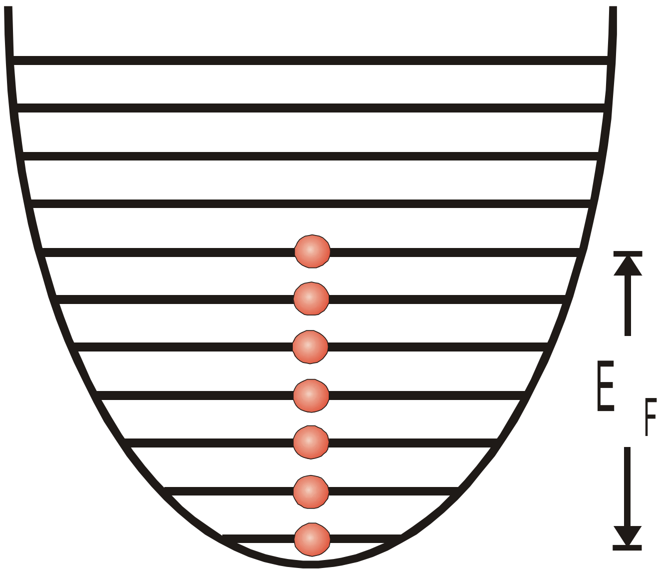
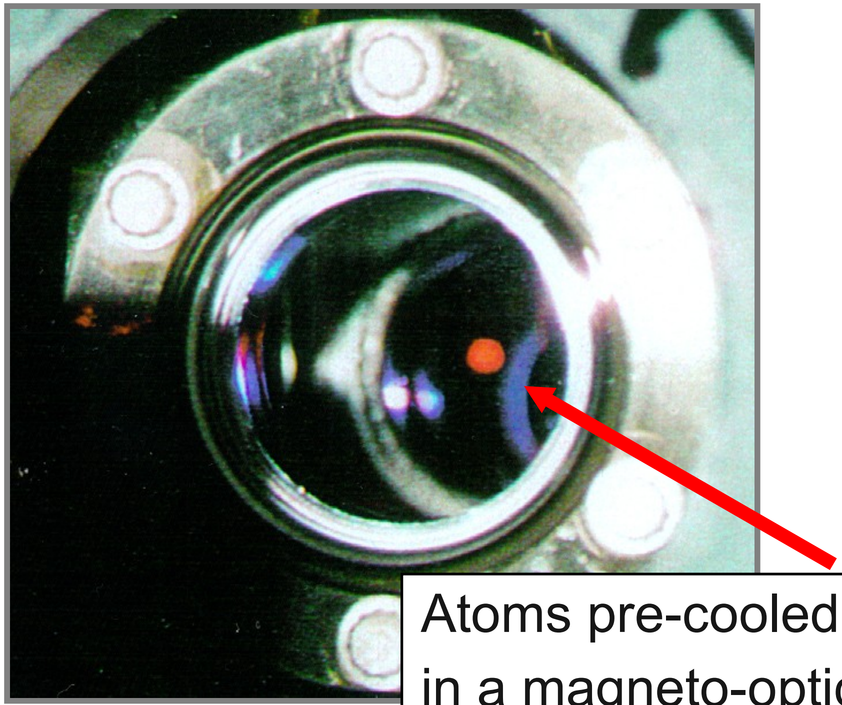
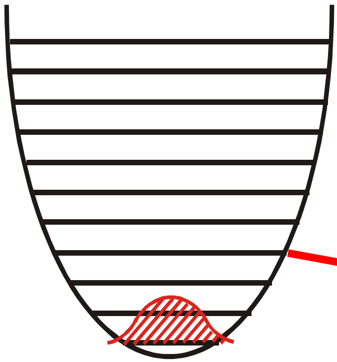
Ultra-Cold Fermions: testing condensed matter theories and getting more



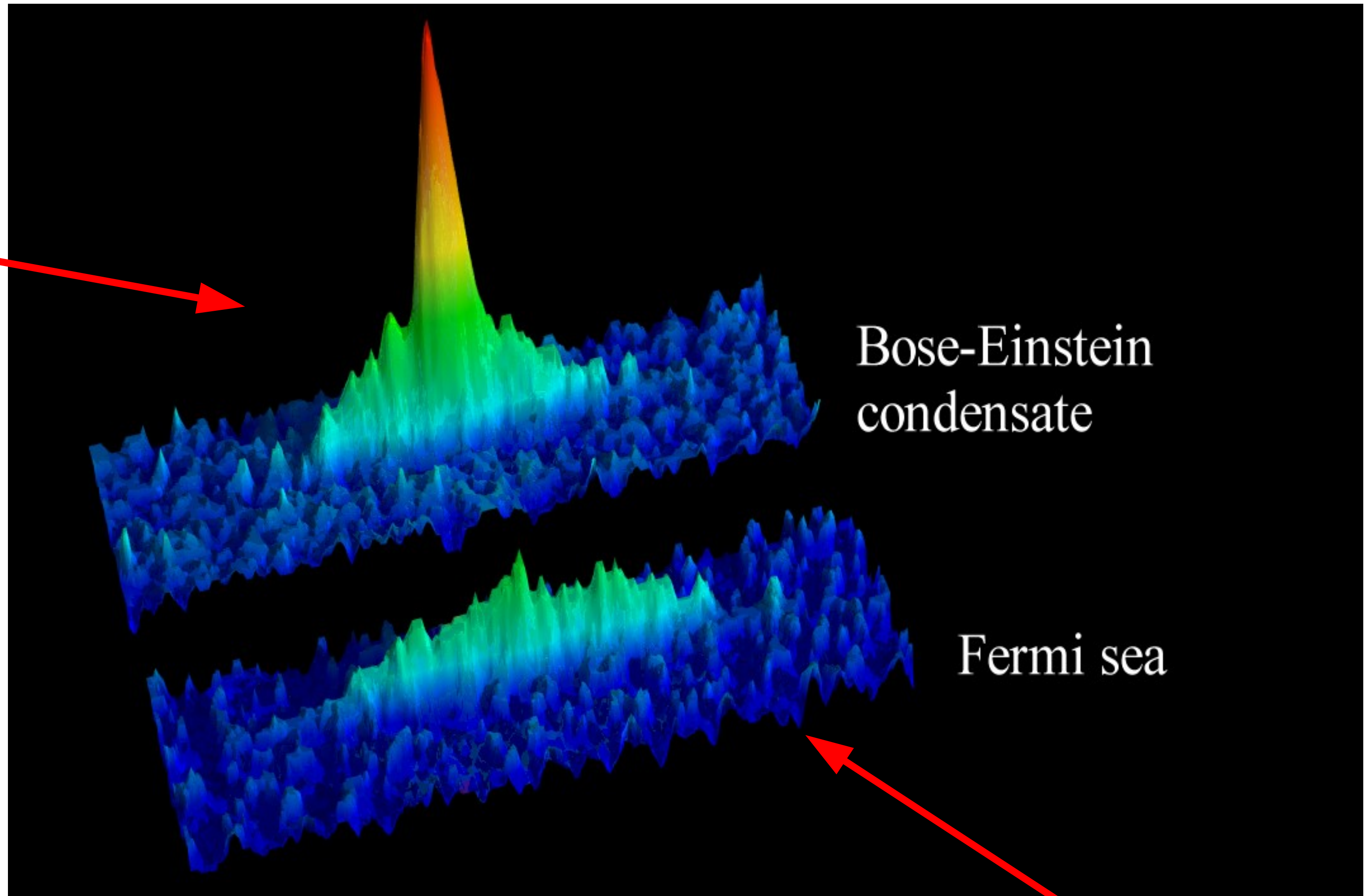


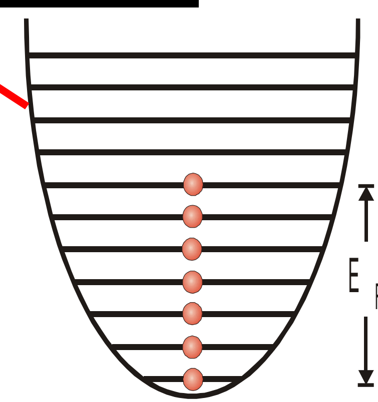
Atoms pre-cooled
in a magneto-optical trap
to 150 μ K

Bose-Einstein condensate and Fermi see



2001
ENS
(Paris)

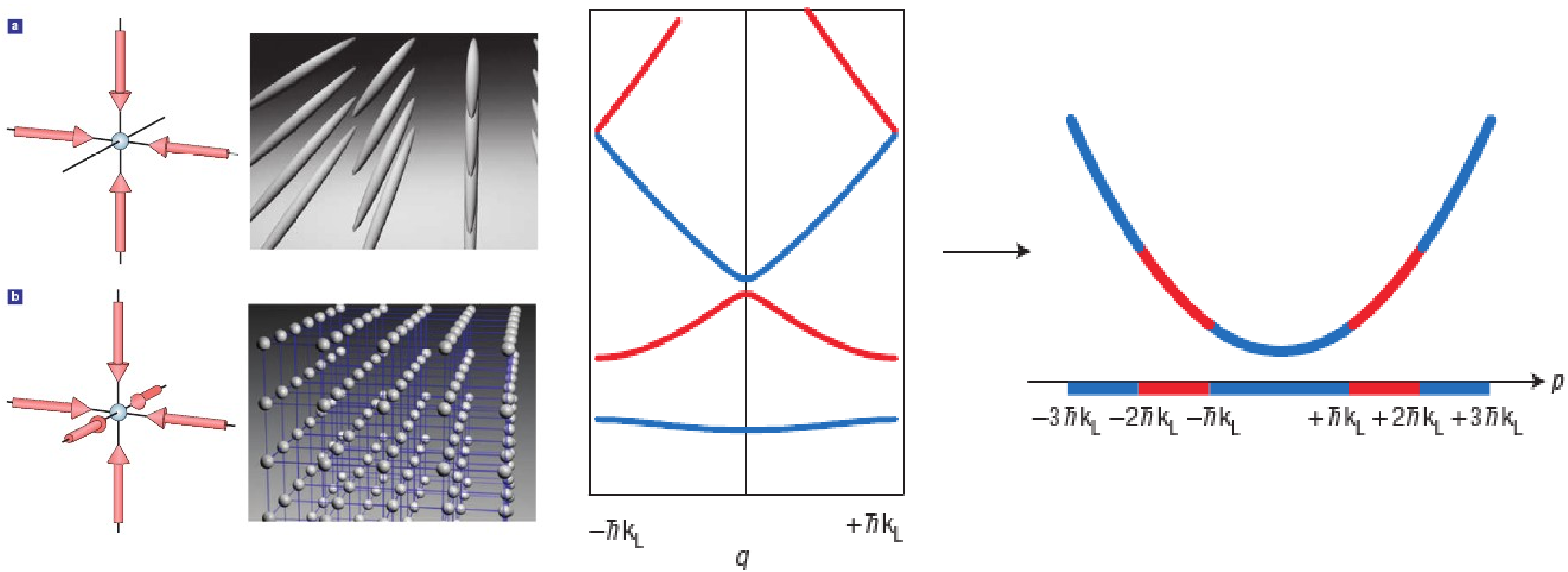


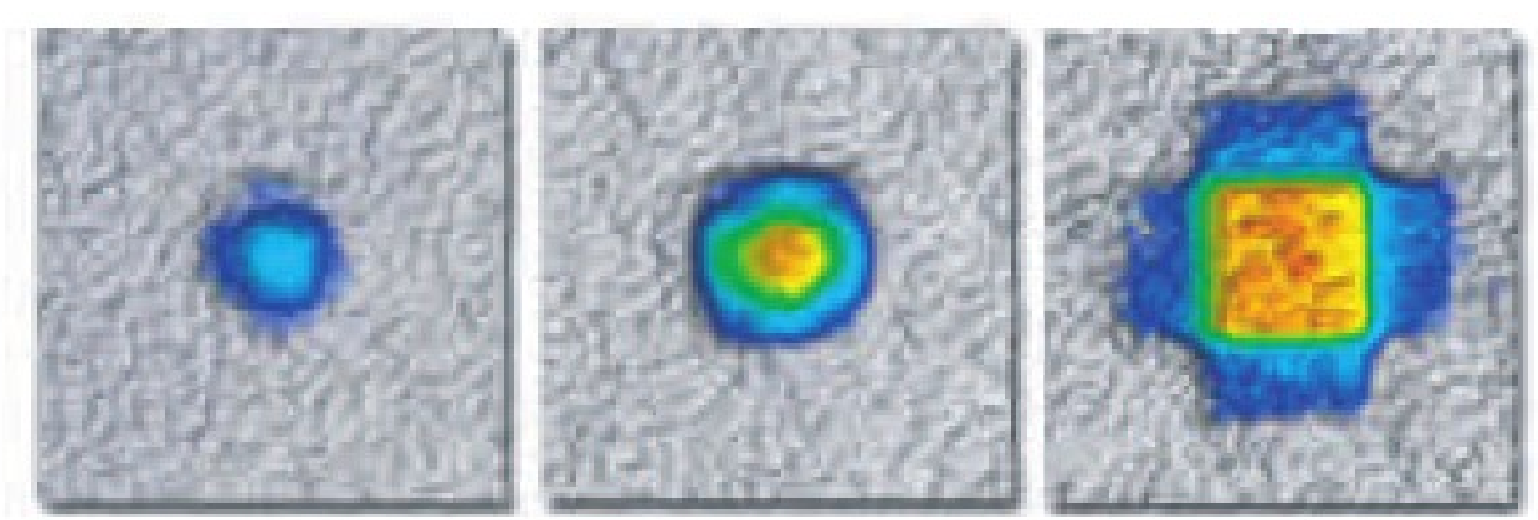


10^4 ^{7}Li atoms, in thermal equilibrium with
10^4 ^{6}Li atoms in a Fermi sea

Quantum degeneracy: $T = 0.28\ mK = 0.2(1)\ T_C = 0.2\ T_F$

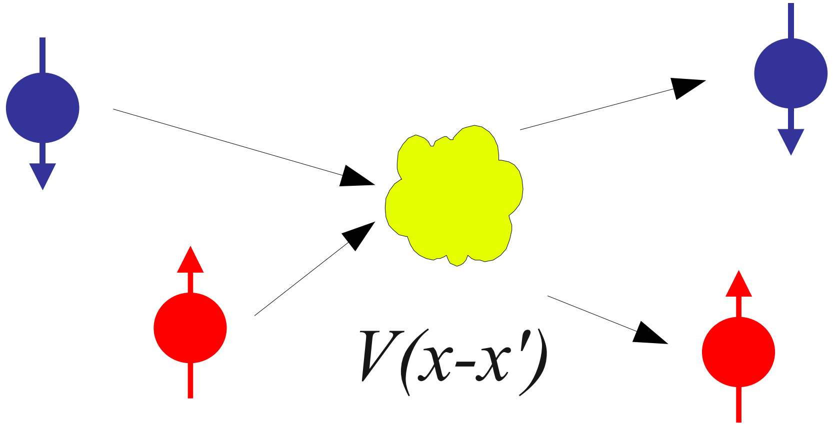
Observing the Fermi Surface





Interaction: s-wave scattering length

At low density and temperature the 2- body interaction is conveniently described by an **effective contact potential** which reproduces the low-energy behaviour of the microscopic potential



$$V(x-x') \rightarrow V_{eff}(x-x') \propto a\delta(x-x')(+reg.)$$

s-wave scattering length

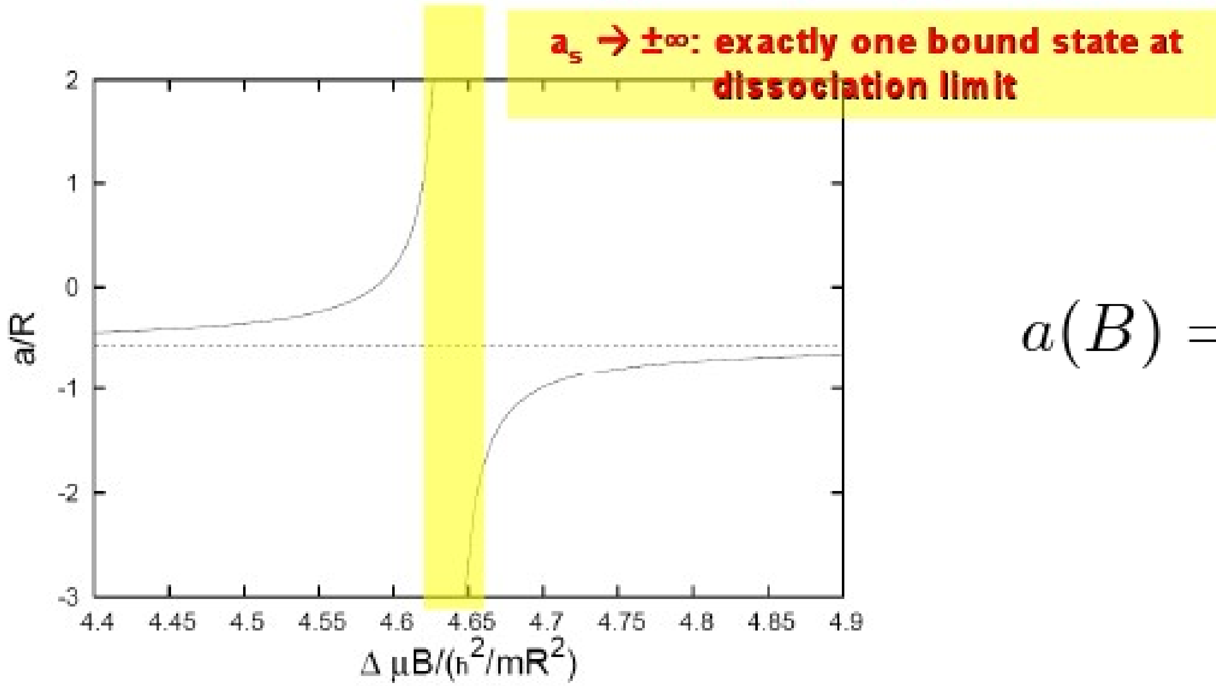
i) *a>0* : positive scattering & a Bound State (D=2,3)

ii) *a<0* : negative scattering & NO Bound State (D=2,3)

