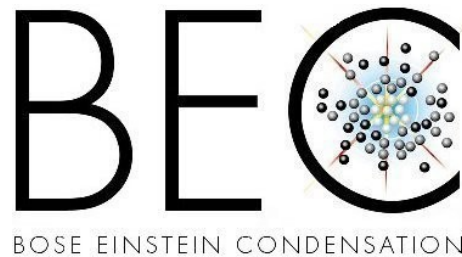


Ultra-cold gases

Alessio Recati

CNR-INFN BEC Center/
Dip. Fisica, Univ. di Trento (I) &
Dep. Physik, TUM (D)



Lectures

L. 1)

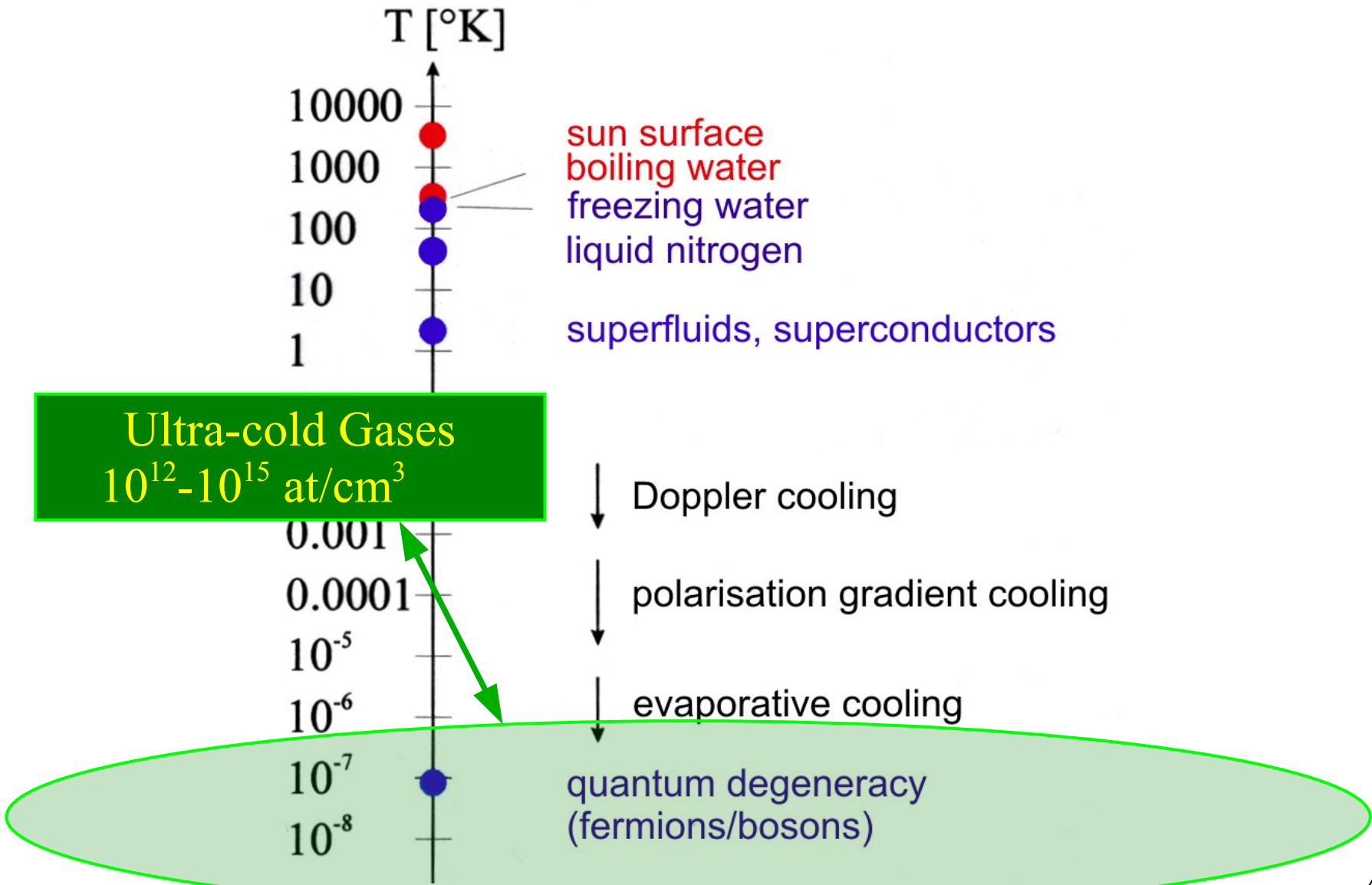
- ◆ Introduction to ultracold gases
- ◆ Bosonic atoms:
 - From weak to strong interacting gases
 - An application to precise measurement (Casimir forces)

L. 2)

- ◆ Feshbach resonance and strongly interacting Fermi gas/the Unitarity limit
- ◆ BCS-BEC crossover
- ◆ Polarized Fermi gases:
 - "a" polaron problem
 - new Fermi-Landau liquid

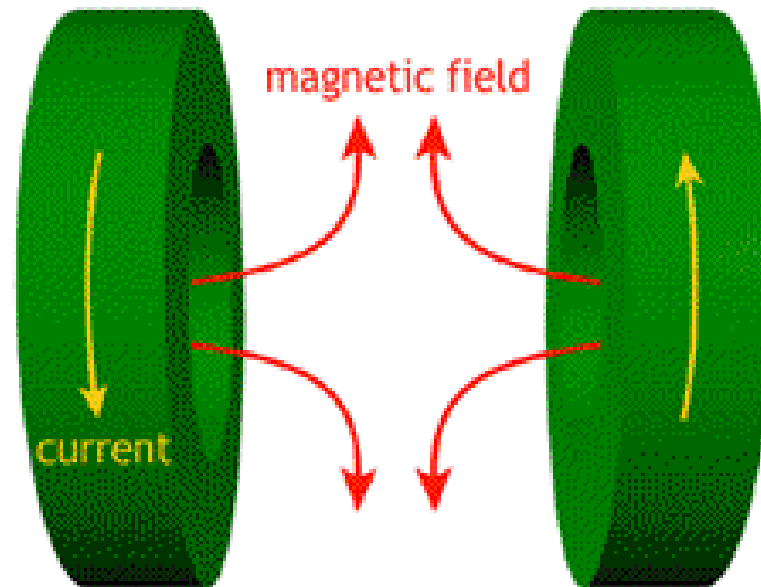
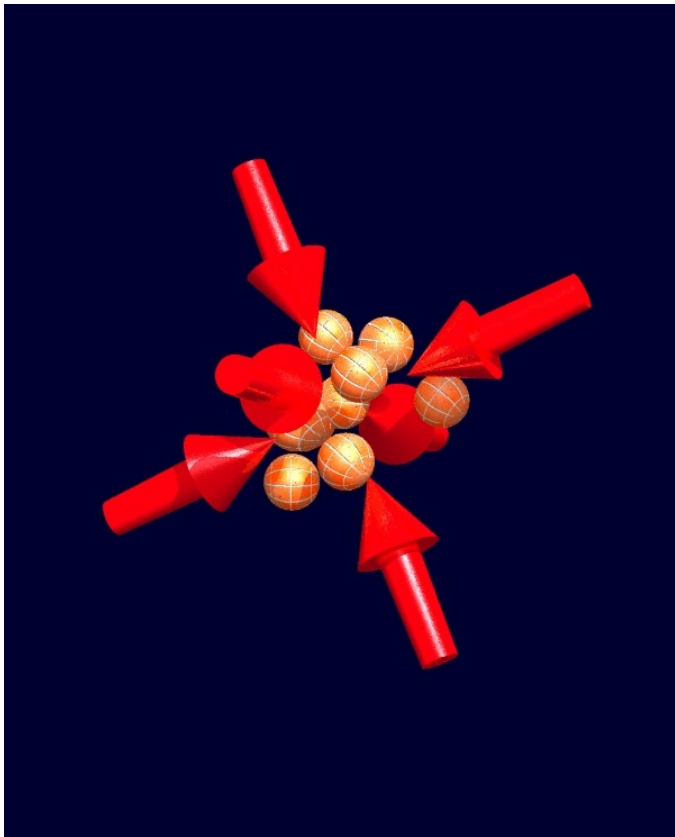
"Why should I care about a bunch of atoms which do what they are expected to do?"

Temperature scale



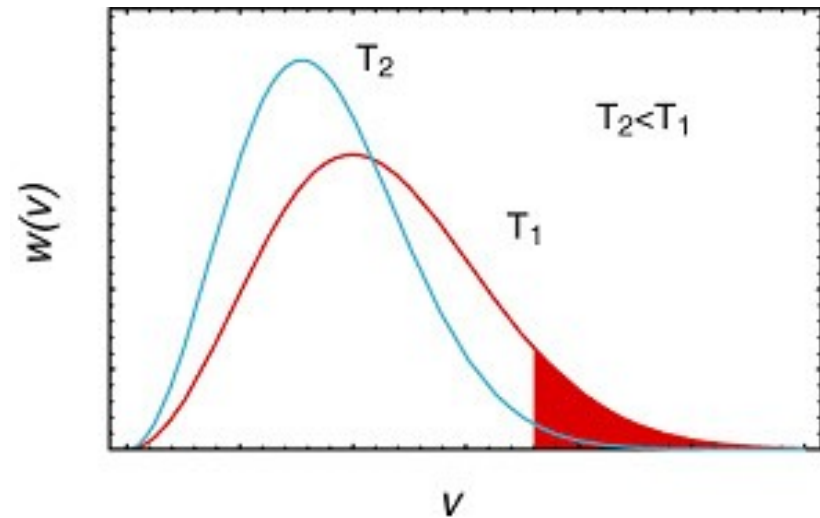
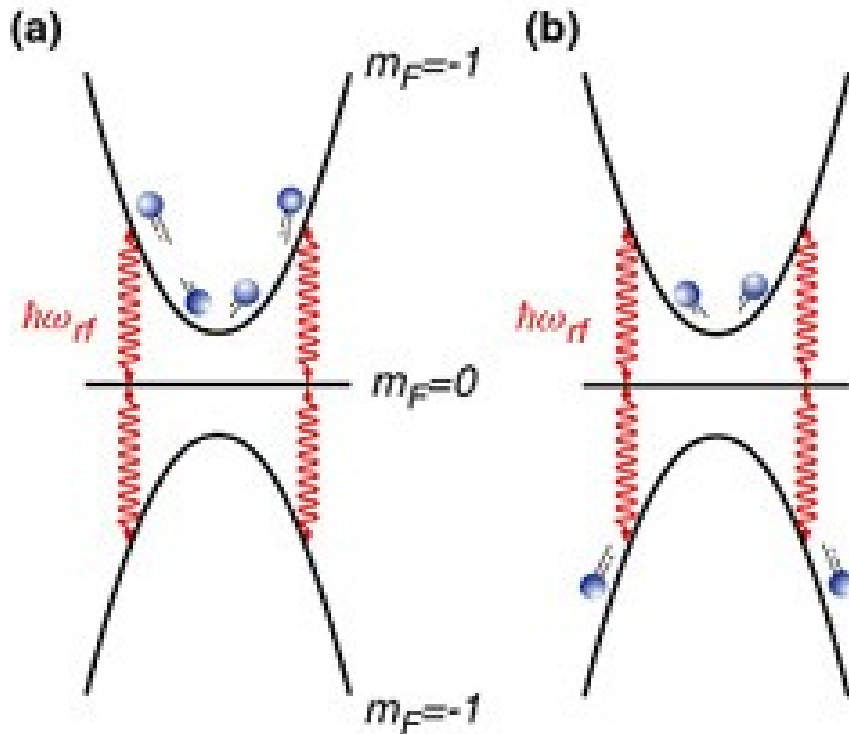
Cooling & Trapping neutral cold gases

1. Laser light pressure (laser cooling)
2. Electric and/or magnetic confinement: harmonic traps
3. Evaporative cooling (a.k.a. cup of coffee cooling)



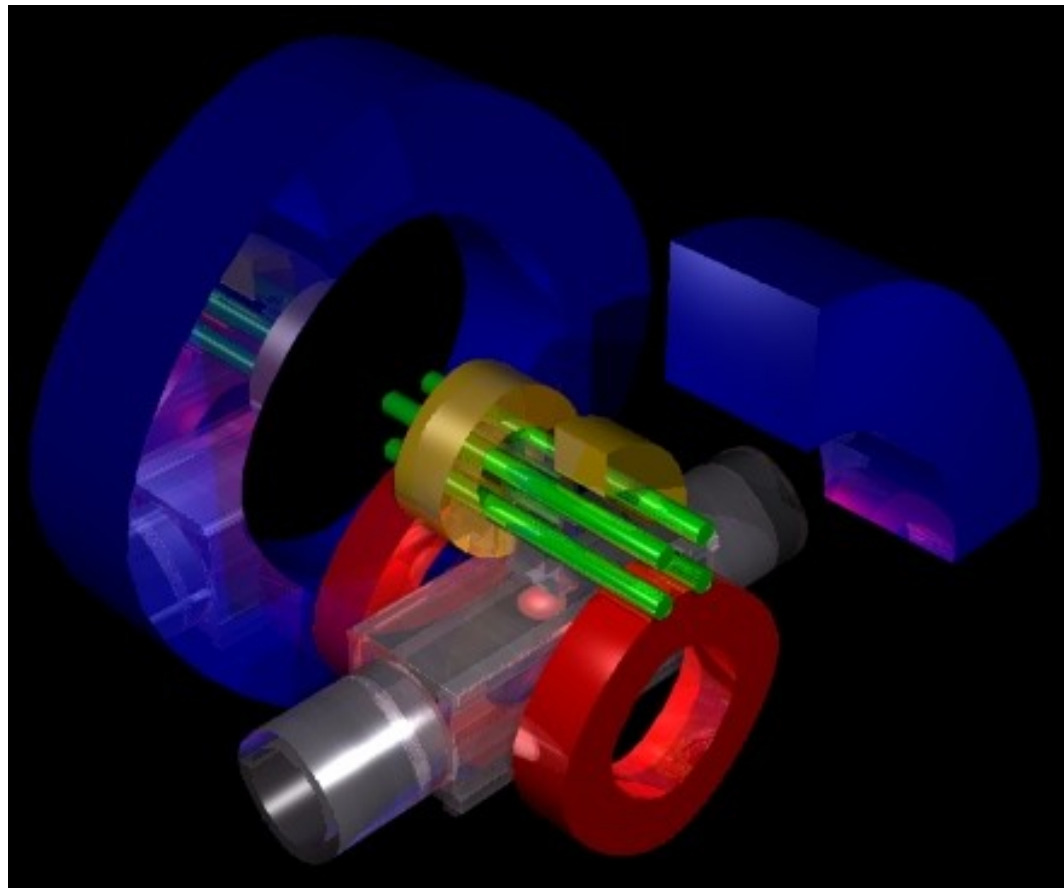
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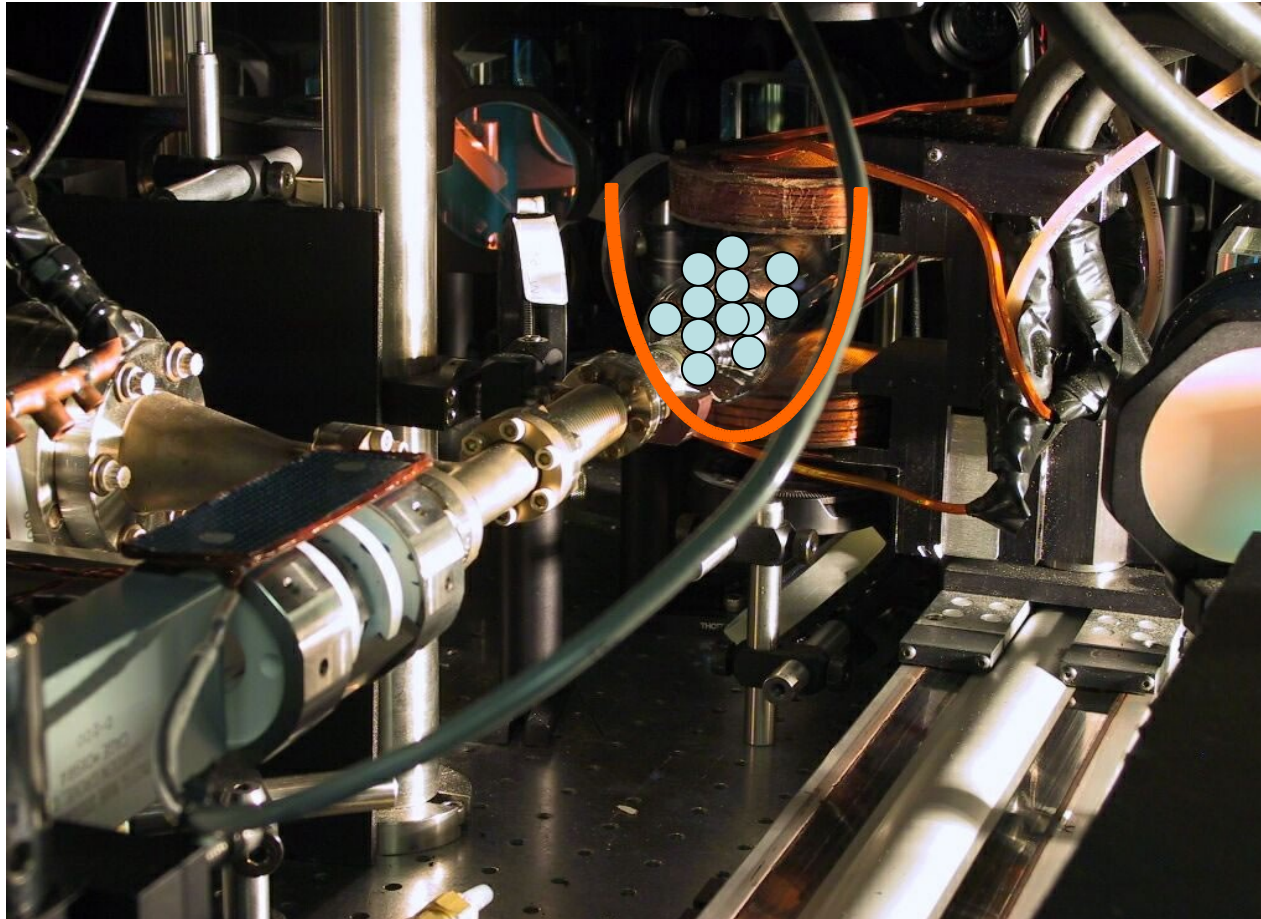
Cooling & Trapping neutral cold gases

