \varphi^\dagger \varphi A_\mu A_\mu$$

If the vacuum expectation value of φ is $\neq 0$, e.g. $\langle\varphi\rangle = \langle\varphi_1\rangle/\sqrt{2}$, the polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory is

$$\Pi_{\mu\nu}(q) = (2\pi)^4 ie^2 \langle\varphi_1\rangle^2 \left[g_{\mu\nu} - \left(q_\mu q_\nu/q^2\right)\right]$$

Therefore the A_μ field acquires a mass $\mu^2 = e^2\langle\varphi_1\rangle^2$ and gauge invariance is preserved, $q_\mu \Pi_{\mu\nu} = 0$.

Electroweak unification

S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interaction. What could be more natural than to unite these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and the electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum.

The NJL model as a low-energy effective theory of QCD

e.g. T. Hatsuda, T. Kunihiro, Phys. Rep. **247**, 221 (1994)

The NJL model has been reinterpreted in terms of quark variables. One is interested in the low energy degrees of freedom on a scale smaller than some cut-off $\Lambda \sim 1$ Gev. The short distance dynamics above Λ is dictated by perturbative QCD and is treated as a small perturbation. Confinement is also treated as a small perturbation. The total Lagrangian is then

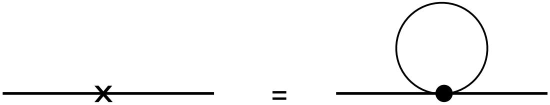
$$L_{\mathsf{QCD}} \simeq L_{\mathsf{NJL}} + L_{\mathsf{KMT}} + \varepsilon \left(L_{\mathsf{conf}} + L_{\mathsf{OGE}} \right)$$

where the Kobayashi–Maskawa–'t Hooft term

$$L_{\mathsf{KMT}} = g_D \det_{i,j} \left[\bar{q}_i (1 - \gamma_5) q_j + \text{h.c.} \right]$$

mimics the axial anomaly and L_{OGE} is the one gluon exchange potential.

Analysis of the mean field approximation



$$m = -\frac{g_0 m i}{2\pi^4} \int \frac{d^4 p}{p^2 - m^2 - i\varepsilon} \, F(p, \Lambda)$$

This equation has the obvious solution $m = 0$ but if $\frac{2\pi^2}{g_0 \Lambda^2} < 1$ there is a second non zero solution which lowers the energy of the vacuum.

The Bogolubov-Valatin transformation

$$a^{(m)}(\mathbf{p},s)=\left[\tfrac{1}{2}(1+\beta_p)\right]^{\frac{1}{2}}a^{(0)}(\mathbf{p},s)+\left[\tfrac{1}{2}(1-\beta_p)\right]^{\frac{1}{2}}b^{(0)\dagger}(-\mathbf{p},s),$$

$$b^{(m)}(\mathbf{p},s)=\left[\tfrac{1}{2}(1+\beta_p)\right]^{\frac{1}{2}}b^{(0)}(\mathbf{p},s)-\left[\tfrac{1}{2}(1-\beta_p)\right]^{\frac{1}{2}}a^{(0)\dagger}(-\mathbf{p},s),$$

$$\beta_p=|\mathbf{p}|/(\mathbf{p}^2+m^2)^{\frac{1}{2}}.$$

Structure of the vacuum

$$\Omega^{(m)} = \prod_{\mathbf{p},s} \{ [\tfrac{1}{2}(1+\beta_p)]^{\frac{1}{2}} - [\tfrac{1}{2}(1-\beta_p)]^{\frac{1}{2}} a^{(0)\dagger}(\mathbf{p},s) b^{(0)\dagger}(-\mathbf{p},s) \} \Omega^{(0)}.$$

$$(\Omega^{(0)},\Omega^{(m)}) = \prod_{\mathbf{p},s} [\tfrac{1}{2}(1+\beta_p)]^{\frac{1}{2}} = \exp\{ \sum_{\mathbf{p},s} \tfrac{1}{2} \ln[\tfrac{1}{2}(1+\beta_p)] \}.$$