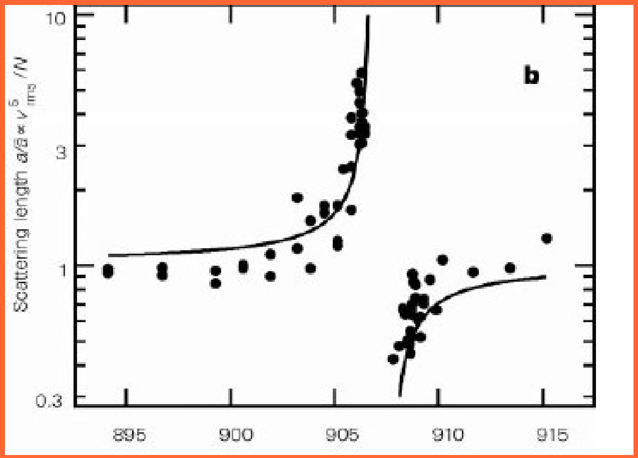
Due to Pauli principle only fermions in different internal states can – at this level- interact

Interaction: s-wave scattering length

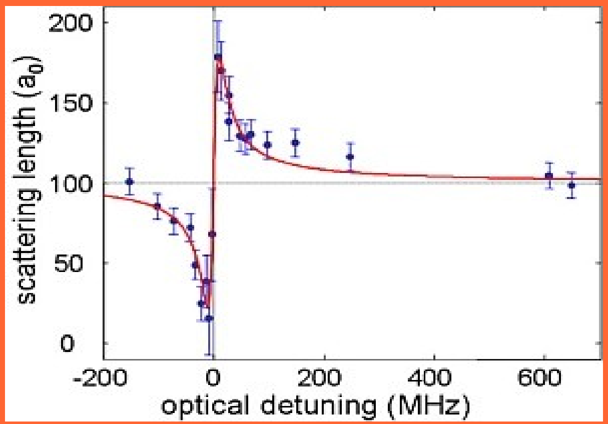
Cold Atoms: possibility of **tuning** the scattering length



$$a(B) = a\left(1 - \frac{\Delta_B}{B - B_0}\right)$$



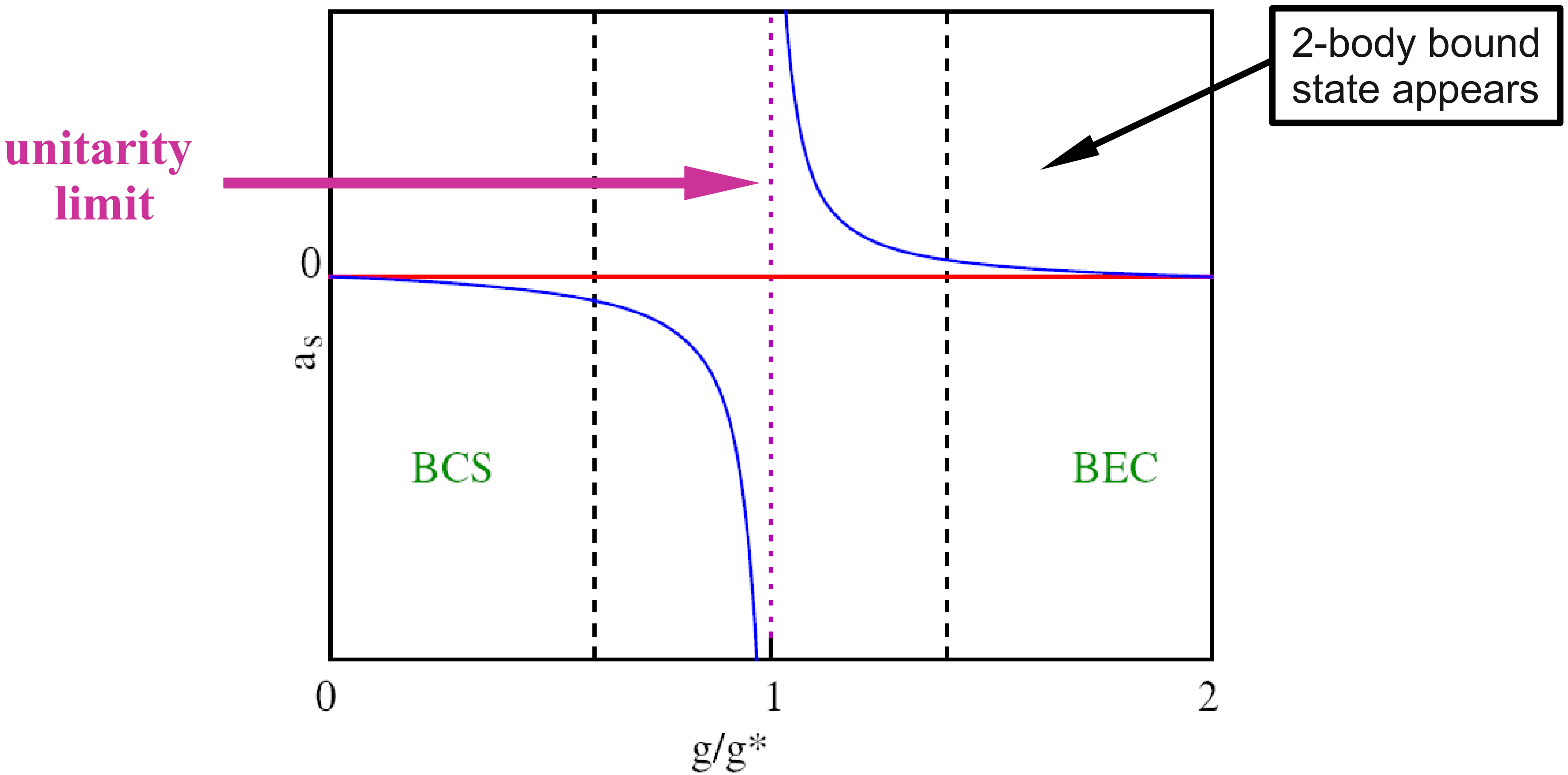
[MIT, Nature **392**, 151 (1998)]



[Innsbruck, PRL **93**, 123001 (2004)]

BCS vs Bose-Einstein Condensation

The behaviour of the Fermionic s-wave scattering length is ***not continous***

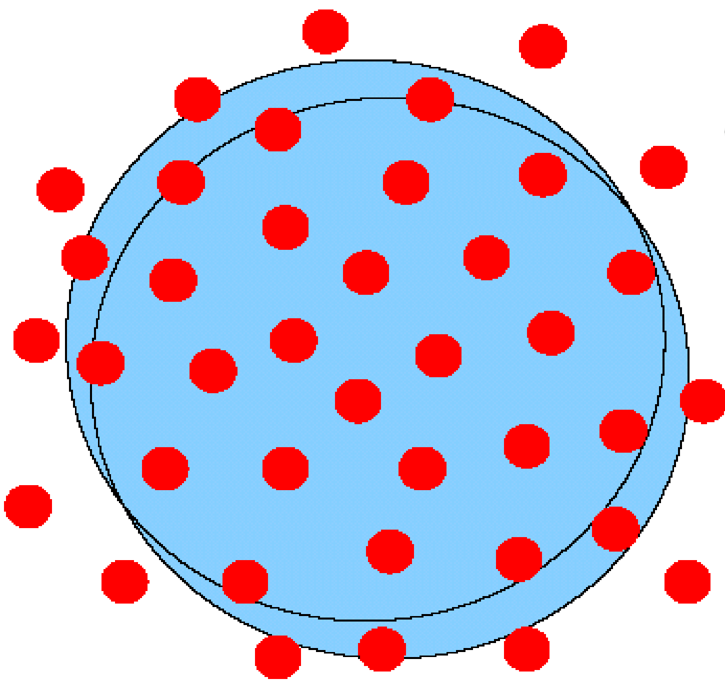


Crossover postulate: even though the scattering length changes abruptly in the many-body problem the ***crossover is smooth***
[Leggett; Nozieres/Schmitt-Rink]

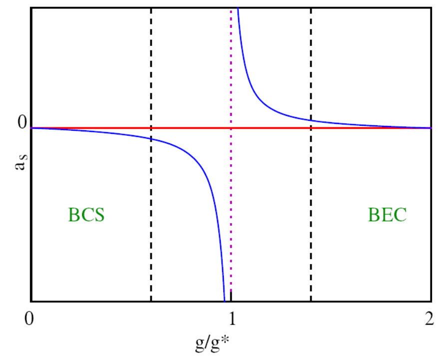
BCS vs Bose-Einstein Condensation

Weak Coupling: $k_F|a_s| \ll 1$
Ovelapping Cooper Pairs

$$\xi_b = \xi_0 = \frac{\hbar v_F}{\pi \Delta_0} \gg k_F^{-1}$$

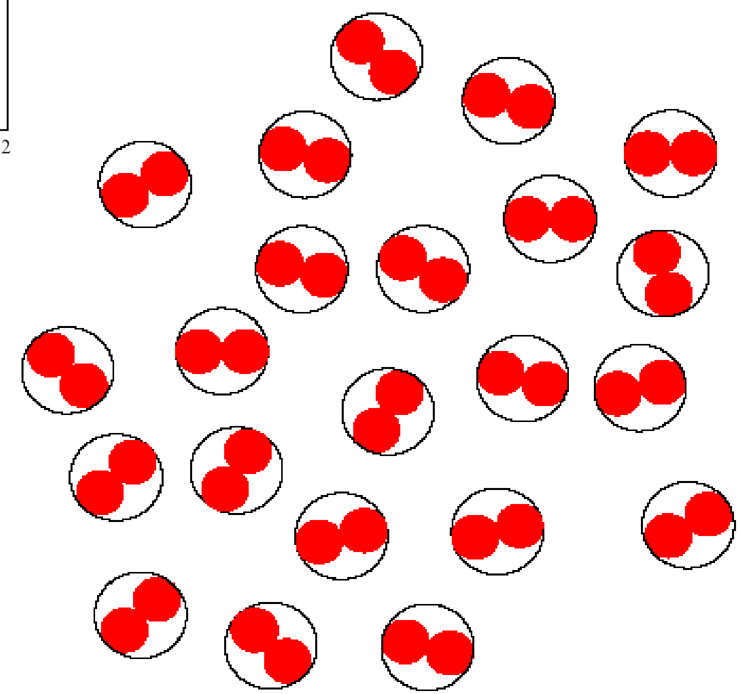


$T^* = T_c^{(BCS)}$



Strong Coupling: $k_F a_s \ll 1$
(Ideal) gas of molecules

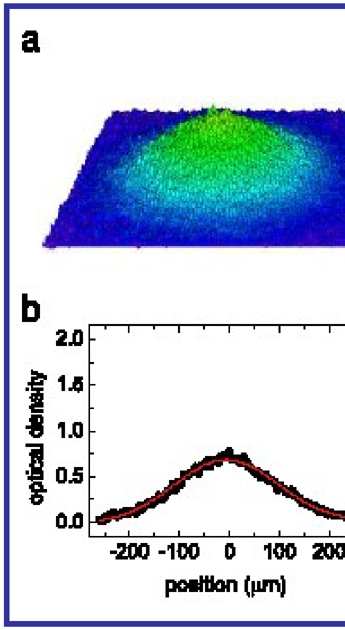
$$\xi_b \sim a_s \ll k_F^{-1} \qquad E_b = \frac{\hbar^2}{m a_s^2}$$

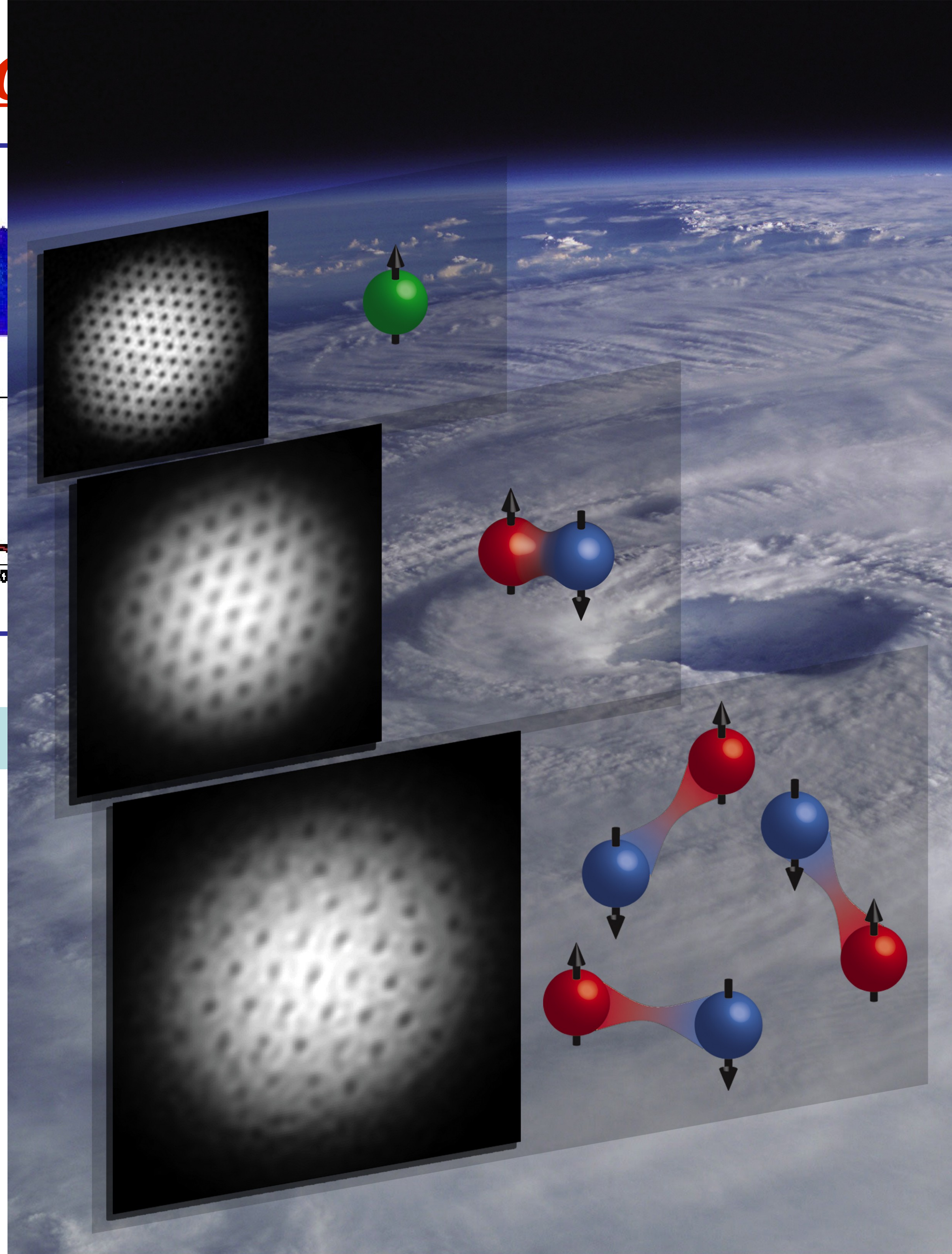


$T^* \gg T_c^{(BEC)}$

Note on finite T: Except for very weak coupling (BCS) pairs form and condense at different temperature, ***T**** and T_c