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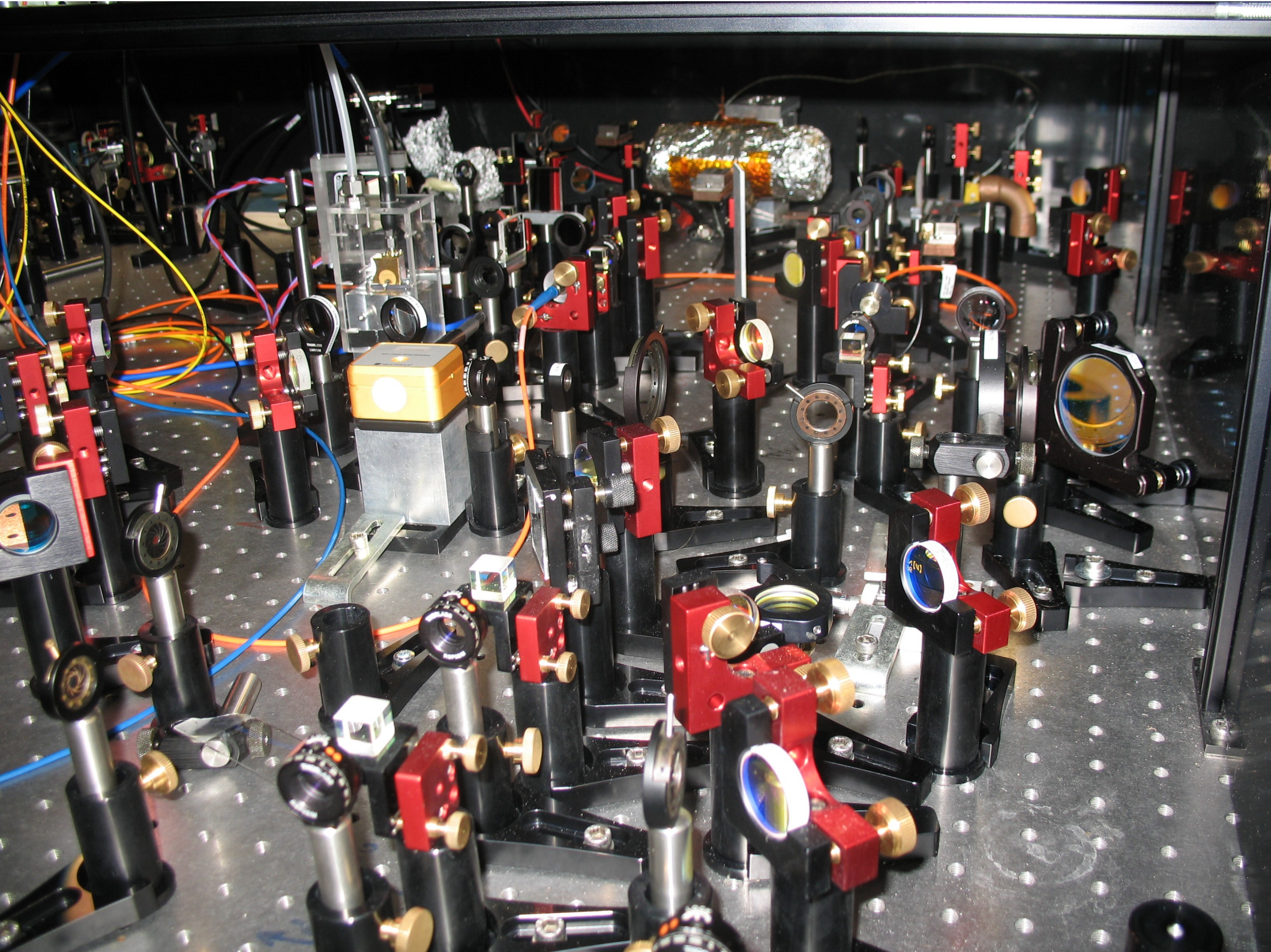


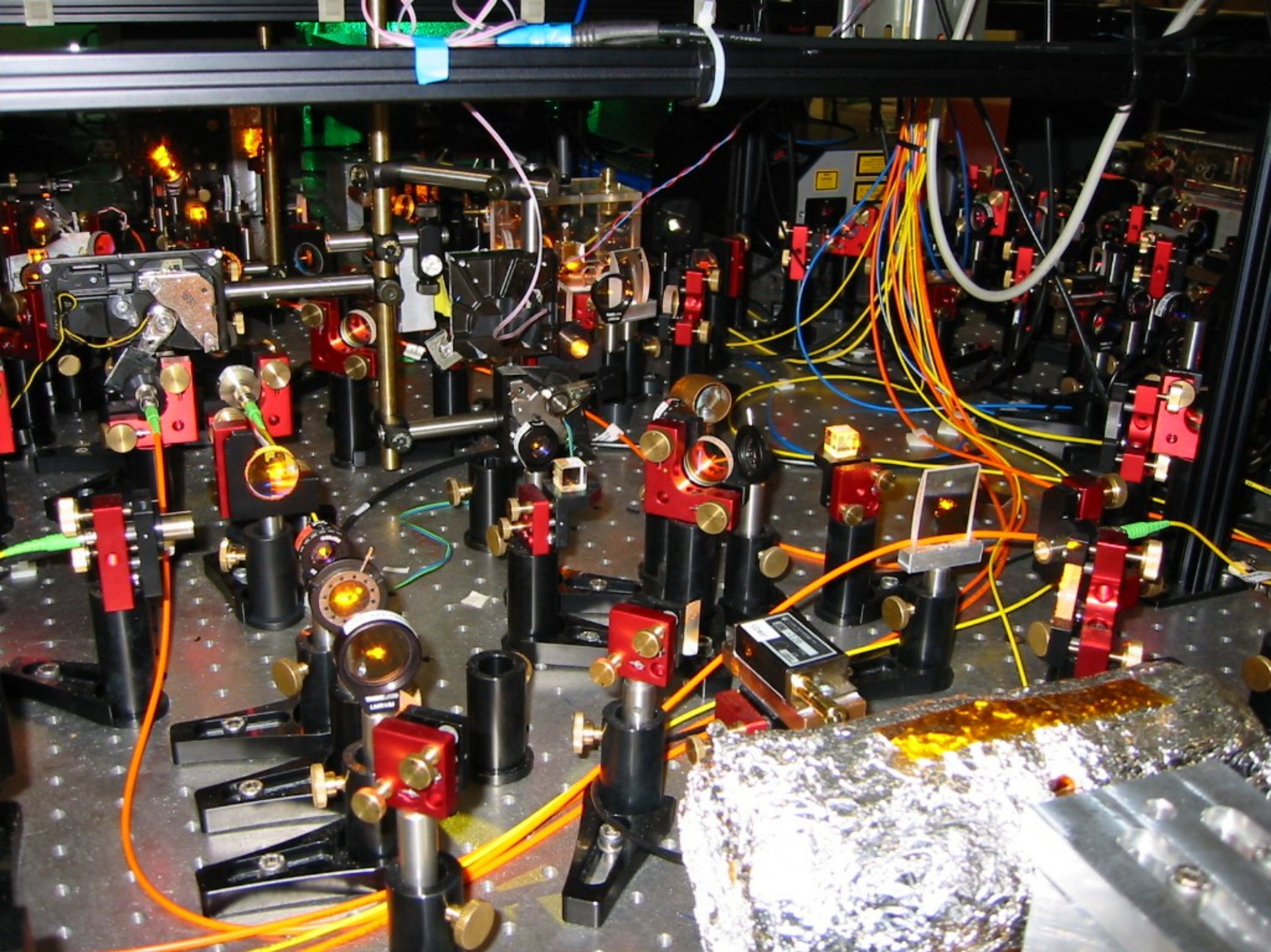
Cooling & Trapping neutral cold gases

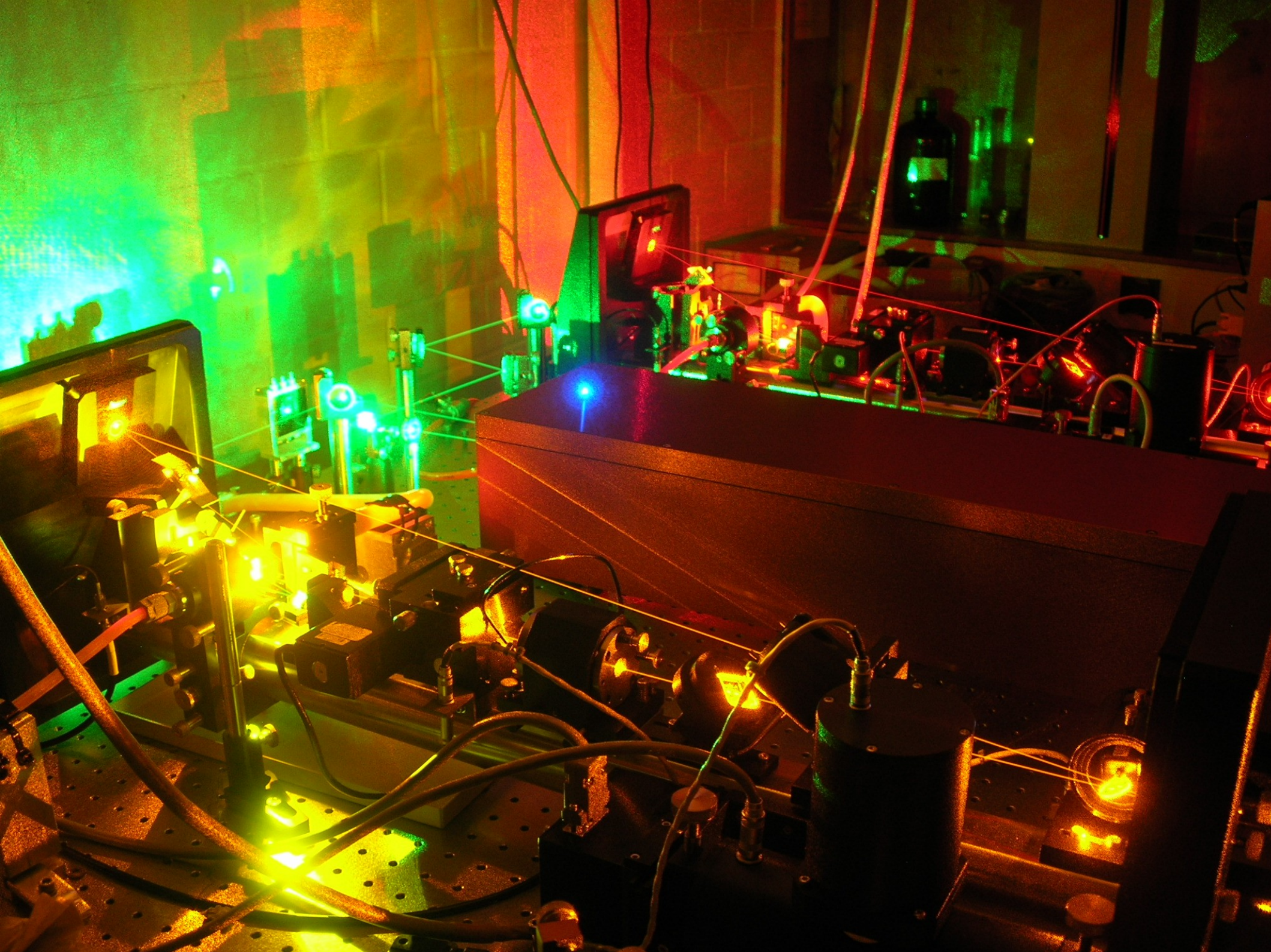
1. Laser light pressure
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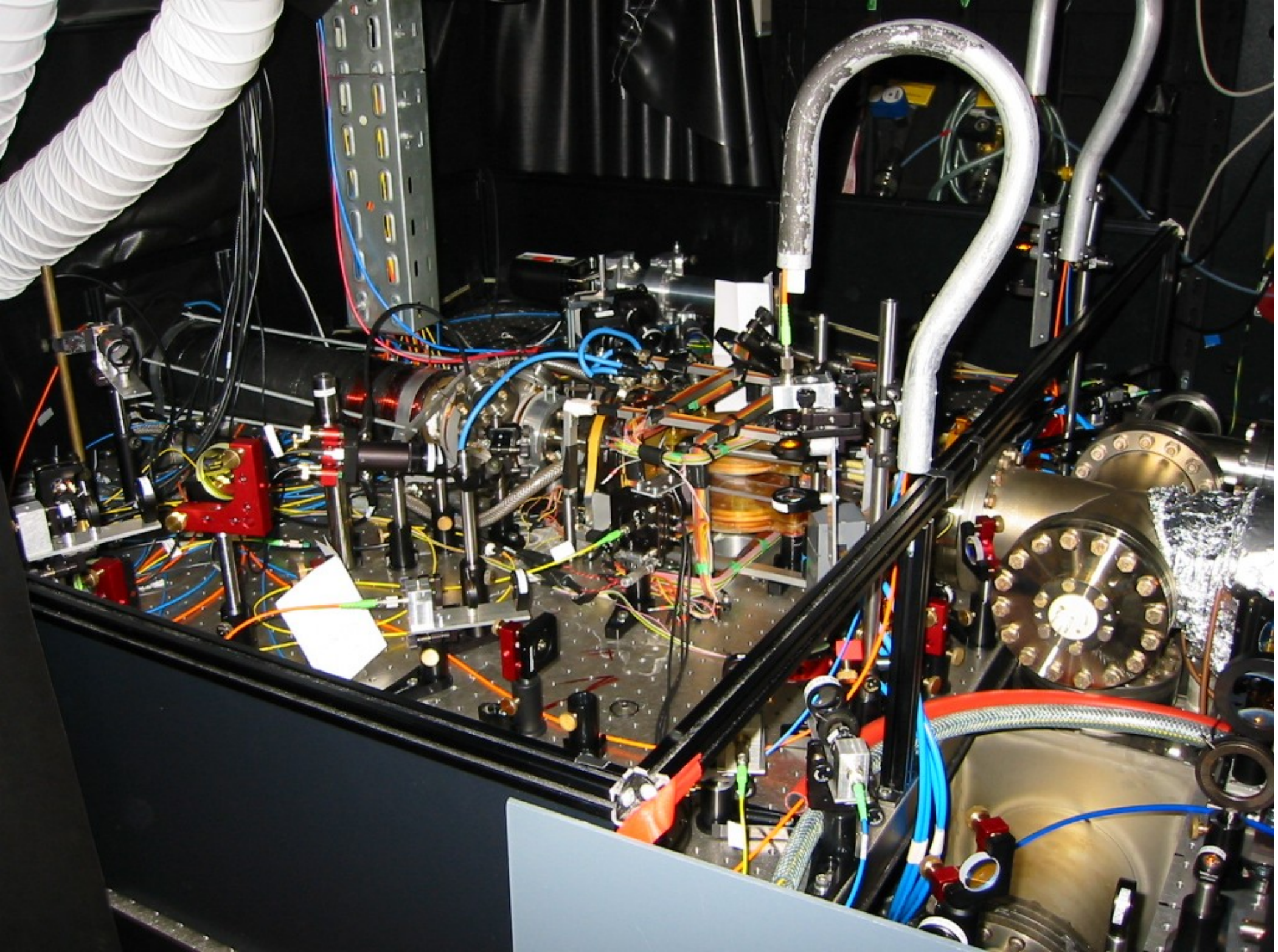


