# Relativistic hydrodynamics,Heavy ion collisions, Perfect liquids

# and all that

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Many thanks for D.Rischke,B.Betz,J.Noronha,M.Gyulassy,I.Mishustin,S.Jeon,G.Moore and many others... first and foremost the organizing committe who invited me here! I hope not to disappoint!

**Il punto di questo talk** e' di spiegare cosa significano alcuni concetti di cui sentirete parlare nei talk (teorici e sperimentali) nelle conferenze. Pertanto, il tutto e' molto generale.

**Incoraggio** domande in qualsiasi momento. Il talk e' modulare, e preferisco rinunciare a qualcosa a vantaggio di spiegare meglio qualchos'altro.

**Il liquido perfetto** e sperimentalmente osservato, ed e' perfettamente capito. Se dimentico di spiegare i dettagli, chiedete alla fine

## Philosophy:
What is hydrodynamics?

## Philosophy

- What is hydrodynamics? How does it relate to thermodynamics?
- Ideal and non-ideal hydrodynamics: A macroscopic "derivation"
- Why do we expect and hope it works at RHIC
- A microscopic derivation: Weak and strong coupling

## Cuisine

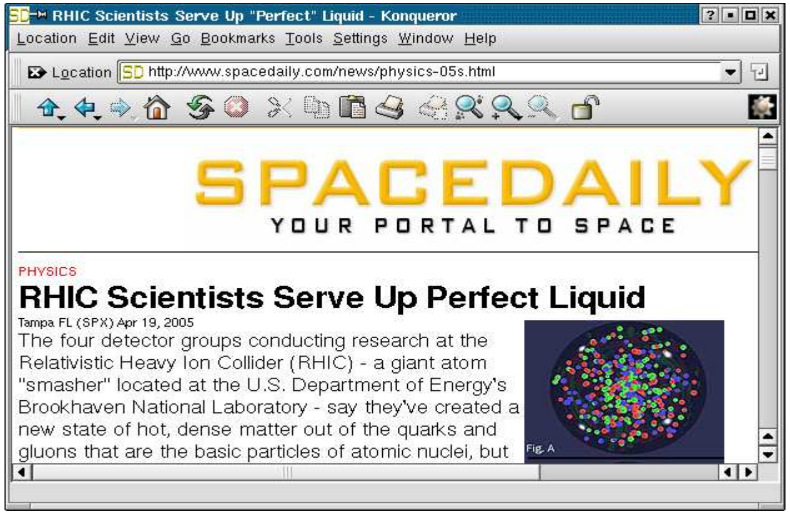
- Numerics
- Initial conditions
- EoS
- Freeze-out

## science

- Spectra
- Elliptic flow
- HBT puzzle
- Mach cones

## conclusions

If you google "perfect liquid", this page comes first:




Creating "the perfect liquid",ie a system that can be described very well by hydrodynamics, was the heavy ion discovery that generated by far most publicity in the non−scientific literature.

On what basis was this discovery claimed? And what does it MEAN?

## What is (ideal) hydrodynamics (part I)?

Infinite system in equilibrium (relativistic) is characterized by Energy density, Pressure and conserved charge density. Pressure is isotropic (equal in all directions). In this case, Its energy momentum content in the rest frame is characterized by the energy-momentum tensor

$$T^{\mu\nu}_{comoving} = \begin{pmatrix} e(p,\rho) & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

where $e(p,\rho)$ are, in terms of the partition function, the usual relations

$$eV = -\frac{\partial \ln Z}{\partial 1/T} \quad , \quad pV = -T \ln Z \quad , \quad \rho V = -\lambda \frac{\partial \ln Z}{\partial \lambda} \quad \left(\lambda = e^{\mu/T}\right)$$

The energy momentum tensor described in the previous page is only valid in one frame (the rest frame). If this frame, however, is moving with a flow-velocity $u^\mu = \gamma(1, \vec{v})$, then one can use a general Lorentz-transformation

$$\Lambda_\mu^\nu = \begin{pmatrix} \gamma & -v_x\gamma & -v_y\gamma & -v_z\gamma \\ -v_x\gamma & 1+(\gamma-1)\frac{v_x^2}{\vec{v}^2} & (\gamma-1)\frac{v_x v_y}{\vec{v}^2} & (\gamma-1)\frac{v_x v_z}{\vec{v}^2} \\ -v_y\gamma & (\gamma-1)\frac{v_y v_x}{\vec{v}^2} & 1+(\gamma-1)\frac{v_y^2}{\vec{v}^2} & (\gamma-1)\frac{v_y v_z}{\vec{v}^2} \\ -v_z\gamma & (\gamma-1)\frac{v_z v_x}{\vec{v}^2} & (\gamma-1)\frac{v_z v_y}{\vec{v}^2} & 1+(\gamma-1)\frac{v_z^2}{\vec{v}^2} \end{pmatrix}$$

to move to a lab-frame co-moving with $u^\mu$ . Then, in the lab frame,

$$T^{\mu\nu} = T^{\alpha\beta}\big|_{rest}\Lambda_\alpha^\mu \Lambda_\beta^\nu = (e+P)u_\mu u_\nu - p g_{\mu\nu}$$

The conserved charge density becomes a current vector $j^\mu = \rho u^\mu$

Conservation of momentum and Charge always gives us 5 Equations:

$$\underbrace{\partial_\mu T^{\mu\nu} = 0}_{4} \quad , \quad \underbrace{\partial_\mu j^\mu = 0}_{1}$$

However, $T^{\mu\nu}$ has 10 independent components ($4X4$ symmetric matrix), and $j^\mu$ has 4. There is generally more to dynamics than conservation laws!

But local equilibrium/isotropy, in some frame, reduces these independent components drastically.

Lets make an approximation: The system is so big w.r.t. the constituents that we can divide it into "infinitesimal volume elements", each of which is infinitely big wrt constituents. Lets furthermore assume that the system expands so slowly wrt the microscopic dynamics that we can disregard microscopic non-equilibrium and just assume that pressure is the only force acting on the system, and the system is always in equilibrium.

In this case, $T^{\mu\nu}$ and $j^{\mu}$ are specified by just 6 parameters ($u_{x,y,z}, p, e, \rho$ )

$$T^{\mu\nu} = (e+p)u^{\mu}u^{\nu} - pg^{\mu\nu} \quad , \quad j^{\mu} = \rho u^{\mu}$$

Together with the equation of state, we have 6 equations with 6 unknowns. In principle, the system can be solved from any initial conditions

A note on entropy Since

$$s = \frac{dp}{dT} = \frac{p + e - \rho}{T}$$

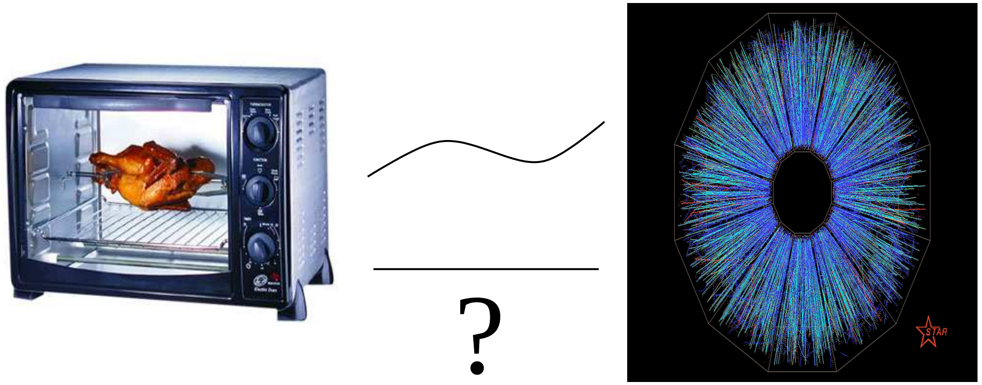
if $e(t), u$ continuus (No shocks or phase transitions!), entropy in an ideal fluid is always conserved, and its possible to rewrite hydrodynamic equations as

$$\underbrace{u^\mu \partial_\mu \left(T u_\nu\right) = 0}_{energy-momentum} \quad , \quad \underbrace{\partial_\mu \left(s u^\mu\right) = 0}_{entropy} \quad , \quad \underbrace{\partial_\mu \left(\rho u^\mu\right) = 0}_{charge}$$

All of hydrodynamics can be rewritten in terms of Speed of sound

$$c_s^2 = -\frac{dP}{de} \quad , \quad s = s(T_0) \exp\left[\int_{T_0}^{T} \frac{dT}{T c_s^2(T)}\right]$$

## Why we hope hydrodynamics works to some extent in Heavy ion collisions

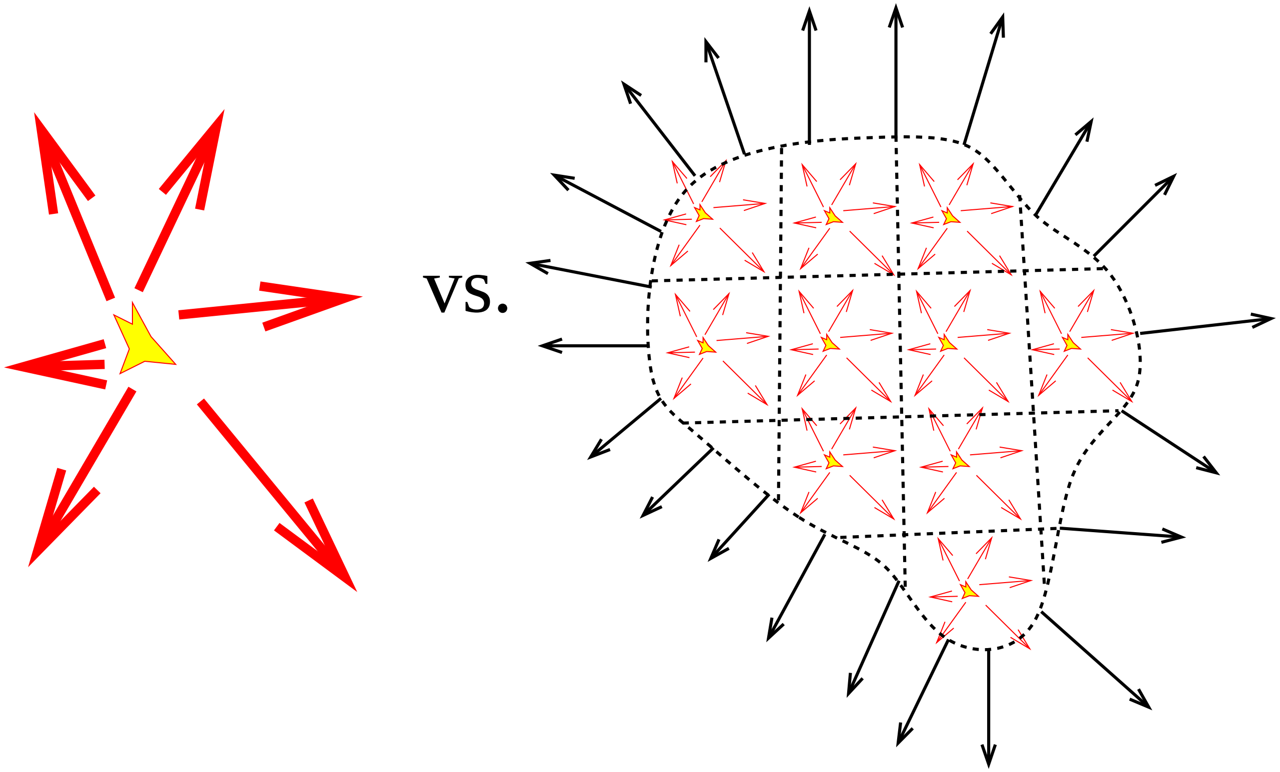



We are in the process of producing and studying the quark gluon plasma, a phase of matter. And of studying the phase transitions and in general the thermodynamics of strongly interacting matter.

But we are creating a very violent and fast explosion of particles. Phase transitions and thermodynamics in general are adiabatic phenomena, changes happen infinitely slowly! The best we can hope for if we want to see QCD thermodynamics is for hydrodynamics to work!

What is <u>not</u> Hydrodynamics:

Equilibration, especially "fake" equilibration, is different from LOCAL equilibration

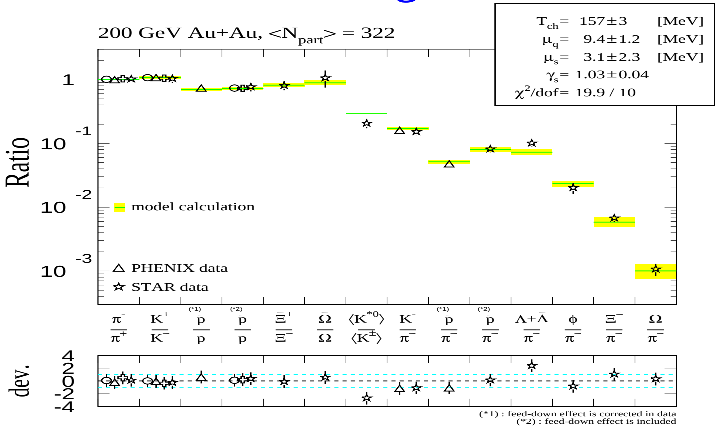


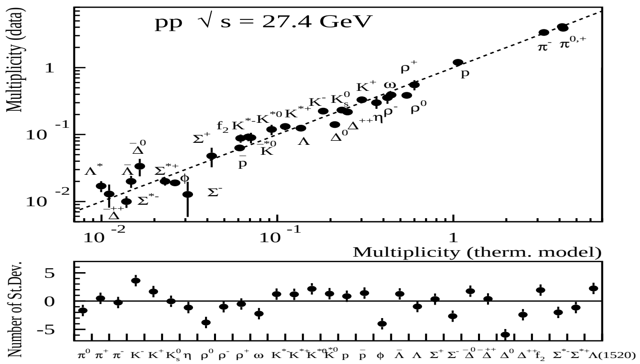
So this: (+Braun-Munzinger,Becattini,Rafelski,GT,...) is not (necessarily) a fluid!

Kaneta,Xu: RHIC Au−Au

Becattini et al:p−p,e+−e−

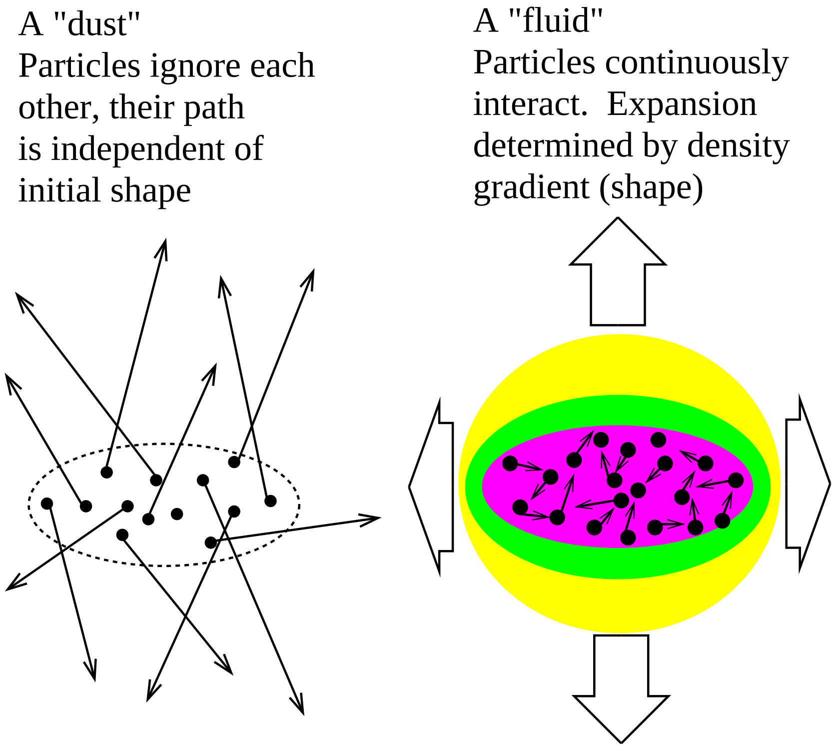
Also Braun−Munziger,Stachel,Rafelski,GT,...





many particle ratios with a wide range of masses described by only a temperature and chemical potential

No one knows what this means, explanations range from the mundane (phase space dominance) to esoteric (Confinement=Black holes!). But for hydrodynamics we need Temperature and flow.




Signature of local thermalization: Pressure $\rightarrow$ collective flow!

Changes in equation of state, viscosity etc. $\rightarrow$ transition

non-ideal hydro: Deviation from equilibrium "small".
Even if Equilibrium not ideal, we can still find a "flow vector" diagonalizing the symmetric $T_{\mu\nu}$. Eigenvalue will be the Energy density.

$$T_{\mu\nu} u^{\mu} = e u_{\nu}$$

In equilibrium, all other member of $T_{\mu\nu}$will be determined by $e, u_{\mu}$(and the Equation of state). Since we are "approximately" in equilibrium, we can integrate out (Coarse-grain) microscopic degrees of freedom. $T_{\mu\nu}$will then depend on $e, u_{\mu}$ and their gradients!

$$T_{\mu\nu} = \underbrace{(p+\rho) u_{\mu} u_{\nu} - p g_{\mu\nu}}_{ideal} + \Pi_{\mu\nu} \left( \partial u, \partial e \partial \rho \right)$$

## The form of $\Pi_{\mu\nu}$

- Since we integrated out microscopic dynamics, $\Pi_{\mu\nu} \sim f\left(\partial u, \partial p, \partial \rho\right)$ First term in gradient expansion: Only one $\partial u$(1 term in Taylor)

- These are not independent: $\partial e, \partial \rho$can be put to 0provided we choose a frame at rest with $e$(Landau Frame) or $\rho$(Eckart frame). For subsequent discussion we shall do it and forget $\rho$(Non-ideal Hydrodynamics with $\rho$ never implemented). Hence $u_\mu \Pi^{\mu\nu} = 0$

- 2nd law of Thermodynamics: $\partial_\mu s u^\mu > 0$

- Lorentz transformations and symmetries: Traceless part ("shear") and Traced part (bulk) have to be independent. Isotropy means that

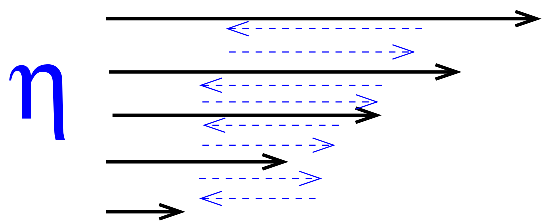
$$\Pi_{\mu\nu} \sim \underbrace{-}_{Friction} \underbrace{\alpha}_{Equilibrium} \underbrace{\sum \partial u}_{Traceless,traced}$$

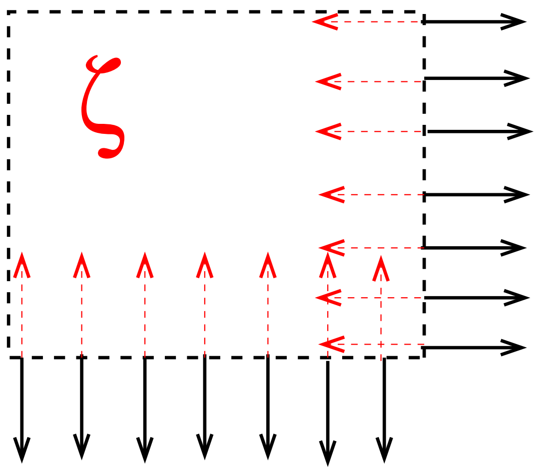
Putting all tese together, we find that the only allowed combination is

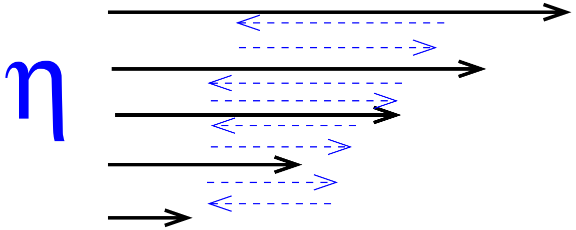
$$\Pi_{\mu\nu} = -\left(\zeta - \frac{2}{3}\eta\right)\partial_\alpha u^\alpha \left(u_\mu u_\nu - g_{\mu\nu}\right)$$

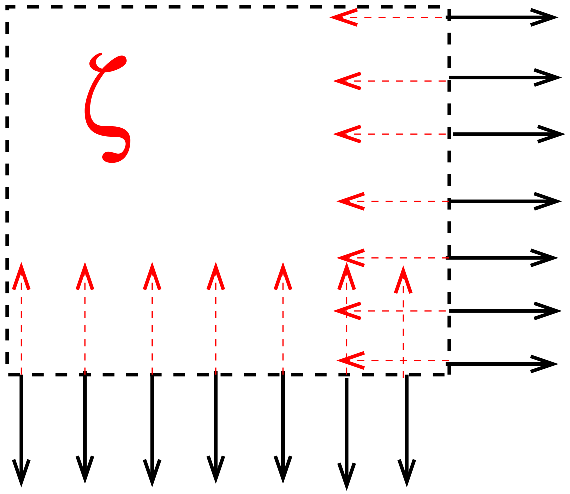
$$-\eta\left(\partial_\mu u_\nu + \partial_\nu u_\mu + u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu\right)$$

where Shear viscosity $\eta$ and bulk viscosity $\zeta$ are new equilibrium parameters!
($6 \to 8$ Equations with $6 \to 8$ unknowns. Complicated but still solvable!)






- Frictions, transforming Gradients into heat
  And hence increase entropy

- Shear viscosity diffusion of momentum, bulk viscosity diffusion across $T_\mu^\mu = e - 3p$(ie EoS)

- For a conformal gas, $\zeta$ (not $\eta$)=0

Sound waves Expanding Navier-Stokes equations around Static background

$$T_{\mu\nu} = Diag[e,p,p,p] + \delta T_{\mu\nu}(\delta p, \delta e, \delta u_L, \delta u_T)$$

yields dispersion relation for sound waves

$$\partial_t \delta e + ik\delta u_L = J^0$$

$$\partial_t \delta u_L + ic_s^2 k\epsilon + \frac{4}{3}\frac{\eta}{e_0+p_0}k^2\delta u_L = J^L$$

$$\partial_t \delta \vec{u}_T + \frac{4}{3}\frac{\eta}{e_0+p_0}k^2\delta \vec{u}_T = \vec{J}^T$$

Sound waves propagate at speed of sound $c_s^2 = dP/de$ , diffuse with a power of $k^2$ and a lenght scale $\sim \eta/(e+p)$ . Since Grand-Canonical energies, pressures uncorrelated, linearized relations can be used to extract viscosities from Energy momentum correlations with Quantum-Field theory techniques
Kubo formulae

$$\eta = \lim_{w \to 0} \frac{1}{2w} \int dt dx e^{iwt} \left\langle \hat{T}_{xy}(x) \hat{T}_{xy}(0) \right\rangle \quad , \quad \zeta = \lim_{w \to 0} \frac{1}{2w} \int dt dx e^{iwt} \left\langle \hat{T}_{\mu\nu}(x) \hat{T}^{\mu\nu}(0) \right\rangle$$

Usually Kinetic calculations (see next) simpler,through Kubo used in AdS/CFT.
...And we have a problem!