BC





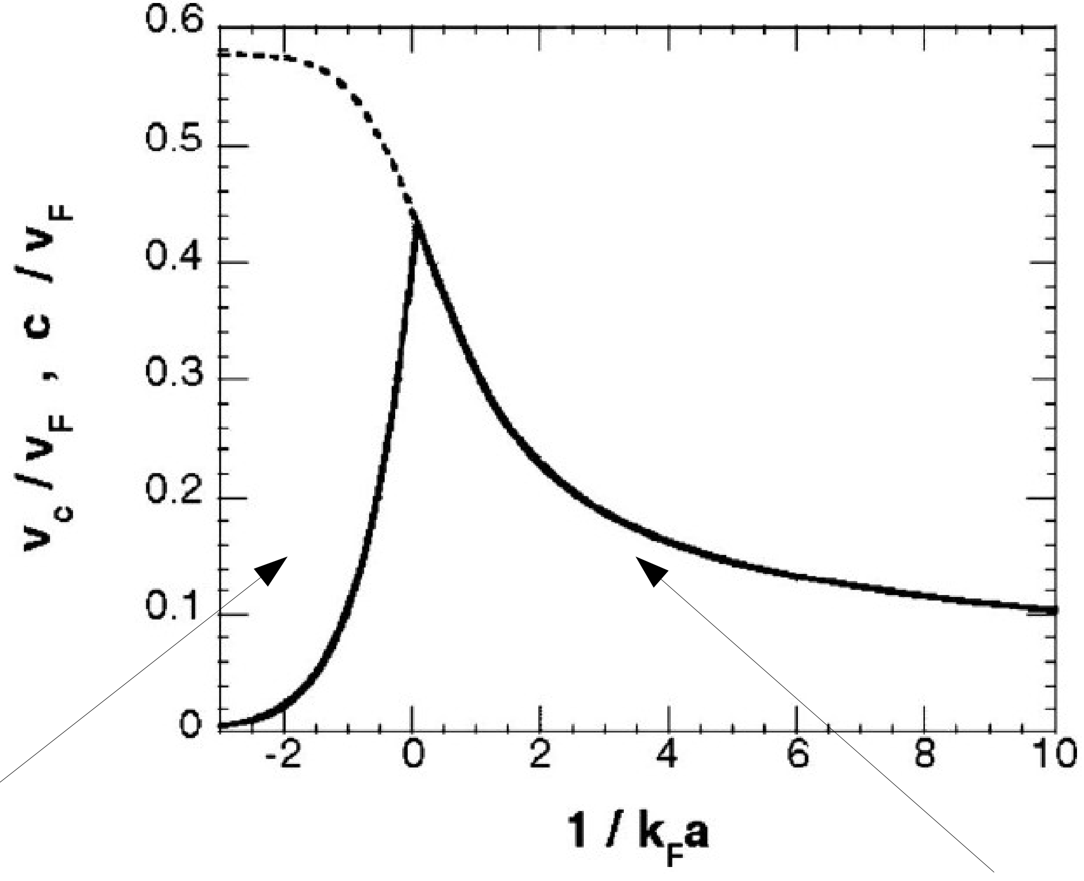
n condensation
ic gas

T, ENS, RICE,

]

BCS vs Bose-Einstein Condensation

Landau critical velocity for a system to give rise to energy dissipation $v_c = \min_q(\hbar\omega_q/q)$



Due to pair breaking

$$m(v_c^{\rm sp})^2 = \sqrt{\Delta^2 + \mu^2} - \mu$$

Due to phonon excitation (as in a BEC (L.1))

Superfluid fermions at unitarity

- The only scales at unitarity are the Fermi energy and the temperature.
- The thermodynamic properties have an “universal” form.

In particular at *T=0*

energy density, pressure, chemical potential are ***proportional*** to the ones of an ideal Fermi gas with a density equal to the superfluid one.

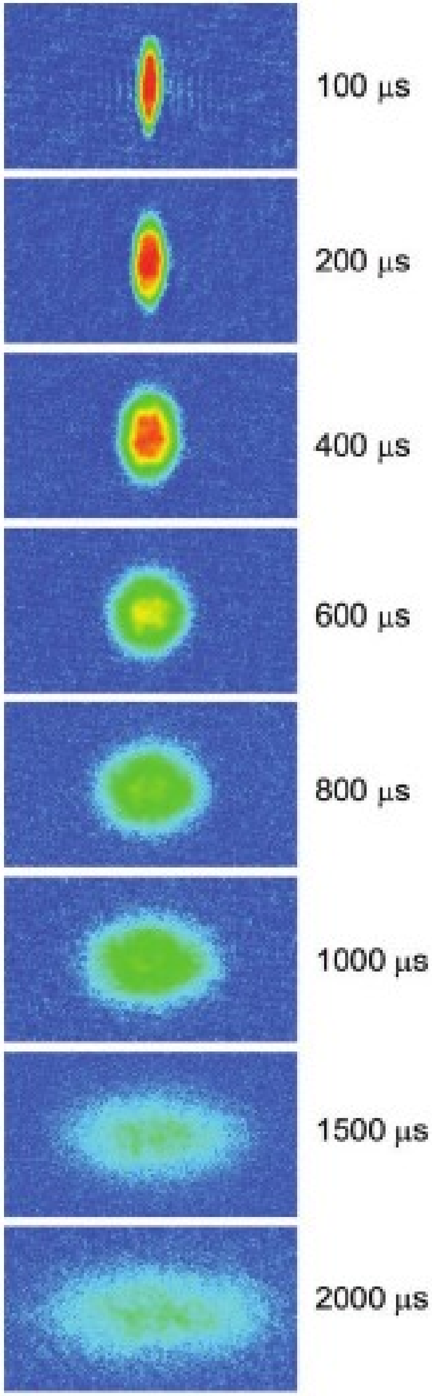
The universal parameter (via Montecarlo & Experiments)

S

$$\xi_S \simeq 0.42$$

$$\frac{E_S}{N_S} = 2\xi_S \frac{3}{5}\frac{\hbar^2}{2m}(6\pi^2 n_S)^{2/3} \equiv 2\epsilon_S(n_S)$$

(After yesterday discussion)
Is a Fermi gas at Unitarity a perfect fluid?



Strongly Interacting ^{6}Li gas T = 10^{-7} K

Hydrodynamic equation for a superfluid
or a perfect (collisional) fluid

$$\frac{\partial}{\partial t} n + \nabla \cdot (n\mathbf{v}) = 0$$

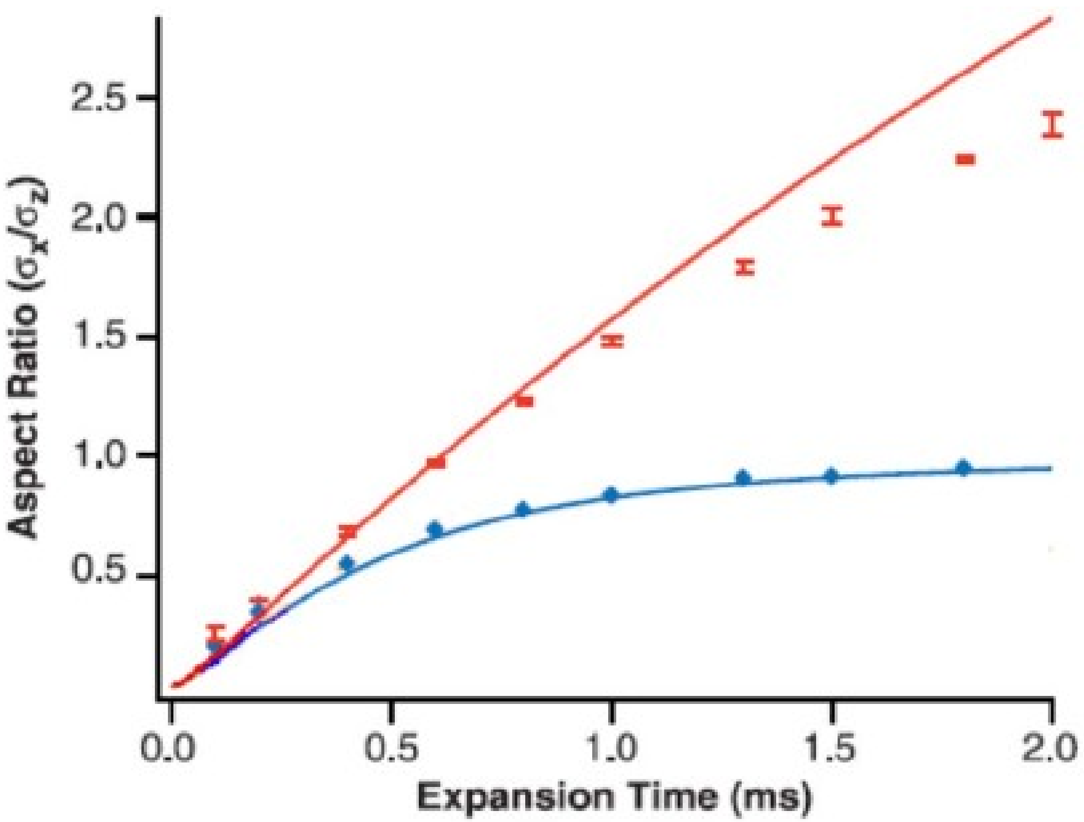
$$m\frac{\partial}{\partial t}\mathbf{v} + \nabla\left(\frac{1}{2}m\mathbf{v}^2 + \mu(n) + V_{\text{ho}}\right) = 0$$

At Unitarity one finds same expansion for $T < T_c << T_F$ and T close to T_F

[Duke, Science (2002)]

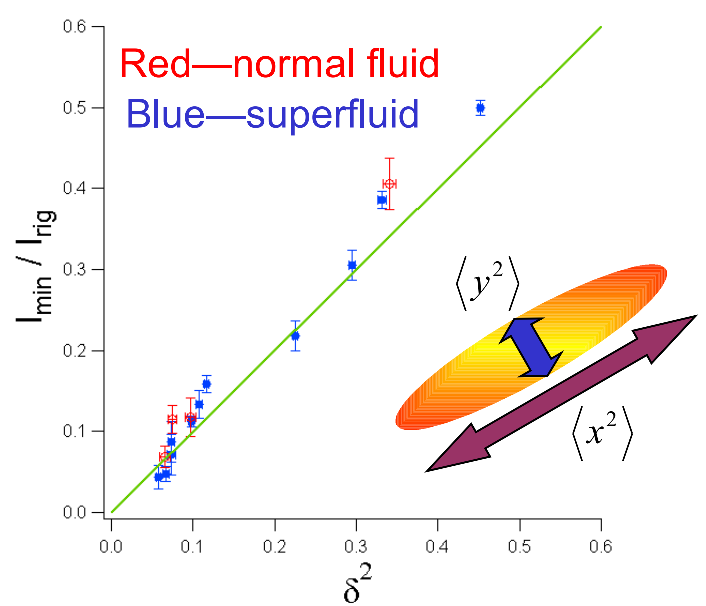
(After yesterday discussion)
Is a Fermi gas at Unitarity a perfect fluid?

At Unitarity one finds same expansion for $T<T_c<<T_F$ and T close to T_F, but different from a wealy interacting Fermi gas



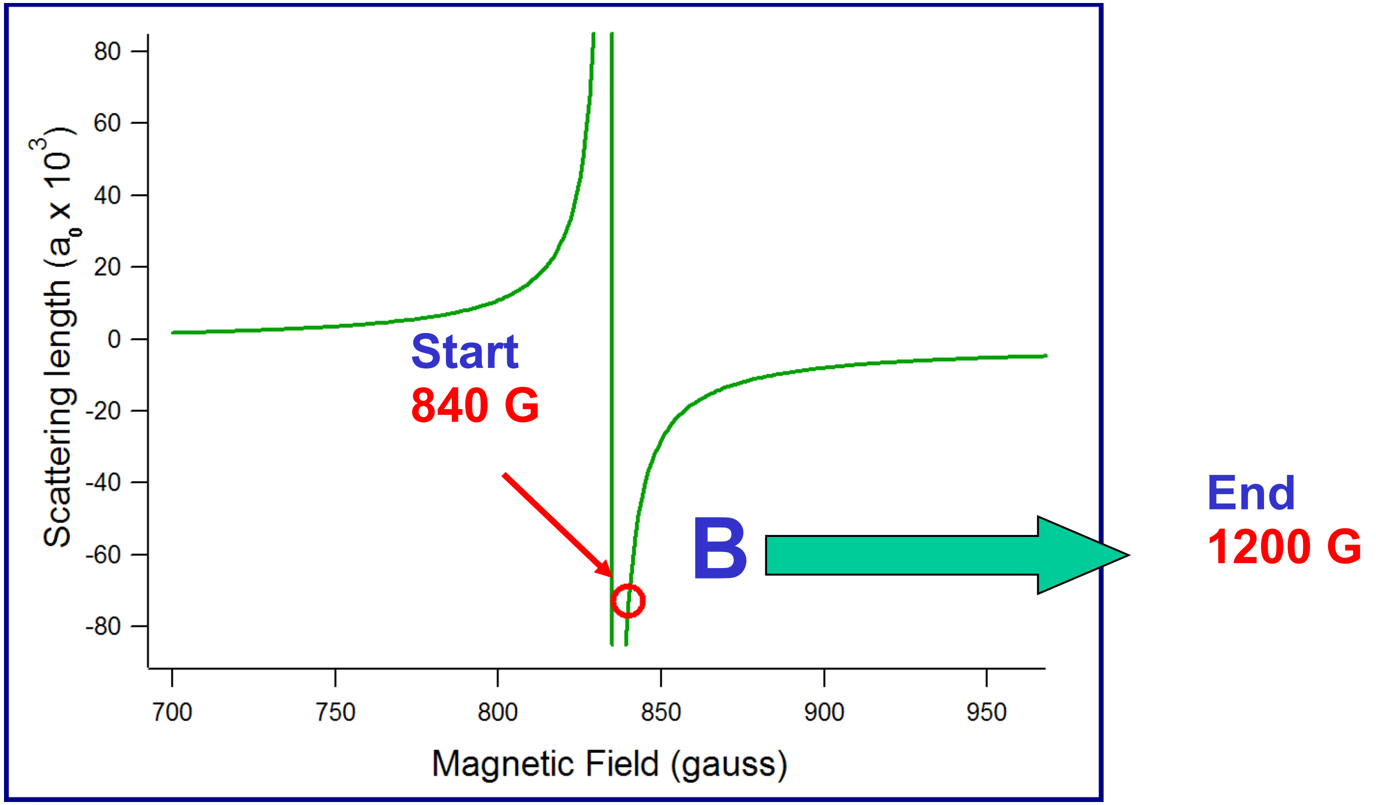
Moment of Inertia

$$\frac{I}{I_{rigid}} = \delta^2 = \frac{\langle x^2 - y^2 \rangle^2}{\langle x^2 + y^2 \rangle^2}$$



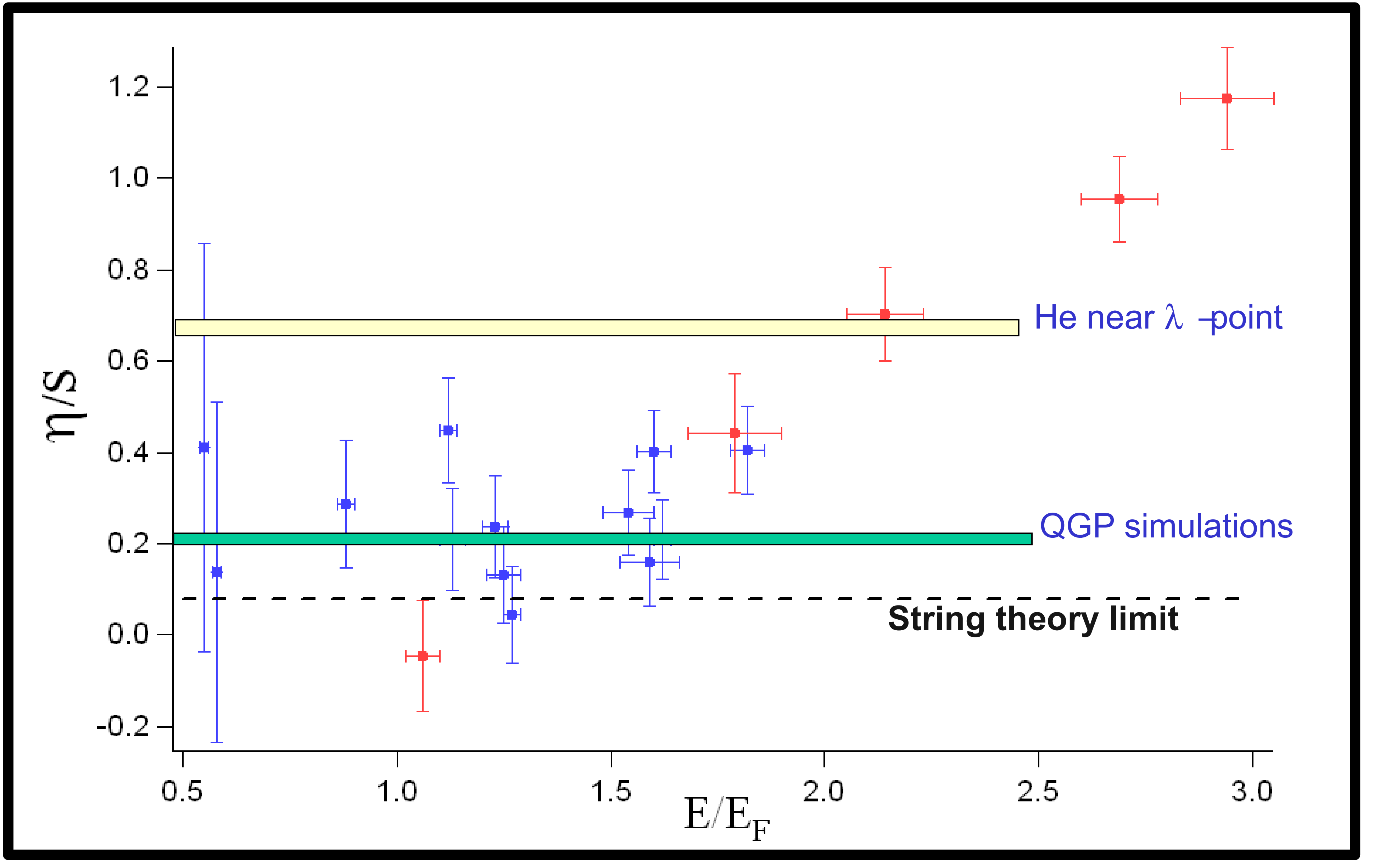
(After yesterday discussion)
Is a Fermi gas at Unitarity a perfect fluid?