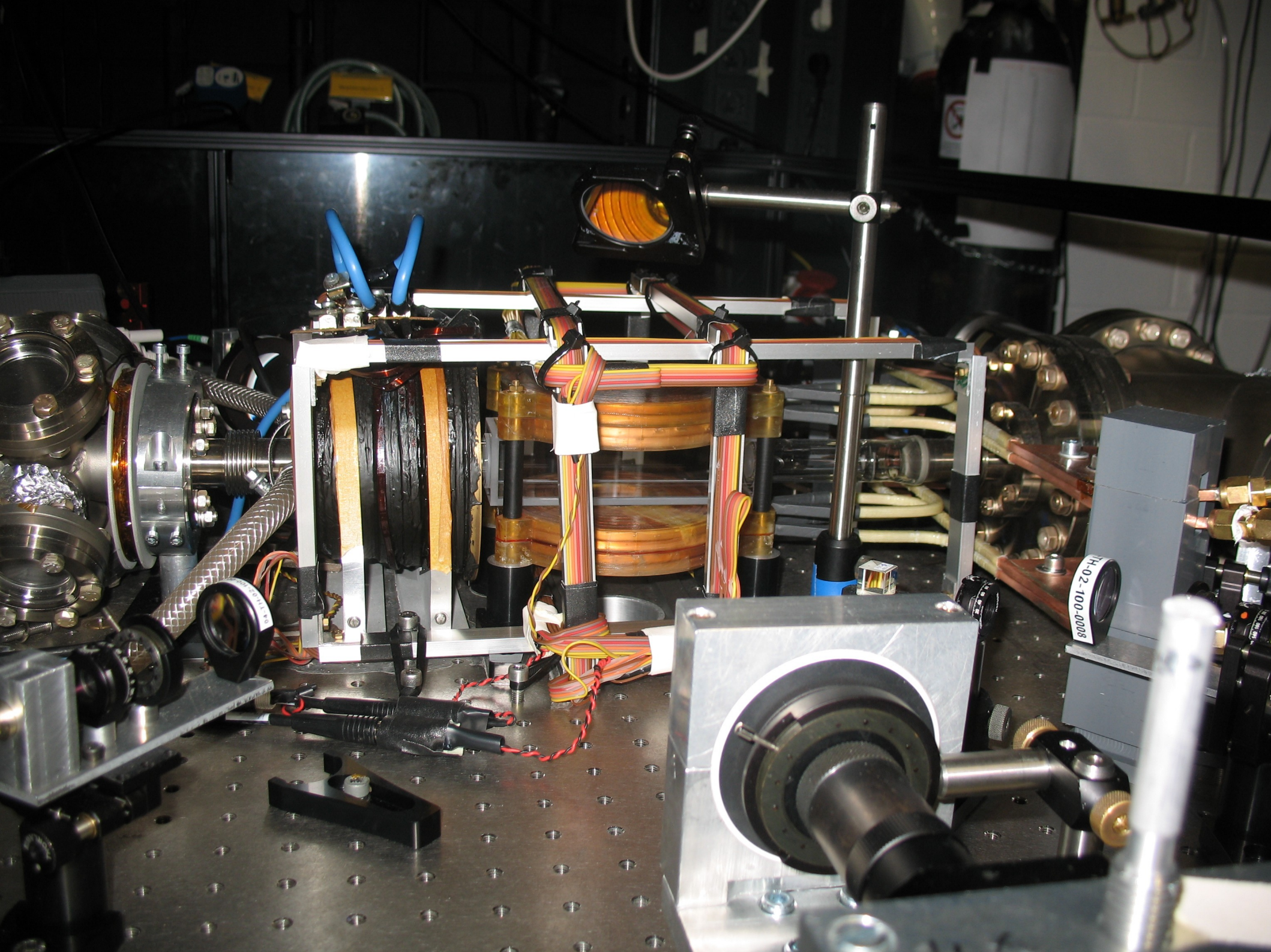
Experimental setup Bose-Einstein condensation at JILA





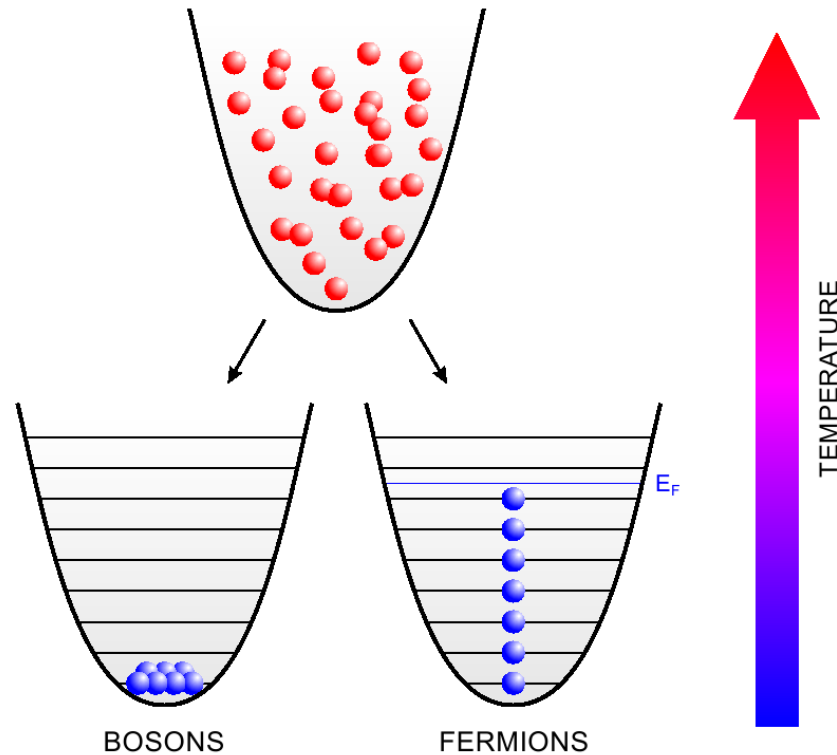






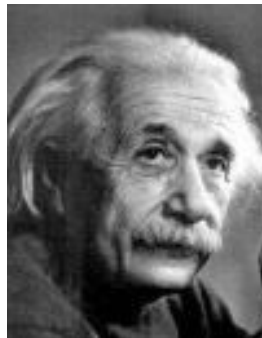
Statistica Quantistica: bosoni e fermioni

“Ai bosoni piace stare insieme, nello stesso stato”



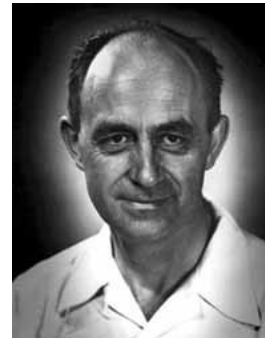
“Ai fermioni non piace stare insieme, nello stesso stato”

● Bose-Einstein statistics (1924)



Dilute gases: 1995, JILA, MIT

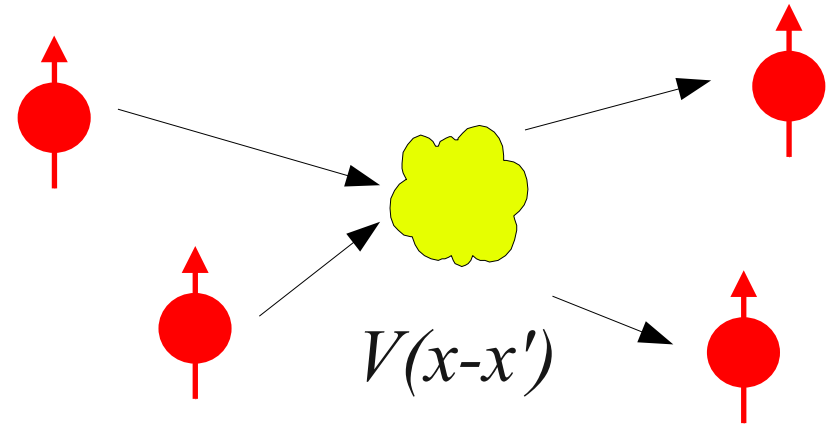
● Fermi-Dirac statistics (1926)



Dilute gases: 1999, JILA

Interaction: s-wave scattering length

At low density and temperature the 2- body interaction is conveniently described by an **effective contact potential** which reproduces the low-energy behaviour of the microscopic potential



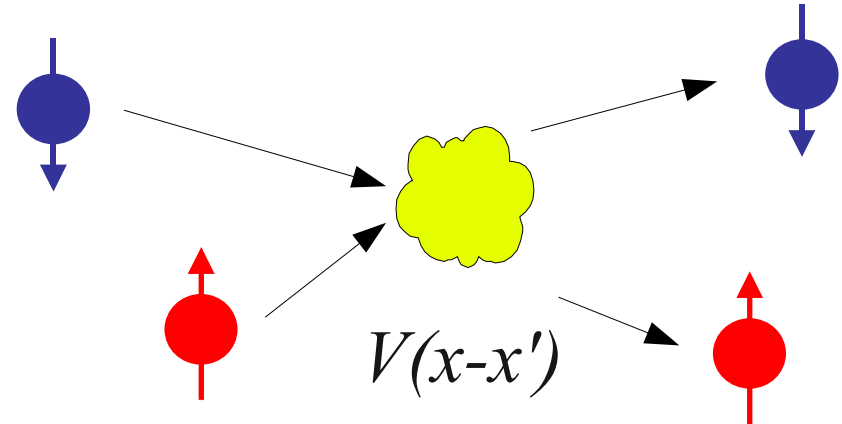
$$V(x - x') \rightarrow V_{eff}(x - x') \propto a\delta(x - x')(+reg.)$$

s-wave scattering length

- i) $a > 0$: positive scattering & a Bound State (D=2,3)
- ii) $a < 0$: negative scattering & NO Bound State (D=2,3)

Interaction: s-wave scattering length

At low density and temperature the 2- body interaction is conveniently described by an **effective contact potential** which reproduces the low-energy behaviour of the microscopic potential



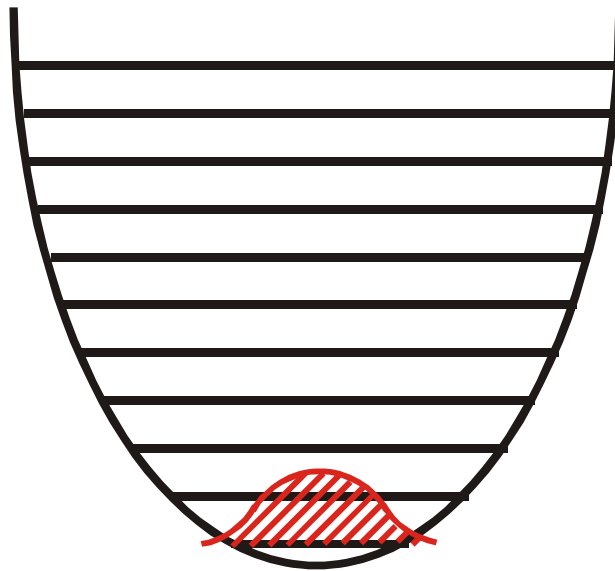
$$V(x - x') \rightarrow V_{eff}(x - x') \propto a\delta(x - x')(+reg.)$$

s-wave scattering length

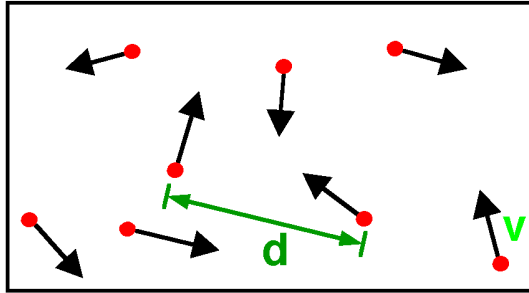
- i) $a > 0$: positive scattering & a Bound State (D=2,3)
- ii) $a < 0$: negative scattering & NO Bound State (D=2,3)

**Due to Pauli principle only fermions
in different internal states can – at this level- interact**

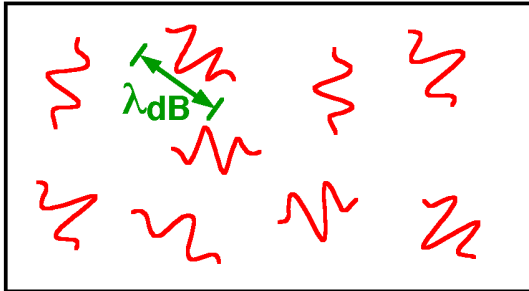
**Ultra-Cold Bosons:
from BEC to
strongly interacting systems**



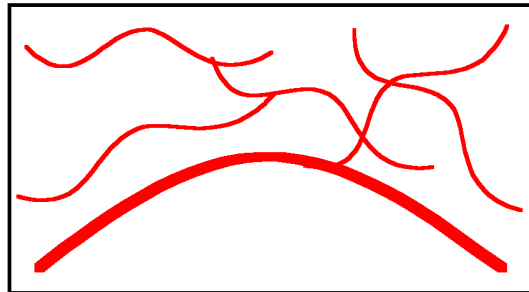
What is Bose-Einstein condensation (BEC)?