Fourier-Transforming

$$\partial_t \delta \vec{u}_T + \frac{4}{3}\frac{\eta}{e_0 + p_0} k^2 \delta \vec{u}_T = \vec{J}^T$$

we get the dispersion relation

$$w = \frac{4\eta}{3(e+p)} k^2$$

makes it clear that diffusion speed $w/k \sim k$ grows to $\infty$ as $k \to \infty$ (wavelength $\to 0$). Our theory has short-wavelength sound waves travelling faster than light. (A common problem to all diffusion-type equations)

Of course this effective long gradient theory should fail for short gradients, but is there a way to see it in effective theory language?

Yes! 2nd order in Gradient fixes the problem

$$\tau_\pi \partial_t^2 \delta \vec{u_T} + \partial_t \delta \vec{u}_T + \frac{4}{3}\frac{\eta}{e_0 + p_0} k^2 \delta \vec{u}_T = \vec{J}^T$$

It is intuitively clear that adding a $\partial_t^2$ (2nd order) term introduces a limiting speed into the dispersion relation that can be made to be $< c$, since then $w^2 + w \sim k^2 + k$ and $w/k \sim k^0$

Navier-Stokes equations, therefore, need to be extended to 2nd order to make then covariant. Effect of this is a time-scale for viscosity to turn on and lots of other complications!

$$
\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= \Pi_{\rm NS} + \tau_{\Pi q}\, q \cdot \dot{u} - \ell_{\Pi q}\, \partial \cdot q - \zeta\, \hat{\delta}_{0,1}\, \Pi\, \theta \\
&+ \lambda_{\Pi q}\, q \cdot \nabla \alpha + \lambda_{\Pi \pi}\, \pi^{\mu\nu} \sigma_{\mu\nu} + \hat{\delta}_{0,2}\, \Pi^2 + \hat{\epsilon}_0\, q \cdot q + \hat{\eta}_0\, \pi^{\mu\nu} \pi_{\mu\nu} \\
\tau_q\, \Delta^{\mu\nu} \dot{q}_\nu + q^\mu &= q^\mu_{\rm NS} - \tau_{q\Pi}\, \Pi\, \dot{u}^\mu - \tau_{q\pi}\, \pi^{\mu\nu}\, \dot{u}_\nu \\
&+ \ell_{q\Pi}\, \nabla^\mu \Pi - \ell_{q\pi}\, \Delta^{\mu\nu}\, \partial^\lambda \pi_{\nu\lambda} + \tau_q\, \omega^{\mu\nu}\, q_\nu - \frac{\kappa}{\beta} \hat{\delta}_{1,1}\, q^\mu\, \theta \\
&- \lambda_{qq}\, \sigma^{\mu\nu}\, q_\nu + \lambda_{q\Pi}\, \Pi\, \nabla^\mu \alpha + \lambda_{q\pi}\, \pi^{\mu\nu}\, \nabla_\nu \alpha \\
&+ \hat{\delta}_{1,2}\, \Pi\, q^\mu + \hat{\eta}_1\, \pi^{\mu\nu}\, q_\nu \\
\tau_\pi\, \dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} &= \pi^{\mu\nu}_{\rm NS} + 2\, \tau_{\pi q}\, q^{<\mu} \dot{u}^{\nu>} \\
&+ 2\, \ell_{\pi q}\, \nabla^{<\mu} q^{\nu>} + 2\, \tau_\pi\, \pi_\lambda{}^{<\mu} \omega^{\nu>\lambda} - 2\, \eta\, \hat{\delta}_{2,1}\, \pi^{\mu\nu}\, \theta \\
&- 2\, \tau_\pi\, \pi_\lambda{}^{<\mu} \sigma^{\nu>\lambda} - 2\, \lambda_{\pi q}\, q^{<\mu} \nabla^{\nu>} \alpha + 2\, \lambda_{\pi\Pi}\, \Pi\, \sigma^{\mu\nu} \\
&+ \hat{\delta}_{2,2}\, \Pi\, \pi^{\mu\nu} - \hat{\eta}_2\, \pi_\lambda{}^{<\mu} \pi^{\nu>\lambda} - \hat{\epsilon}_2\, q^{<\mu} q^{\nu>}
\end{aligned}
$$

D.Rischke,B.Betz,Henkel,Niemi,Muronga
Romatshke,Choudhuri,Song,Heinz,...

MANY coefficients not studied at all

Understood fully in conformally invariant theories (Romatschke,Son,...) and partially in pQCD (G. Moore)

Involved (+10 simultaneous equations)

$$
\zeta, \eta, \kappa, \quad \underbrace{\tau_\Pi, \tau_q, \tau_\pi}_{\text{relaxation times}} \;, \underbrace{l_{\Pi q}, l_{q\Pi}, l_{q\pi}, l_{\pi q} \dots}_{\text{coupling lenghts}}
$$

## Theory:

What is hydrodynamics really?

Its an effective theory! of what?

Microscopic picture: Boltzmann equation ( neglecting quantum correction):

$$\left(\frac{1}{m}p^\mu \frac{\partial}{\partial x^\mu} + F^\mu \frac{\partial}{\partial p^\mu}\right) f(x,p) = C^{2body}[f] + C^{3body}[f] + ...$$

$$C^{2body} = \int d^3[X,X',P,P']\sigma(P,P' \Leftrightarrow p,p')\left[f(X,P)f(X',P') - f(x,p)f(X',P')\right]$$

**Ideal hydro:** $C = 0$ (Gain=Loss) $f = \Upsilon e^{-p_\mu u^\mu/T}$ always,($T, u_\mu$ change)

**Non-ideal:** Expand $C[f]$ around $f - f_{eq}$ ,$\equiv$ Knudsen n.$K = l_{mfp}\partial_\mu u_\nu$

**Free-streaming:** $C[f] = 0$ (As $\sigma = 0$ ),

$$f(x^\mu, p^\mu) = \int d\tau dx'^\mu dp'^\mu f\left(x'^\mu p'^\mu\right)\delta\left[\frac{p'^\mu}{m}\tau - \left(x_\mu - x'_\mu\right)\right]$$

So the small parameter for hydro is the Knudsen Number $K = l_{mfp} \partial_\mu u_\nu$
Ideal hydro $O(K^0)$ ,Navier-Stokes $O(K^1)$ ,Israel-Stewart $O(K^2)$ . Note $K$ "really" a "tensor". (Grad expansion):

$$f = f_{eq}\left[\frac{u^\mu p_\mu}{T}\right]\left[1 + \underbrace{\epsilon}_{O(K^1)[\zeta]+higher} + \underbrace{\epsilon_\mu}_{O(K^1)[\eta]+higher} p^\mu + \underbrace{\epsilon_{\mu\nu}}_{O(K^2)+higher} p^\mu p^\nu + ...\right]$$

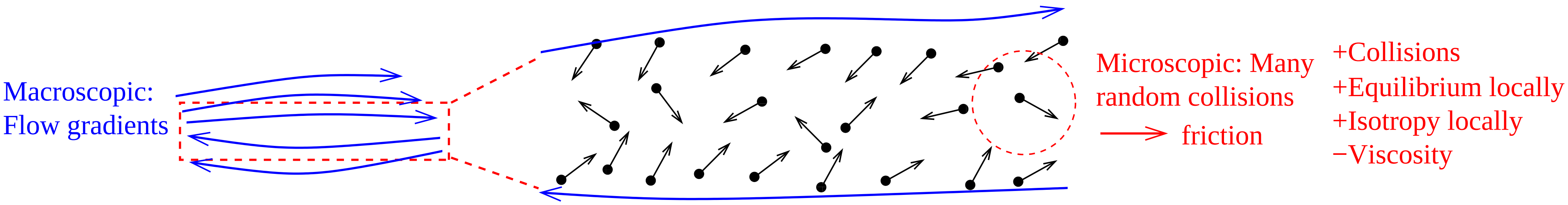
Plug into Boltzmann equation use $H$-theorem and obtain $\epsilon_\mu$ in terms of $\eta \partial u$ etc.. For first order, we can show that

$$\eta = \frac{1}{5}\langle p \rangle s l_{mfp} \quad , \quad \zeta = \left(c_s^2 - \frac{1}{3}\right)\eta$$

Last relation relies on 1 reaction, broken if elastic and inelastic collisions
equivalent to Kubo formulae in perturbative case!

$$So \qquad \eta \sim e l_{mfp} \sim sT l_{mfp}$$

Note: This means that $\eta/s$ is a "pure" number in natural units (no scale)! It reflects the "readiness of thermalization" of the system, the speed at which the the degress of freedom $\sim s$ rethermalize when disturbed (by a flow gradient). ( NB:Superfluid has low $\eta$ but also low $s$ .)




It might be counter-intuitive that a low $l_{mfp}$ (ie, a lot of reinteractions) mean low $\eta$ . But viscosity is a "diffusion" of momentum due to the finiteness of $l_{mfp}$ . When $l_{mfp}$ small, MANY collisions prevent diffusion

## $\eta$ and perturbation theory

Perturbation theory means,generally, weak coupling constant. Ie, a large mean free path and a large viscosity

$$\frac{\eta}{s} \sim e l_{mfp} \sim \frac{T}{\sigma_{crossection}} \sim \left.\frac{1}{\alpha^2 \ln \alpha}\right|_{perturbation\ \ theory} \sim_{any\ \ \underbrace{sensible}_{>1}\ \ \alpha}$$

$\eta/s < 1$ would require a $\alpha$ too large for calculation to work!

Attempts to lower this by many-body effects ($3 \leftrightarrow 2$ collisions, Plasma instabilities ). But low experimental viscosity (see later!) encourages us to look beyond perturbation theory

## Beyond weak coupling I

What happens when coupling is strong (non-perturbative)?
In the non-perturbative limit

- We can not anymore use the Scattering approximation, and hence molecular chaos. Microscopic degrees of freedom are strongly correlated.

- 3 particle interactions will be more likely than 2-particle, 4 particle more likely than 3 particle and so on...

Hence the use of the Boltzmann equation not justified.

## Is hydrodynamics justified at strong coupling?

PROBABLY: Remember the "Hydro as an effective field theory" derivation, relying on the gradient expansion of conserved number densities (Energy,momentum,charge,...),ie local averages of coarse-grained systems. Strongly interacting fields, since they... interact strongly, should always be approximately in a locally maximum entropy state. Hence, in local equilibrium. Hence, their dynamics should be approximately that of an ideal fluid.

## Is hydrodynamics justified at strong coupling?

Some people regard hydrodynamics as a limiting theory of the Boltzmann equation (and hydrodynamics people as "too stupid/lazy to do transport"). not quite true:Hydrodynamics is a limit of the Boltzmann equation, but it also applies to many other systems. any system where

- The second law of thermodynamics and causality apply (system is local and entropy increases!)
- the equilibration time is small wrt evolution of the local density ($\sim K$ in weak coupling).

These requirements are more general than those satisfied by the Boltzmann equation. There are systems where hydrodynamics applies and the Boltzmann equation is lousy. eg Water!

How low can the viscosity be? Lets forget we cant use the Boltzmann equation at strong coupling!A rough estimate: (Danielewicz and Gyulassy, 1987)

$$l_{mfp} \geq \langle \lambda_{debroglie} \rangle \sim 1/\langle p \rangle$$

If one plugs this into the Boltzmann equations and calculates viscosity the usual way, a lower limit is obtained

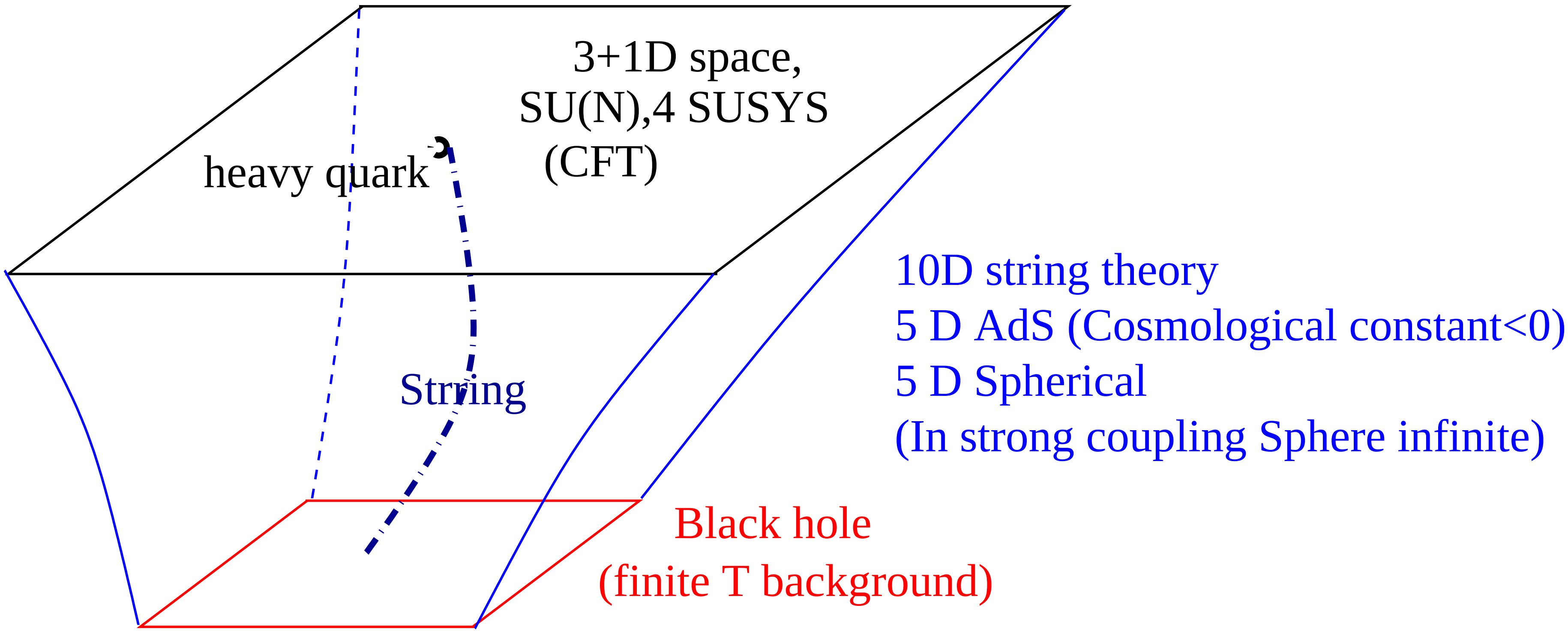
$$\eta/s \geq 1/(15\pi)$$

but this procedure is less than rigurous:Remember, we cant use Boltzmann!

## A way to make this (a bit!) more rigurous: Hydrodynamics (and viscosity) from AdS/CFT

The AdS-CFT correspondence: Every $\left\langle \hat{O}_{CFT} \right\rangle$ a 4D $N_{susy} = 4$ Gauge theory with $N_c$ colors and T'hooft coupling $\lambda$ , can be calculated by translating to a 10D string theory, with 5 Anti-DeSitter ($\Lambda < 0$ ) dimensions, 5 dimensions compactified on a sphere, and a string coupling constant of $g_s = \lambda/(4\pi N_c)$

- dictionary between $\hat{O}_{CFT}$ and $\hat{O}_{ADS}$ can be worked out
- Links strongly coupled CFT to weakly coupled perturbative string theory. Infinitely strongly coupled CFT $\Leftrightarrow$ classical supergravity.




$g_{\mu\nu}|_{asymptotic} \Leftrightarrow T_{\mu\nu}$

Finite $T$ background $\Leftrightarrow$ Black hole in AdS space

$\lambda \to \infty$ $\Leftrightarrow$ Classical geometry (Einstein's equations for $g^{\mu\nu}$ )

A BIG note of caution: This is NOT QCD (4 SUSYs, no quarks, $N_c, \lambda \to \infty$). This has the potential of introducing qualitative subtle differences.

**CFT** The theory is conformally invariant. No running coupling, no phase transition, no hadrons, no bulk viscosity

**QCD** Is approximately conformally invariant at weak coupling, big-time non-invariant at strong coupling

But we just want to check that hydrodynamics works in a strongly coupled theory, so thats OK as a "toy-model" (still: CFT is a symmetry QCD does not have. And its a conjecture. So Caveat Emptor!).

**Entropy density** Can be extracted from the entropy of the Black hole: $s = \frac{3}{4} s_{SB}$

$\eta$ Can be gotten with the Kubo formula, via the linearized theory of perturbations of a Black hole in AdS-space $\eta \sim \lim_{w \rightarrow 0} e^{iwx} \langle h_{\mu\nu}(0) h_{\mu\nu}(x) \rangle$. Plugging in the numbers we get the famous "limit"

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

(Compare with Kinetic theory limit of $1/15\pi$ ).
NB: It seems the bound is violated for more complicated dual theories. not clear if $\eta/s$ can go to 0.

Hydrodynamics can be investigated by perturbations on the black hole. It seems that strongly coupled system can indeed be described by Israel-Stewart equations (Janik,Peshanski,Kovchegov,Minwalla,... ). All coefficients compatible with CFT worked out (Baier,Romatschke,Son,... )! Usual hydrodynamic phenomena (Sound waves, Mach cones) are there and are very similar to expectations from Navier-Stokes equations (eg Chesler+Yaffee,Yarom+Pufu+Gubser,Noronha+Torrieri,...)

NB:AdS/CFT more general than hydrodynamics. No equilibrium assumption present, $\langle T_{\mu\nu} \rangle$ calculated from "quantum field theory". Higher order calculations (eg $\langle T_{\mu\nu} T_{\alpha\beta} ... \rangle$ ) possible Ab initio (unlike hydrodynamics).

NB2:all AdS/CFT calculations up til now, too idealized to be reliably compared to experiment directly. But a fast-developing field

## A recap on the theory

Hydrodynamics is an effective theory, where the "small parameter" is the thermalization time (as opposed to the macroscopic evolution of the system).

It is expected to arise as a limiting case of any system which is local, and where energy conservation and the 2nd law of thermodynamics apply.
This includes a wide variety of systems (Boltzmann equation, strongly coupled theories with string duals, water,...)

The ingredients (Equation of state, transport coefficients) are calculable from <u>equilibrium thermodynamics</u>. Thus, hydrodynamics is the "best way" to link statistical physics to evolving systems. If we want to study the statistical properties of QCD, we <u>want</u> hydrodynamics to be a good approximation!

## Cuisine:

What are the ingredients of a Hydrodynamic model?

Ideal Hydro equations

$$\frac{\partial}{\partial t}\left[(P+e)\gamma u^{\nu}-P\delta_0^{\nu}\right]=-\frac{\partial}{\partial x_i}\left[(P+e)\gamma\vec{v_i}u^{\nu}+P\delta_i^{\nu}\right]$$

$$\frac{\partial}{\partial t}\left[(\rho_{B,S})\gamma\right]=-\frac{\partial}{\partial x_i}\left[\rho_{B,S}\gamma\vec{v_i}\right], P=\left[-T\ln\left(Z_{GC}\right)\right](e,\rho_B,\rho_S)$$

solvable $N_{equations}=N_{unknowns}\ (\gamma,e,P,\rho_{B,S})$ but

non-linear (all unknowns functions of $x,t$) but

Flux-conserving $\frac{\partial U}{\partial t}=-\frac{\partial}{\partial x_i}\left(U\vec{v_i}+f_i\right)$ but

Expensive (disentangling $\vec{v_i},e,P,\rho B,S$ from $U$, due to non-linear terms in $\gamma$,EOS)

Non-linear Eulerian hydrodynamics:Solve

$$\frac{\partial U}{\partial t} = -\frac{\partial}{\partial x_i}(U\vec{v_i} + f_i)$$

on a lattice from initial conditions

$$U \to U_i^t = U_i^{t-dt} + dt\frac{dU^t}{dt}$$

$$\frac{\partial}{\partial x} \to \frac{U_{i+1} - U_i}{\Delta x}$$

N dimensions $\Rightarrow$ Operator splitting (N 1D steps)

Lagrangian hydrodynamicsGrid moves with fluid. Sometimes used, will not discuss it here

Shocks/discontinuities from Non-linearity of EoS and sharp initial conditions Euler method may fail, through it works unexpectedly well for cross-over transition! (Romatscke et al,Chojnacki et. al.).

Many algorithms, with advantages/cons. Excellent papers by Rischke et al describing and comparing them. See also review by Marti', Muller

**Godunov-type methods** (PPM,HLLE,...) Shuryak,Hirano,...

Based on analytical "step" solution of hydro equations, each square propagated using this solution

**FCT** (SHASTA,LPFCT,...) Kolb,Heinz,Rischke...

Runga-Kutta+A correction step for numerical diffusion based on Flux conservation

**SPH** Kodama,Grassi,...

Fluid discretised into particles

Bottom line check,check,check...

- Does it reproduce well-known analytical solutions?
- Does it conserve entropy/produce appropriate amount of entropy? (Ie, is numerical viscosity "under control"?)
- Does it reproduce correct dispersion relations for sound?
- Do different groups reproduce the same solution given initial conditions, EoS?

Caveat Emptor! (But check out the tech-QM collaboration!
https://wiki.bnl.gov/TECHQM/index.php/Bulk_Evolution

## Hydro Cuisine

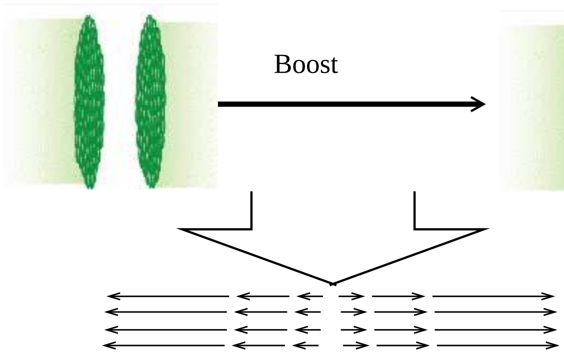
Now, we just need to know what happens...