Entropy Measurement
by *Adiabatic* Sweep of Magnetic Field B

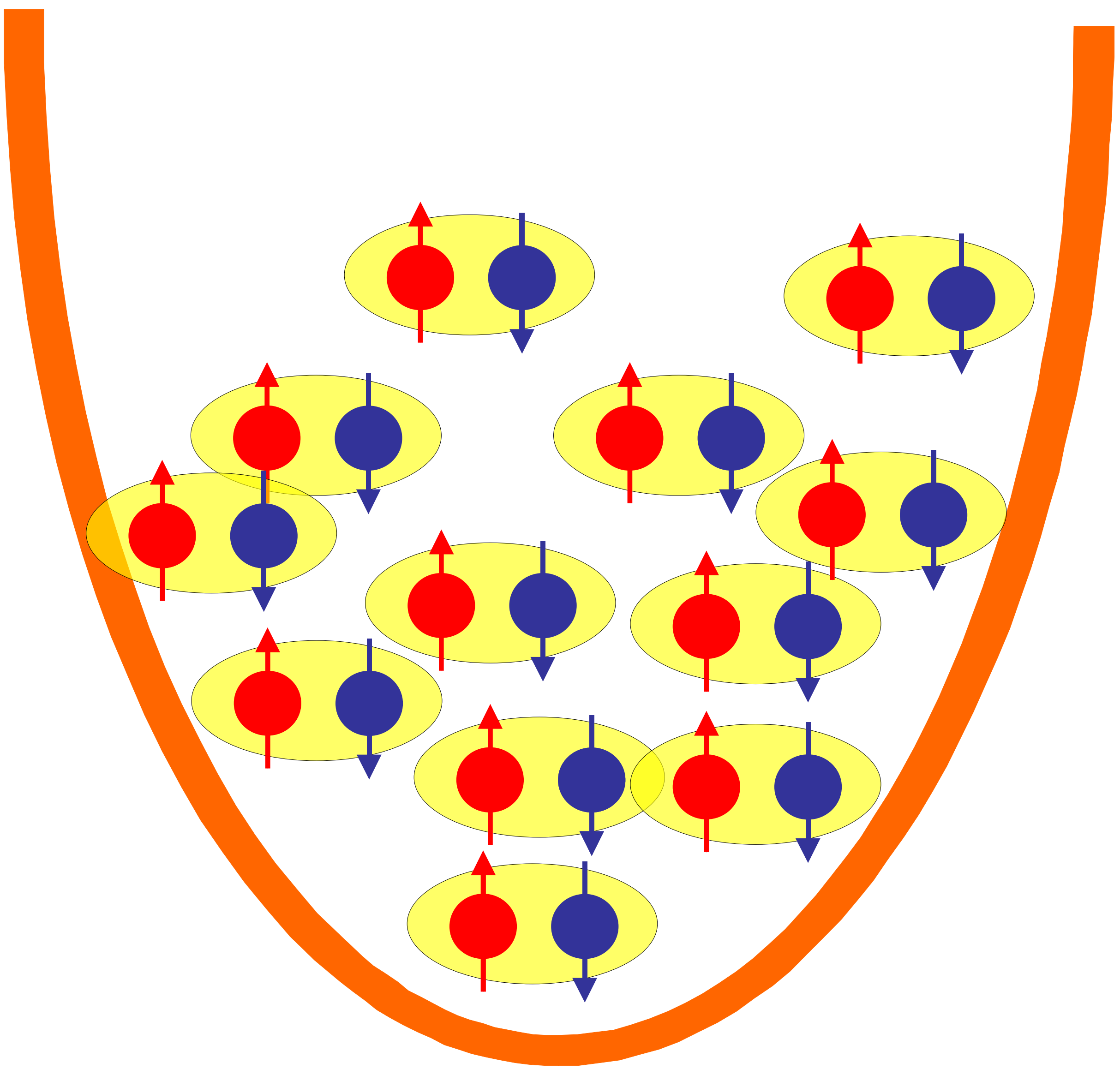


Weakly interacting:
Entropy at 1200 G known from cloud size — Ideal Fermi gas

(After yesterday discussion)
Is a Fermi gas at Unitarity a perfect fluid?



Balanced Fermi gases at unitarity



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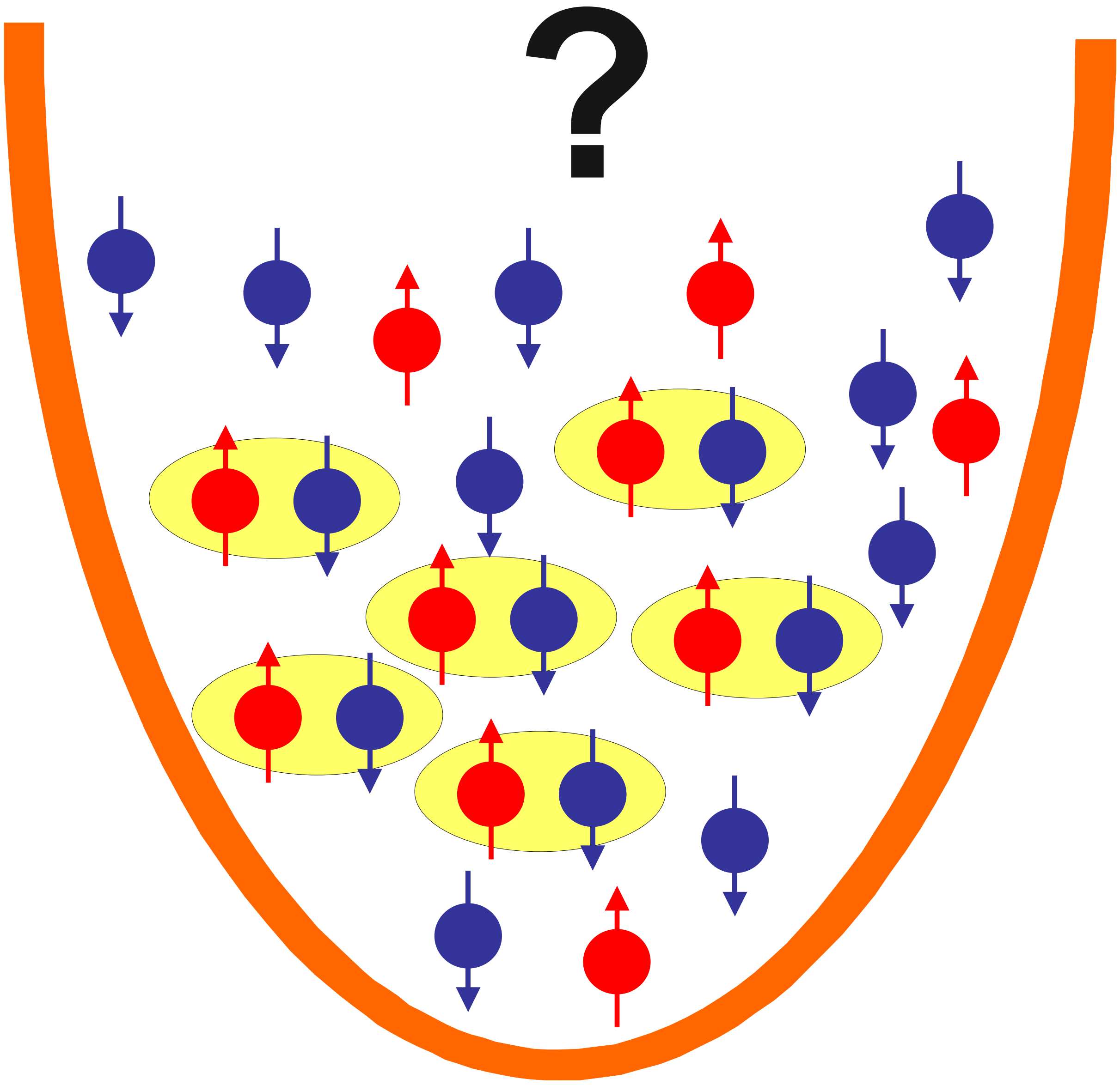

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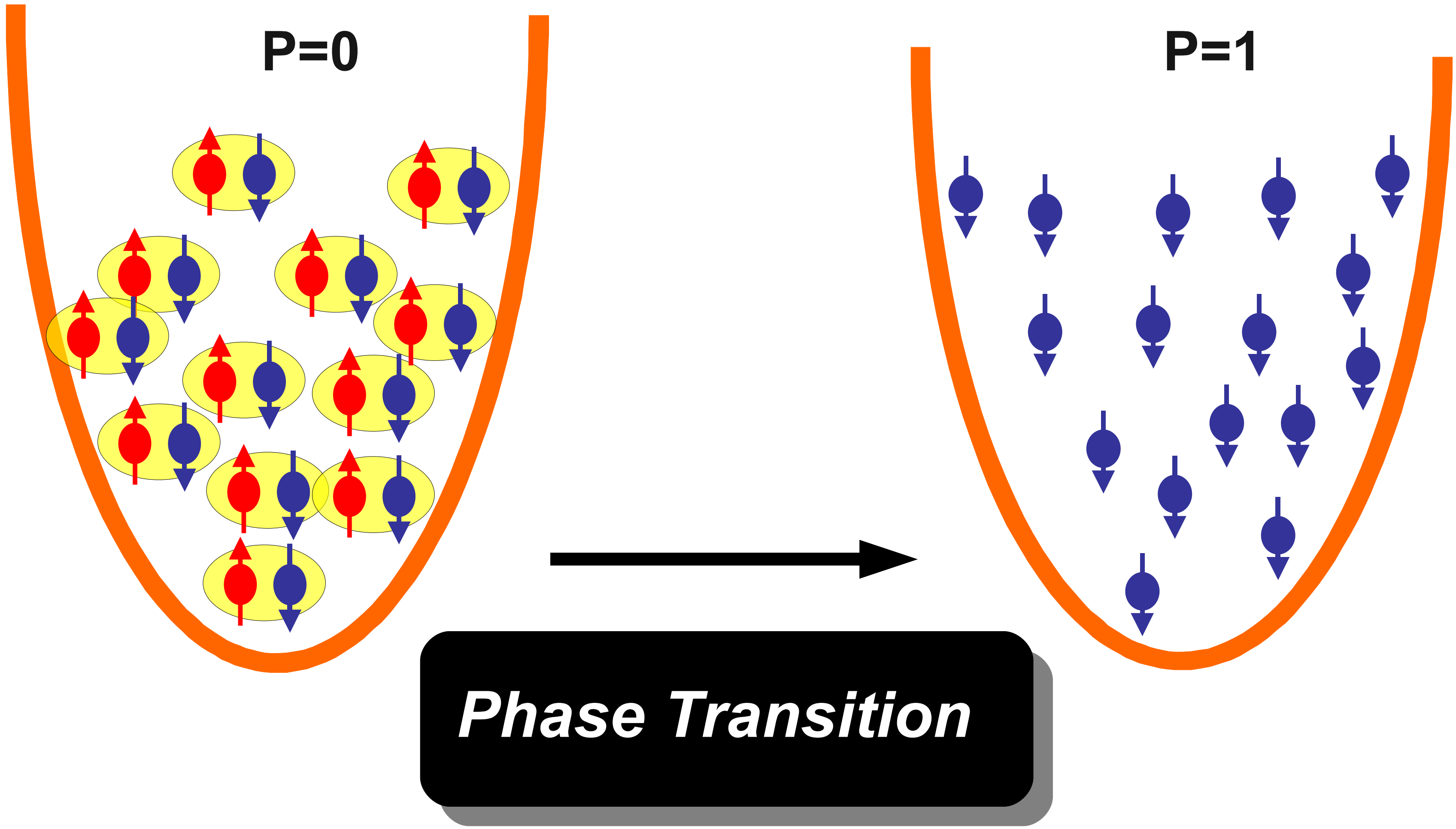
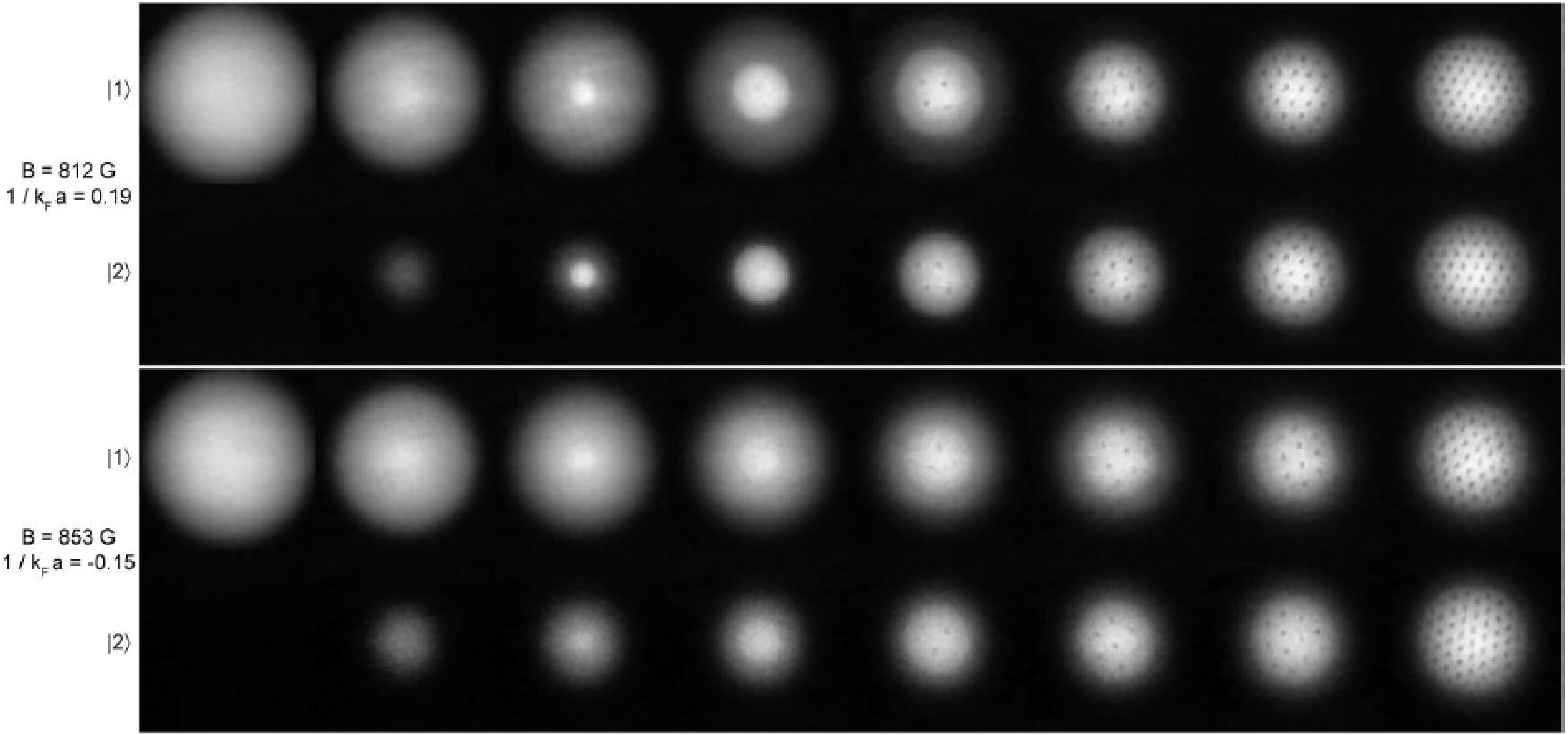