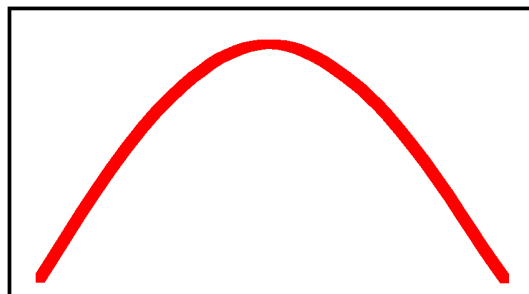
High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"



Low Temperature T:
De Broglie wavelength
 $\lambda_{dB} = h/mv \propto T^{-1/2}$
"Wave packets"



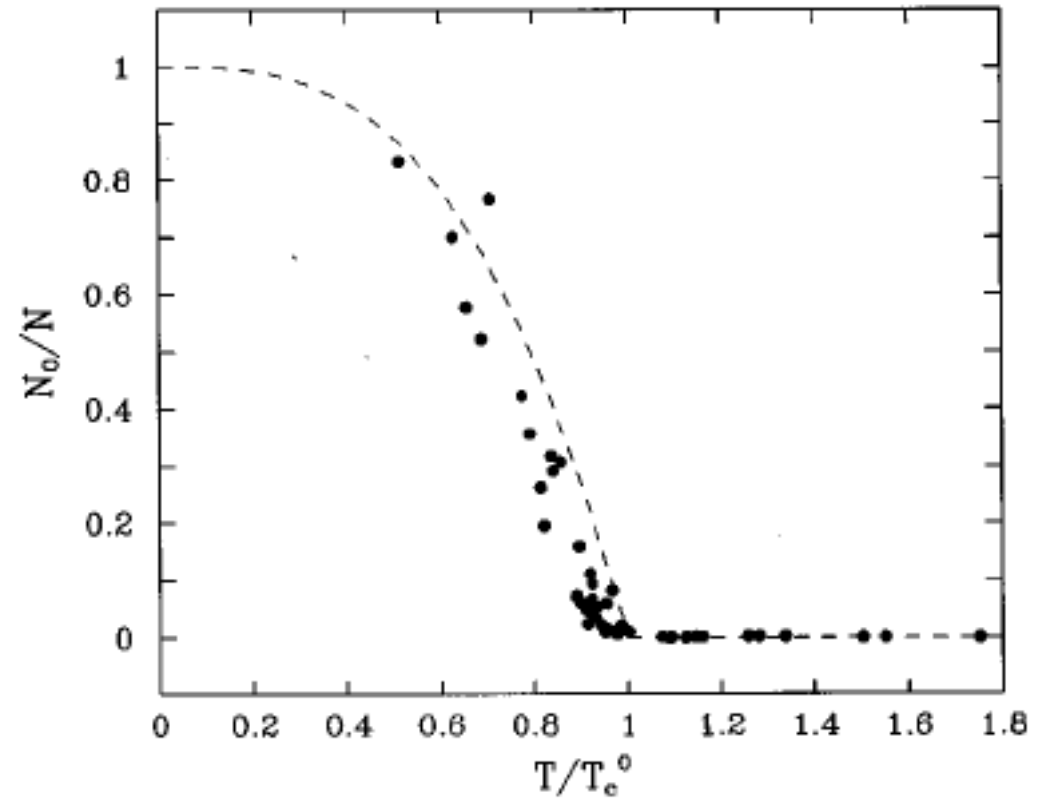
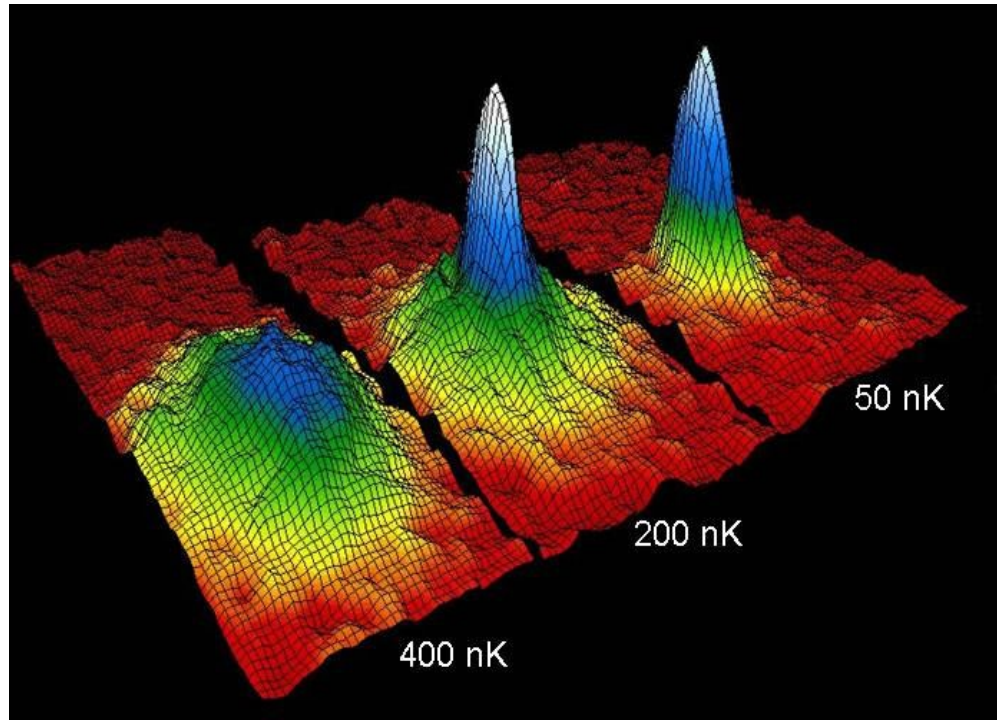
$T = T_{crit}$:
Bose-Einstein
Condensation
 $\lambda_{dB} \approx d$
"Matter wave overlap"



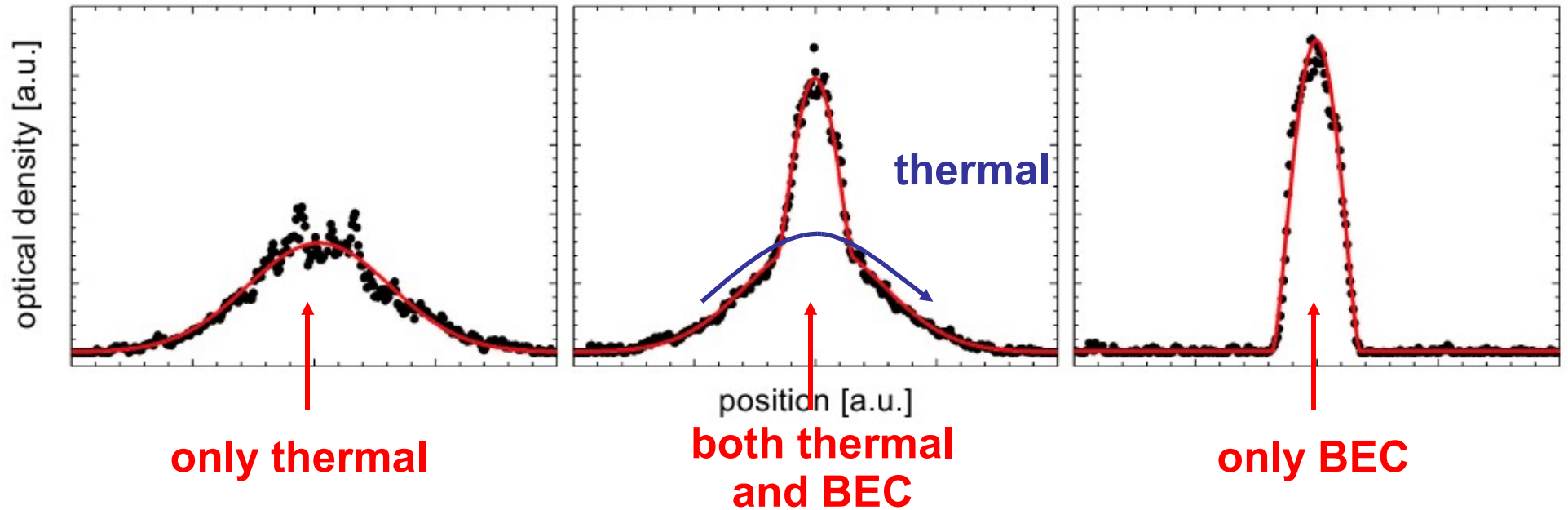
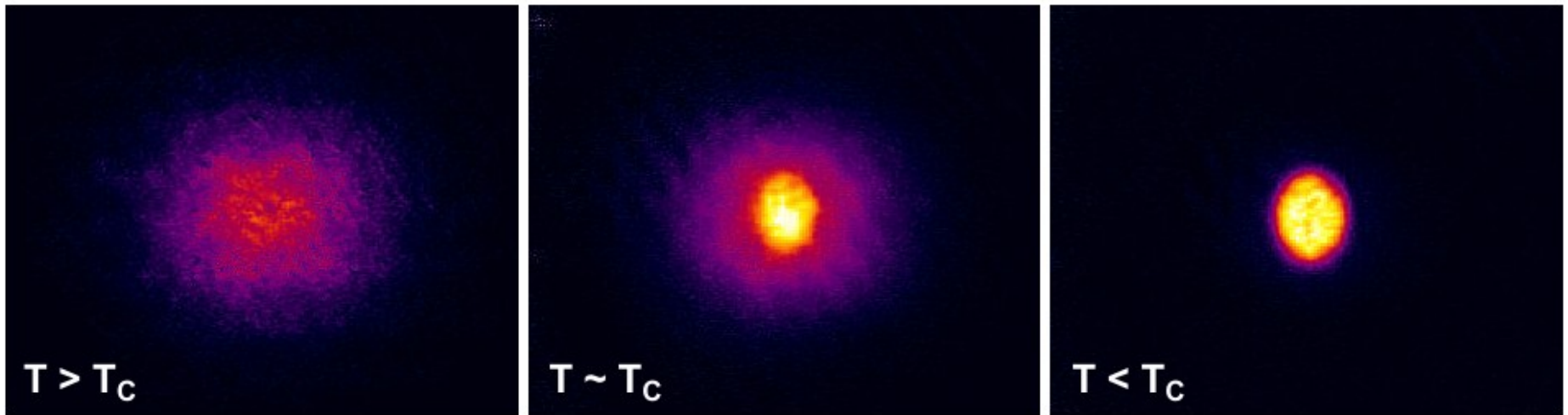
$T = 0$:
Pure Bose
condensate
"Giant matter wave"

New state of matter...

One of the first BEC images (JILA 1995)



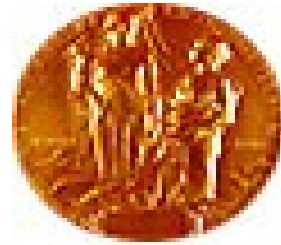
Below a certain temperature a macroscopic atomic fraction occupies the lowest energy state



Temperature measured through
thermal component tails

Physics Nobel Laureates

1997



"for development of methods to cool and trap atoms with laser light"



Steven Chu

**Claude
Cohen-
Tannoudji**

**William D.
Phillips**

2001

"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

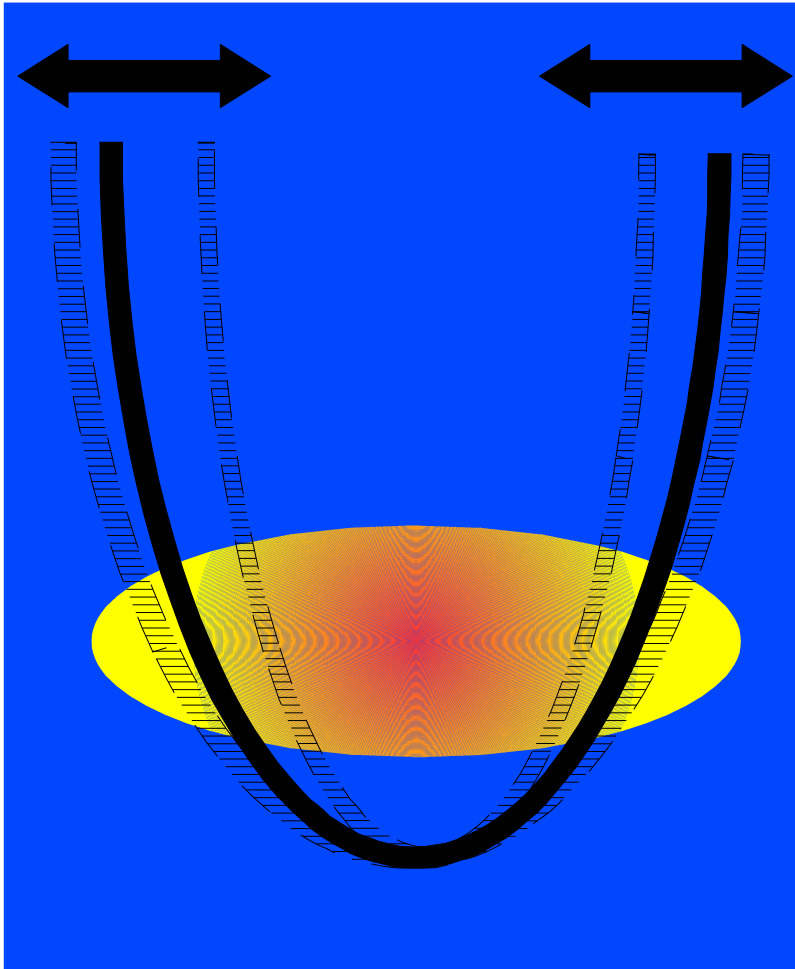


**Eric A.
Cornell**

**Wolfgang
Ketterle**

**Carl E.
Wieman**

Collective and Josephson like oscillations/Interference

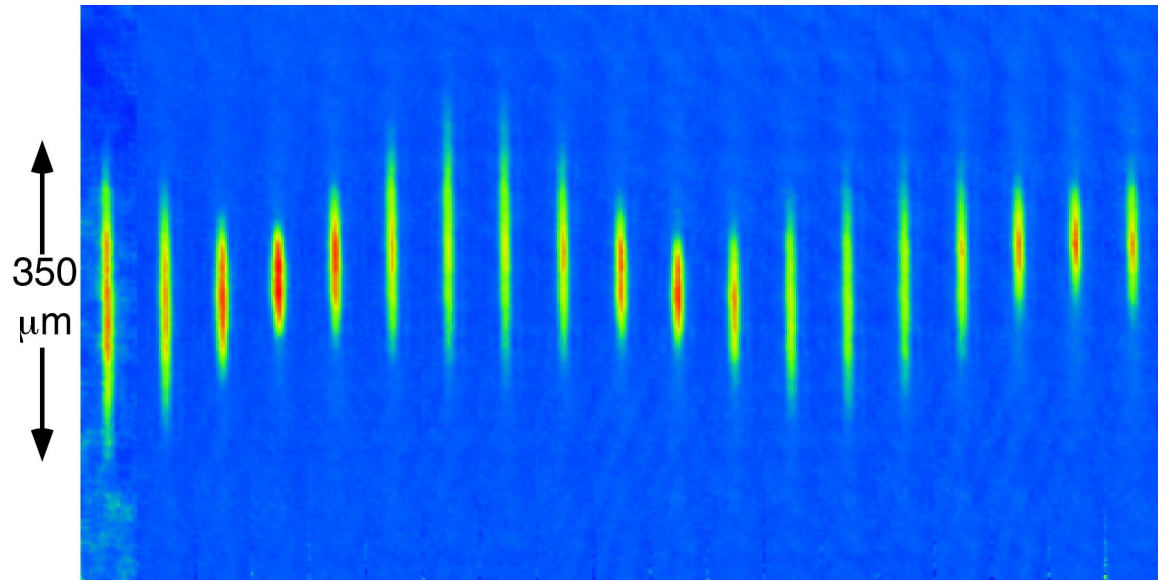


Collective Oscillations:

exp. (MIT '97): $\omega = 1.57\omega_z$

teory (HD): $\omega = 1.58\omega_z$

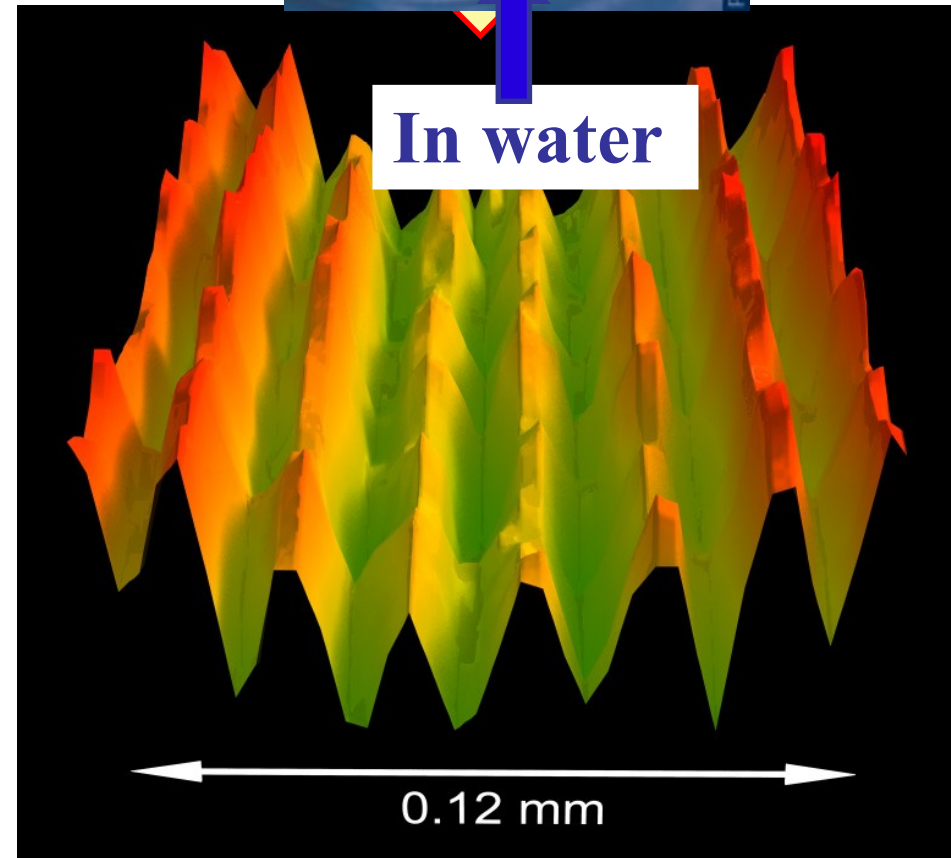
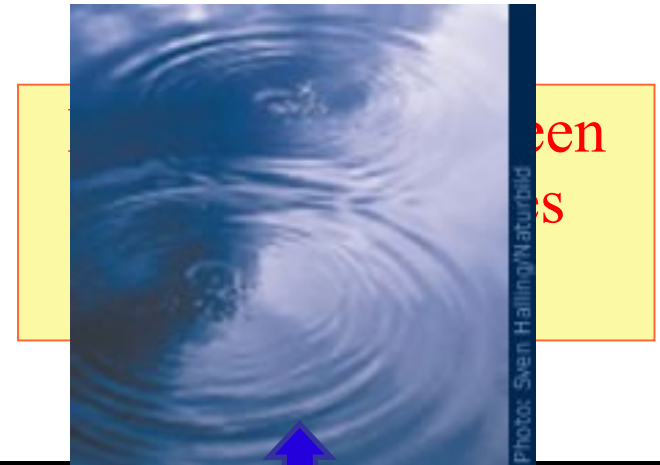
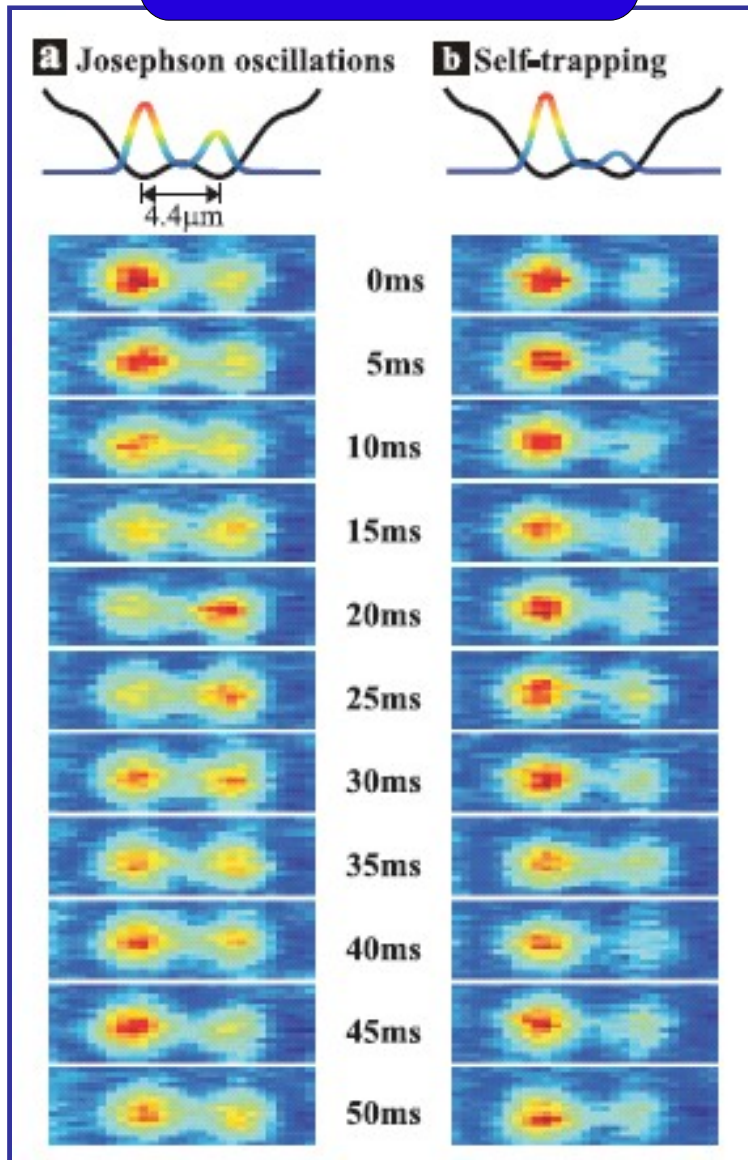
ideal gas: $\omega = 2\omega_z$



5 milliseconds per frame

Collective and Josephson like oscillations/Interference

**Double well
(Heidelberg 2004)**



Gross-Pitaevskii equation

$$i\hbar\partial_t\Psi(\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\Delta + V(\mathbf{x}) + \frac{4\pi\hbar a}{m}|\Psi(\mathbf{x})|^2 \right) \Psi(\mathbf{x})$$

$\mu\Psi(\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\Delta + V(\mathbf{x}) + \frac{4\pi\hbar a}{m}|\Psi(\mathbf{x})|^2 \right) \Psi(\mathbf{x})$

Chemical potential

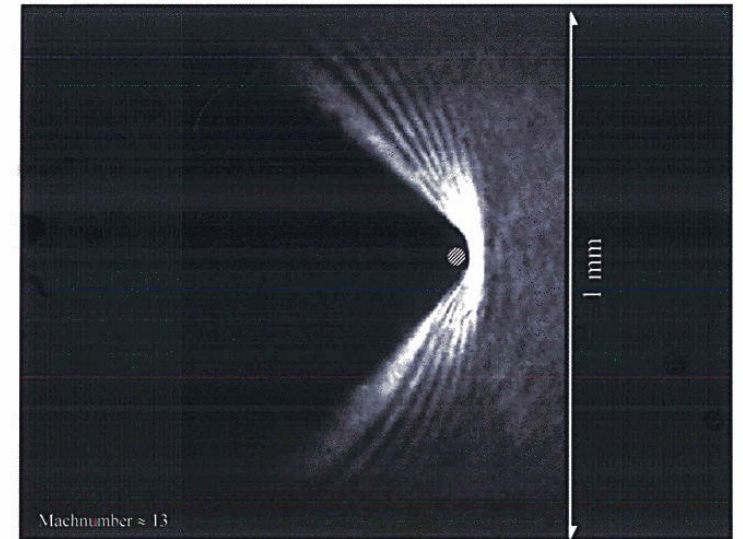
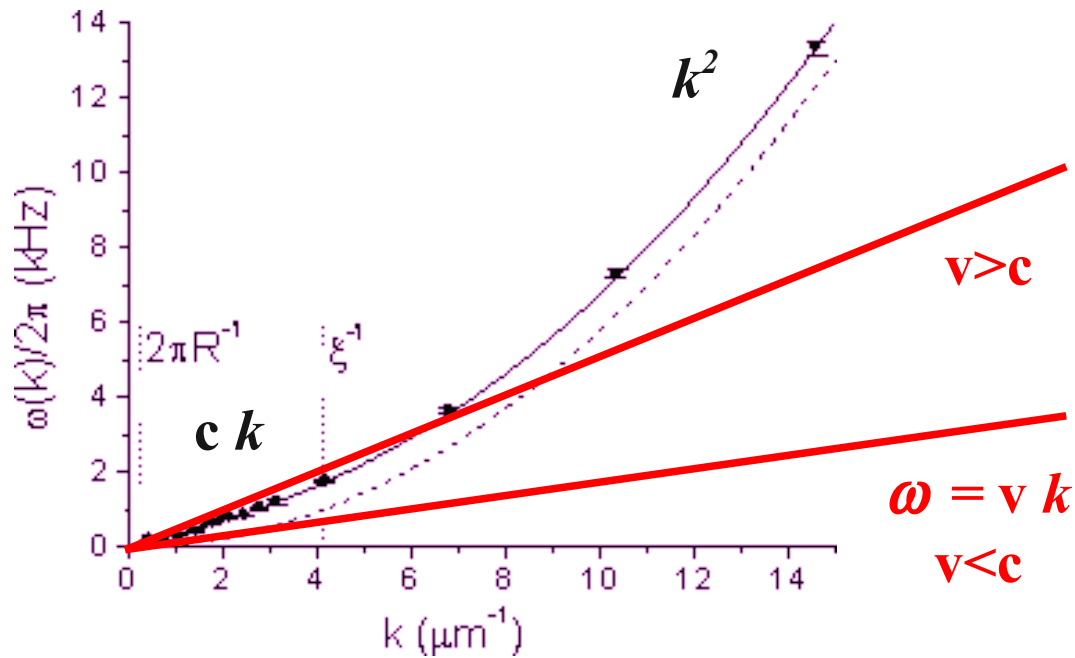
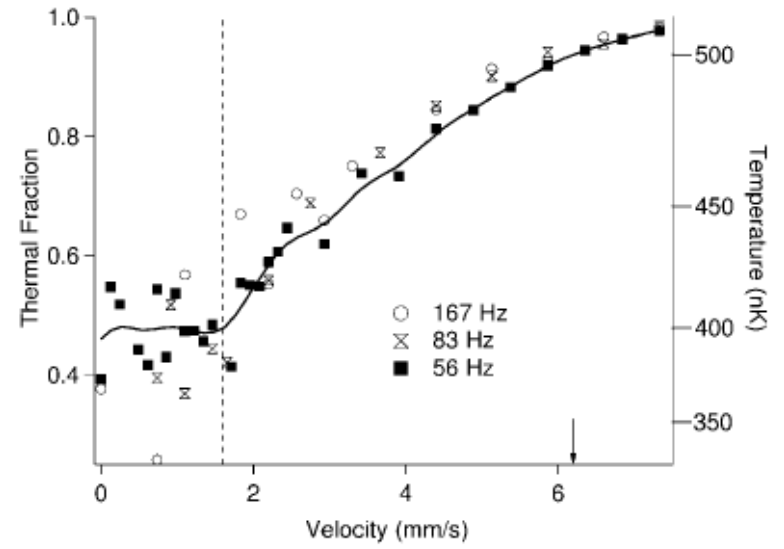
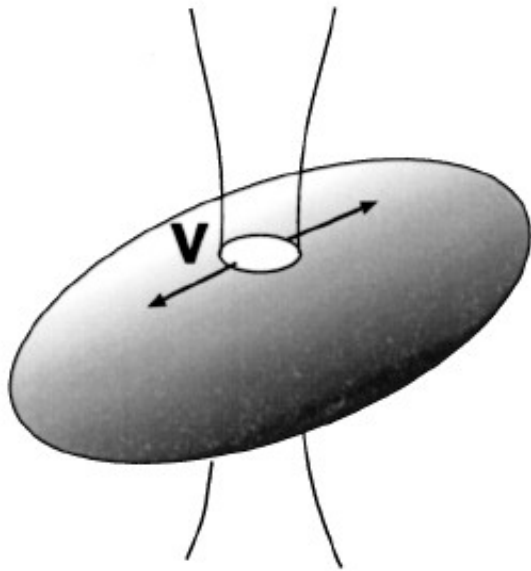
Kin. Energy