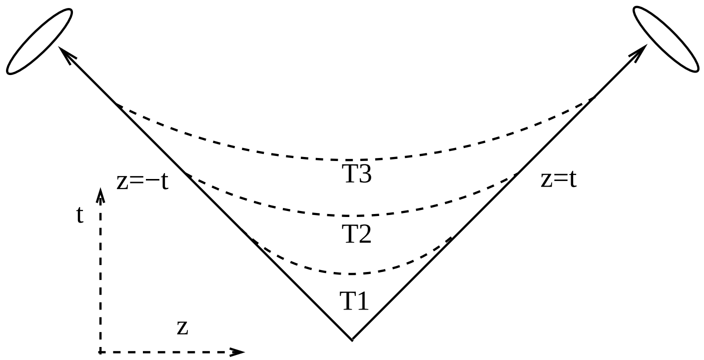
**Before** (Initial conditions)

**During** (Equation of state)

**After** (Decoupling)

## A useful coordinate system: Bjorken hydrodynamics




Best to reparametrize $t, z$ coordinates into

$$\alpha = \frac{z+t}{z-t} \quad , \quad \left( y = \frac{p_z+E}{p_z-E} \right) \quad , \quad \tau = \sqrt{t^2 - z^2} \quad , \quad \left( m_T = \sqrt{E^2 - p_z^2} \right)$$

Perfect Boost invariance: Physics independent of $y, \alpha$,only function of $\tau$

Boost-invariance⇔Transparency,so higher $\sqrt{s} \rightarrow$ more boost-invariance

Hydrodynamic equations in transversely homogeneus Bjorken equation reduce to

$$\frac{dP}{d\tau} + \frac{e+p}{\tau} + \frac{\zeta + 4\eta/3}{\tau^2} = 0 \quad , \quad \frac{d\rho}{d\tau} + \frac{\rho}{\tau} = 0$$

1D equivalent to Hubble equations for flat space

Boost-invariant flow is an "attractor":Even in "Landau" Hydrodynamics (initial condition a small "Brick" in $z$), dynamics at $|y| \sim 0 \ll |y_{+,-}|$ resembles Bjorken after a few $fm$.

Nevertheless, It is unclear how boost invariant the system is in reality and how it varies with $\sqrt{s}$ (More on this later).

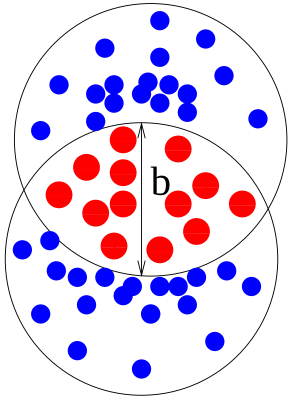
Bjorken hydrodynamics exactly solvable.

$$\int_{e_0}^{e_{freezeout}} \frac{de}{e+p} = \int_{\tau_0}^{\tau_{freezeout}} \frac{d\tau}{\tau} + f_{characteristic}\left(\zeta + \frac{4}{3}\eta, \tau\right)$$

At $\tau_0 \to 0$ equations diverge (not surprising).
If $\tau_0$ known, ideal 1D hydrodynamics gives rise to the famous Bjorken formula.

$$\frac{dE_T}{dy} = \underbrace{e(T_0)}_{Initial\ \ e} \left(\pi A^2 \tau_0\right)^{-1}$$

What could $\tau_0$ be? Naively, $\sim l_{mfp}$ or bounded by uncertainity principle $\tau_0 \sim 1/T_0$.




# Transverse initial conditions: The Glauber model

- Independent superimposed collisions $N_{coll}$ (Geometry)
- Each "Wounded nucleus" ($> 1$ collision) gives off energy

$$\frac{dN}{dy} = aN_{part} + bN_{collisions}$$

$a, b$ fitted to data (Cant calculate energy released into $y = 0$ region)

## The Color-Glass condensate:an alternative initial condition

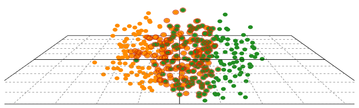
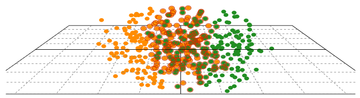
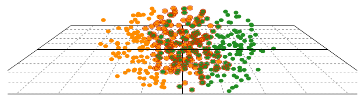
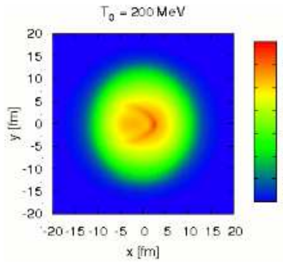
High in $\sqrt{s}$ (RHIC?) soft particle production dominated by Gluons at low $x$ ("Saturation scale"): $Q_s = x_s \sqrt{s}$ set by balance between gluon splitting and fusion). One can argue that in this regime gluon field

- Random (Neighbouring Color vertices point in random directions)
- Classical, solvable by

$$\partial_\mu F^{\mu\nu} = J^\nu|_{random \quad source}$$

This model has been used to generate initial conditions for hydro. Gradients steeper than in Glauber. Hence, if CGC valid, $\eta/s$ needs to be bigger to compensate.In general, initial conditions and viscosity correlated

NB: A note on fluctuations

, , ,.... $\neq$ 


At most < + + +..>=

Initial conditions in all models vary a lot e-by-e. So, for example $\left\langle \epsilon^2 \right\rangle \neq \left\langle \epsilon \right\rangle^2$, needs to be accounted for in $v_2$ calculation

Really need to produce ensemble of events and evolve them with hydro. But expensive, and effect of non-ideal conditions on fluctuations unclear! (If viscosity too low, Turbulence (Enhanced fluctuations)
Viscosity damps fluctuations, but at large $K$ microscopic interactions enhance them. Not Clear! GT,Vogel,Bleicher,nucl-th/0703031

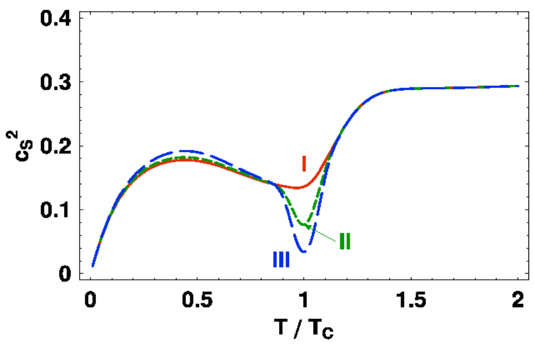
Elliptic flow, Mach cones susceptible to this, see later

## Equation of state

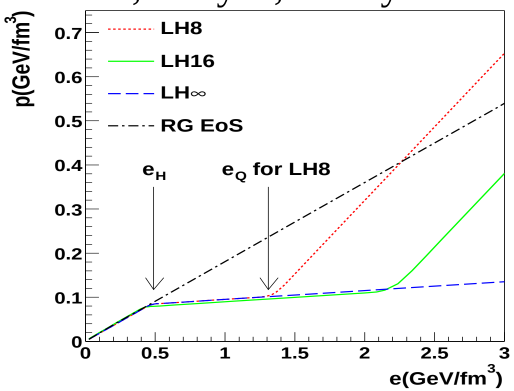
$T < T_c$ Resonance-gas model (RG) ,$T >> T_c$ $P = \alpha P_{SB}^{N_f,N_c}$

**Mixed** : First order hydro (Maxwell construction) or smooth Cross-over (Interpolation)

Chojnacki,Florkowski

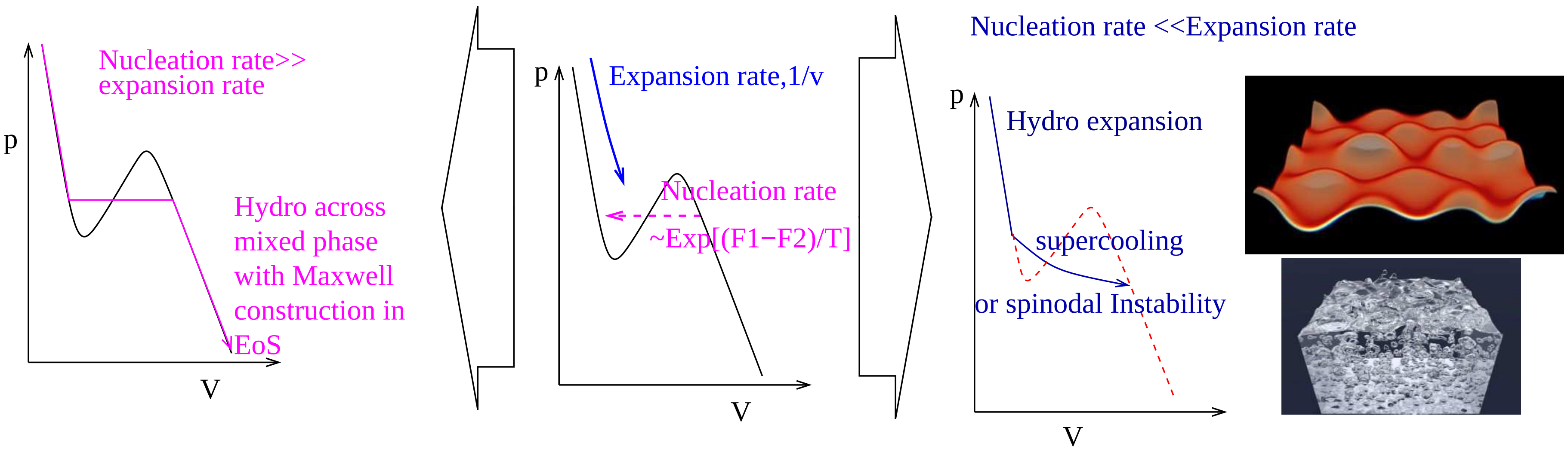



Lauret,Shuryak,Teaney




Lattice: $At small \rho$ Cross-over,large $\rho$ 1st order.

## Nature of phase transition important for




- HBT (See Later!), Numerics (Careful with shocks!)
- Nucleation? Another "macroscopic" scale: Time of transition between Coexisting phases! If large, Hydro <u>not</u> valid (nucleation,supercooling,Spinoidal,... etc.)

## Freeze-out ?????????????????????

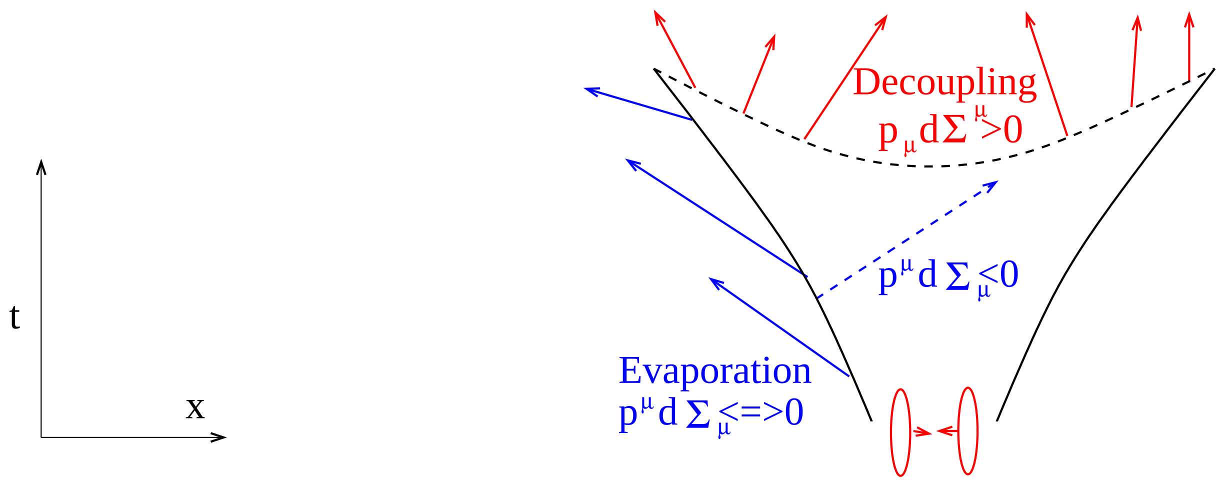
Approximation: $l_{mfp}$ goes $\eta/(sT) \to \infty$ instantaneusly according to some local criterion (T,K,... ), Conservation of $p^\mu, s \to$ Cooper-Frye formula

$$\left( E\frac{dN}{d^3p} \right)_i = \int d\underbrace{\Sigma_\mu p^\mu f(p_\mu u^\mu, T, \mu)}_{ideal} \underbrace{\left[ 1 + \frac{p_\mu p_\nu \Pi^{\mu\nu}}{2T^2(e+p)} \right]}_{viscosity} + \underbrace{\left( E\frac{dN}{d^3p} \right)_{j \to i}}_{resonances}$$

$d\Sigma_\mu$:Spacetime, + a local criterion$\to$ 3D Hypersruface $\Sigma_\mu$ parametrizable in terms of 3 parameters $u, v, w$(eg, $t = t_f(x_f, y_f, z_f)$or $t = t_f(\tau_f x_f, y_f, \eta_f)$). Then, by Stokes's theorem

$$d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma} \frac{\partial^\alpha}{\partial u} \frac{\partial^\beta}{\partial v} \frac{\partial^\gamma}{\partial w}$$

Self-evidentProblem:What if $d\Sigma_\mu p^\mu < 0$?




Physically: Particles emitted into the fluid. Need backreaction of fluid to emission to analyze properly.

Bugaev: Add $\Theta(\Sigma_\mu p^\mu)$ to Cooper-Frye, but this introduces a small violation of $\langle p^\mu \rangle$, entropy. To do better, "Post freeze-out" Transport? Escape probability? Mean fields? Lots of papers but no consensus! important observables not (?) so sensitive to freeze-out (except 1!)

## A cuisine recap

- Hydrodynamic numerics is non-trivial. Any numerical solution needs to be thoroughly checked.

- Initial conditions have to be known before transport properties can be said to be under control. This is a systematic uncertainity of present viscosity estimates $\eta/s$ can change from 0 to $\sim 2$

- Freezeout not understood on a conceptual level

# Science: What hydrodynamics can and cant describe

# Flow: Transverse and Elliptic ($v_2$)

## Science

So, we have everything. What can we calculate?

And how are we doing?

- Spectra (transverse flow) OK, but...
- $v_2$ (Elliptic flow) Too well!
- HBT radius (collision shape) not good enough!
- Mach cones????

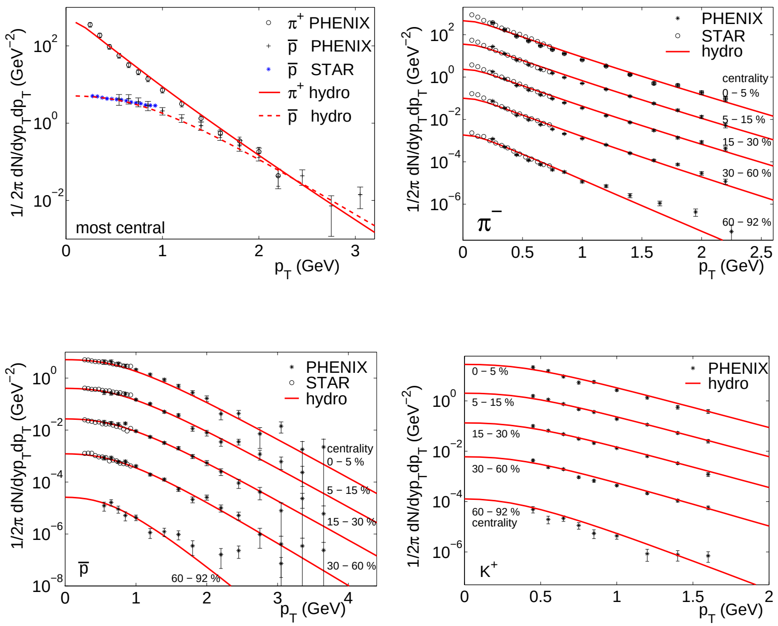
## A general consideration

Hydro cannot fit data, since, given initial condition and equation of state, hydro is deterministic. To fit data, use hydro-inspired models

$$E\frac{dN}{dy p_T dp_T} = \int_r dr \left( 1 - \left.\frac{dt}{dr}\right|_{freeze-out} \right) \exp\left[ -\frac{\gamma(E - v_T p_T)}{T} \right]$$

Where $\frac{dt}{dr}, v_T, T, ...$are fit parameters
Eg, Blast-wave (Heinz, Shnedermann, experiments....: $dt/dr = 0$) or "burning log" $dt/dr < 0$
Parametrize dependence of $T, v_T$on $r, y$ $\rightarrow$MANY parameters!
(Also resonances, separate chemical and thermal f.o.,....)
Bottom line:A hydro-inspired fit is nice, but to understand the bulk equation of state at early times, we need hydro!

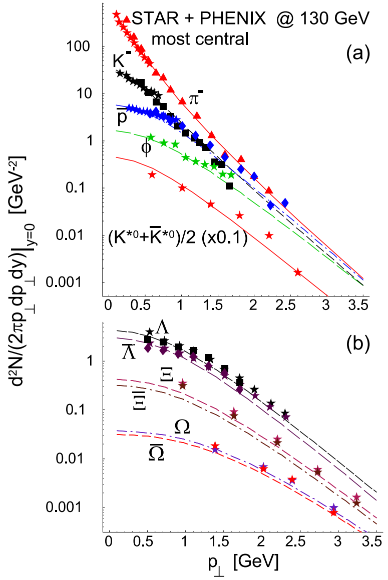
# Spectra (transverse flow)




From a critical temperature or one spectrum, we can get spectra of all particles at all centralities

All hydro-inspired models achieve similar fit quality. which is not good news

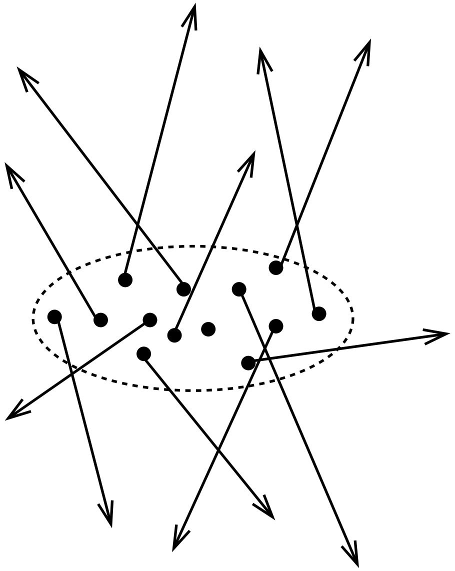
This description is not unique. Most hydro assumes decoupling temperature of $\sim$ 100 GeV and neglects resonances.
Florkowski, GT, Rafelski,... : hydro-inspired model w. resonances and high-T freeze-out (140 or 170 MeV) also works. So where is the freeze-out?



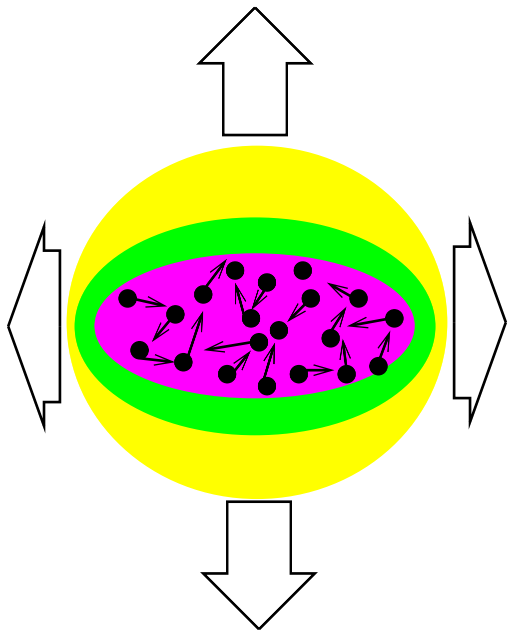

If freeze-out really at $\sim$ 100 GeV, most flow generated at later stages of collision. Does NOT constrain earlier interesting stage (Gyulassy...)

# Anisotropy

A "dust"
Particles ignore each other, their path is independent of initial shape



A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape)



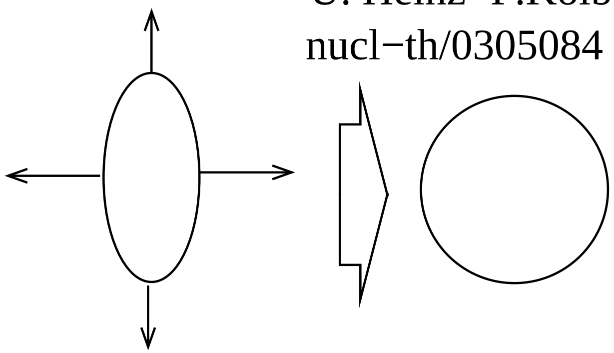
Initial Space Anisotropy $\Rightarrow$ hydro $\Rightarrow$ flow anisotropy
Ollitraut: Good observable for early dynamics
Poskanzer: a good way to Parametrize

$$E\frac{dN}{d^3p} = E\frac{dN}{dydp_T}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi)\right]$$

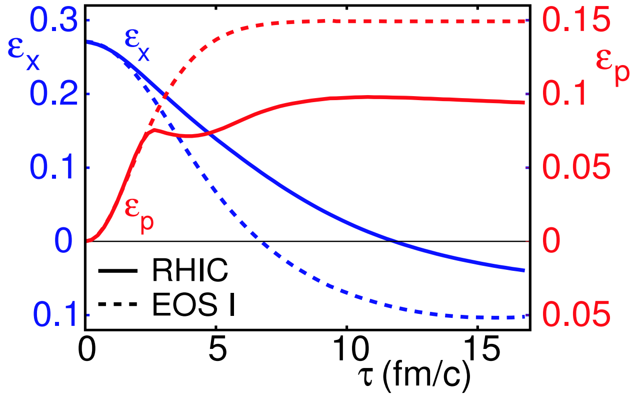
$v_1$ called directed flow, $v_2$ elliptic flow.
Important note:If Cooper-Frye holds

$$v_n \sim \int \cos(2\phi) \exp\left[-\frac{E - p_T v_T (1 + \sum_n \delta u_m \cos(m\phi))}{T}\right]$$

So each harmonic in the flow $\delta u_m$ influences all $v_n$ with a weight $I_{n-m}(p_T \delta u_m / T) \neq 0$ . Hence, fluctuating initial conditionsintroduce uncertainity in all $v_n$ (and Mach cones, see later!). Work to be done here!