[Phase Transition to a normal phaase for large magnetic field
B. S. Chandrasekhar (1962), A. M. Clogston (1962)]

Recent Experiments on imbalanced Fermi gases at unitarity



MIT, Science **311**, 492 (2006)

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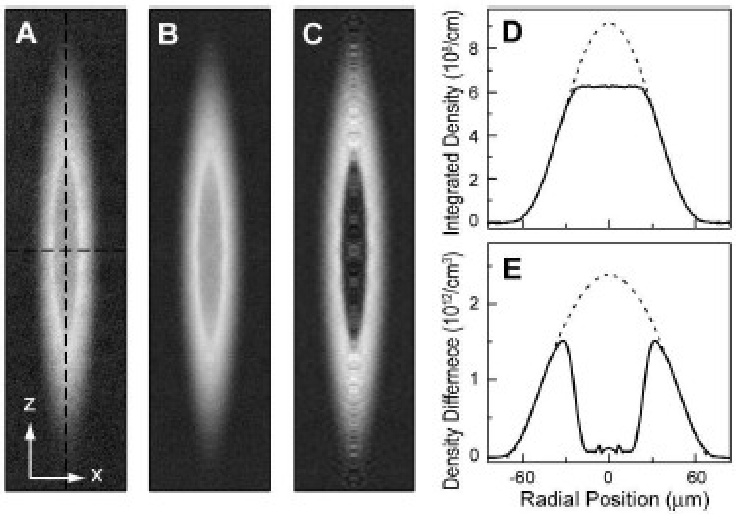
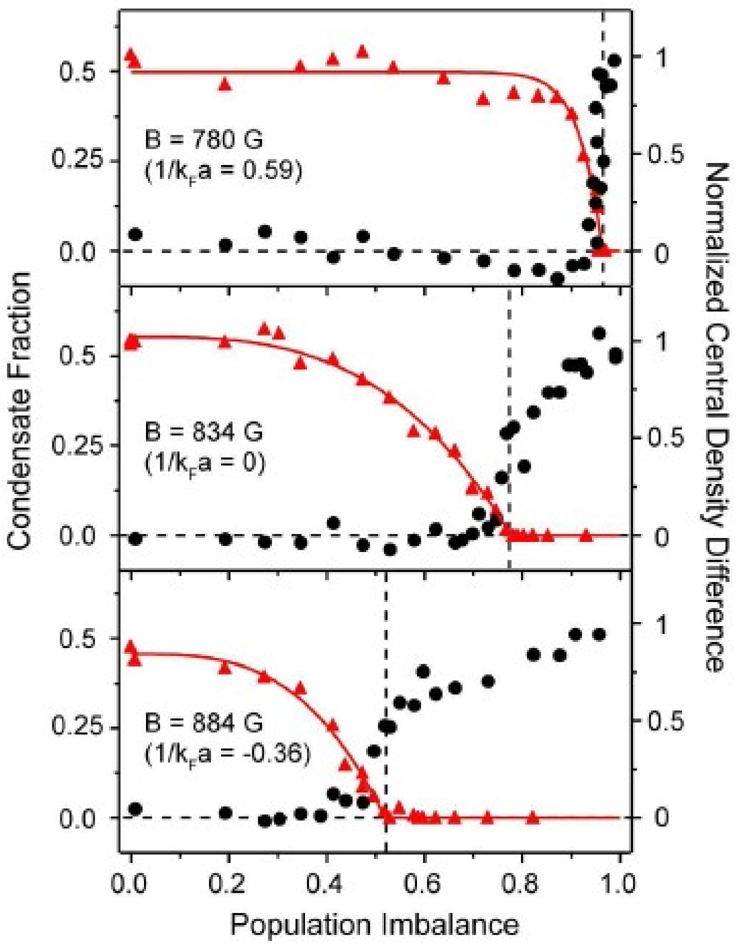




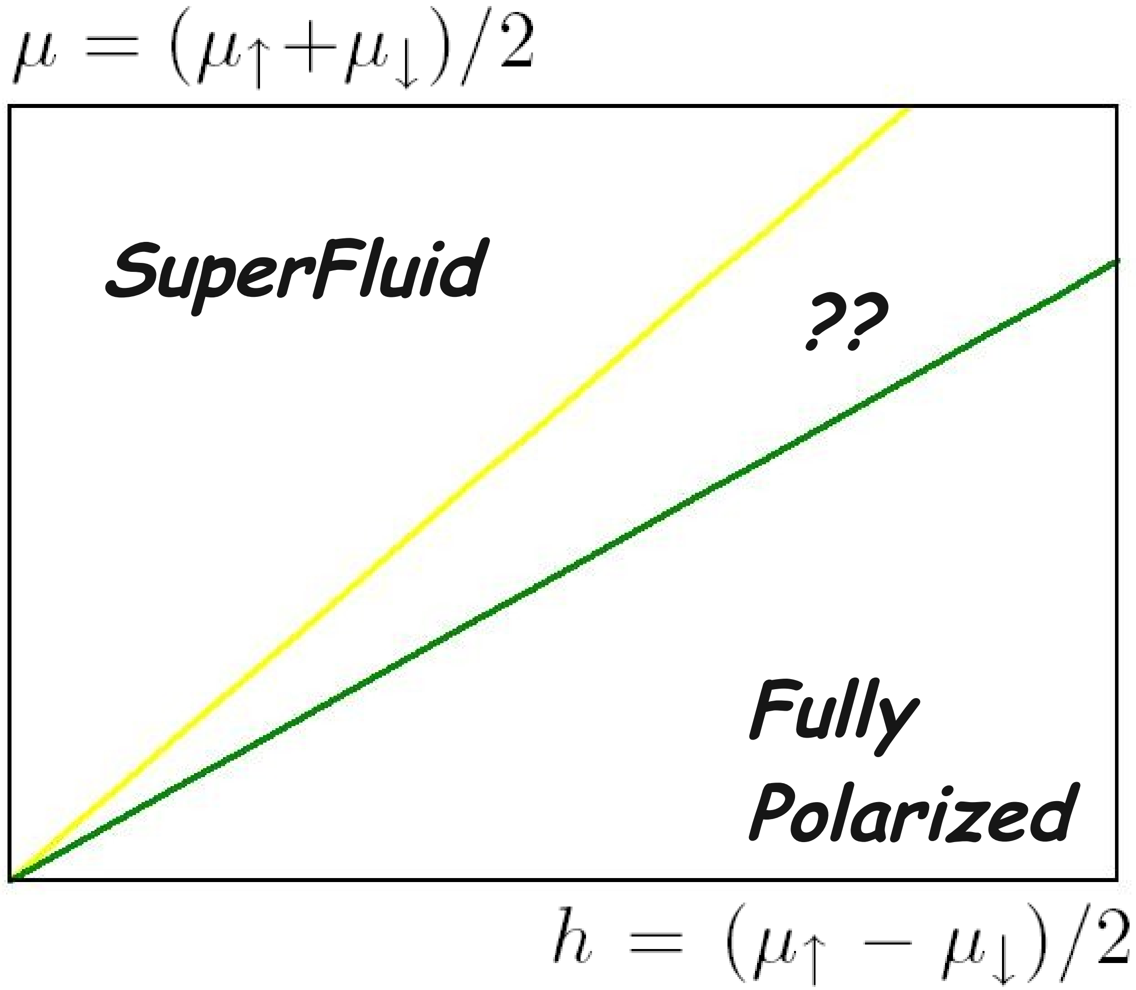
BEC

Unitarity

BCS

[MIT, Phys. Rev. Lett. **97**, 030401 (2006)]

Phase diagram



E.g., only
SF and P phase

$$P_p = P_s \Rightarrow \mu = \frac{(2\xi_S)^{3/5}}{2-(2\xi_S)^{3/5}} h$$

Normal phase of polarized Fermi gas at unitarity

Assumption:
at high polarization homogeneous phase,
NORMAL FERMI LIQUID: consider a very dilute mixture of spin-↓ atoms immersed in non-interacting gas of spin-↑ atoms

Energy expansion for small concentration $x = \frac{n_\downarrow}{n_\uparrow}$

$$\frac{E(x)}{N_\uparrow} = \frac{3}{5}\epsilon_{F\uparrow}\left(1 - Ax + \frac{m}{m^*}x^{5/3} + \cdots\right) \equiv \frac{3}{5}\epsilon_{F\uparrow}\epsilon(x)$$

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Non interacting gas

single-particle energy

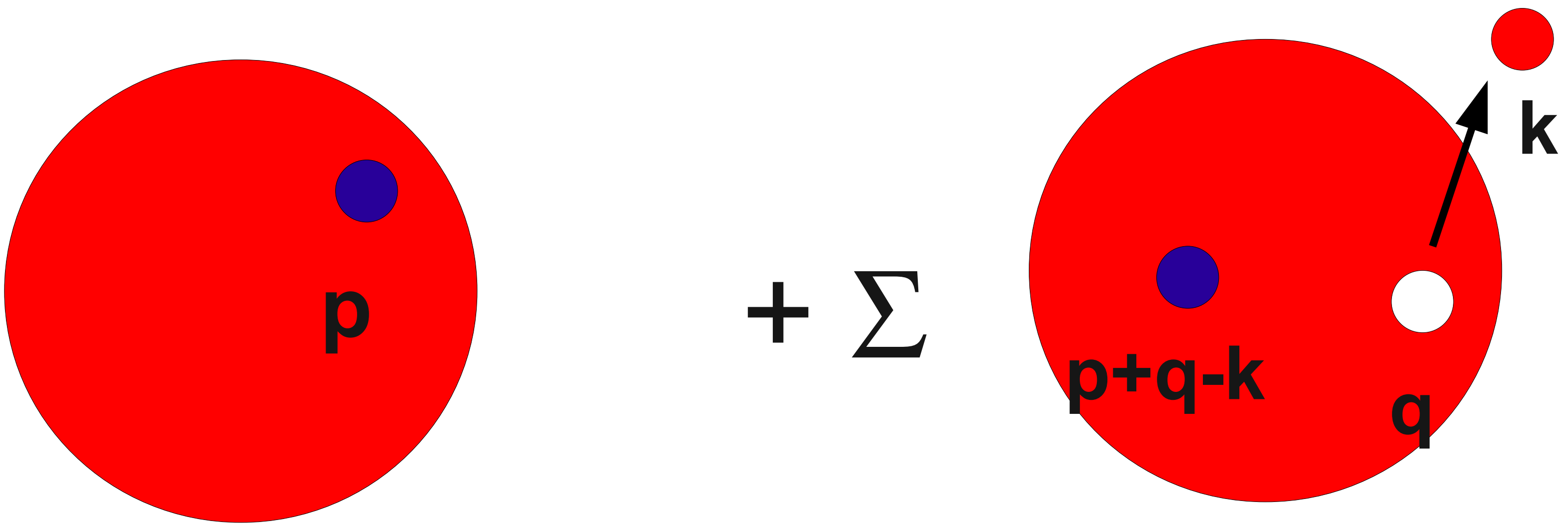
quantum pressure
of a Fermi gas of quasi-particles
with an effective mass

Normal phase of polarized Fermi gas at unitarity

Consider a SINGLE down atom interacting with an ideal Fermi gas (up-atoms).

Variational Ansatz (single particle hole excitations):

$$|\psi\rangle = \phi_0 |\mathbf{p}\rangle_\downarrow \; |0\rangle_\uparrow + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}} |\mathbf{p}+\mathbf{q}-\mathbf{k}\rangle_\downarrow \; c^\dagger_{\mathbf{k}_\uparrow} \, c_{\mathbf{q}_\uparrow} \, |0\rangle_\uparrow$$



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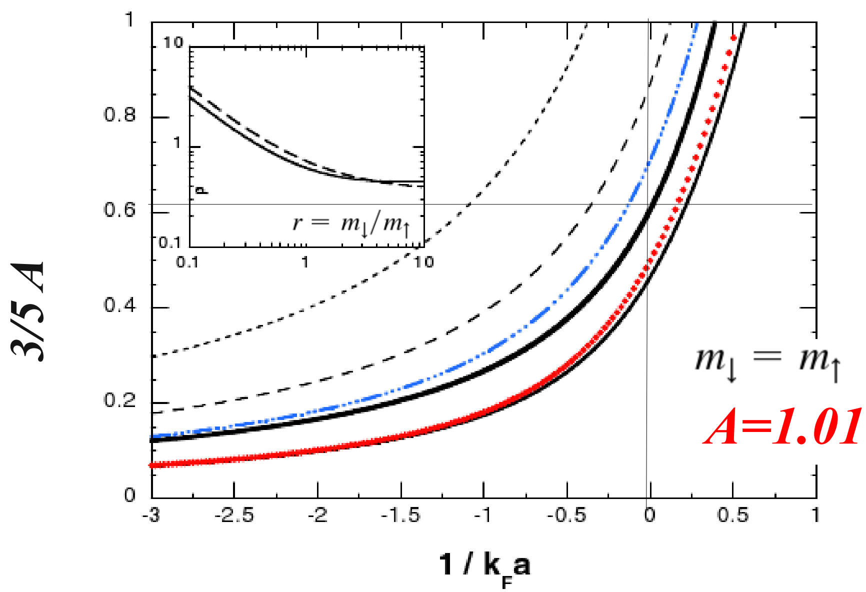

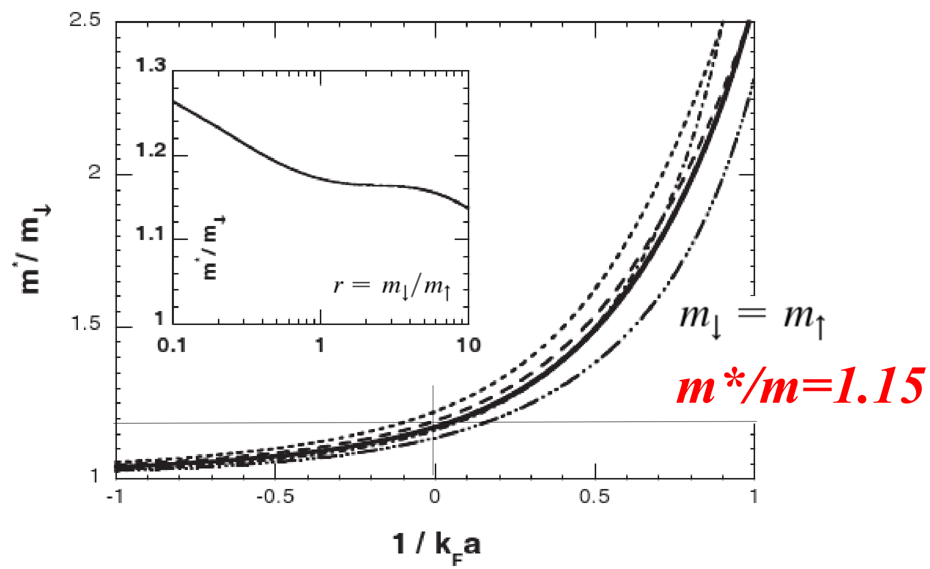

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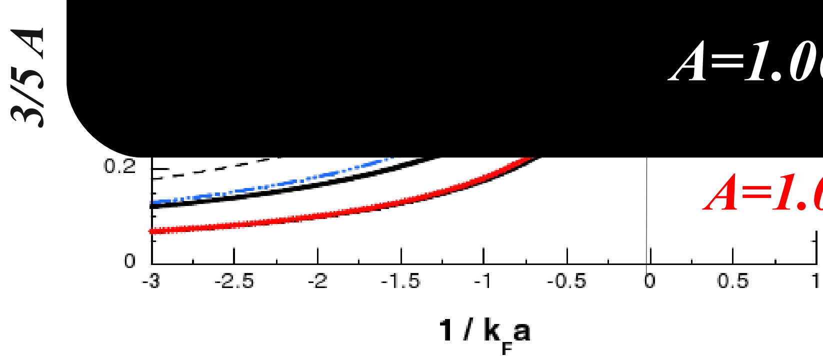
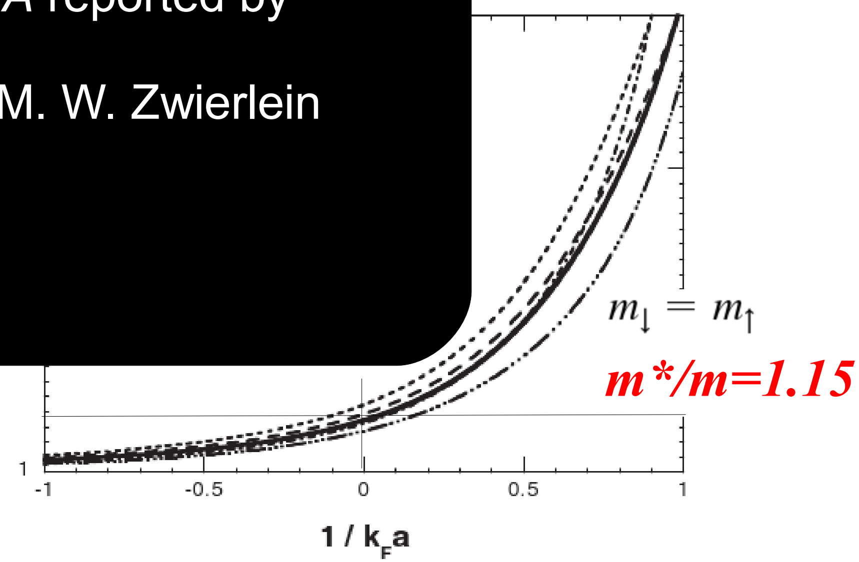
First measurements of the coefficient *A* reported by