Ext. Potential energy

2-body interaction

Order Parameter $\Psi = \sqrt{n}e^{iS}$

Superfluidity of a BEC

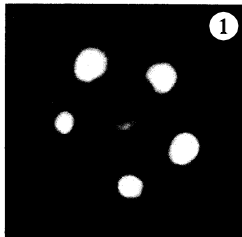


Cherenkov phonons

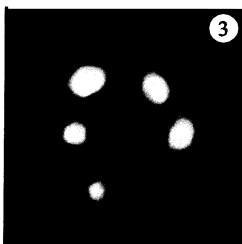
Vortices in a BEC

In rotating condensate vortices are produced

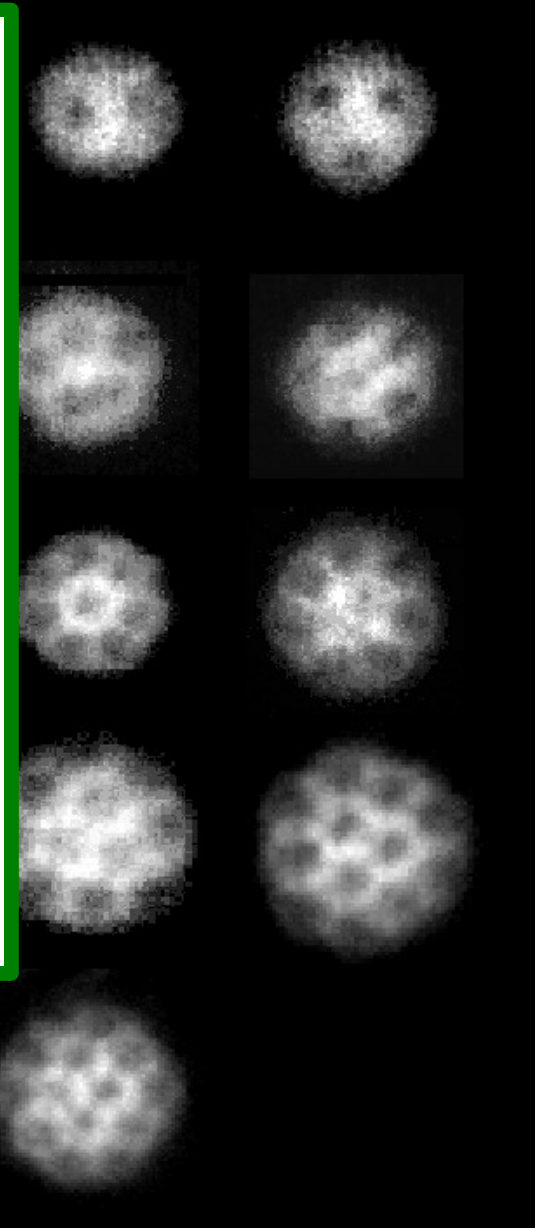
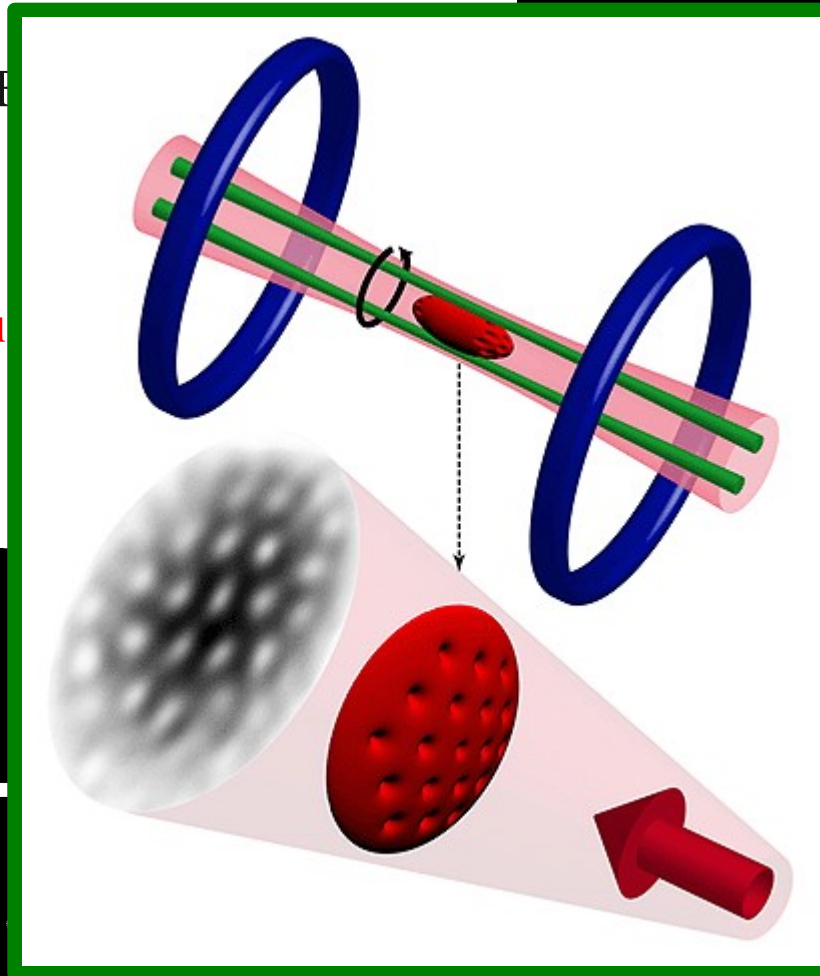
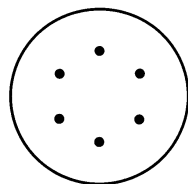
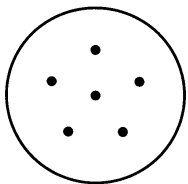
Vortices in a BEC
condensate
superfluid helium



1



3



Vortices in a BEC

In rotating condensate vortices are produced
“quantized”

In a superfluid the motion is irrotational (free vortex)

$$\Psi = \sqrt{n} e^{iS}$$



$$v_{\theta} = \frac{\hbar}{m} \nabla S = \frac{C}{r}$$



Where C is an integer multiple, n , of a minimum value:

$$C = \left(\frac{\hbar}{m} \right) n$$

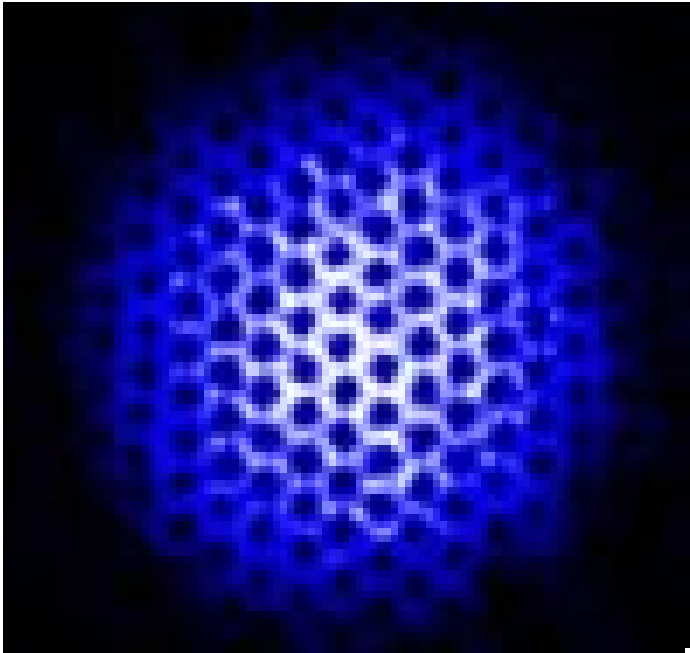
quantum of circulation

Indeed by definition one must have:

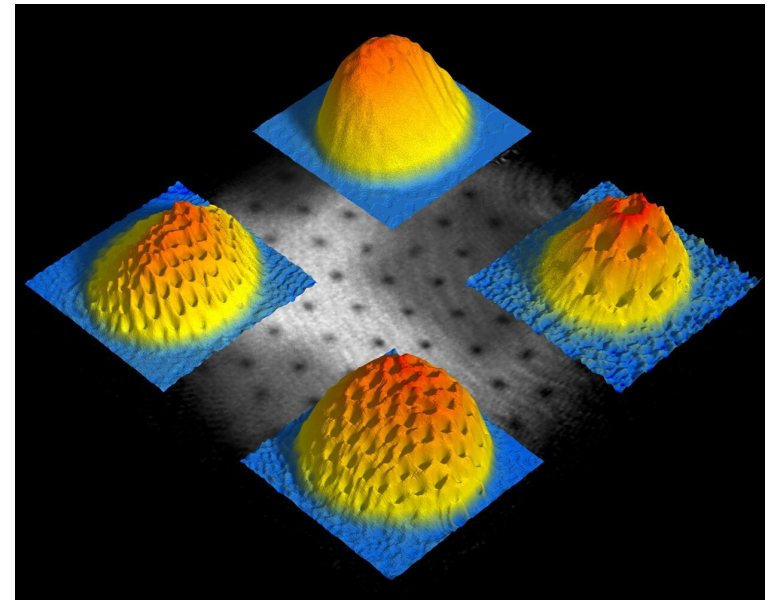
$$\oint \mathbf{v} \cdot d\mathbf{l} = \frac{\hbar}{m} 2\pi n$$

Vortices in a BEC

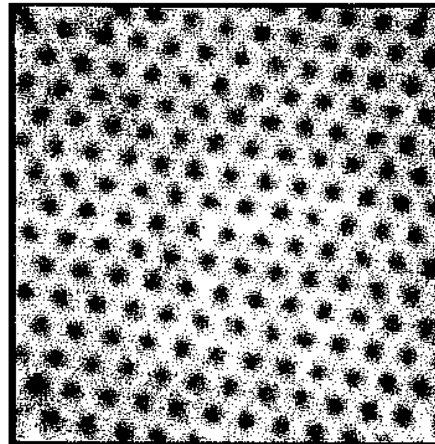
In rotating condensate vortices are produced
“quantized” and cristallize (Vortex Lattice)



[JILA, 2002]



[MIT, 2002]



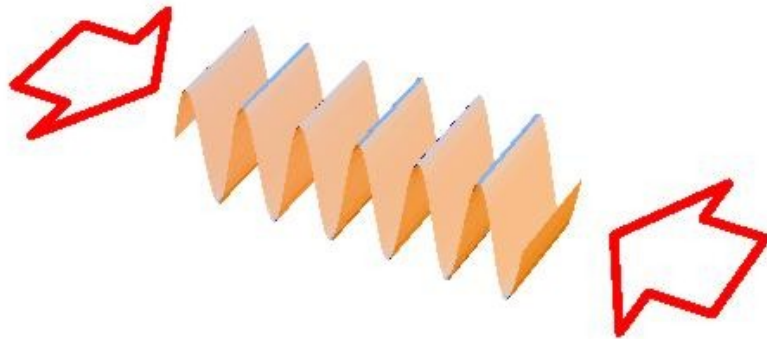
Vortices in NbSe₂ defined by scanning tunneling microscopy (STM).

Optical Lattices

Periodic potential via the dipole force

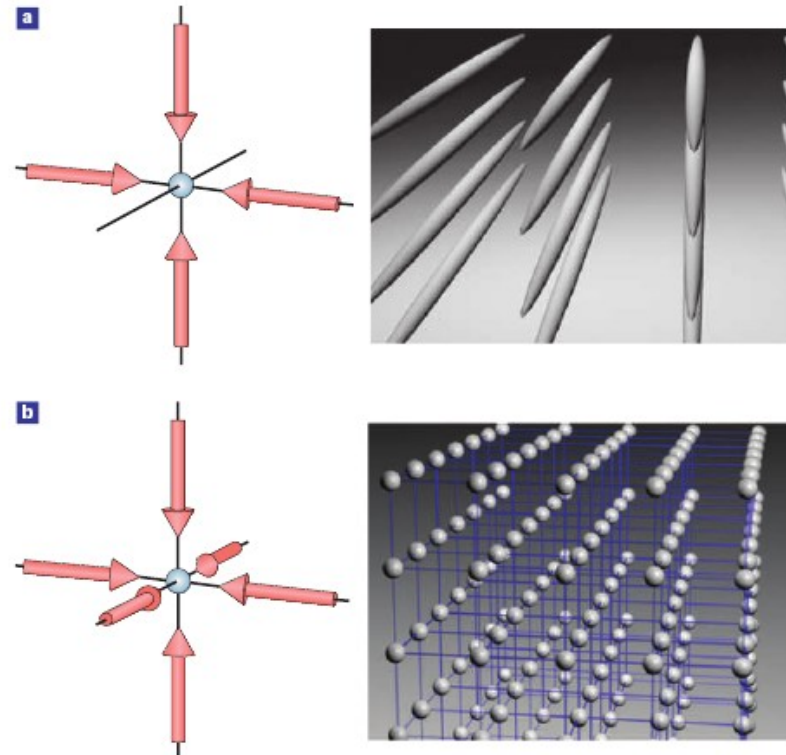
$$\mathbf{F}(\mathbf{x}) = -(\alpha(\omega)|\mathbf{E}(\mathbf{x})|^2/2)$$

Laser standing wave



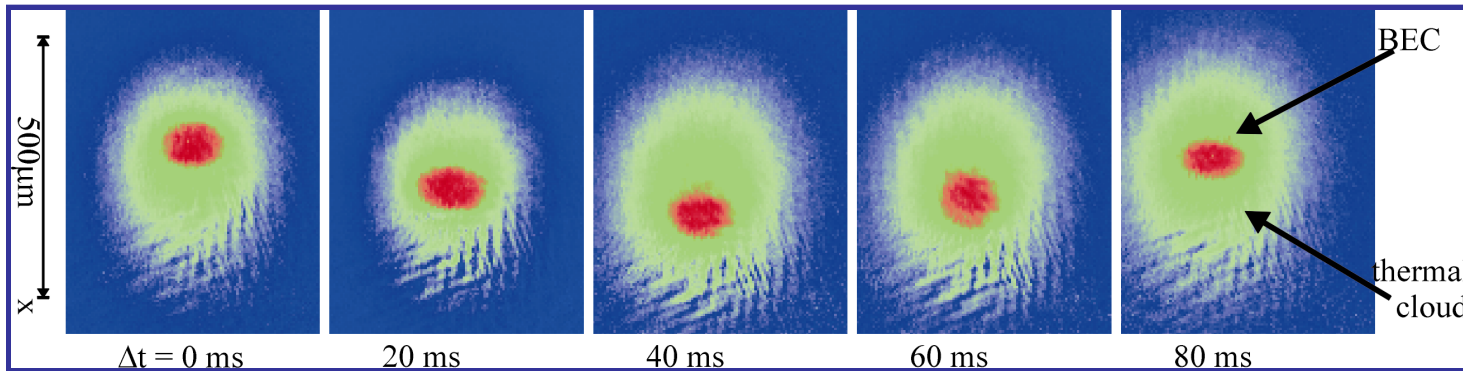
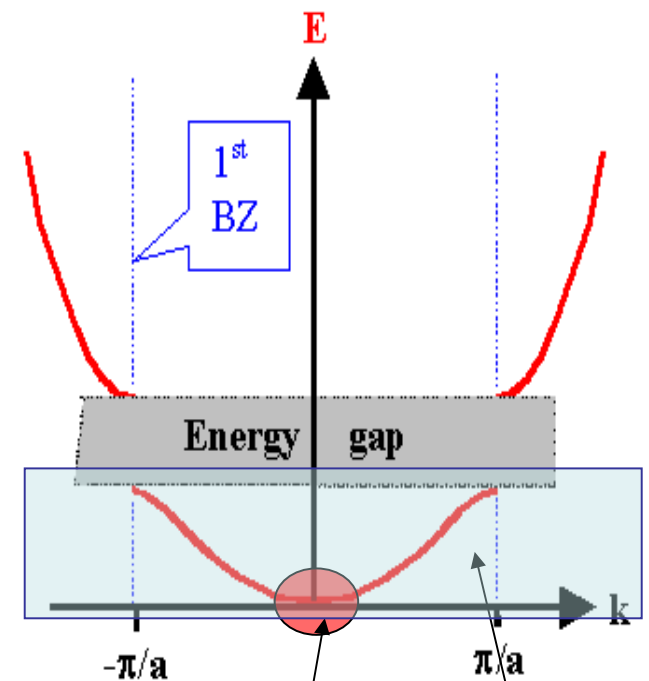
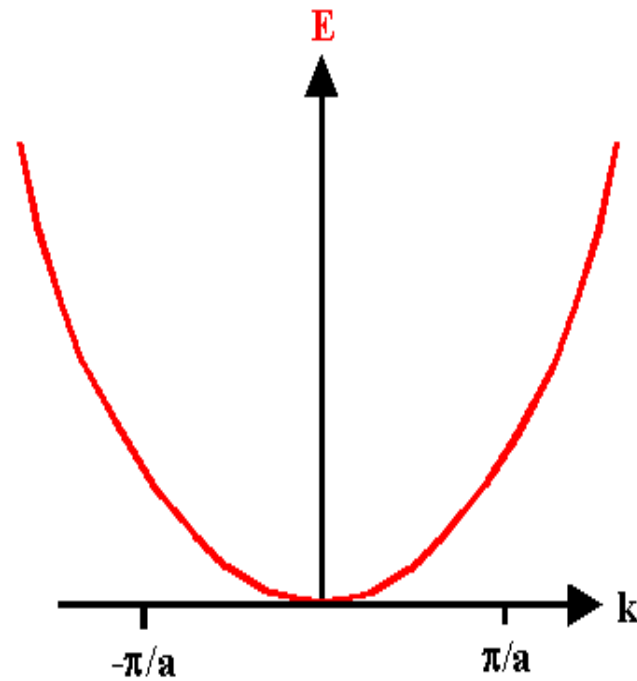
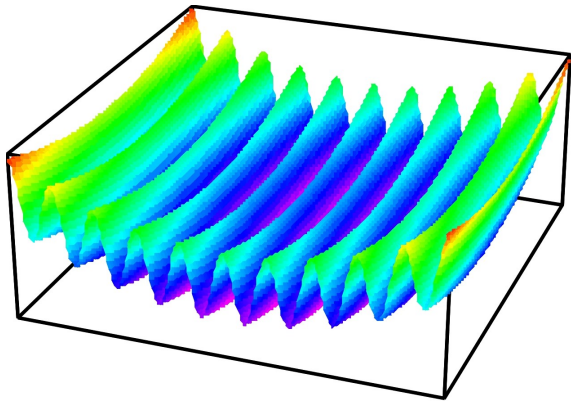
1D (optical) lattice

$$V(\mathbf{x}) = sE_r \sin^2(kx)$$



1D dynamics, (Fermi/Bose) Hubbard model description, Mott insulating phase...