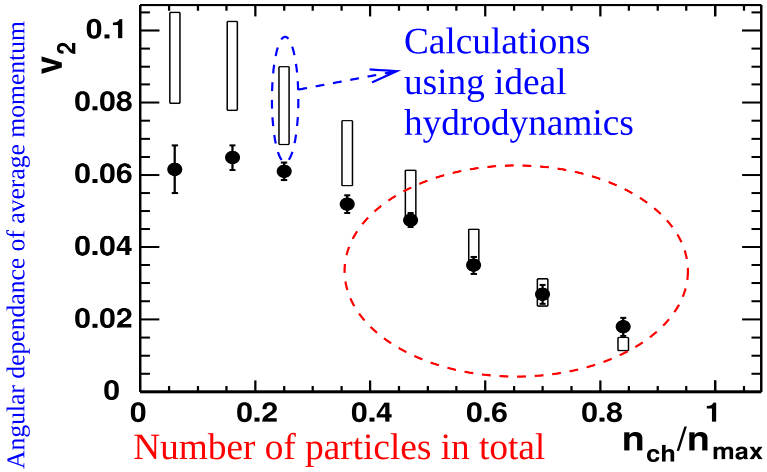
U. Heinz P.Kolb
nucl−th/0305084



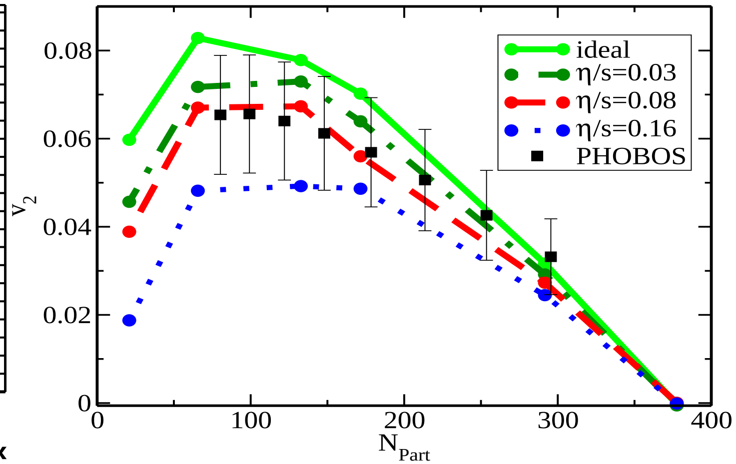

- $v_2$ "self-quenching": As it is formed, system becomes more spherical (and dilute). Hence, $v_2$ forms quickly and saturates. Because of this, it is sensitive to the early stages of the collision, and less sensitive to freeze-out (good!)

- It is a gradient, and viscosity, as we saw, transforms gradients into heat. Hence, a lot of viscosity kills $v_2$

P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.




P.Romatschke,PRL99:172301,2007



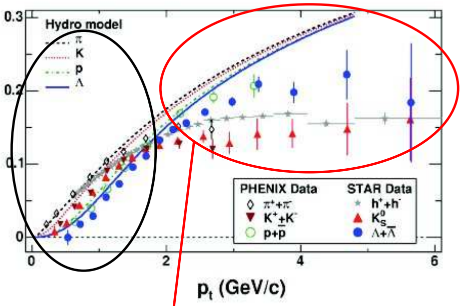

$v_2$: Too good at RHIC!

- Ideal hydro holds for all high centrality bins
  Heinz, Kolb:early thermalization "Puzzle"

- Teaney: Shear viscosity would make things worse.
  Shuryak: "Sticky Molasses", better than liquid He

Compilation by STAR collaboration

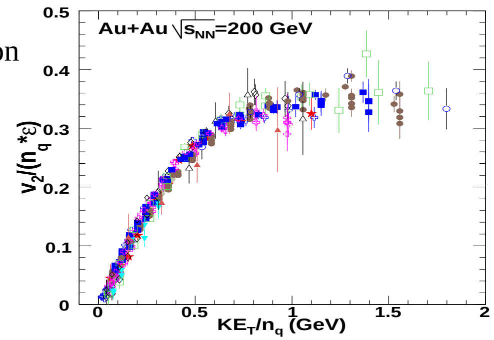
Hydro +Cooper−Frye applies (mass scaling)




Hydro stops applying (Saturation)

VS

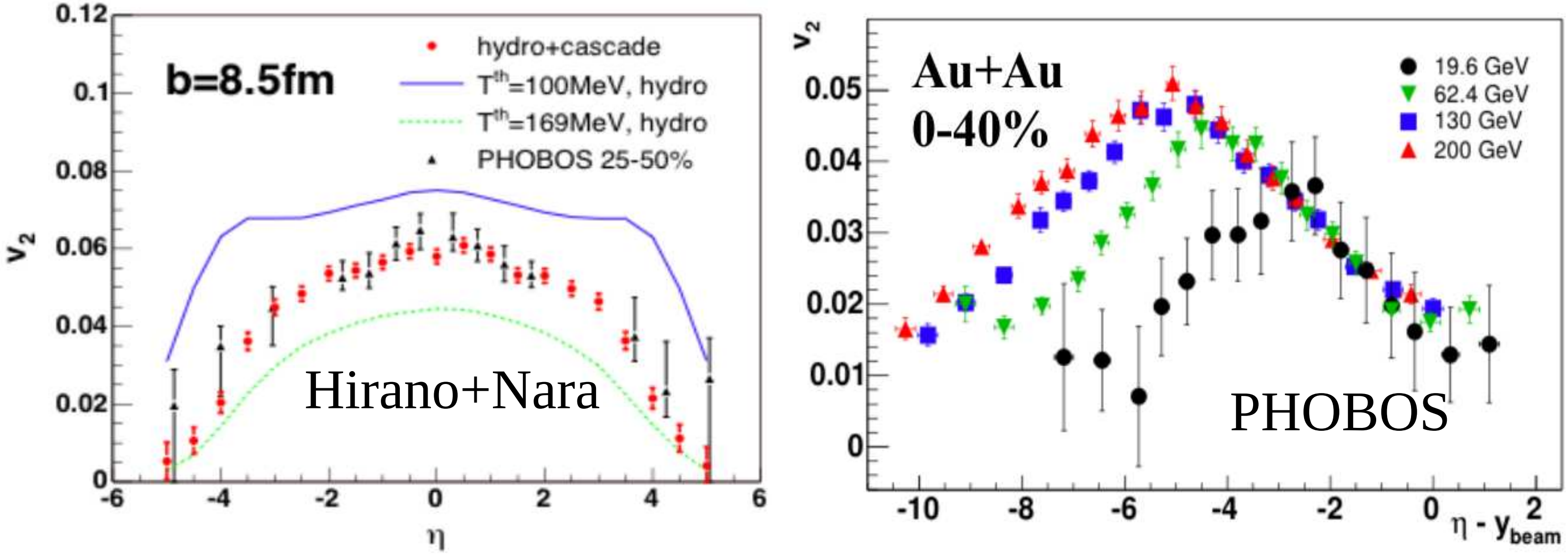
PHENIX Collaboration PRL 98 162301 (2007)




- At low $p_T$ hydro does a good job at accounting for $v_2$ of most particles
- $v_2$ Mass dependance, expected from hydro, works well
- At intermediate $p_T$ this fails. Meson/Baryon scaling takes over $\rightarrow$ COALESCENCE? At what point does coalescence stop working?

If coalescence works at all momenta, conclusions from hydro have to be revises, as partonic flow $\neq$ medium flow. Big systematic uncertainity.

BUT: $v_2$ dependence on rapidity far from Boost-invariant

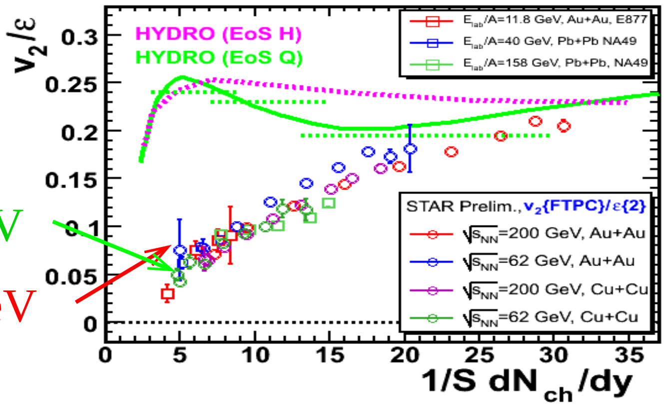



Hydro can fit this with reasonable $y$ dependence on initial conditions. But scaling ($\sim$ Universal fragmentation) looks waay too simple! No one knows (and it would be great to find out!) how much such simple scaling constrains hydro!

Compilation by STAR collaboration,QM06

This is Cu−Cu@200 GeV

This is Au+Au@11.8 GeV




A nice way to compare different energies, centralities is to plot $v_2/\epsilon$($\epsilon$ =eccentricity of initial almond) vs $\frac{dN}{Sdy}$($S$ =Surface of almond).

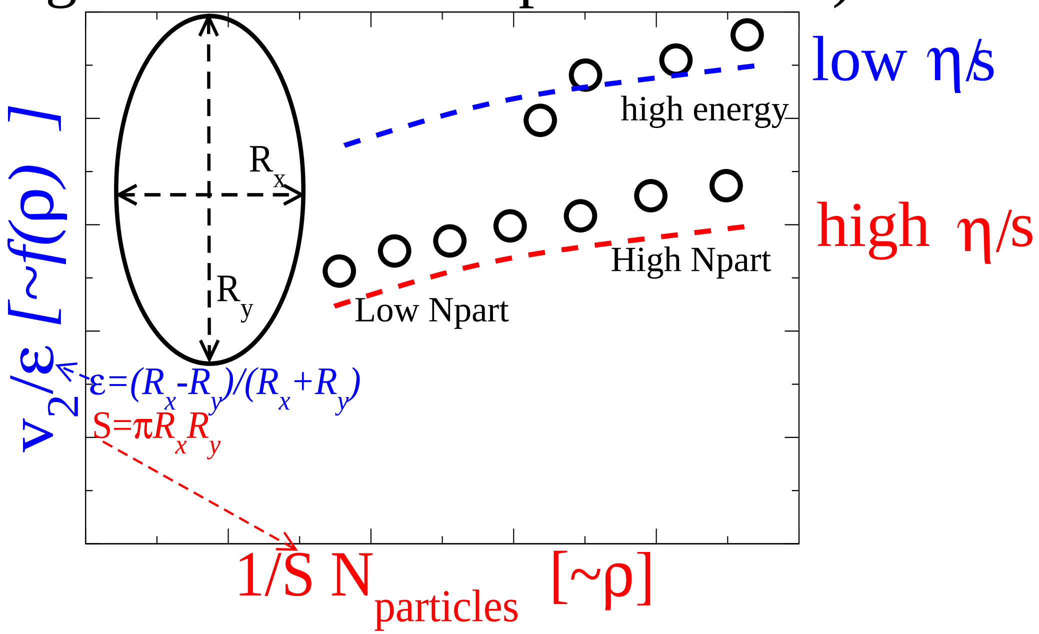
$$v_2 \sim \epsilon(1 - O(1)\frac{\eta}{s}) \quad , \quad \frac{1}{S}\frac{dN}{dy} \sim s\left(1 + \frac{1}{s}\frac{ds}{sy}\right)$$

Transition from viscous to good liquid should signal a break in scaling. Scans in energy and system size allow us to compare systems with same $1/SdN/dy$, very different $\sqrt{s}$ ($\sim T_0, ds/dy$)

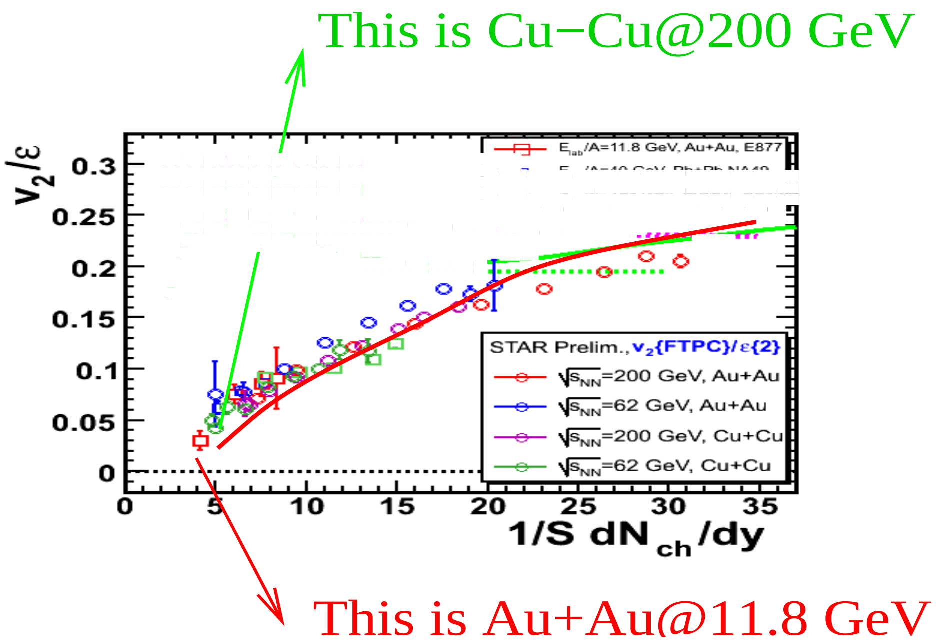
But, when (energy, system size) does this perfect fluid form?

GT, Phys.Rev.C76:024903,2007: QGP transition should mean a change in the speed of sound and drop in the mean free path

Expectation (If high v2 at RHIC signals transition to perfect fluid)



What does experiment say?



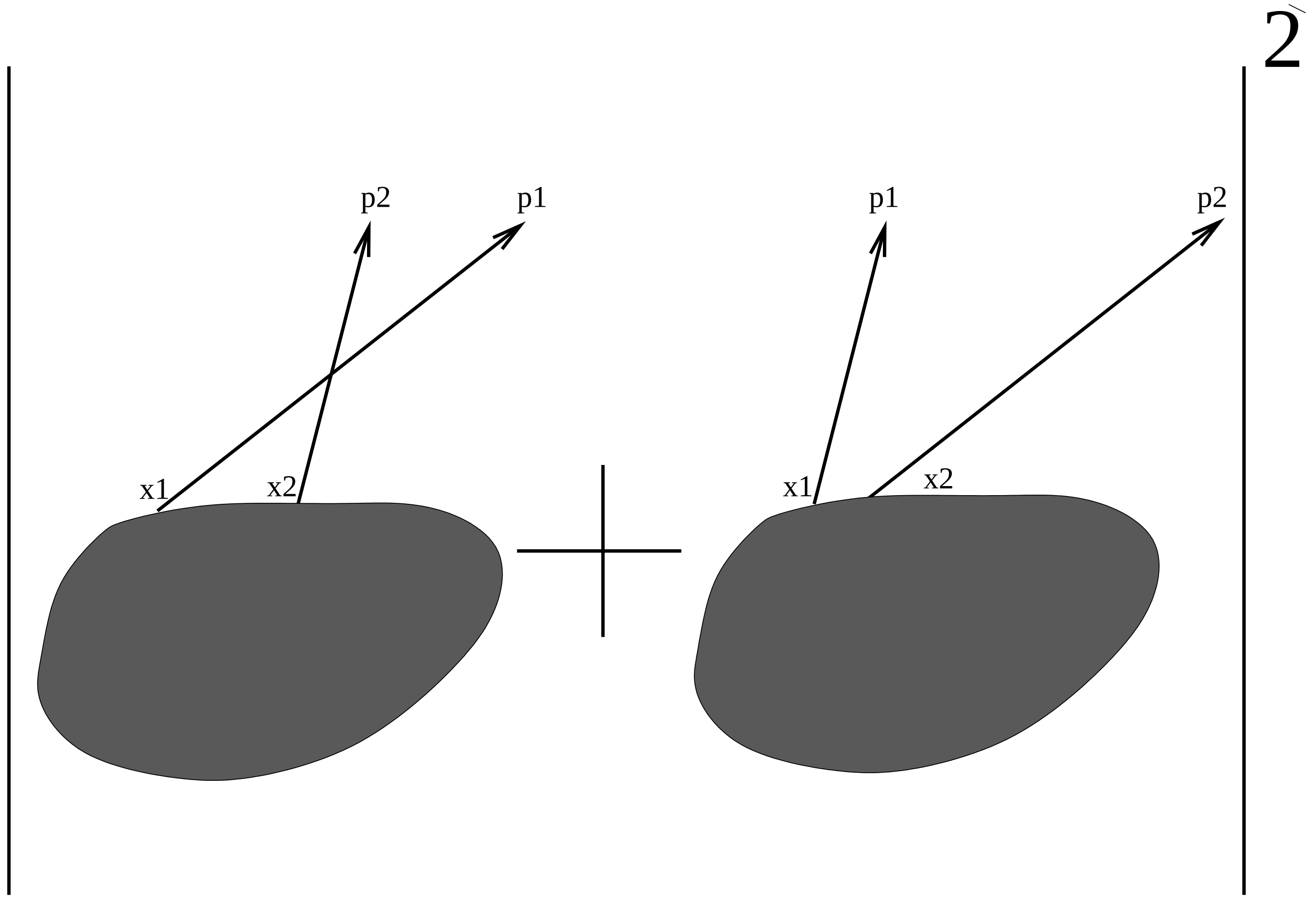

Scaling holds in the same way, smoothly, for all energies system sizes examined so far. When does the "perfect liquid" form?

## A recap on $v_2$

- $v_2$ very sensitive to viscosity in the early stages of the evolution
- It is therefore very interesting that hydrodynamics works. Evidence of a good fluid.
- Scaling with rapidity, system size difficoult to explain

# HBT: The spacetime picture

HBT: classical source emitting quantum free particles



$$\Psi(x_{1,2},p_{1,2}) = \frac{1}{\sqrt{2}}\Big(S(x_1,p_1)S(x_2,p_2)e^{i(p_1x_1+p_2x_2)} \pm S(x_2p_1)S(x_1p_2)e^{i(p_2x_1+p_1x_2)}\Big)$$

Measurement of $C(p_1,p_2)$ gives handle on $S(x,p)$

$$C(p_1,p_2) \sim |\tilde{S}(p_1-p_2,p_2)|^2$$

Where the <u>momentum correlation coefficient</u> $C(p_1,p_2)$ is

$$C(p_1,p_2) = \frac{\rho(p_1,p_2)-\rho(p_1)\rho(p_2)}{\rho(p_1)\rho(p_2)}$$

And $\tilde{S}(k,q) = \int d^4xS(x,q)e^{ikx}, S(x,p) = d\Sigma_\mu p^\mu f(p_\mu u^\mu, T)$ given by the differential Cooper-Frye formula

Usually $\tilde{S}(q,p) \sim$ Gaussian $\Rightarrow$ parametrization in terms of $R_{out}, R_{side}, R_{long}$

$$S(\underbrace{k}_{p_1+p_2}, \underbrace{q}_{p_1-p_2}) \simeq N(k)\exp\left[R_o^2(k)q_o^2 + R_s^2(k)q_s^2 + R_l^2(k)q_l^2 + R_{ij}(k)q_iq_j\right]$$

S.Pratt, PRD33, 1314 (1986), G. F. Bertsch, NPA498, 173c (1989).

**"long"** Beam direction $(\vec{z})$

**"out"** $(\vec{p}_1 + \vec{p}_2) \times \vec{z}$

**"side"** "out" $\times$ "long"

$k_{side} = 0$ by construction

This parametrization is useful because...