A. Schirotzek, C. Wu, A. Sommer, and M. W. Zwierlein
arXiv:0902.3021

A=1.06(7)





Note: it is equivalent to a T-matrix approach

$$\omega - \epsilon_{\downarrow k} + \mu_\downarrow - \Sigma(k,\omega) = 0 \;\Rightarrow\; \mu_\downarrow = \Sigma(0,0) \quad \& \quad \frac{m^*}{m_\downarrow} = \frac{1 - \frac{\partial \Sigma}{\partial \omega}}{1 + 2m_\downarrow \frac{\partial \Sigma}{\partial k^2}}$$

Superfluid-Normal phase coexistence at unitarity

$$\frac{E(x)}{N_\uparrow} = \frac{3}{5}\epsilon_{F\uparrow}\left(1 - Ax + \frac{m}{m^*}x^{5/3} + Bx^2\right) \equiv \frac{3}{5}\epsilon_{F\uparrow}\epsilon(x)$$

interaction between quasi-particles

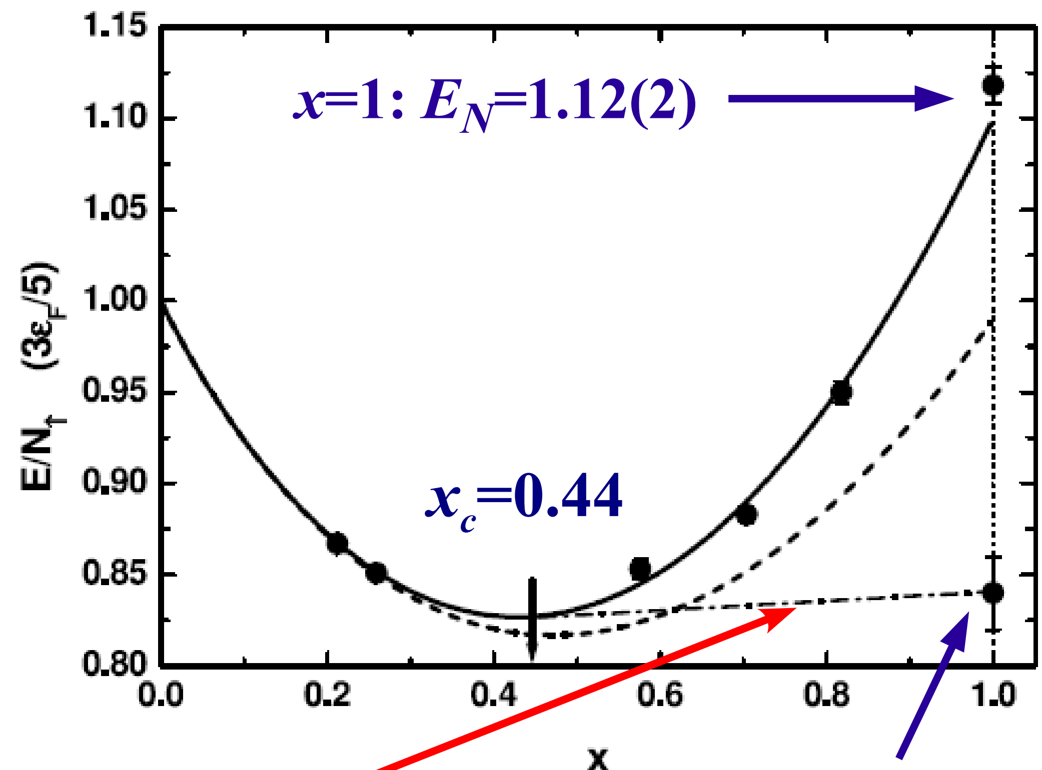
Most recent values using FN-QMC

***A* = 0.99(2)**

***m*/m* = 1.09(3)**

***B* = 0.14**

*[S. Pilati and S. Giorgini, Phys. Rev. Lett. **100**, 030401 (2008)]*



Coexistence line

x=1: E_S=0.84(2)

Critical concentration x_c:

$$P_{SF} = P_N$$

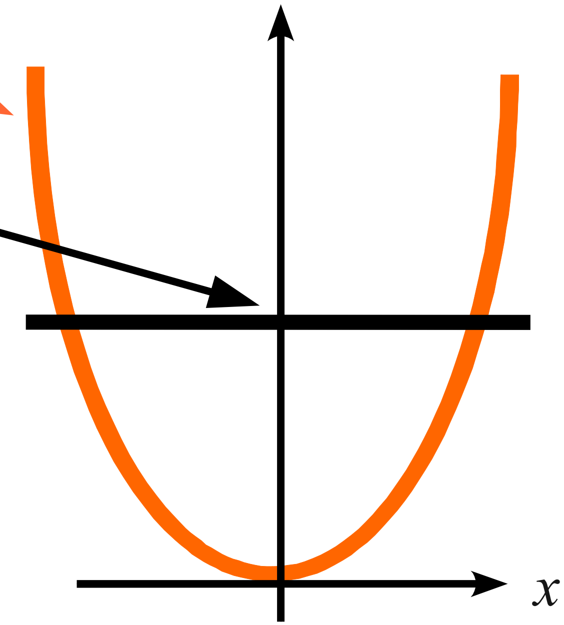
$$\frac{\epsilon'(x_c)}{\epsilon(x_c)} = \frac{5}{3}\frac{\epsilon(x_c)^{3/5} - (2\xi_S)^{3/5}}{x_c - 1}$$

SF	N with x_c=0.44

Phase Separation

Exploring Phase diagram in the Trap: LDA

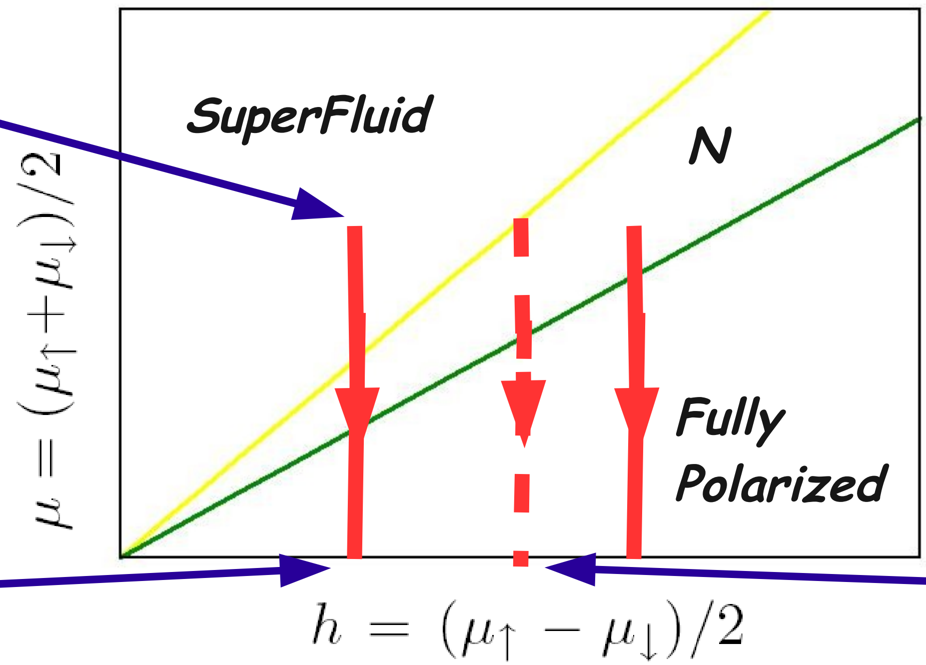
LDA : $\mu_\sigma(\mathbf{x}) = \mu_\sigma^0 - V(\mathbf{x}) = \mu_\sigma^0 - \frac{1}{2}m\omega x^2$



$\mu(\mathbf{x}) = \mu^0 - \frac{1}{2}m\omega x^2$	Decreasing outward
$h(\mathbf{x}) = h^0$	Constant also inside the trap

By the total number of atoms

By the imbalance



Critical imbalance

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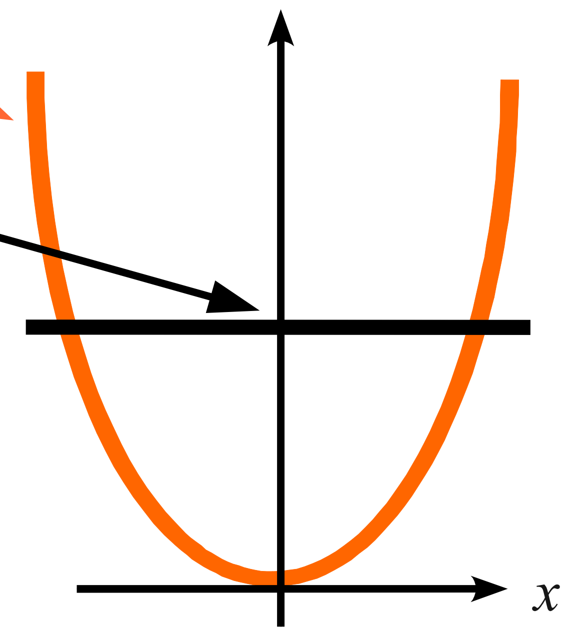

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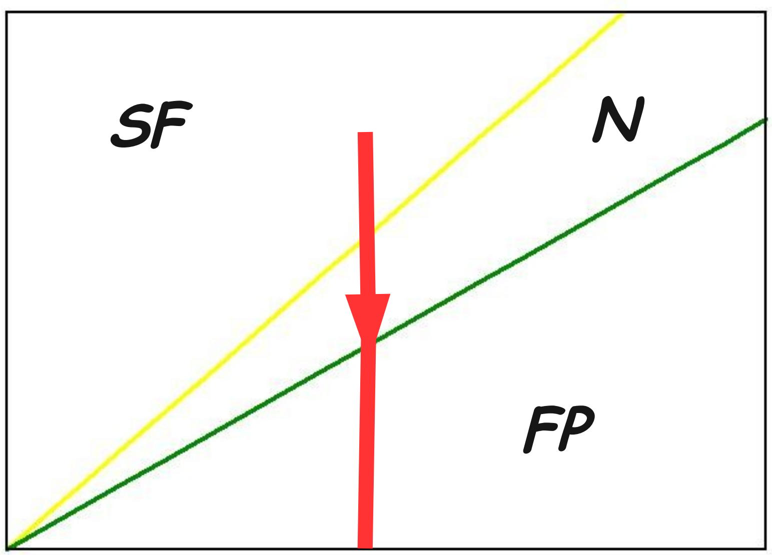
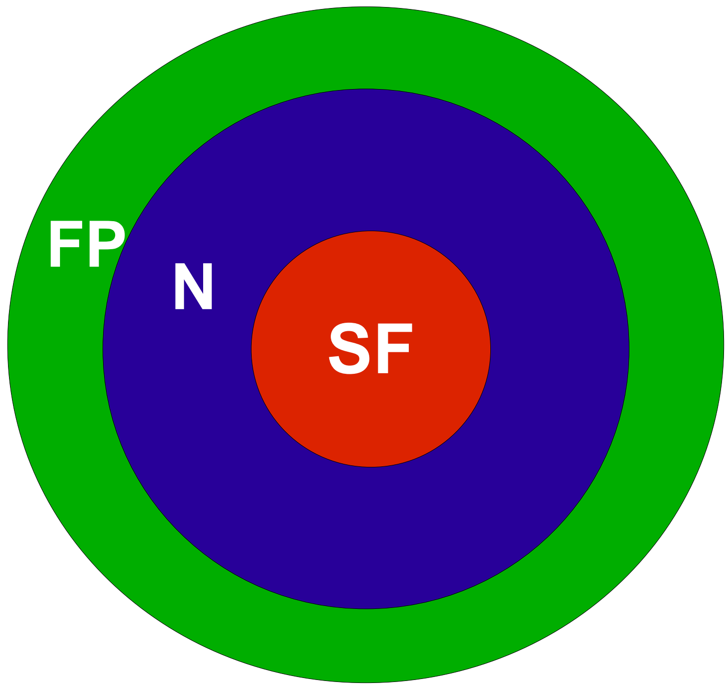
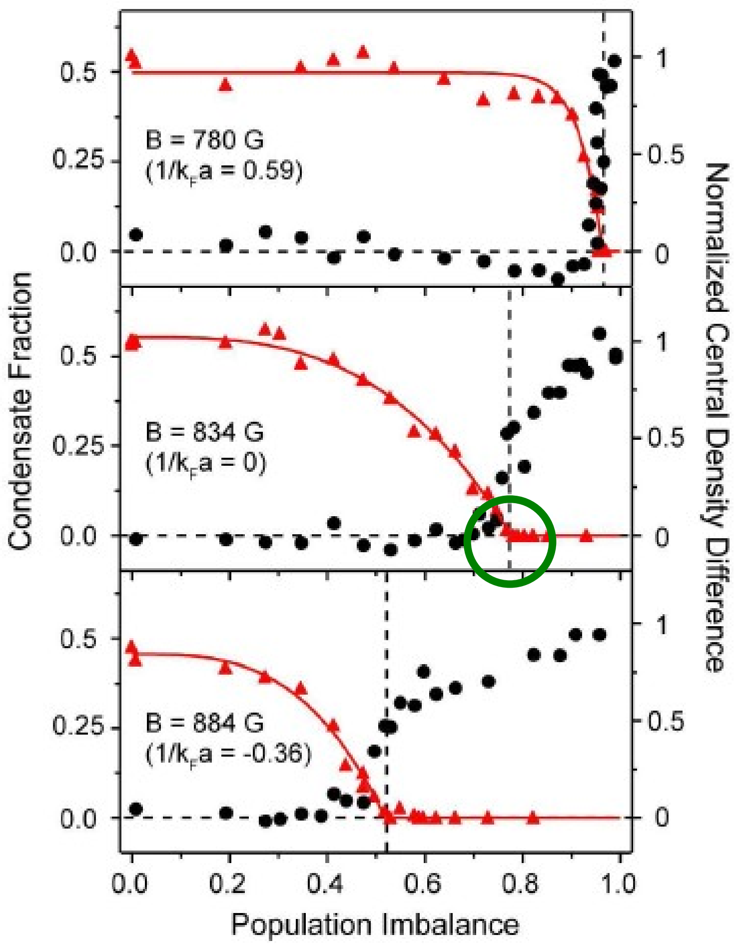