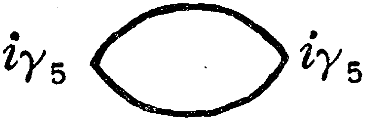
Since the sum in expoment is negative and divergent we have $(\Omega^{(0)}, \Omega^{(m)}) = 0$

Chiral transformations of the vacuum

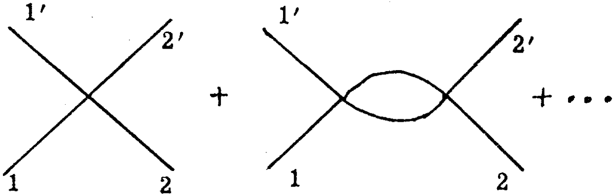
$$
\begin{aligned}
(\Omega_{\alpha}{}^{(m)}, \Omega_{\alpha'}{}^{(m)})
&= \prod_{p,\pm} \left[\tfrac{1}{2}(1+\beta_p) - e^{\pm 2i(\alpha'-\alpha)}\tfrac{1}{2}(1-\beta_p)\right] \\
&= \prod_{p,\pm} \left[1 + (e^{\pm 2i(\alpha'-\alpha)} - 1)\tfrac{1}{2}(1-\beta_p)\right] \\
&= \exp\Big\{ \sum_{p,\pm} \ln\left[1 + (e^{\pm 2i(\alpha'-\alpha)} - 1)\tfrac{1}{2}(1-\beta_p)\right]\Big\}.
\end{aligned}
$$

the sum in the exponent is negative and divergent so that $(\Omega_{\alpha}^{(m)}, \Omega_{\alpha'}^{(m)}) = 0$

Nucleon-nucleon scattering



(a)



(b)

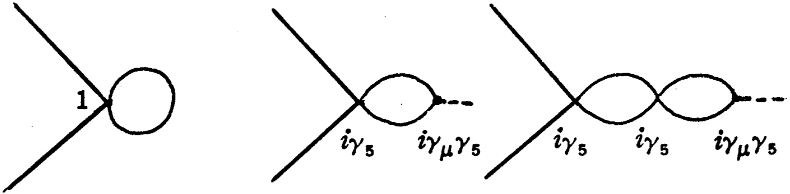
$$J_P(q) = -\frac{2ig_0}{(2\pi)^4} \times \int \frac{4(m^2+p^2)-q^2}{[(p+\frac{1}{2}q)^2+m^2][(p-\frac{1}{2}q)^2+m^2]} d^4p.$$

$$2g_0 i\gamma_5 \frac{1}{1-J_p(q)} i\gamma_5,$$

Due to the mass self-consistency equation $J_P(0) = 1$

Summary

According to our model, the pion is not the primary agent of strong interactions, but only a secondary effect. The primary interaction is unknown. At the present stage of the model the latter is only required to have appropriate dynamical and symmetry properties, although the nonlinear four-fermion interaction, which we actually adopted, has certain practical advantages.



Chirality conservation and soft pion production

Y. Nambu, D. Lurie, Phys. Rev. **125**, 1429 (1962).

An effective model consisting of a nucleon field ψ of mass m and a massless pseudoscalar field $var\phi$(pion) coupled through

$$L_{\text{int}} = -m\bar{\psi}\exp[(ig/m)\gamma_5\phi]\psi,$$

In addition, we shall introduce an external vector (or axial vector) potential $V_\mu(A_\mu)$,

$$L_{\text{ext}} = i\bar{\psi}\gamma_\mu\psi V_\mu \quad (\text{or } i\bar{\psi}\gamma_\mu\gamma_5\psi A_\mu). \tag{2.2'}$$

The entire Lagrangian is invariant under the γ_5 transformation,

$$\psi \to e^{i\alpha\gamma_5}\psi, \tag{2.3a}$$

and

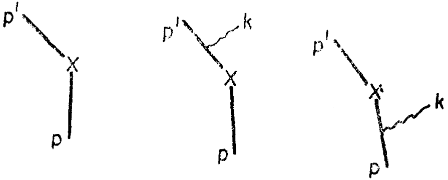
$$\phi \to \phi - (2m/g)\alpha, \tag{2.3b}$$

Chirality is defined by

$$\chi = -i\int j_4 d^3x = \int \bar{\psi}\gamma_4\gamma_5\psi d^3x + (2im/g)\int \partial_4\phi d^3x$$

$$= \chi_N + \chi_\pi.$$

FIG. 1. Lowest order diagrams to be considered for chirality conservation in potential scattering. The point ✕ is where the potential acts.