If

$$\left\langle (\Delta x^\mu)^2 \right\rangle (p) = \int d^4x S(x,p)(x - \langle x \rangle)^2$$

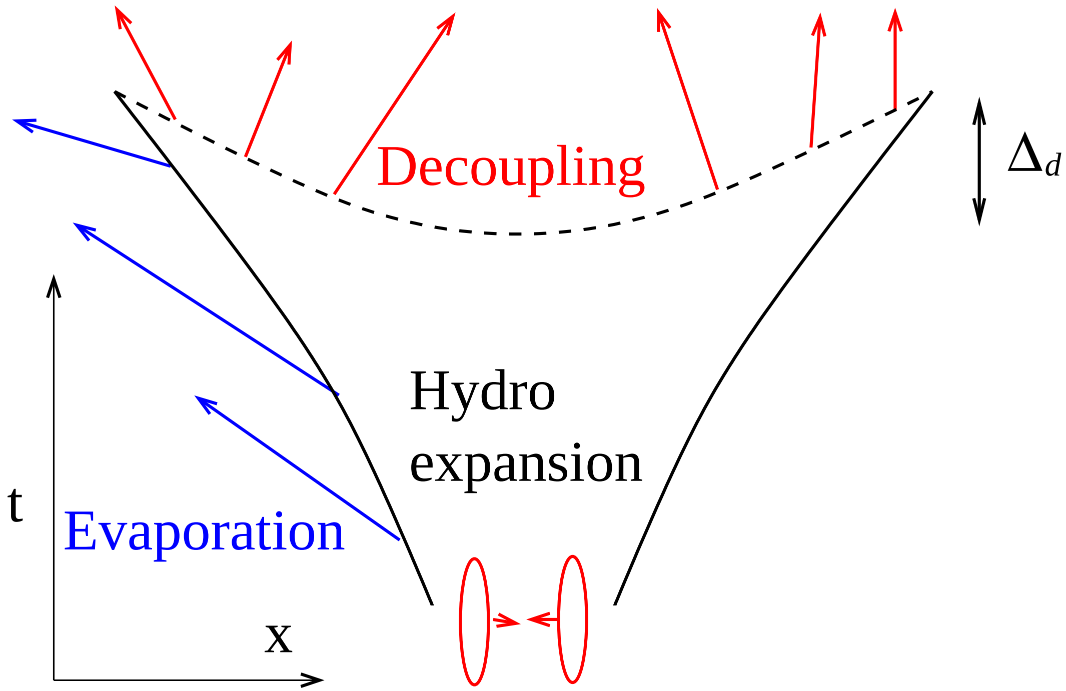
then

$$R_o^2 = \left\langle \left( \Delta r - \frac{k_o}{k_0} \Delta t \right)^2 \right\rangle$$

$$R_s^2 = \left\langle (\Delta r)^2 \right\rangle$$

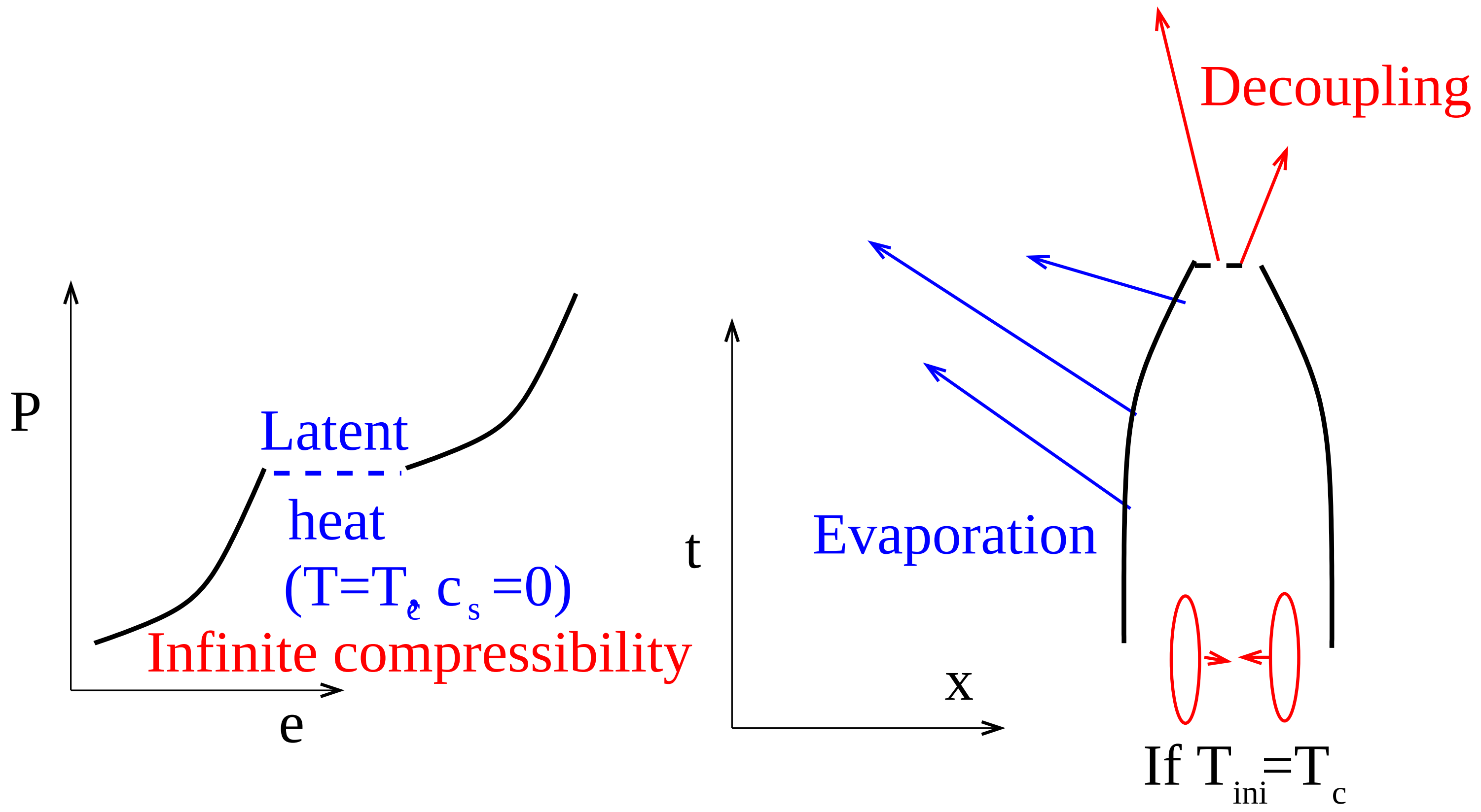
Comparing $R_0$ and $R_s$ $\longrightarrow$ emission time. This was "the" signature for deconfinement!

“generic” fireball (starting energy away from $T_c$), evolution by hydrodynamics, $d\Sigma^{\mu}$ given by critical $T \sim 100$ MeV

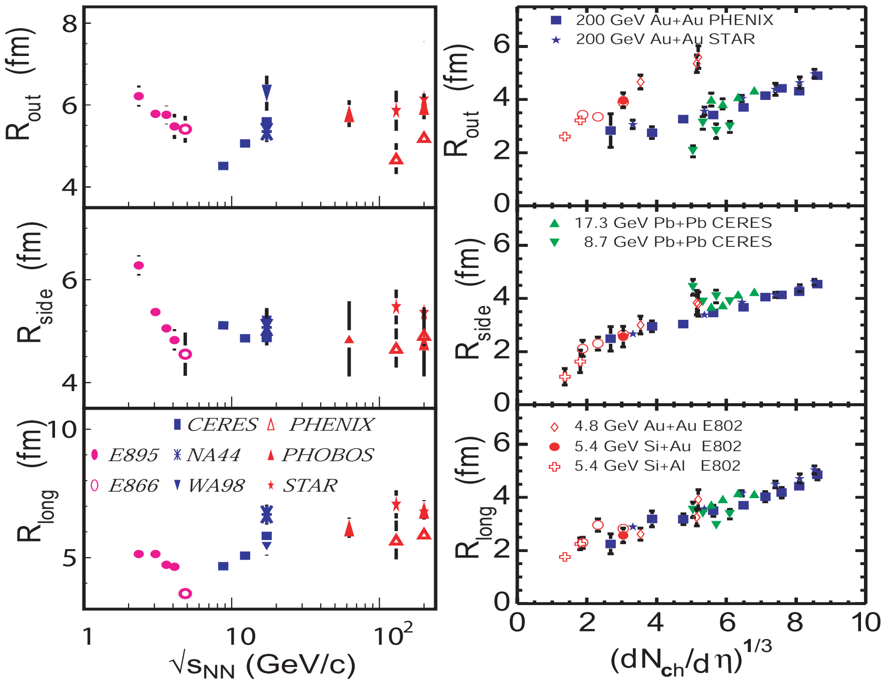



Evaporation suppressed w.r.t. decoupling, , so $\left\langle (\Delta t)^2 \right\rangle \sim \Delta_d$. Higher $\sqrt{s}(\sim T_{initial})$,larger $\left\langle (\Delta x)^2 \right\rangle$ , $\left\langle (\Delta t)^2 \right\rangle$. $R_0$ and $R_s$ increase, but $R_o$ more.

But if $T_{initial} \simeq T_c$ and there is a 1st order phase transition, things get interesting!



The HBT puzzle I We should have hit the transition temperature, but nothing interesting happens to $R_o$



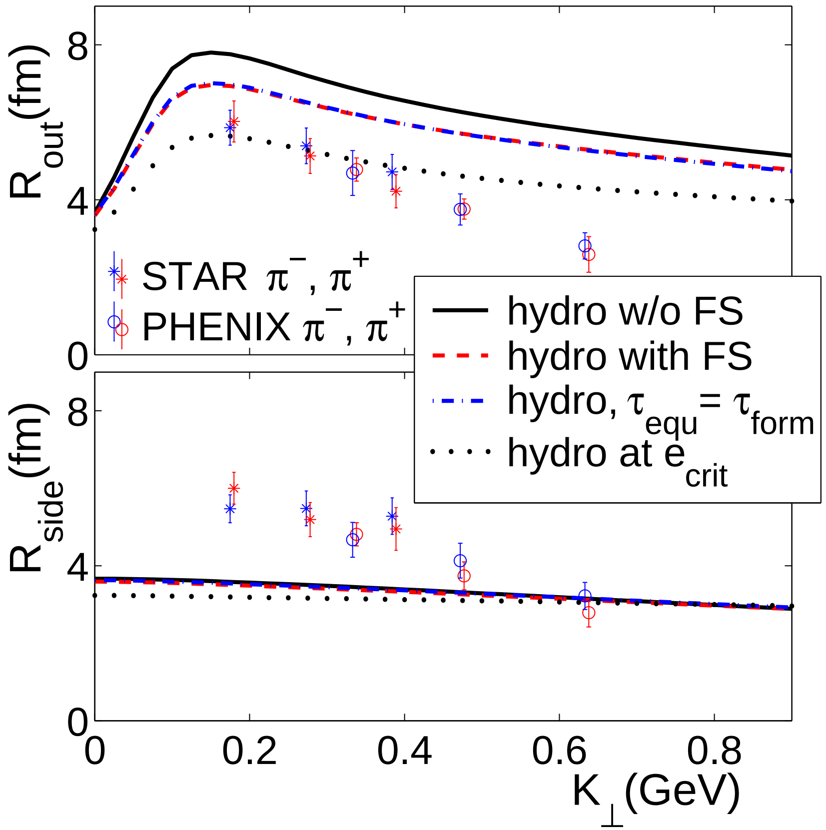

All radiuses constant with energy

(Scale well with $(dN/dy)^{1/3}$)

M. Lisa nucl−th/0701058

We now know (think?) that it's a corss-over, but an increase in $R_o/R_s$ should still happen

The HBT puzzle II Parameters describing flow do <u>not</u> fit HBT!




Freeze-out proceeds <u>too fast</u>

Does this mean:

**(a)** HBT is complicated (Gaussian approximation, homogeneity regions, reinteractions,...) let's not care too much if we get it wrong.
"Consensus" at QM09:HBT solution a "conspiracy" of pre-Equilibrium flow, No Mixed phase, and viscosity!
(This way $R_{out}/R_{side} \sim 1.1$. But scaling not resolved!)

**(b)** Our physics understanding is basically correct. But something is missing that would allow us to understand freeze-out.

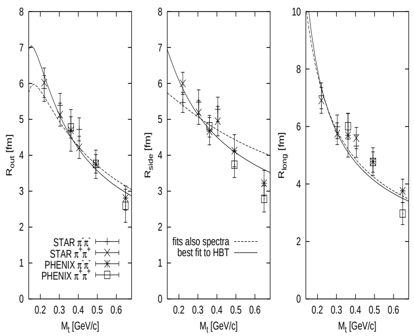
**(c)** Panic! We don't have a clue! (whole model wrong)

# Why not (c) (don't panic) II

HBT has been described, together with $v_2$ and spectra, by "Hydro-inspired models" with flow and size as fit parameters

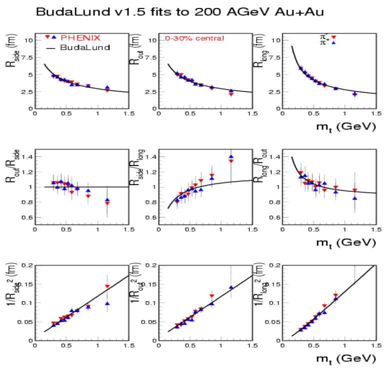
These are NOT "explanations" but FITS. But they SUGGEST where to look for an explanation

"Blast wave"
Flow+Sudden freezeout
(Il lab frame)
put artificially



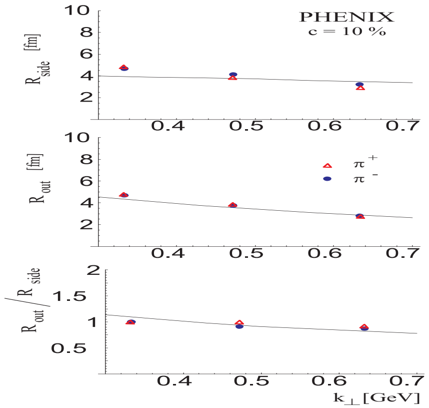
Frodermann et al,nucl−th/0602023

"Buda−Lund"
Hot (>Tc) core
+Colder halo




Csorgo et al, nucl−th/0510027

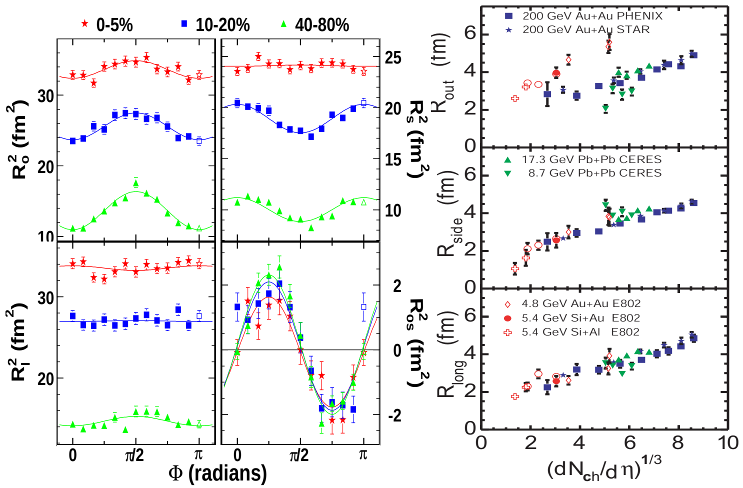
"Krakow model"
Hubble expansion and high (chemical) T freeze−out




Baran et al,nucl−th/0212053

Problem: these models very different, but all fit the data. Generally not consistent hydro solution

Why not (c) (don't panic!): HBT in some ways as expected

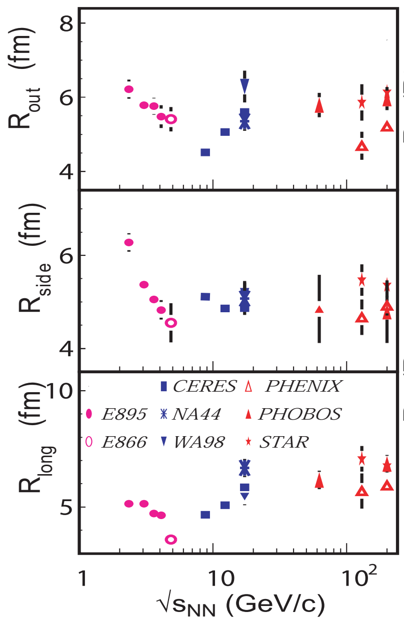


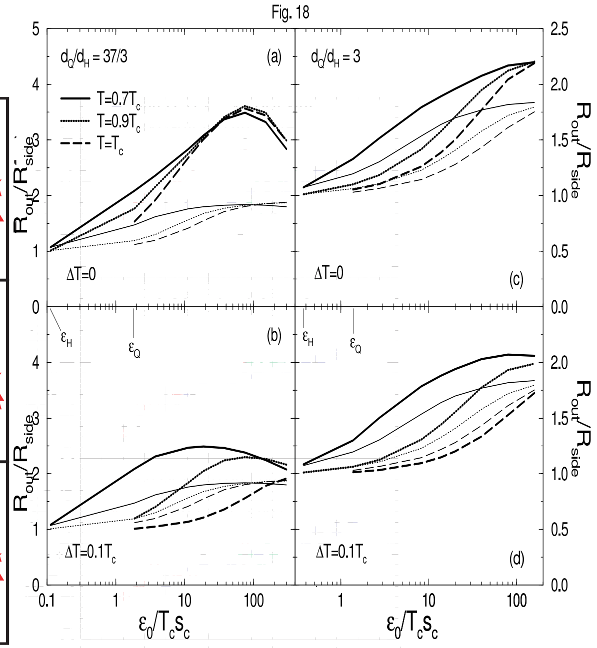

- The scaling with $(dN/dy)^{1/3}$ is just what one would expect for a gas that expands isotropically to a critical average density, and instantaneusly breaks apart.
- Comparing angular HBT with $v_2$, we see that the time-scale of the collision measured in the two approaches matches.

## Why not (a) (don't get complacent!)

- That instantaneusly (in lab frame!) is problematic to model within hydro, no matter how many refinements (viscosity, pre-existing flow,afterburner,...) one adds
- Its not just that it fails, its how it fails

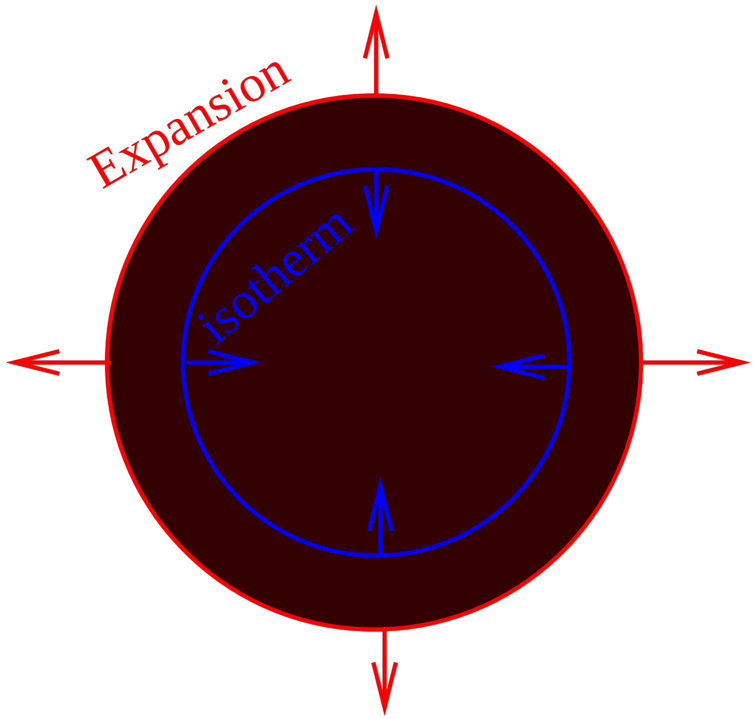
$$R_o \sim \left\langle (\Delta R)^2 \right\rangle - 2\frac{k_o}{k_0}\left\langle (\Delta R)(\Delta t) \right\rangle + \left\langle (\Delta t)^2 \right\rangle \qquad , \qquad R_s \sim \left\langle (\Delta R)^2 \right\rangle$$



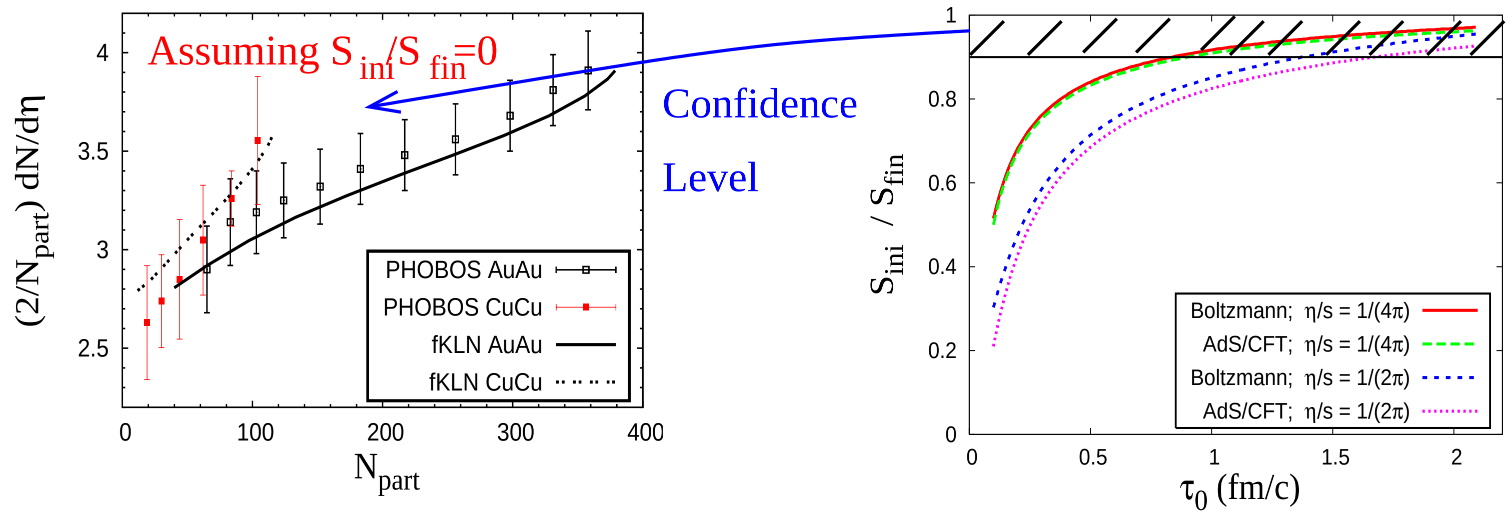

Higher $\sqrt{s}$ $\longrightarrow$, longer the lifetime $\langle (\Delta t)^2 \rangle$ , $\longrightarrow$ higher $R_o/R_s$ (especially in mixed phase). Early freeze-out might help , but why should early freeze-out happen? ( additional effects typiccaly lenghten interacting stage) And yet not only $R_o/R_s \sim 1$, it's $\sim$ constant with $\sqrt{s}$.

Isotherms usually travel “inwards”
so $\langle \Delta t \Delta x \rangle < 1$ ,further increasing $R_o/R_s$ . Flow (Lorentz time-dilation) helps, but only so much, at least with approximate boost-invariance.