Bose-Einstein (superfluid) vs. Normal component oscillations

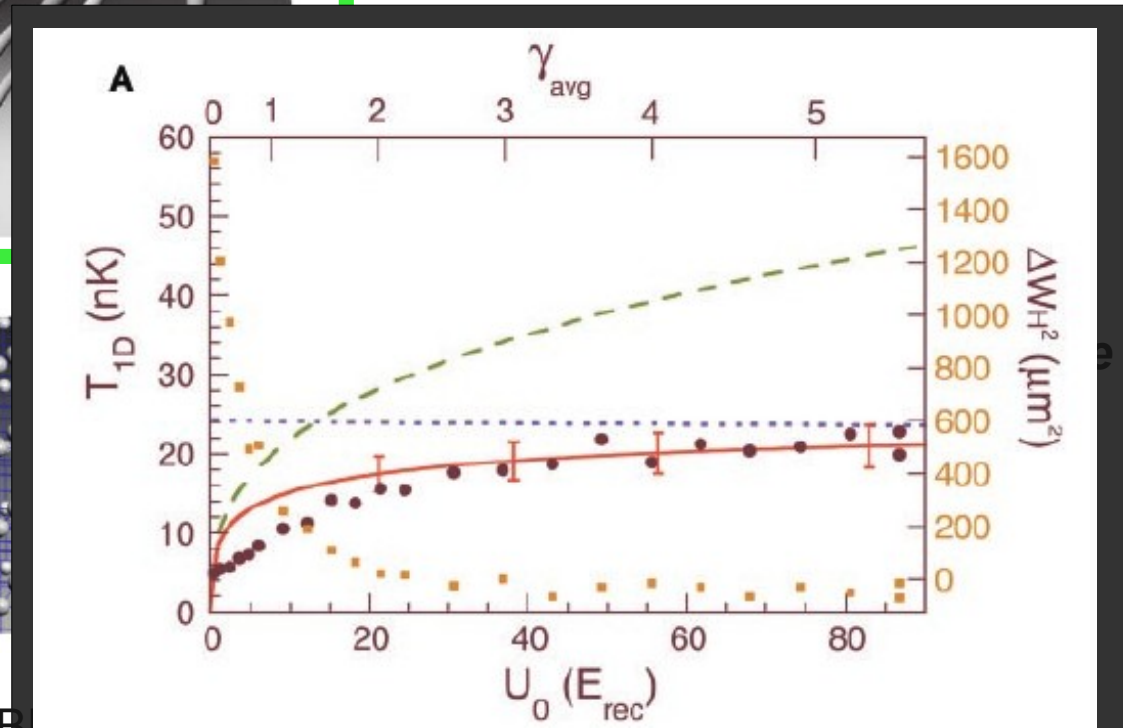
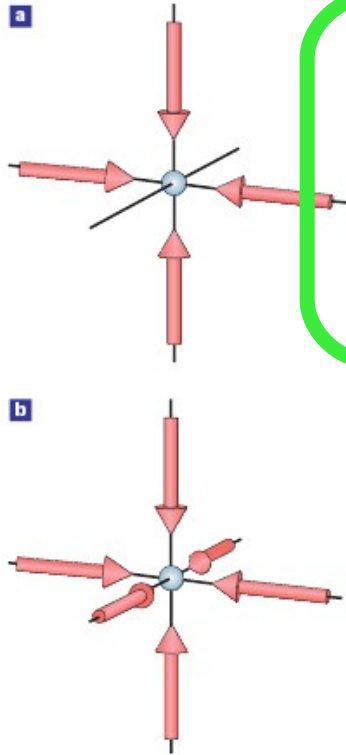


BEC
Thermal cloud

Periodic potential
(Florence 2001)

Only condensate coherently tunnels through the barriers

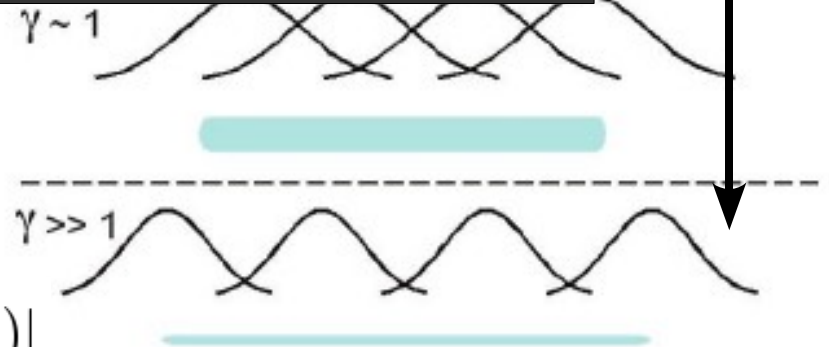
1D Tonks-Girardeau gas (or how to fermionize bosons)



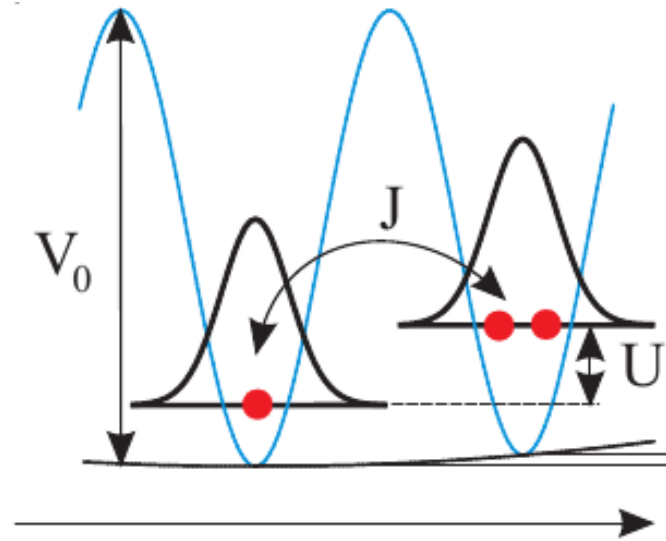
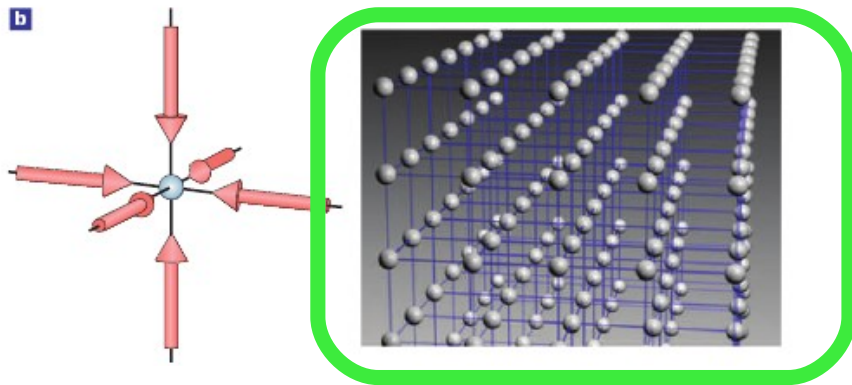
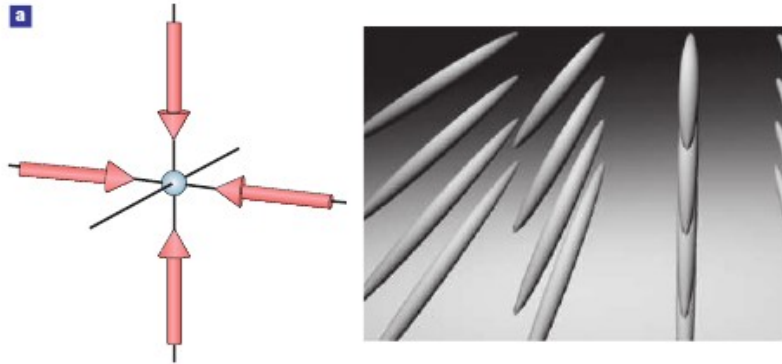
David Weiss' group (2004)

Tonks-Girardeau gas

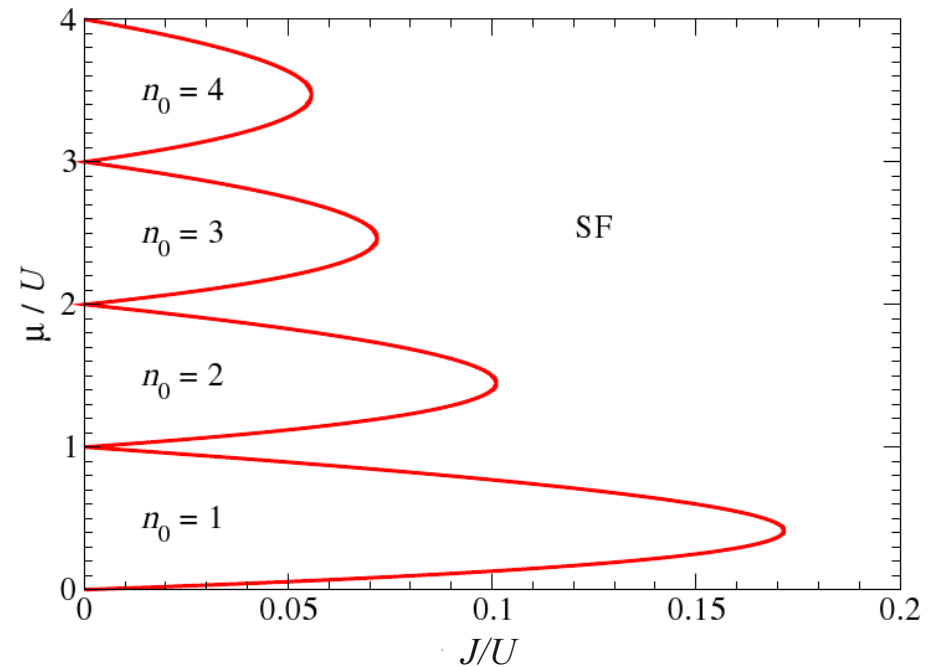
$$\Psi_B(x_1, x_2, \dots, x_N) = |\Psi_F(x_1, x_2, \dots, x_N)|$$



Bose-Hubbard Hamiltonian



$$H = \sum_{i,j} J_{ij} b_i^\dagger b_j - \mu \sum_i n_i + U \sum_i n_i (n_i - 1)$$

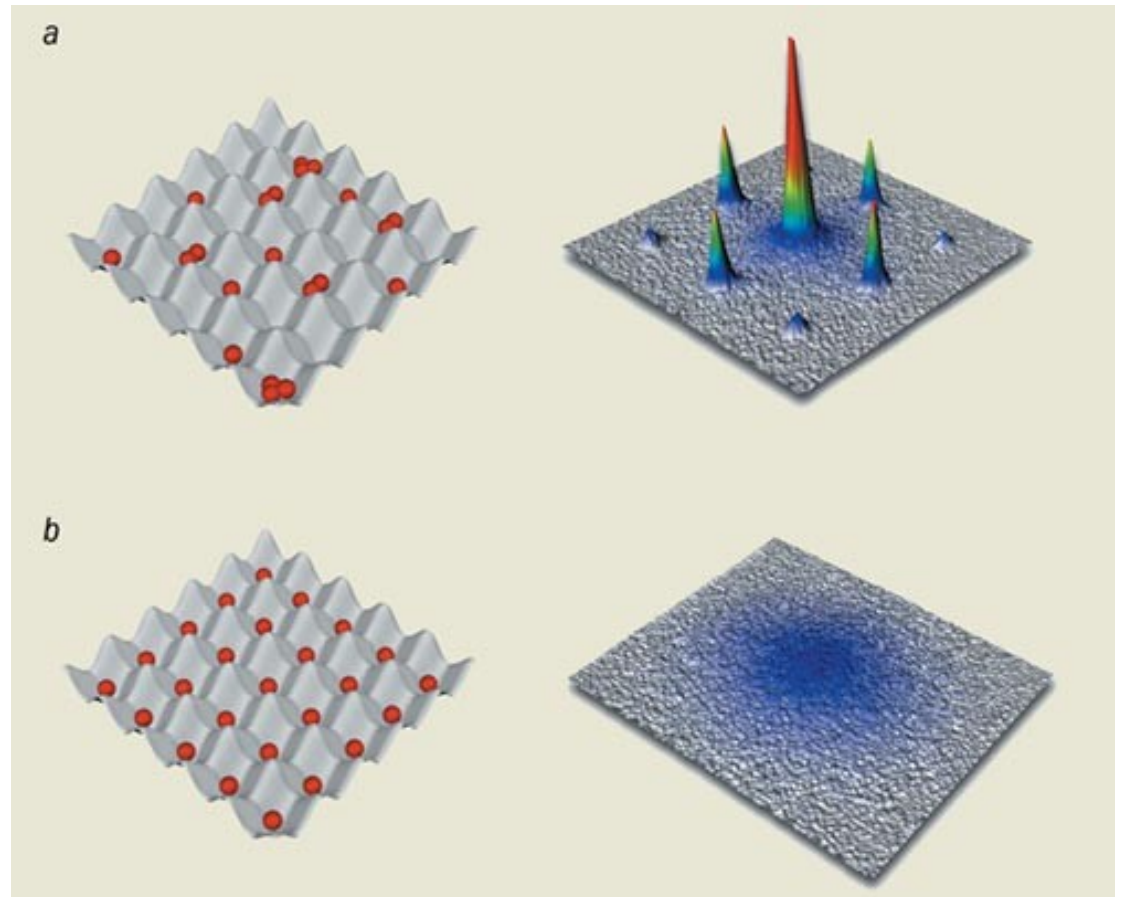


Quantum phase transition from superfluid to Mott insulator

Atoms move around,
delocalized on
many lattice site

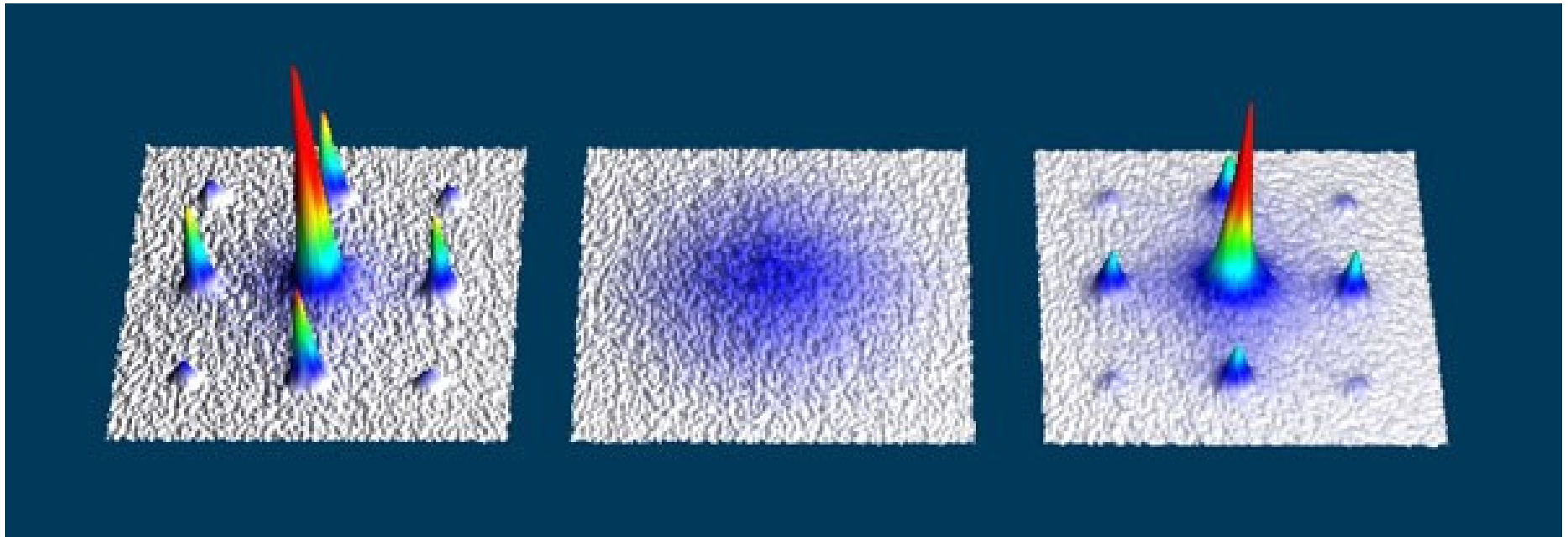
Increasing the
lattice strength

Atoms are
localized on
a single lattice site



[I. Bloch et al. (2002)]