One verifies that $< in|\chi|in >=< out|\chi|out >$

A small fermion bare mass

For $m_1{}^0 = 0$, we had originally

$$1 = J_p(0) = \frac{g_0{}'}{4\pi^2} \int_{4m^2}^{\Lambda 2} d\kappa^2 \left(1 - \frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}},$$

which should now be replaced by

$$1 = \frac{m_1{}^0}{m_1} + \frac{g_0{}'}{4\pi^2} \int_{4m_1{}^2}^{\Lambda 2} d\kappa^2 \left(1 - \frac{4m_1{}^2}{\kappa^2}\right)^{\frac{1}{2}},$$

For the observed value of $\mu^2/4m_1^2 \simeq 1/200$ we have $m_1^0 \simeq 5Mev$.

The effective action

G. Jona-Lasinio, Nuovo Cimento **34**, 1790 (1964)

Define the *partition function*

$$Z[J] = \langle 0|T \exp i[\int dx(L_I + \sum J_i\Phi_i)]|0\rangle$$

where the fields Φ_i transform, e.g., according to the fundamental representation of the orthogonal group. Then

$$G[J] = -i \log Z[J]$$

is the generator of the time ordered vacuum expectation values (in statistical mechanics G is the free energy in the presence of an external field J)

$$\frac{\delta G}{\delta J} = \langle \Phi \rangle = \phi$$

The effective action is the dual functional $\Gamma[\phi]$ defined by the Legendre transformation

$$\frac{\delta \Gamma}{\delta \phi} = -J$$

The vacuum of the theory is defined by the variational principle

$$\frac{\delta\Gamma}{\delta\phi} = 0$$

$\Gamma[\phi]$ is the generator of the vertex functions and can be constructed by simple diagrammatic rules. Its general form is

$$\Gamma[\phi] = L_{\mathsf{cl}}[\phi] + \hbar Q[\phi]$$

Proof of the Goldstone theorem

Consider an infinitesimal transformation of the group $\delta\phi = t_{ij}\phi_j$. Due to the invariance of the effective action Γ we find

$$\Delta^{-1}_{ij}(q=0)t_{jk}\phi_k = 0$$

which implies

$$\det[\Delta^{-1}_{ij}(q=0)] = 0$$

The mass hierarchy problem

Y. Nambu, *Masses as a problem and as a clue*, May 2004

- Unlike the internal quantum numbers like charge and spin, mass is not quantized in regular manner
- Mass receives contributions from interactions. In other words, it is dynamical.
- The masses form hierarchies. Hierarchical structure is an outstanding feature of the universe in terms of size as well of mass. Elementary particles are no exception.

Einstein used to express dissatisfaction with his famous equation of gravity

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

His point was that, from an aesthetic point of view, the left hand side of the equation which describes the gravitational field is based on a beautiful geometrical principle, whereas the right hand side, which describes everything else, ... looks arbitrary and ugly.

... [today] Since gauge fields are based on a beautiful geometrical principle, one may shift them to the left hand side of Einstein's equation. What is left on the right are the matter fields which act as the source for the gauge fields ... Can one geometrize the matter fields and shift everything to the left?

Hierarchical spontaneous symmetry breaking

Y. Nambu, *Masses as a problem and as a clue*, May 2004

> *The BCS mechanism is most relevant to the mass problem because introduces an energy (mass) gap for fermions, and the Goldstone and Higgs modes as low-lying bosonic states. An interesting feature of the SSB is the possibility of hierarchical SSB or "tumbling". Namely an SSB can be a cause for another SSB at lower energy scale.*
>
> *. . . [examples are]*
>
> *1. the chain crystal–phonon–superconductivity. . . . Its NG mode is the phonon which then induces the Cooper pairing of electrons to cause superconductivity.*
>
> *2. the chain QCD–chiral SSB of quarks and hadrons–π and σ mesons–nuclei formation and nucleon pairing–nuclear π and σ modes–nuclear collective modes.*

Chiral molecules