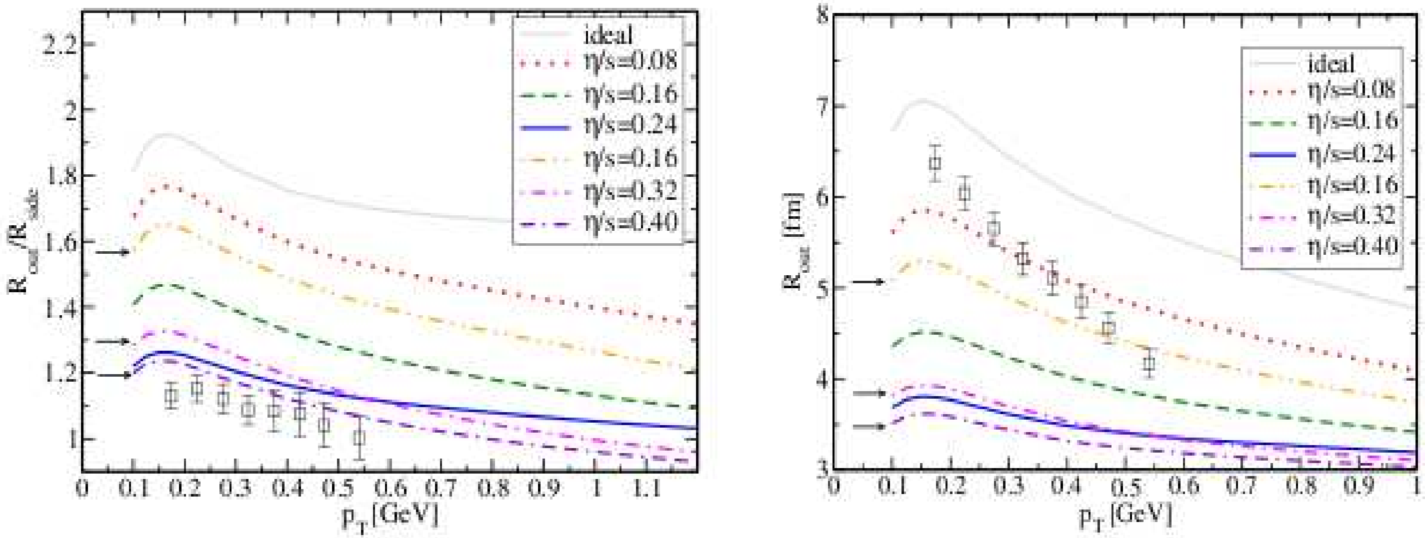
Glauber,CGC etc. model $dN/dy$ as a function of $N_{part}$ well.
These assume all entropy generated at beginning of collision.

Molnar,Dumitru and Nara, 0706.2203



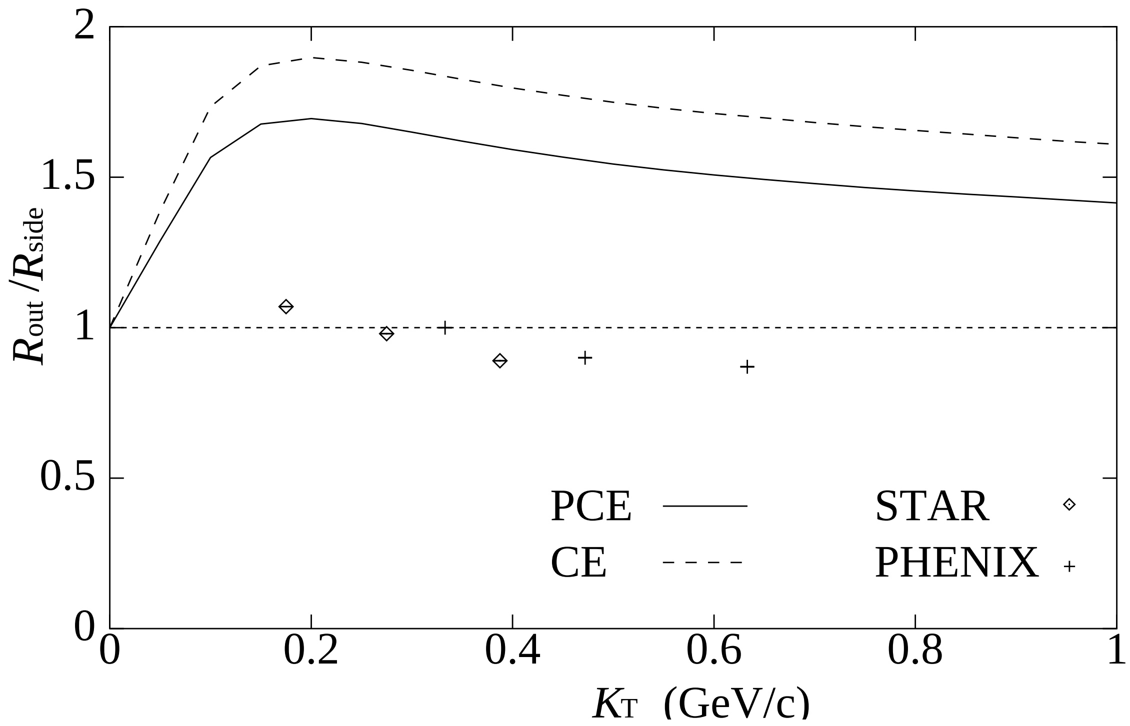

But viscosity $\rightarrow$ Entropy generation $\Delta S \sim \zeta \left(\partial_\mu u_\nu\right)^2$
Any increase in viscosity towards freeze-out will generally lead to deviations from $N_{part}$ vs $dN/dy$. Experiment constrains this

P. Romatschke, nucl−th/0701032




Shear viscosity does <u>not</u> help: It can fix $R_o$ <u>or</u> $R_s$ but <u>not</u> both. Not surprising, as freeze-out time increases in viscous medium

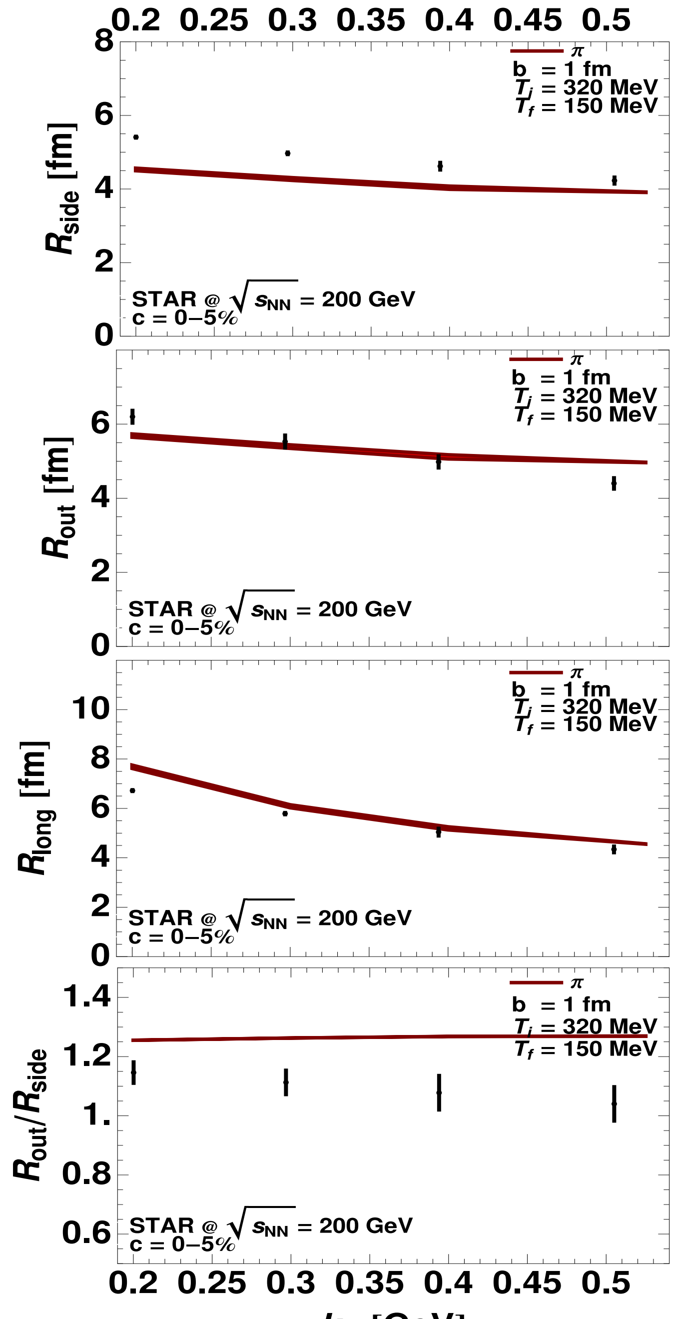
Two "obvious" improvements: Full 3D, and introducing a Hadronic Kinetic afterburner to Hydro, fail (Hirano, nara. Also Soff, Teaney, Shuryak,Bleicher,Steinheimer,... Plot from Hirano, nucl-th/0208068)




Note that CE (No hadronic rescattering) does better than PCE

Recently, agreement between SOME hydro models and HBT markedly improved

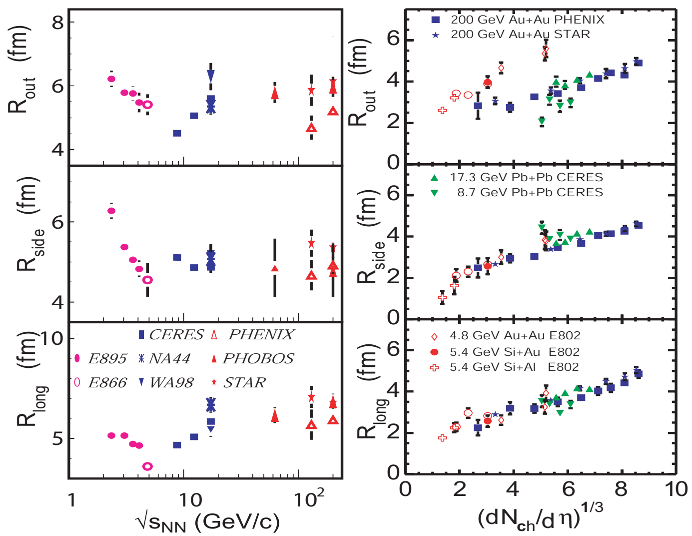
M.Chojnacki et. al. 0712.0947



Recent hydro calculation solves HBT, provided $T_{f.o.}$ high and resonances taken into account. Are we done? Perhaps nearly, but tot quite!

- Hadrons at this temperature should interact! Why dont they?

- What about scaling with $dN/dy$ at all energies? Does the cross-over to sQGP really not affect HBT radii at all?




All radiuses constant with energy

(Scale well with $(dN/dy)^{1/3}$)

M. Lisa nucl−th/0701058

## A recap of HBT

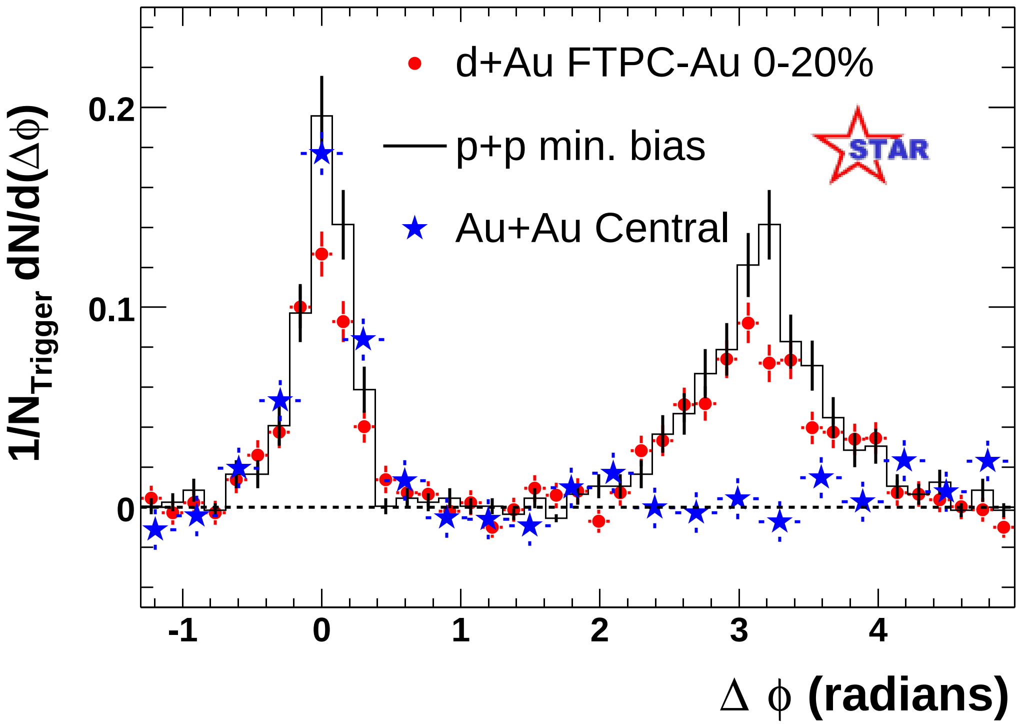
2-particle correlations provide a way to measure the "spacetime" distribution of the collision.
This used to be considered a popular way of detecting a 1st order phase transition, due to the softening of the EoS.

However, data said otherwise!
HBT radii scale very well with energy, and this scaling is not reproduced within hydro. Furthermore, HBT freeze-out <u>times</u> look too <u>sudden</u>

As far as Im concerned, problem still unsolved: Remember, we still dont understand freeze-out

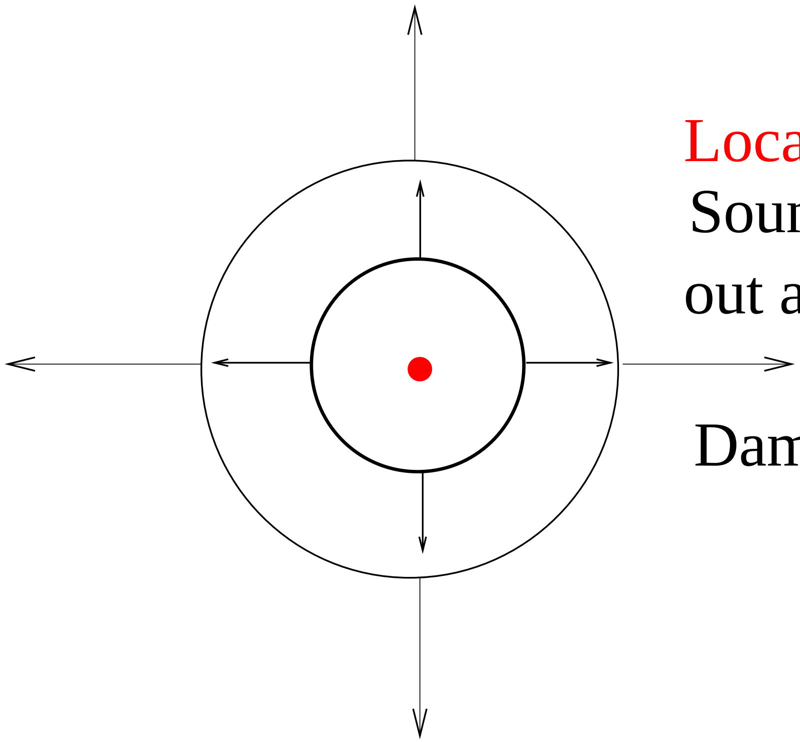
# Mach cones

# Mach cones, or hydrodynamics and jet energy loss




Jets in heavy ion collisions are known to be suppressed, showing that the fluid is opaque. What happens to the jet energy absorbed by the fluid?

If Hydro linear



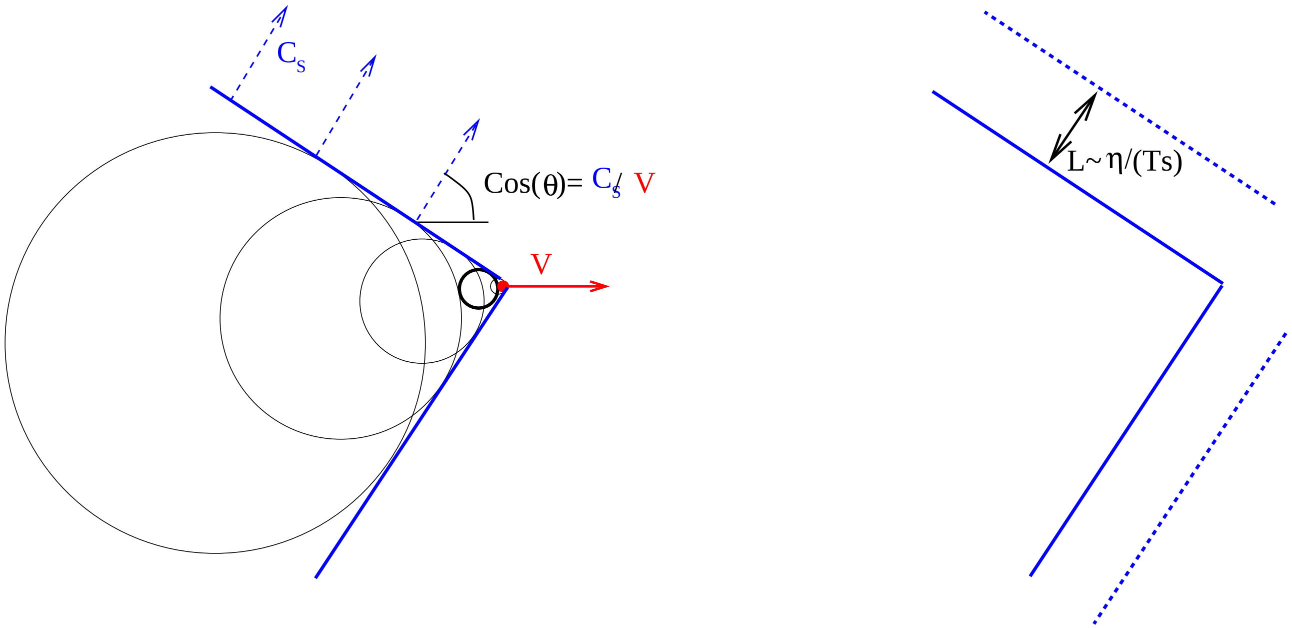
Locally deposited energy:

Sound wave expanding

out at speed $c_s^2 = dp/de$

(Link to EOS!:QGP,HG,Mixed?)

Damping at scale $4\eta/(e+p)$

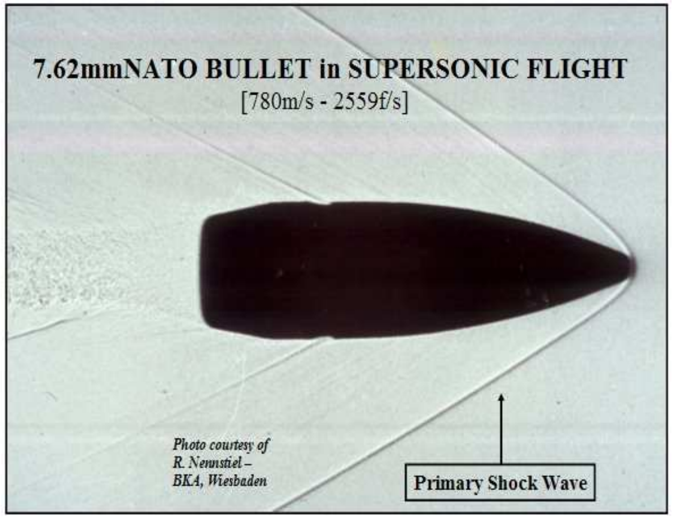



**Mach cone angle** Sensitive to EoS, $\cos\theta = c_s/v$

**Cone killed** by viscosity exponentially, $A(x) \sim A(0)e^{-k^2\Gamma x}, \Gamma \sim \eta/(Ts)$

IF we see this, we confirm fast thermalization and study fluid's $EoS$!

This phenomenon is well known



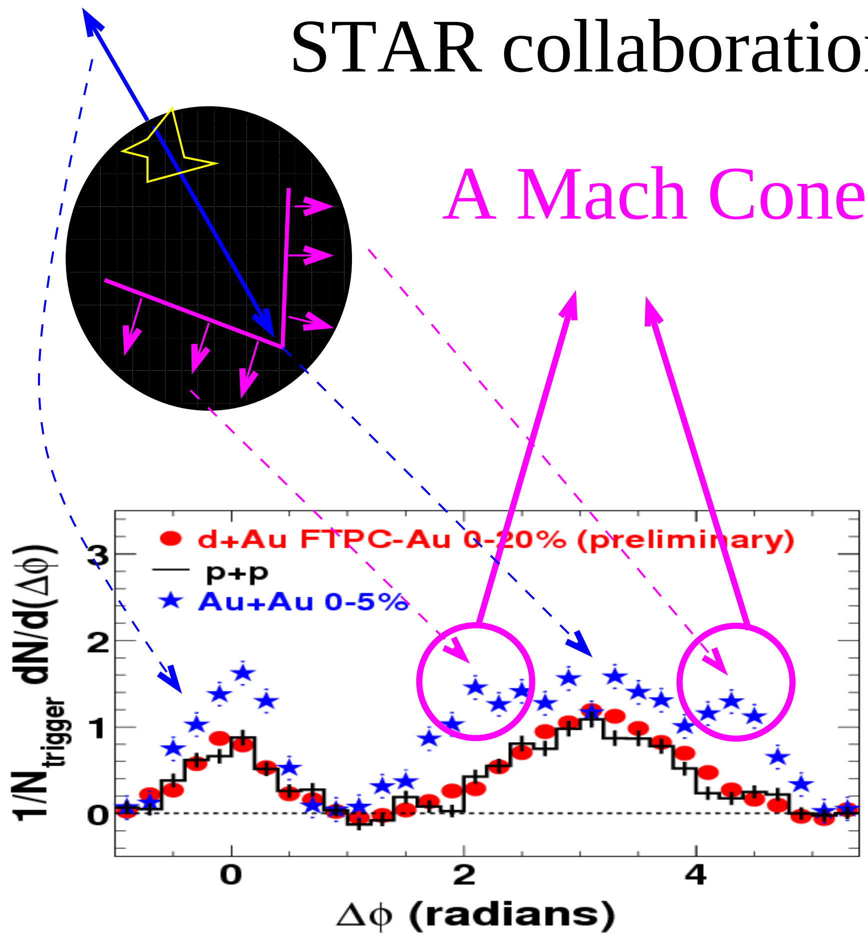

But is it relevant and observable in heavy ion collisions?

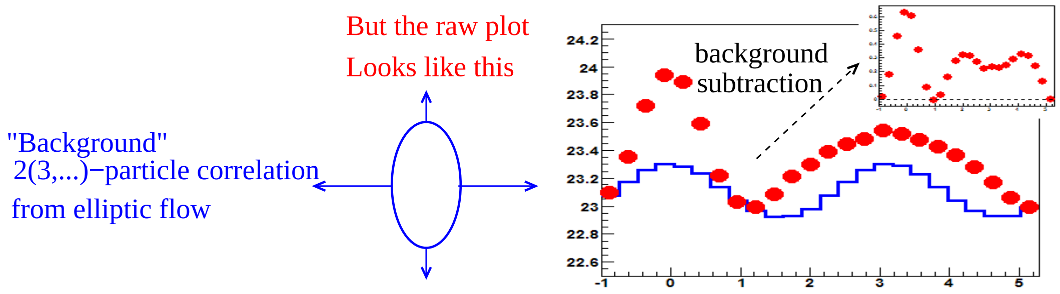
First suggested by Horst Stoecker, W. Scheid, W. Greiner,... ,1975

Experiment:If we lower trigger, away-side peak reappears and...