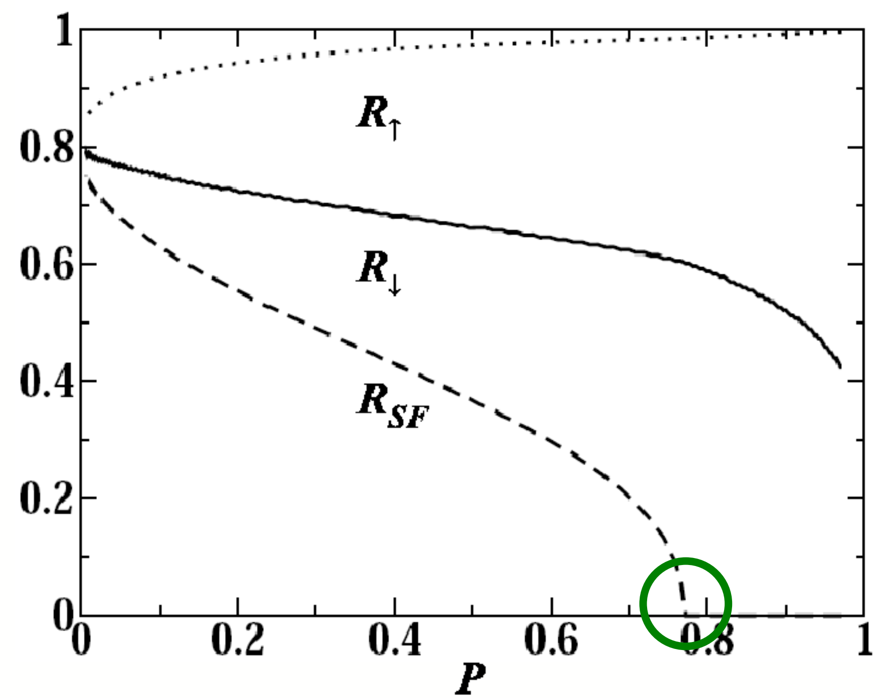


Normal phase of polarized Fermi gas at unitarity: TRAP

1) Critical Polarization (IN TRAP): $P_C = 0.77$

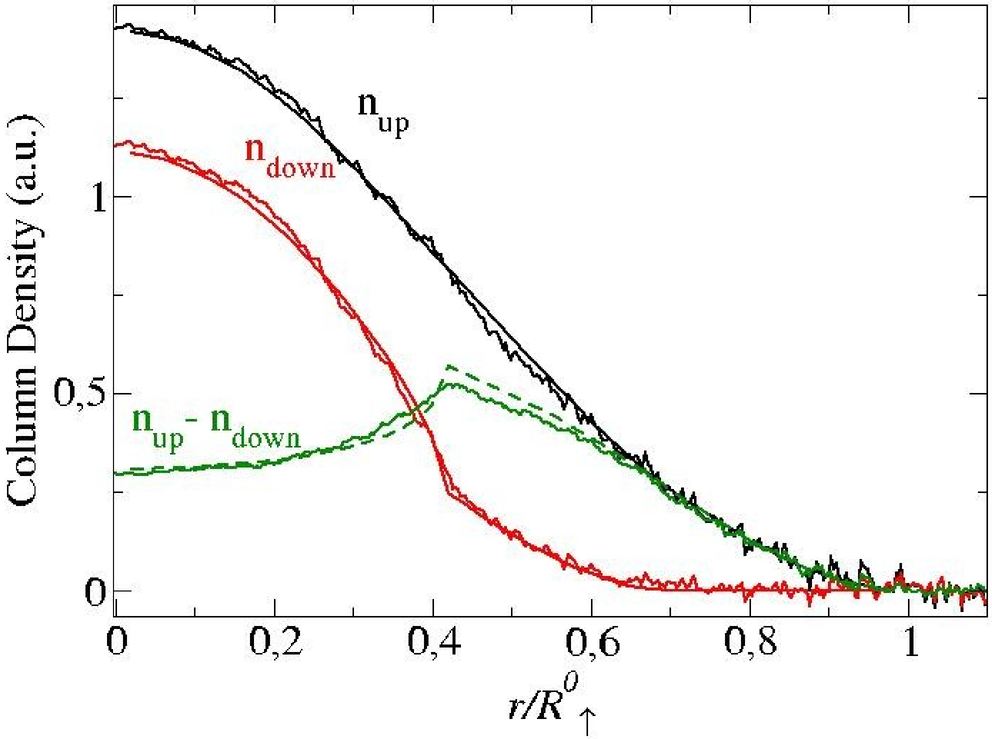
(very good agreement with MIT exps)

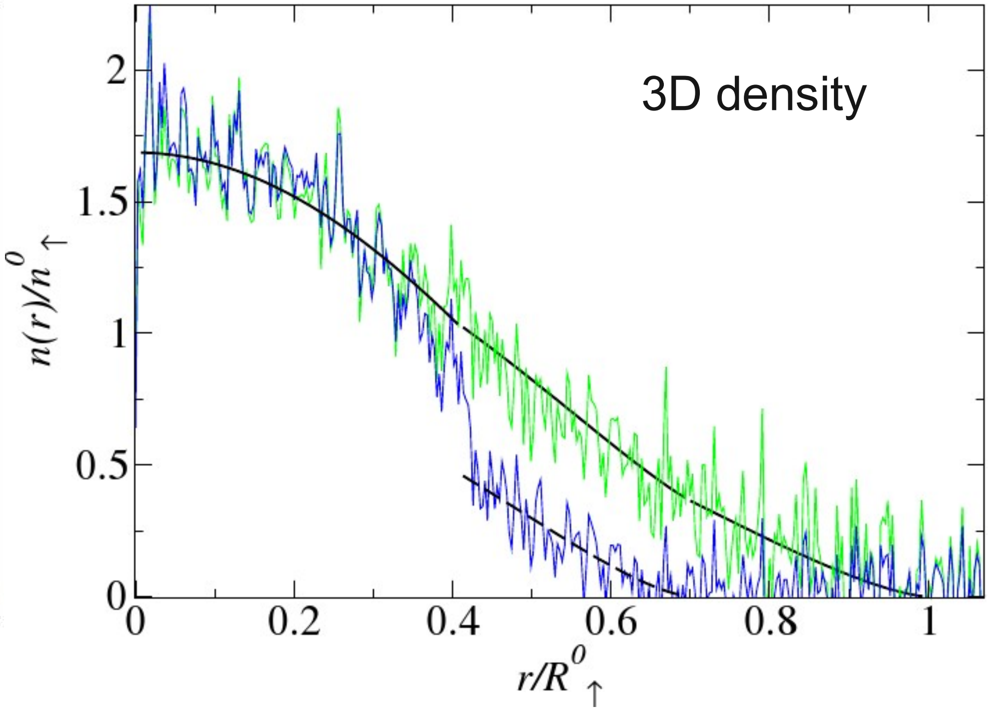




Normal phase of polarized Fermi gas at unitarity: TRAP

2) Density profiles





[Exp. Data from Yong Shin (MIT) compared with theory in A.R., C.Lobo and S. Stringari PRA (2008)]

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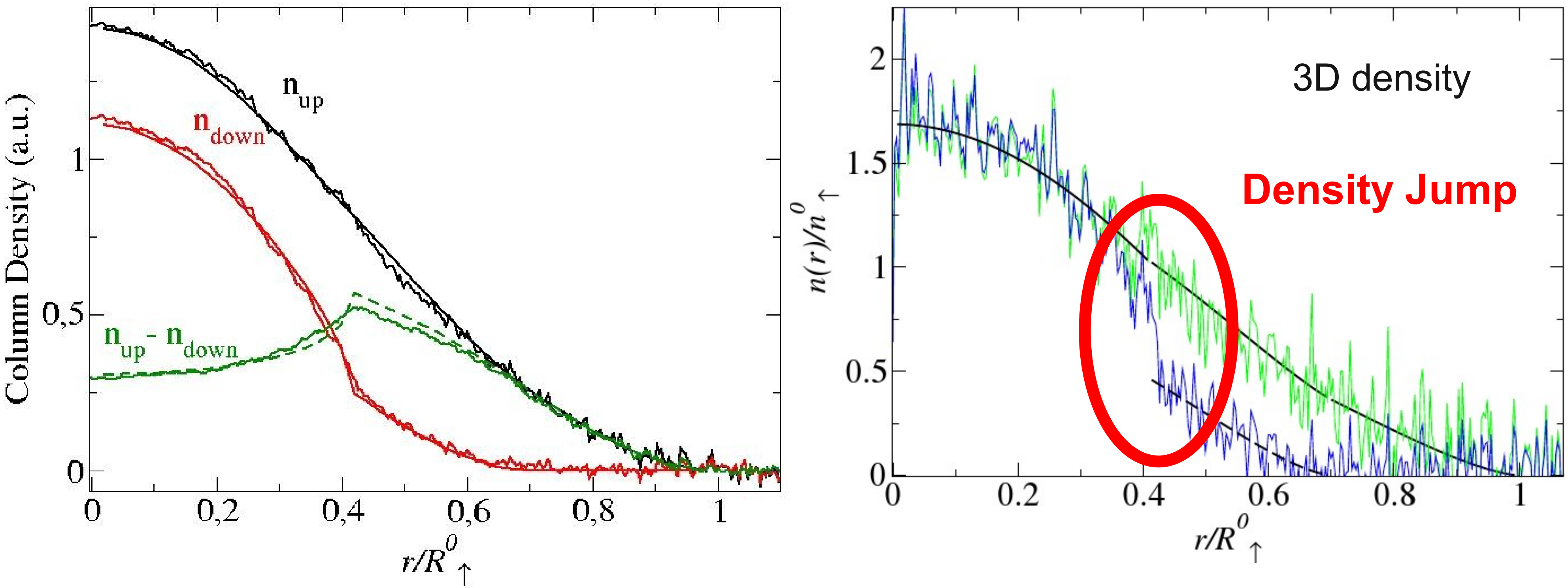


[Exp. Data from Yong Shin (MIT) compared with theory in A.R., C.Lobo and S. Stringari PRA (2008)]

Some Insight into the highly polarized Normal phase

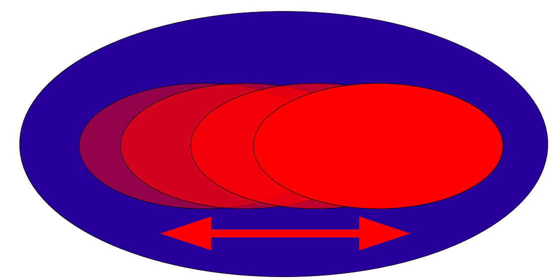
Dipole frequency at high polarization:

the majority component is not affected, the minority can be still think as a non-interacting gas but with *renormalized mass* and *trapping potential*

$$H_{sp} = \frac{p^2}{2m^*} + V(\mathbf{r})\left(1 + \frac{3}{5}A\right)$$

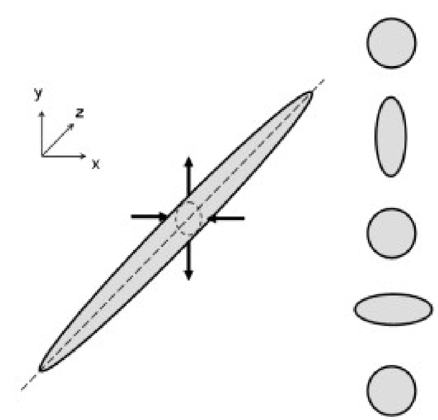
Spin-dipole mode

$$\omega_D^{(s)} = \omega_i \sqrt{\frac{m}{m^*}(1+(3/5)A)} \simeq 1.26\omega_i$$



Spin-radial-quadrupole mode

$$\omega_Q^{(s)} = 2\omega_\perp \sqrt{\frac{m}{m^*}(1+(3/5)A)}$$



Altmeyer (IBK)

Decaying time of the collective modes

We consider the *momentum relaxation* of an homogeneous highly polarized Fermi gas.

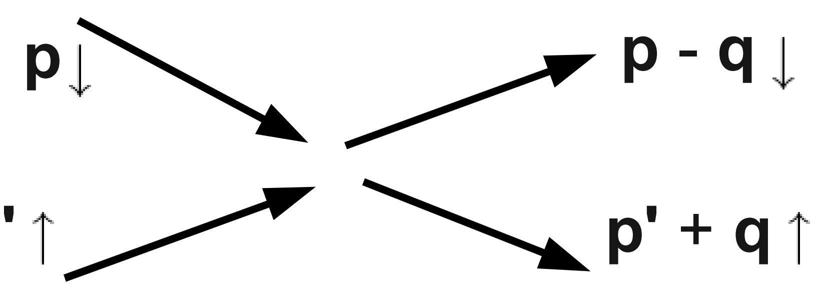
$$\frac{d\mathbf{P}_\downarrow}{dt} = -\frac{\mathbf{P}_\downarrow}{\tau_\mathbf{p}}$$

The minority component have a mean momentum **k** with respect to the majority one:
total momentum per unit volume $\mathbf{P}_\downarrow = n_\downarrow \mathbf{k}$

$$\frac{d\mathbf{P}_\downarrow}{dt} = -2\pi \frac{|U|^2}{V^3} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \mathbf{p}[n_p n_{p'}(1-n_{\mathbf{p}-\mathbf{q}})(1-n_{\mathbf{p}'+\mathbf{q}}) - n_{\mathbf{p}-\mathbf{q}} n_{\mathbf{p}'+\mathbf{q}}(1-n_p)(1-n_{p'})]\delta(\epsilon_p + \epsilon_{p'} - \epsilon_{\mathbf{p}-\mathbf{q}} - \epsilon_{\mathbf{p}'+\mathbf{q}})$$

$$n_{\mathbf{p}\downarrow} = f[\beta(\epsilon_{p\downarrow} - \mathbf{p}\cdot\mathbf{v} - \mu_\downarrow)]$$

$$\epsilon_{p\downarrow} = p^2/2m^*_\downarrow$$



$$\epsilon_{p'\uparrow} = p'^2/2m_\uparrow$$

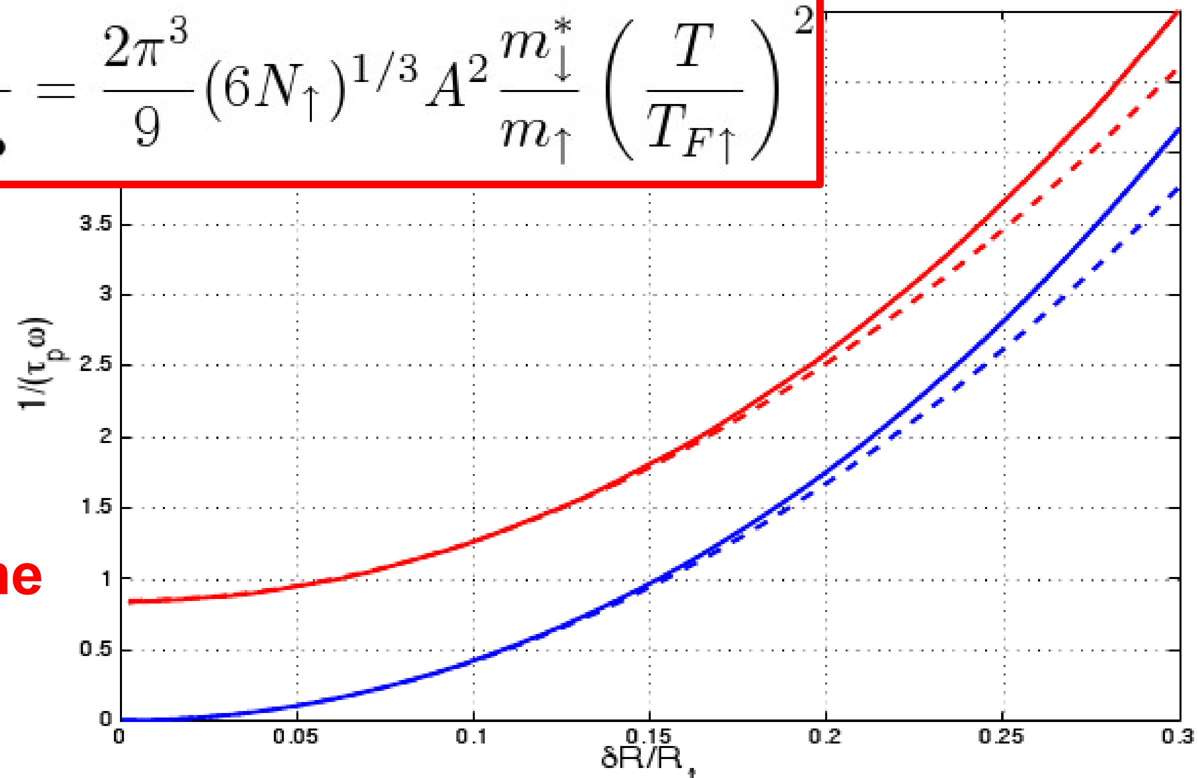
$$n_{p'\uparrow} = f[\beta(\epsilon_{p'\uparrow} - \mu_\uparrow)]$$