*Quantum phase transition
from superfluid to Mott insulator*



superfluid

Mott

superfluid

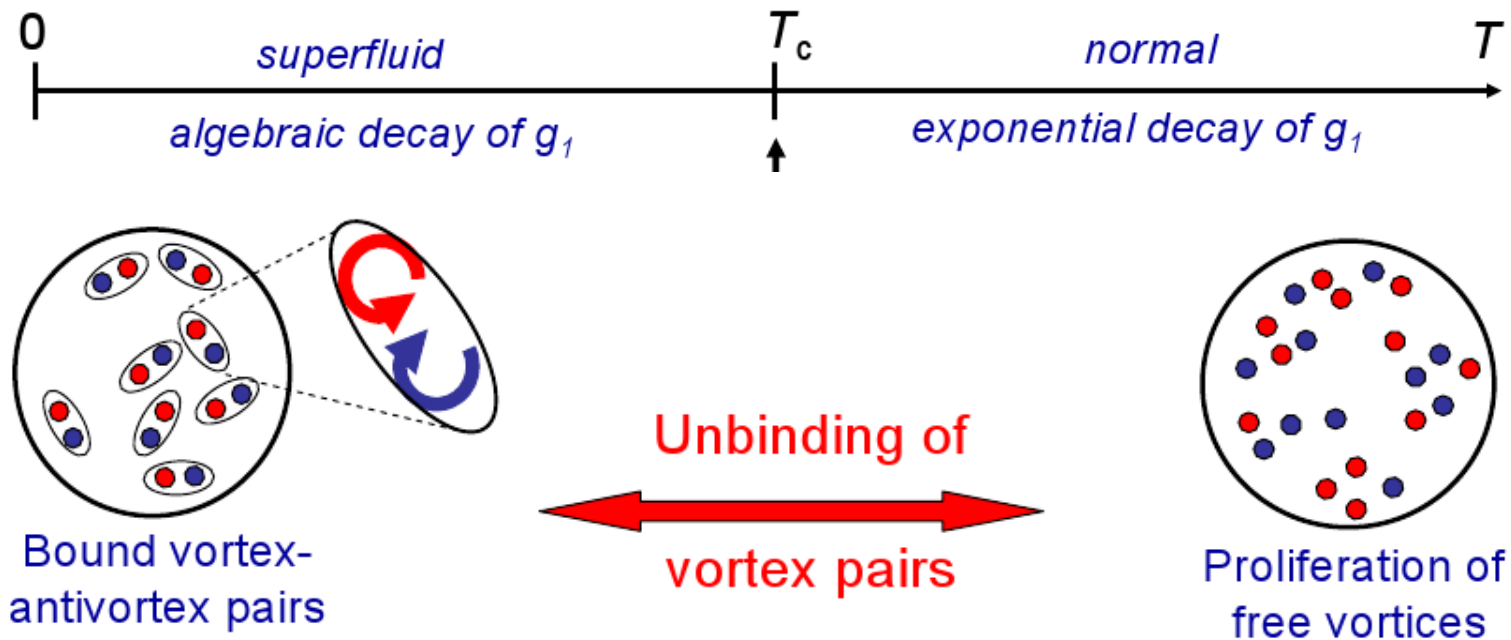


SF – Mott and SF again: a coherent path through a quantum phase transition

BKT phase transition in 2D BEC

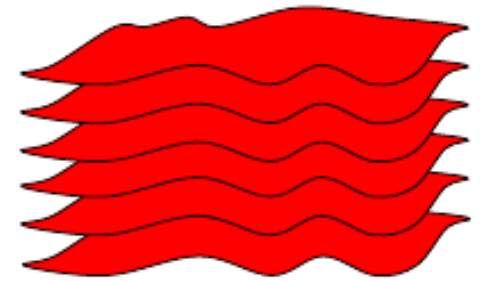
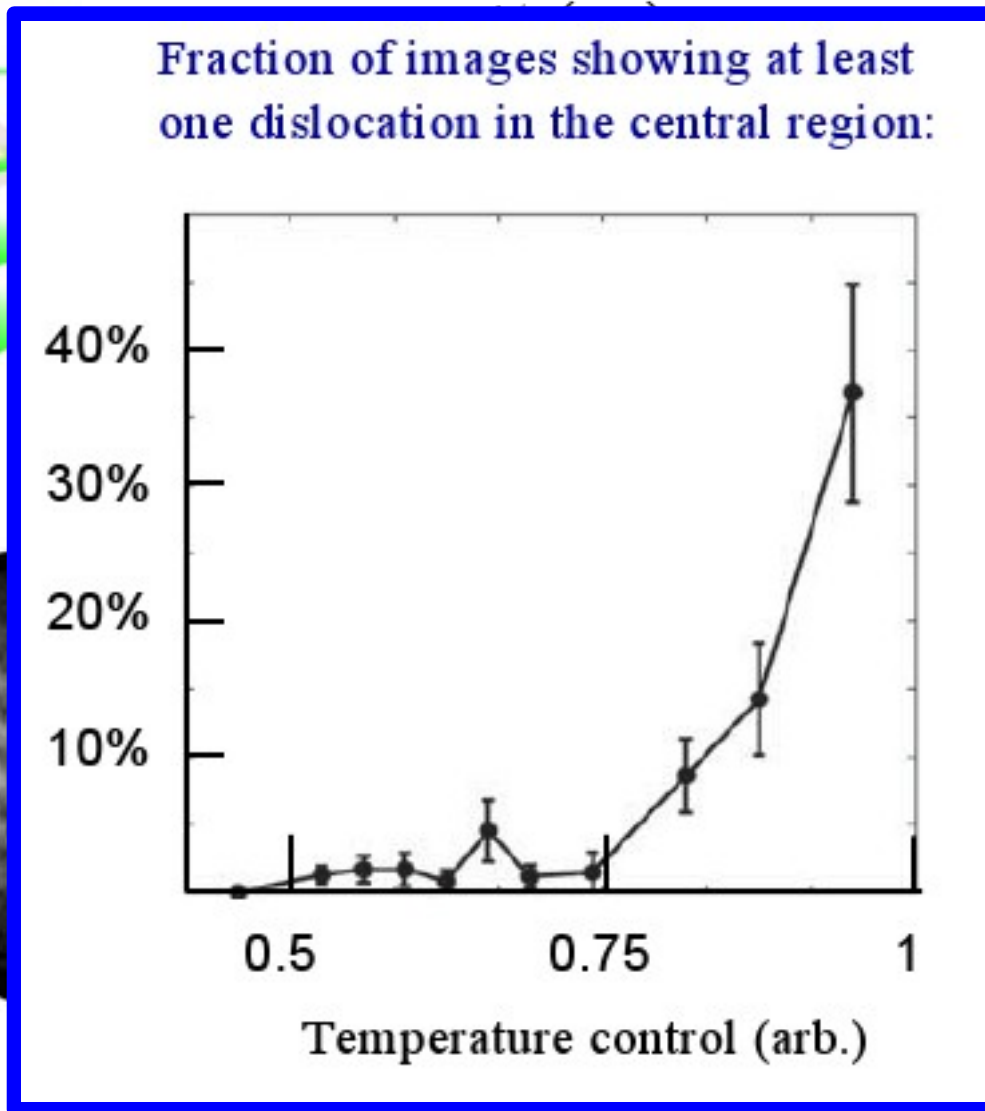
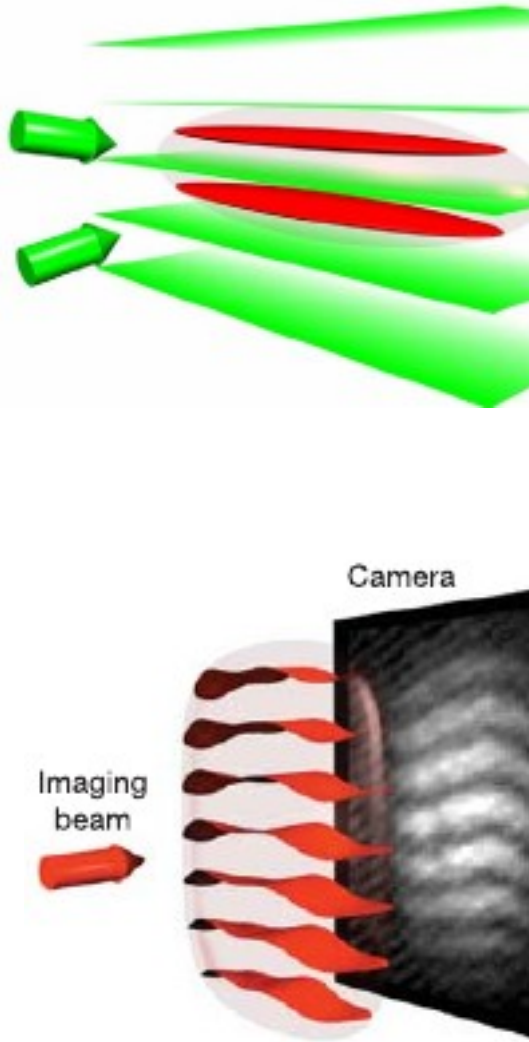
Mermin-Wagner-Hohenberg Theorem:

No true long range order at any finite temperature in 2D

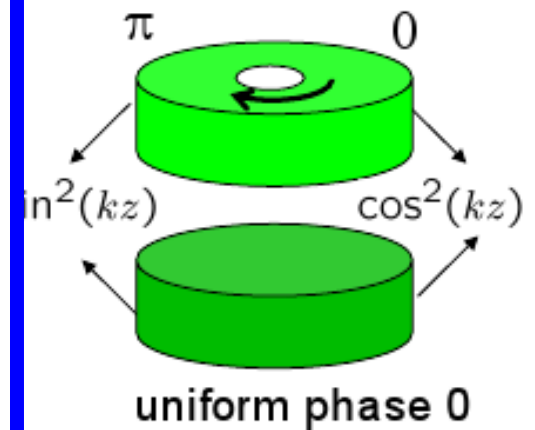


Berezinskii-Kostrelitz-Thouless Transition

BKT phase transition in 2D BEC



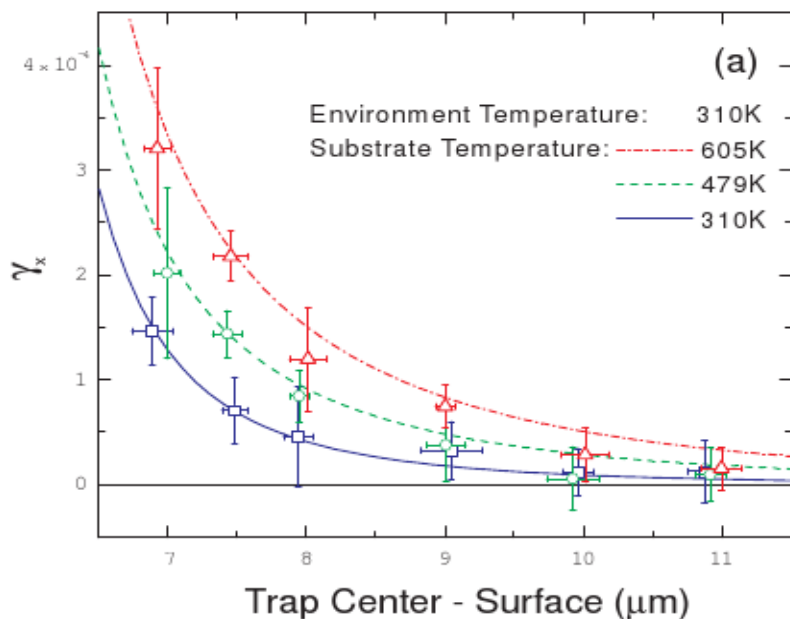
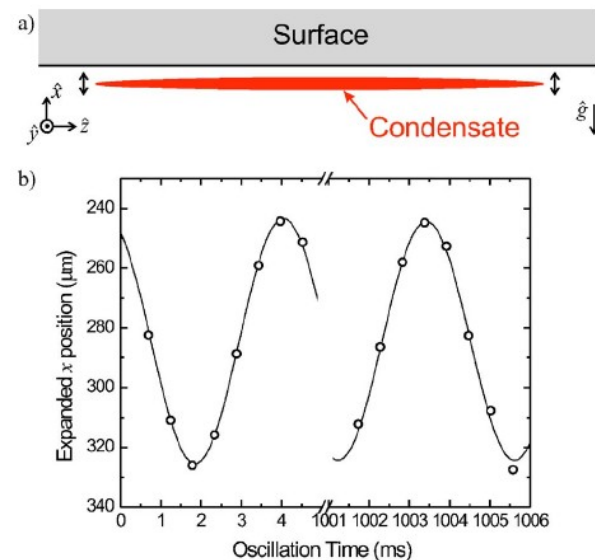
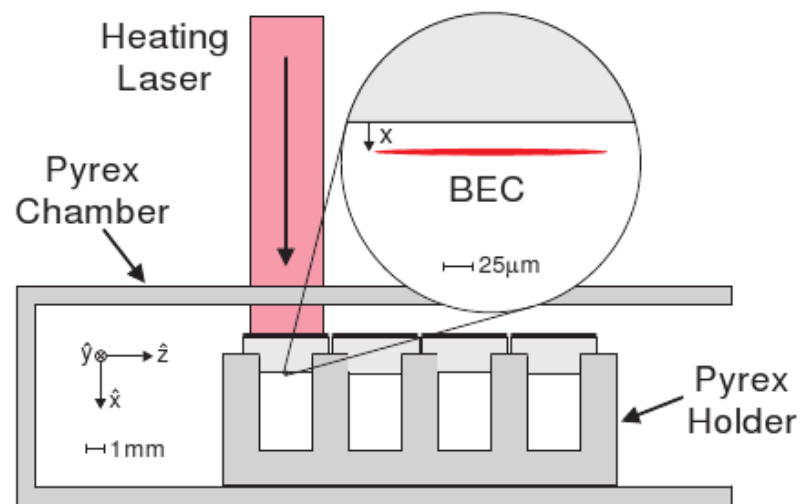
ortex



variation dislocation

[Dalibard's group (2006)]

BEC to measure the temperature dependence of Casimir-Polder forces



$$\gamma_x \equiv \frac{\omega_o - \omega_x}{\omega_o} \simeq \frac{1}{2m\omega_o^2} \langle \partial_x F_{CP} \rangle$$

[From Eric Cornell, 2006]