STAR collaboration

A Mach Cone?



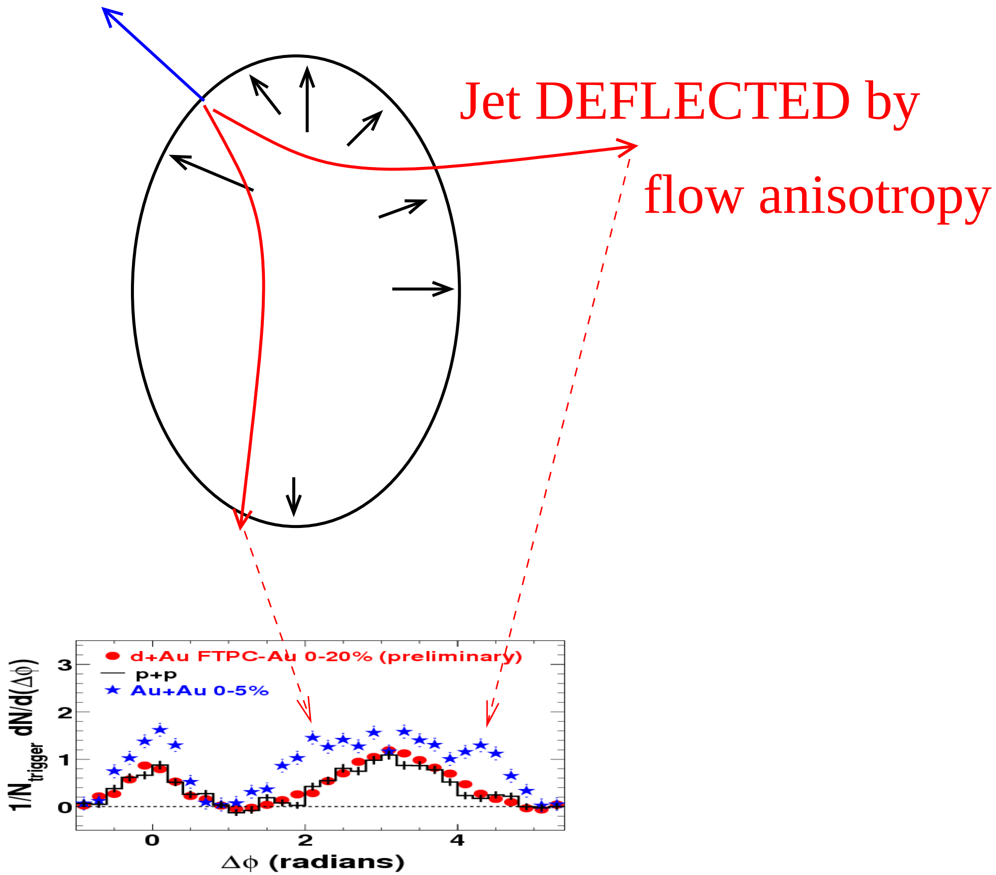



Assume correlations from flow anisotropy and from jet uncorrelated (ZYAM). This is lousy! Even in linear hydro, freeze-out introduces correction (remember that all harmonics in flow go to all $v_n$ . But we dont have anything better.

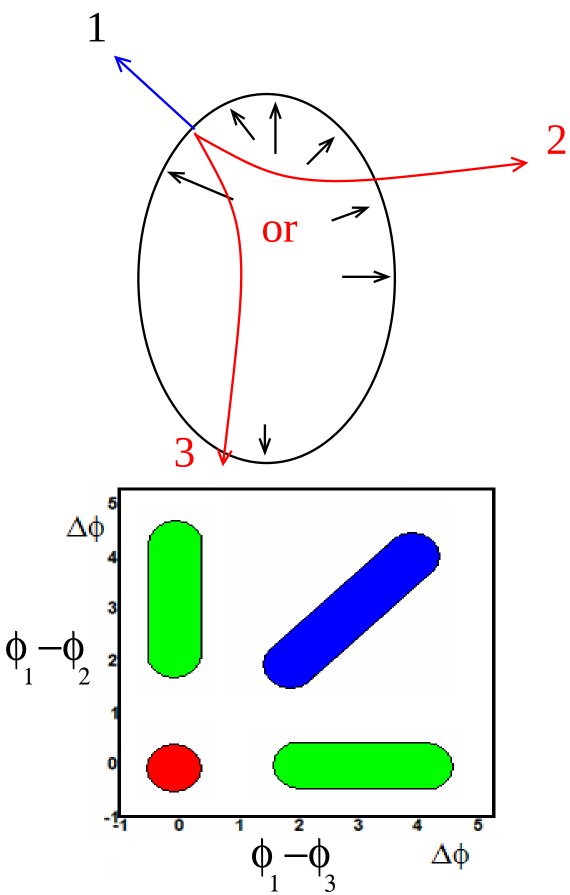
Is ZYAM systematic error enough to produce "peak"?

# Other explanation possible

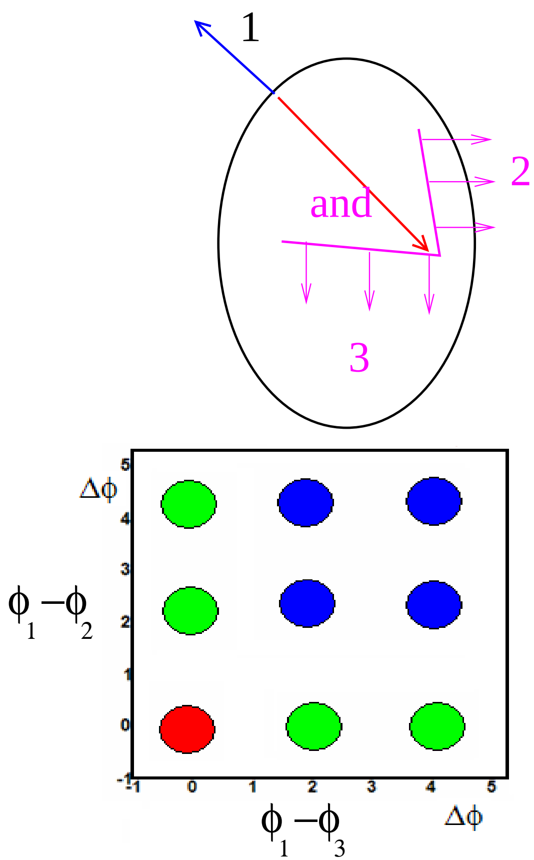
Armesto,Salgado,Wiedemann,PRL93:242301,2004



# But distinguishable:3-particle correlations

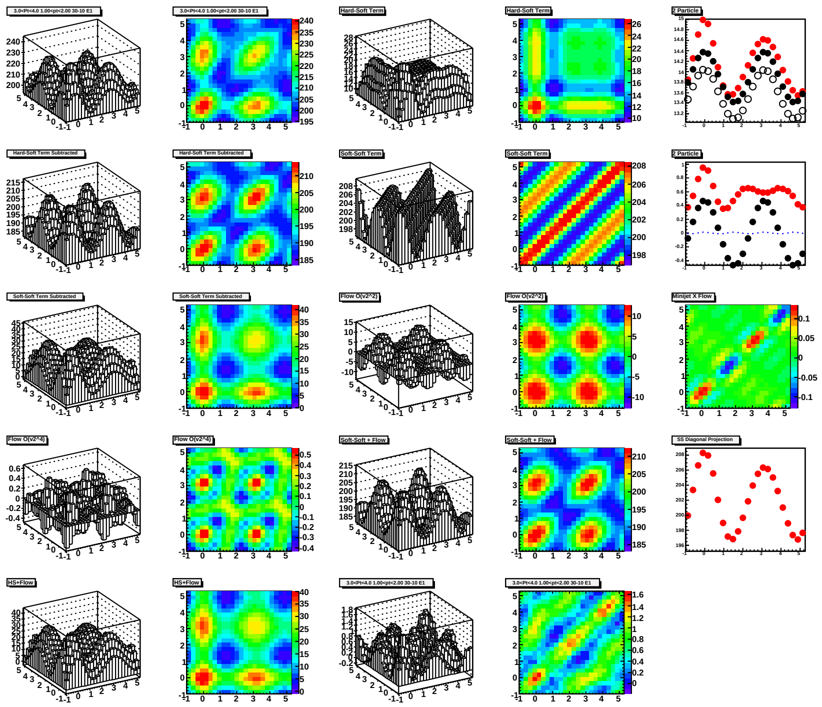



Either 1 or other peak




Both peaks there

Background becomes more tricky... Still use ZYAM to resolve all combinations (Jet$\times$flow,Flow$\times$Flow etc.)

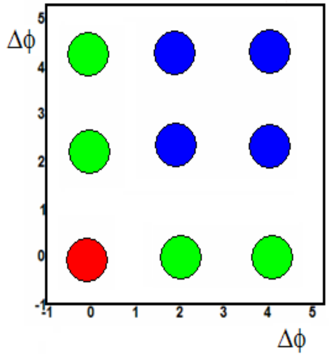


(J.Ulery,PhD thesis)

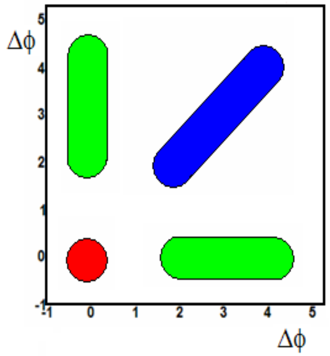
Results look like mixture of Mach and deflected (and why not?)

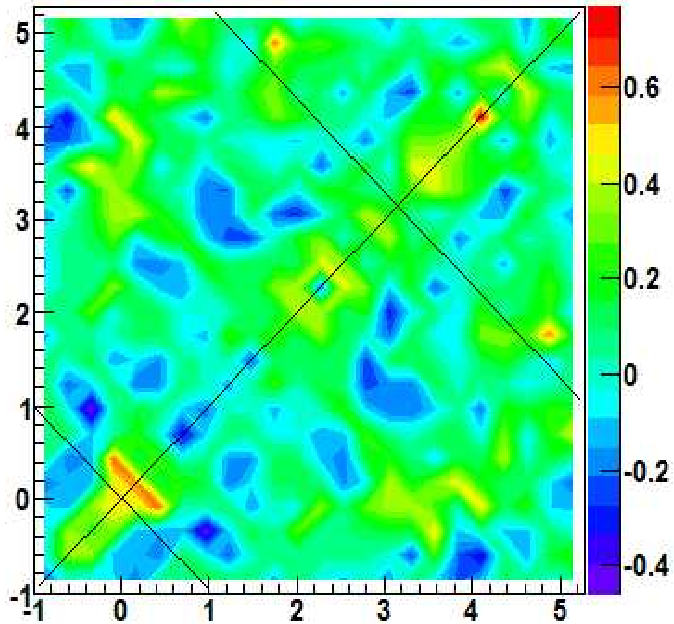
STAR collaboration (PHENIX similar)

Mach

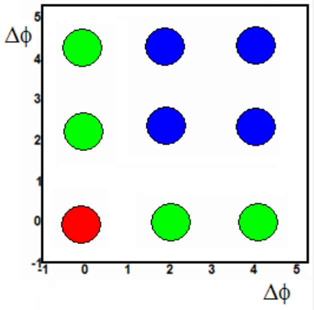



Deflected

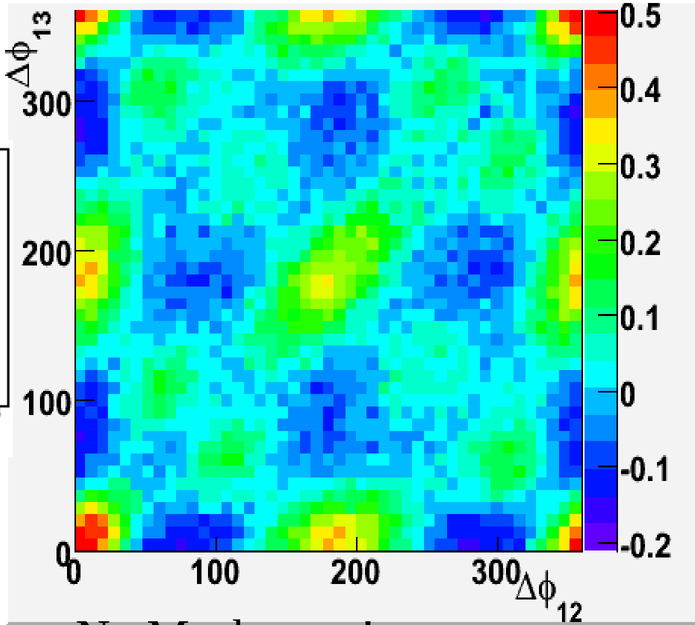


Method based on cumulants, not background subtraction, finds nothing...

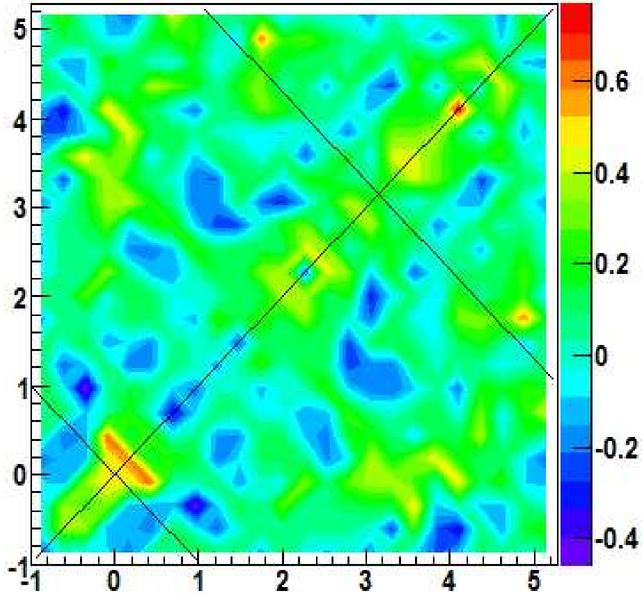



Background subtraction by
Cumulants and Mixed event




No Mach cone!

Background subtraction by
2−Component background (ZYAM)




Mach cone!

## Theory:Why heavy ion collisions $\neq$ "textbook"

- Background non-trivial (flowing, phase transition)
- Non-linear hydrodynamics
- Energy-momentum deposition <u>not</u> trivial, and not well understood.
- Freeze-out: We don't see fluid, but particles

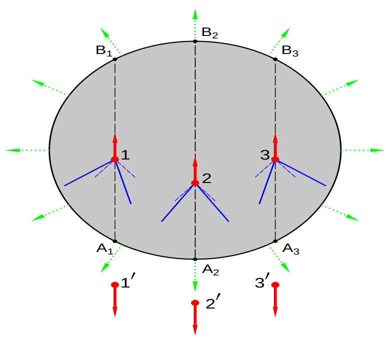
We need something more sophisticated:Full hydro+freeze-out

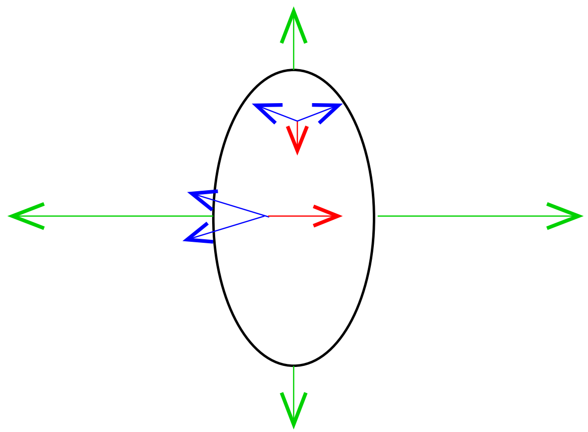
Effect of flow : Usual relationships with frame co-moving with flow (Satarov, Stoecker,Mishustin,PLB627(2005))

In linearized limit, $\theta = \sin^{-1}\left(c_s^{comoving \quad frame}\right) \rightarrow \sin^{-1}\left(c_s\sqrt{\frac{1-v^2}{1-v^2c_s^2}}\right)$

**Transverse flow** should “smear” angle

**elliptic flow** should correlate $\theta_{mach}$ to $\phi_{jet}-\phi_{reaction}$ (Unless neck signal?)



What is $J^\mu$? Well, we don't know!

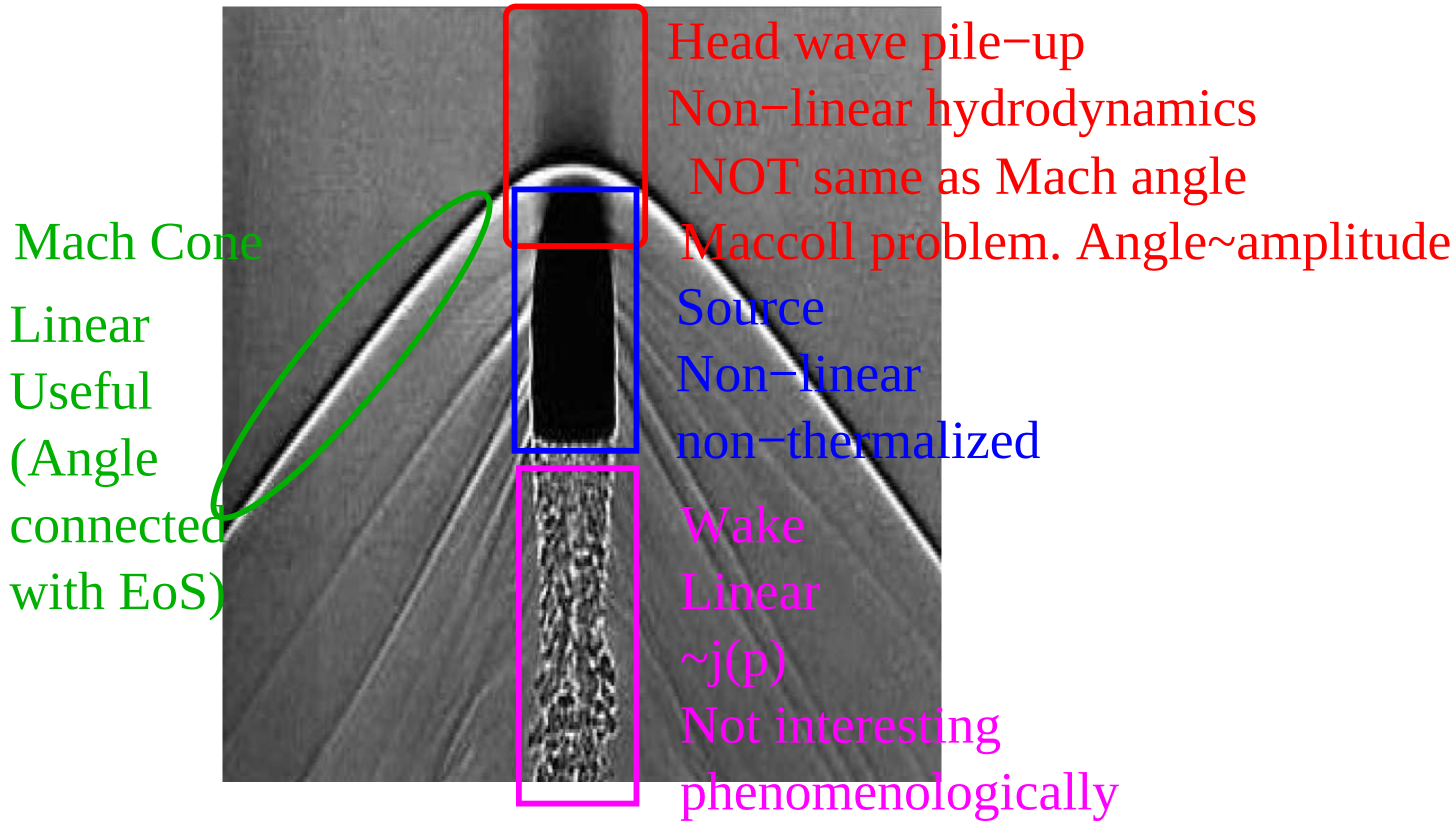
**Textbook** $J^\mu = (e,0,0,0)\delta(\vec{x}-\vec{v}t)$

**On-shell:** $J^\mu = (e, e\vec{v}/|v|)\delta(\vec{x}-\vec{v}t)$But parton does not have to be on-shell:

**Weakly coupled jet-medium** (NB: not inconsistent with hydro: for hydro medium has to be strongly coupled,jet-medium can be anything!)
$J^\mu \sim \frac{dE}{dz} \sim L$ for dense medium ($l_{coherence} > l_{scattering}$ )

**Need** consistent picture of the system, interpolating between fully unthermalized jet and thermalized strongly coupled medium. And it's a non-perturbative non-equilibrium non-linear problem!

Is linearized hydro good? <u>probably not</u>



Source usually (a la Lifshitz-Landau) local

$$J^\mu \sim J_0^\mu \delta(x - vt)$$

For an infinite $\delta$-function, linearization $\delta T^{\mu\nu}/T^{\mu\nu} \ll 1$ badly broken.
Of course, the $\delta$-function approximation of smeared non-equilibrium distribution

$$\delta(x - vt) \simeq f\,(x - vt, \sigma)$$

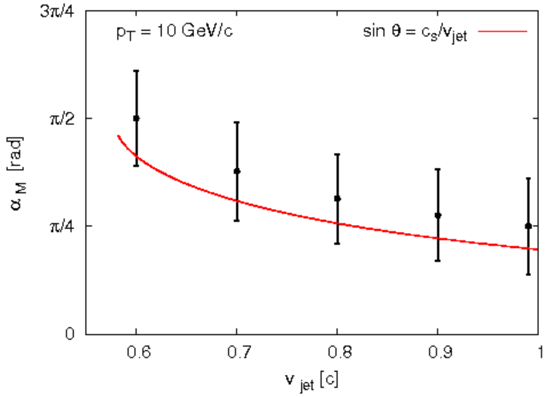
Because full hydrodynamics is non-linear, form of $f$ where $\delta T^{\mu\nu}/T^{\mu\nu} \sim 1$ can have effects in the linearized $(x \gg \sigma, \delta T^{\mu\nu}/T^{\mu\nu} \ll 1)$ region.

Perhaps when $x \gg \sigma$ these effects go away, but this might be too big.
( In AdS/CFT Far-away dynamics does depend on weather source is a heavy quark or a meson. So near-side dynamics changes far-away result)

# Explore range of $J^\mu$s systematically with full hydro;$\sim$ conical, but...

Betz,Gyulassy,Stoecker,Rischke,Torrieri,QM2008 presentation,coming paper
Also J.Casalderrey−Solana, E.V. Shuryak,PRD74 (2006) 085012

(Probably) invisible T pattern independent of source

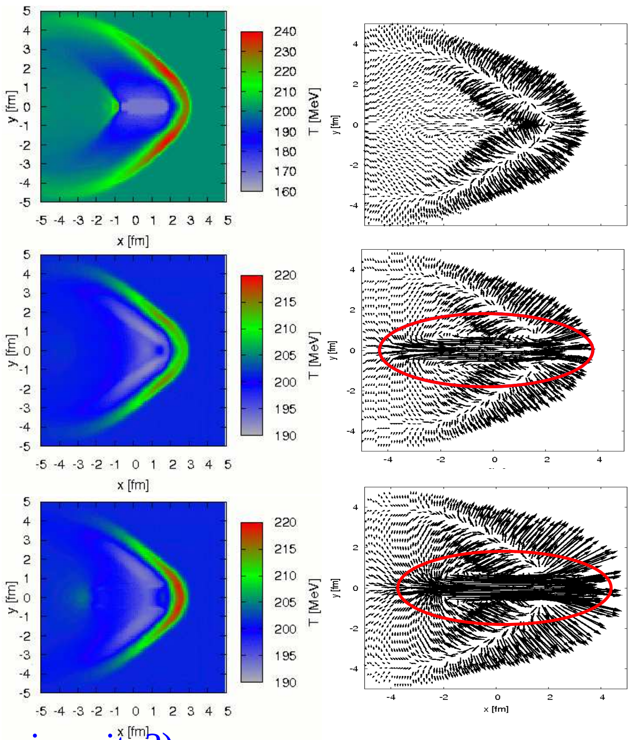



T $\vec{v}$

Pure Energy

Mixed

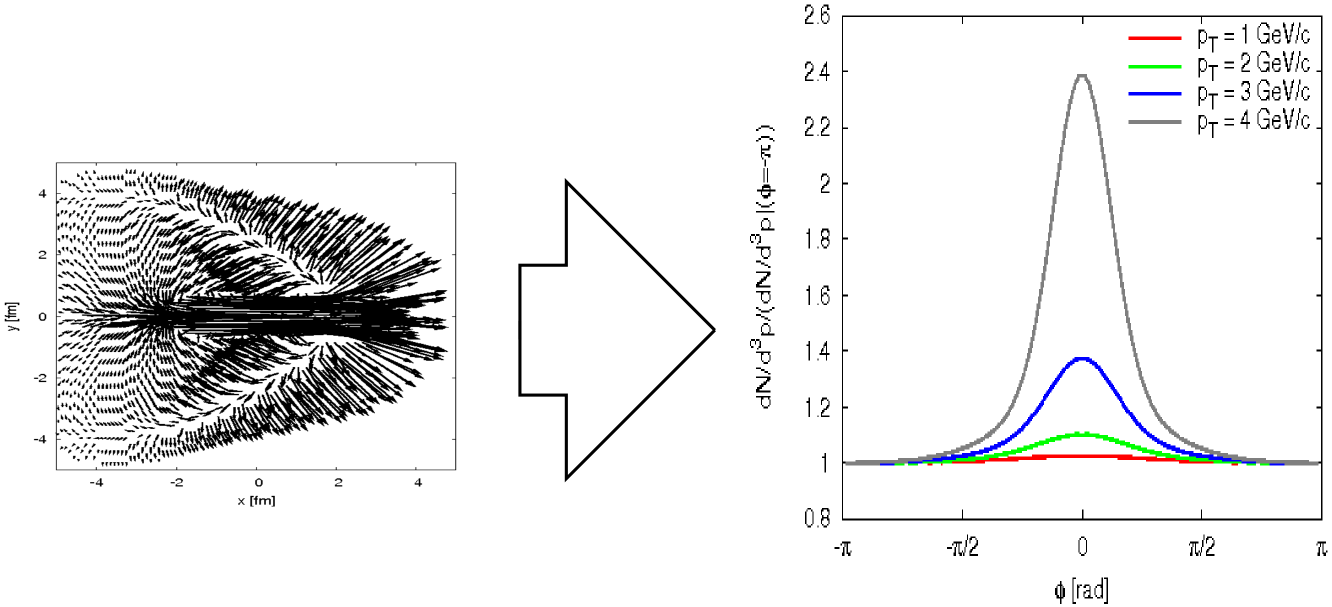
Pure $\vec{p}$



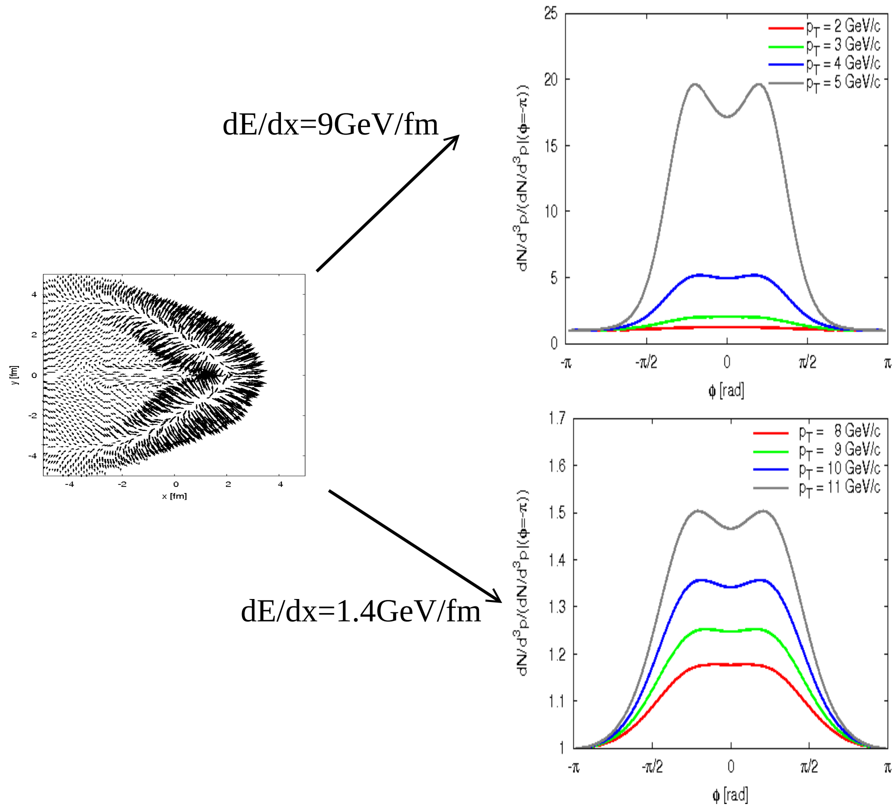

But flow pattern depends on it A LOT!
Momentum deposition creates un−conical "diffusion shock", taking most of the source’s energy/momentum

Mach cone angle survives in full hydro (Non linearities no problem. Numerical viscosity?)

"Realistic" GLV/BDMPS calculation forthcoming; LPM effect also likely to spoil Mach signal



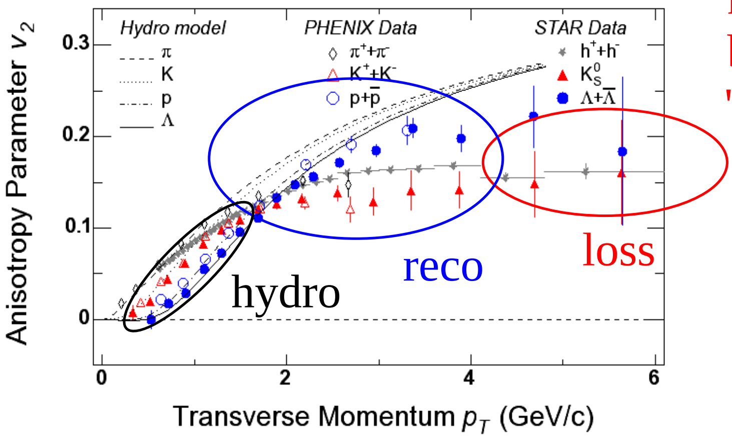

Betz, Gyulassy, Stoecker, Torrieri: As expected, diffusion wakes are phenomenologically useless! Yield a generic "peak" indistinguishable from any other jet energy loss mechanism!



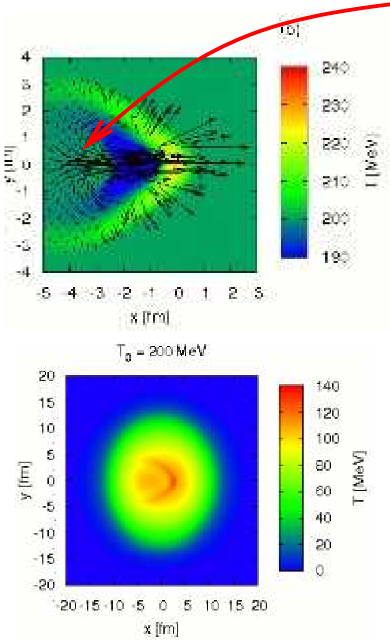

Energy deposition works better: Cone structure, correct angle. Signal increases with $p_T$ (Blue-shift), only strong at very high away-side $p_T$

But... $p_T$ of "soft" associated particle needs to be huge unless jet energy deposition is large! Since $\langle \sigma \rangle \sim 1/\langle Q \rangle^n$, harder particles less thermalized, (medium is more transparent to them)

away−side should be "firmly" in "hydro region

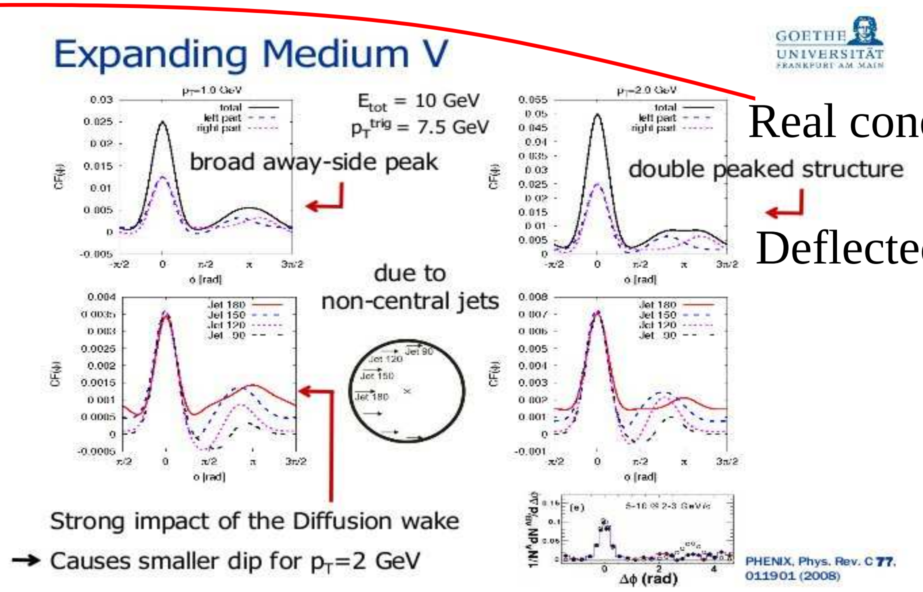



near−side should be "firmly" in "loss" region




Barbara Betz
QM09

on−shell
deposition
in expanding
medium



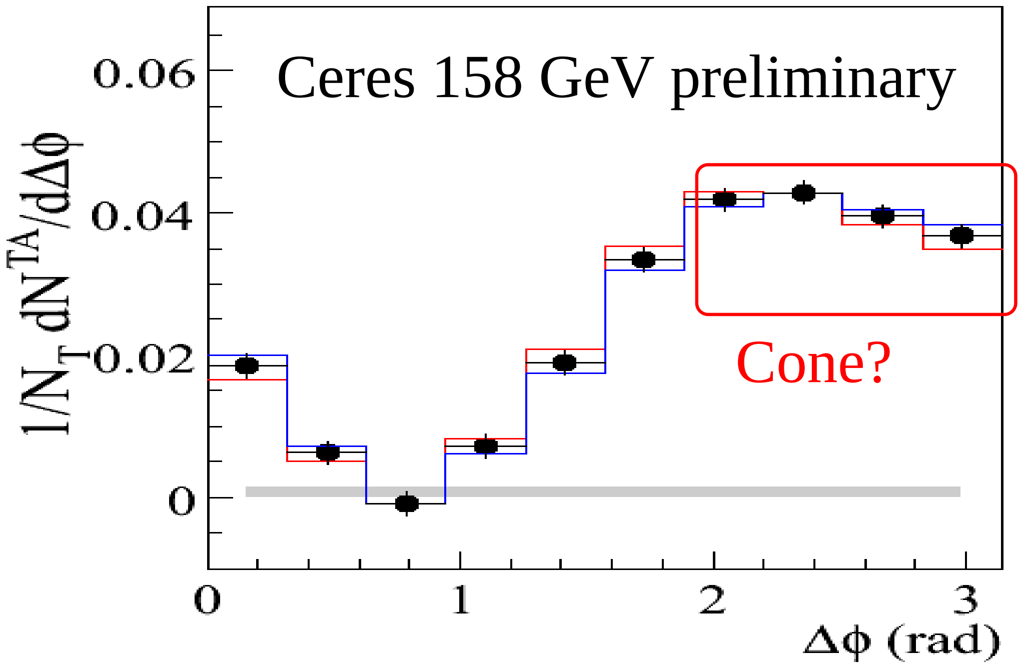

Real cone or

Deflected wake?

Flow restores cone (But is it cone or deflected wake? Angle also changed!)

Need Cone-$v_2$ coupling (How does cone change with reaction plane)

CERES (20 GeV SPS): Mach cone signal clearer! (same angle)



## This is weird

Hydrodynamic approximation works better for observables correlating more particles. So it should work

**best** for transverse flow (not many collisions necessary to make system expand, arises at all shapes)

**Less well** for $v_2$ (sensitive to shape details)

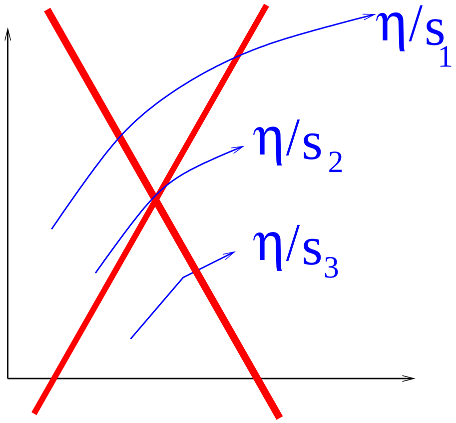
**Less well still** for Mach cones (process only involves few particles at freeze-out).

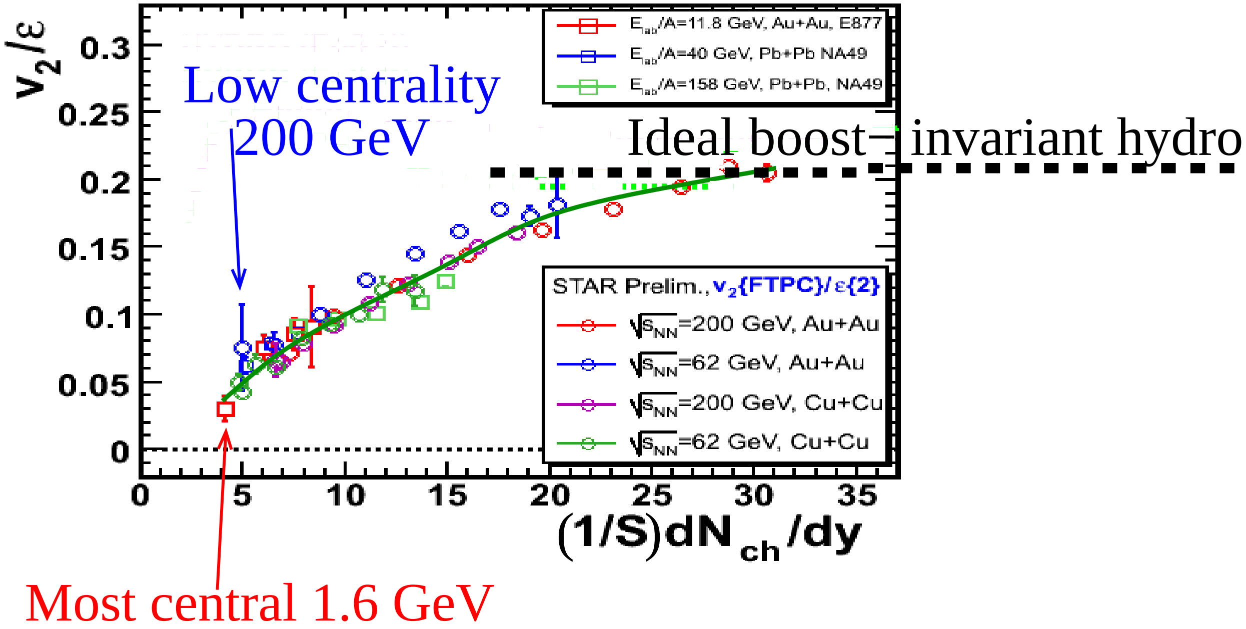
Yet here SPS signal (where $v_2$ is smaller) as good, if not better, than RHIC.

Either not "true" Mach cone or we don't understand $v_2$

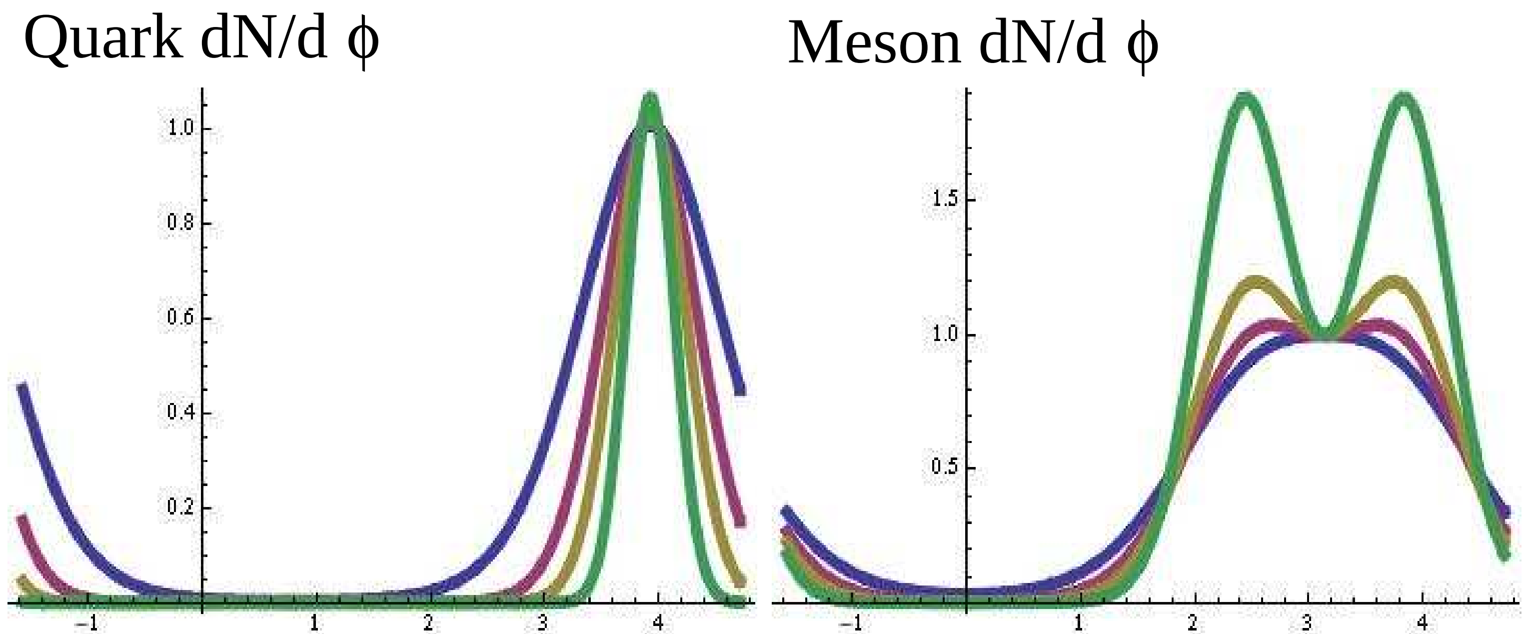
On the other hand, no "turning on" of $v_2$ either!

Song, Heinz
arXiv:0805.1756
GT
PRC76:024903,2007



Mach cones from coalescence?




Blue−red−cyan−green: pT=1,1.5,2,3 GeV
Normalization:Peak=1 GT,Greco,Noronha,Gyulassy: QM09

Coalescing a broad away-side peak → Fake cone. Freeze-out uncertainity again

## A recap of Mach cones

Would confirm hydrodynamic behaviour, and allow a window to look into the EoS.

Background subtraction non-trivial,systematic errors possible

Energy dependence puzzling

Conclusion: The PERFECT LIQUID is well understood (at least by me)

E se ci sono volontari, sono disponibile a continuare la discussione INFORMALMENTE ,davanti a un bicchiere di quello che e PROVATO essere liquido perfetto, dopo questa sessione

But the liquid created at RHIC is not!

We still do not understand many crucial aspects of the system created in heavy ion collisions

- How to disentangle effects of viscosity, EoS,Initial conditions?
  Never mind 10+ Israel-Stewart coefficients!

- How does the "perfect fluid" turn on?
  How do viscosity, initial conditions, EoS change with energy and system size?
  Scaling for a lot of observables suspiciously simple wrt a complicated model such as hydrodynamics

- Freeze-out not understood on a conceptual level (how does a "fluid" transform into "particles" and on a phenomenological level (HBT puzzle, Mach cones, systematics of $v_2$)

## Where to go next?

**Experimentalists** Look for scaling across energy and system size for all your observables. Scaling can be used to <u>counteract</u> models with lots of free parameters

**Theorists** Dont concentrate on one energy range. Do not assume a prescription (eg Freeze-out) is right "just because everyone else is using it".