Decaying time of the collective modes

$$\begin{cases} \omega_D \tau_{\mathbf{p}} \gg 1 & \text{Collisionless regime: possible to see the dipole mode} \\ \omega_D \tau_{\mathbf{p}} \ll 1 & \text{Hydrodynamic regime: the dipole mode overdamped} \end{cases}$$

$$\delta R/R_{\downarrow} \ll T/T_{F\downarrow} : \frac{1}{\omega \tau_{\mathbf{p}}} = \frac{2\pi^3}{9}(6N_{\uparrow})^{1/3} A^2 \frac{m^*_{\downarrow}}{m_{\uparrow}} \left(\frac{T}{T_{F\uparrow}}\right)^2$$

MIT regime



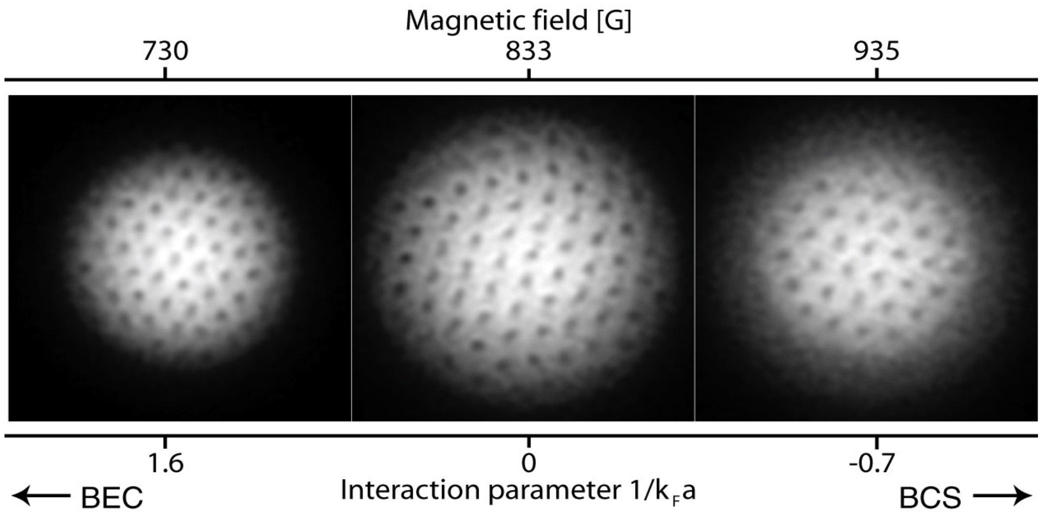
$$T = 0 : \frac{1}{\omega \tau_{\mathbf{p}}} = \frac{8\pi}{25}(6N_{\uparrow})^{1/3} A^2 \frac{m^*_{\downarrow}}{m_{\uparrow}} \left(\frac{T_{F\downarrow}}{T_{F\uparrow}}\right)^2 \left(\frac{\delta R}{R_{\uparrow}}\right)^2$$

Destroying superfluidity by rotation

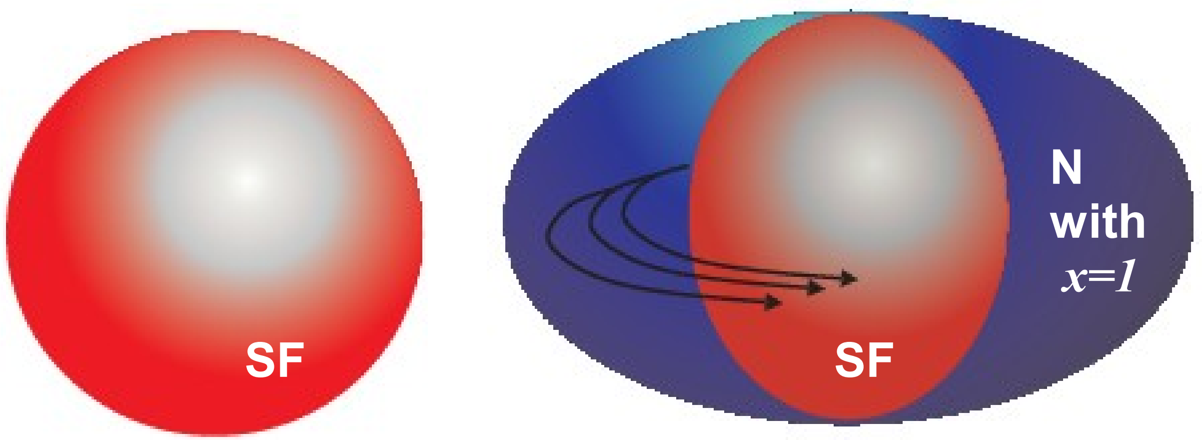
What does it happen if we "apply a rotation / rotate" to the system?

Already seen: **Vortices**

the superfluid lower its energy
by allowing some rotation in the
form of vorticity – BUT
topological defects, energy barrier



The normal part can rotate...
why not ***phase separating***
in order to minimize the energy?
A normal phase with concentration
x=1

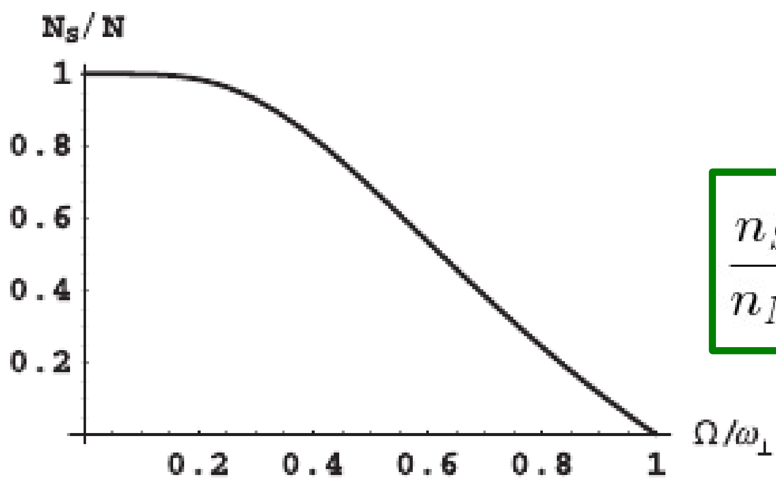


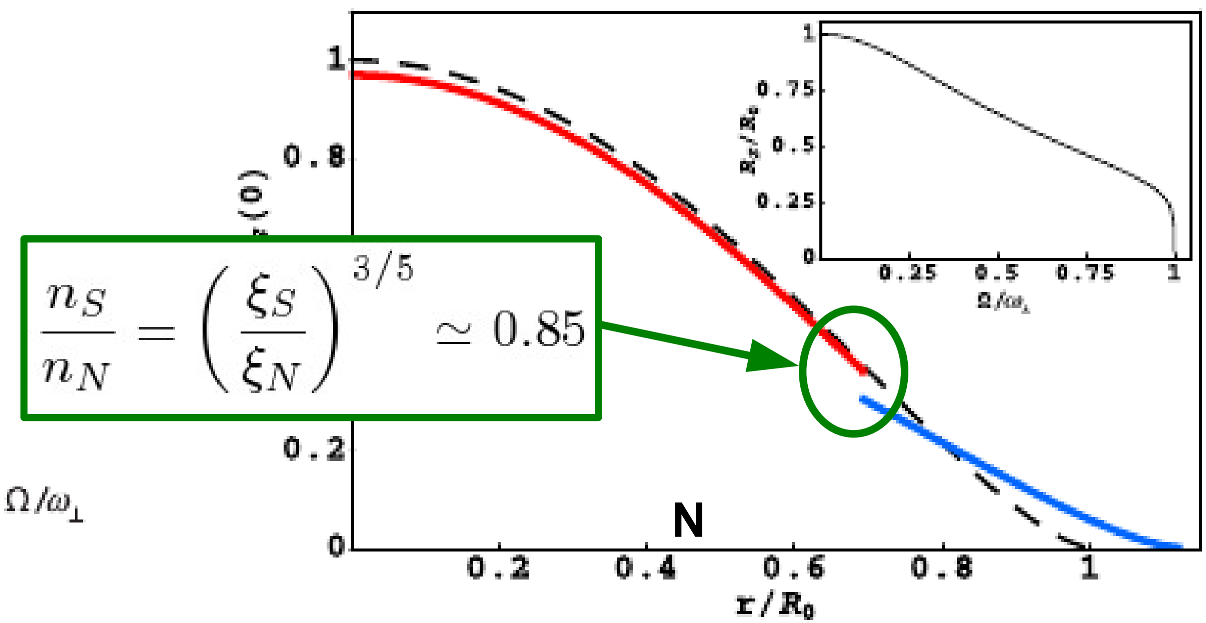
Destroying superfluidity by rotation

Normal phase with concentration ***x=1*****: Strongly interacting Landau-Fermi Liquid**

$$\epsilon_S = \xi_S \frac{3}{5}\frac{\hbar^2}{2m}(6\pi^2 n)^{5/3} \quad < \quad \epsilon_N = \xi_N \frac{3}{5}\frac{\hbar^2}{2m}(6\pi^2 n)^{5/3}$$

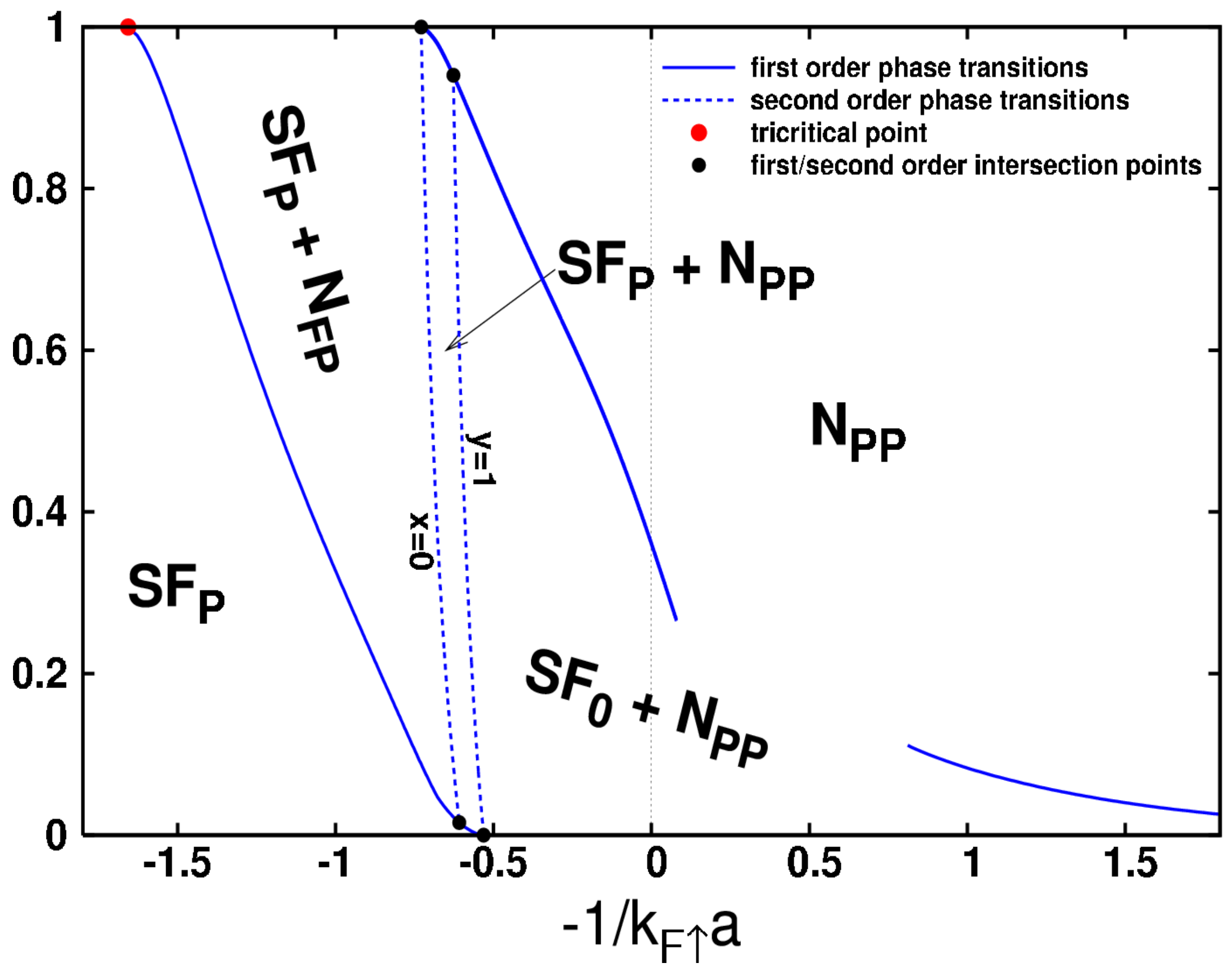
BUT, **normal phase gains energy in the rotating frame** $-m\Omega(\mathbf{r}\times\mathbf{v})_z$





A Bogoliubov-De Gennes approach (quantitavely wrong at unitarity) shows the presence of a third phase at the interface: a superfluid with broken pairs.
[M. Urban, P. Schuck, PRA (2008)]

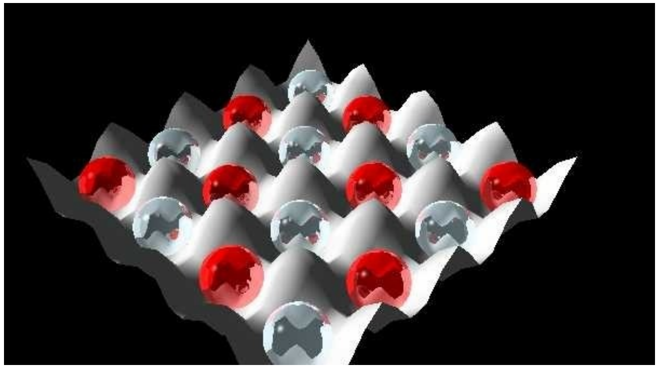
Outside the unitarity regime

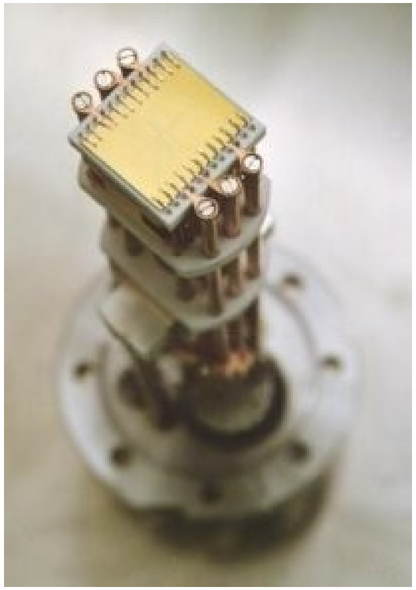


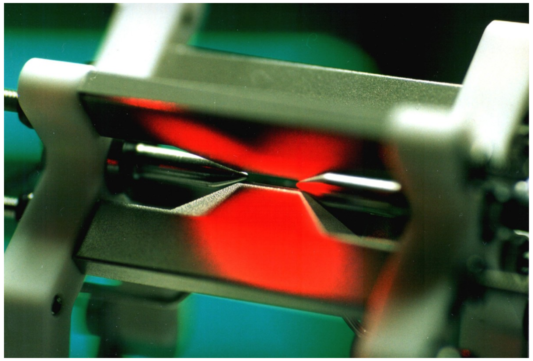
[S. Pilati and S. Giorgini, Phys. Rev. Lett. **100**, 030401 (2008)
G. Bertaina and S. Giorgini, Phys. Rev. A **79**, 013616 (2009)]

...and more...

- More exotic phases (polarized Superfluid, FFLO, Sarma...)
- More than 2 species (analogies with color superfluidity?))
- Bose-Fermi mixtures
- Include disorder and noise
- Cold and Dipolar Molecules
- Low dimensional systems
- Antiferromagnetic order: Néel transition
- Quantum Hall effect
- Cold gases on atom chip
- Cold gases in Cavity QED
- Trapped ions
- Quantum Information/Computation
- Cold gases as photonic crystals







Why are Cold Gases interesting?

• *Diluteness:* atom-atom interactions described by 2-body and 3-body physics
At low energy: a single parameter, the scattering length

• *Comparison with theory:* Gross-Pitaevskii, Bose and Fermi Hubbard models, search for exotic phases,…

• *« Simplicity of detection »*

New way to address some pending questions in the physics of interacting many-body systems

Link with condensed matter (high Tc superconductors, magnetism in lattices), nuclear physics, high energy physics (quark-gluons plasmas), astrophysics…

Experimental tunability of almost all the parameters which enter in the physics of the system under study!

Why are Cold Gases interesting?

Experimental tunability of almost all the parameters which enter in the physics of the system under study!

Control of sign of interaction and of trapping parameters:

- weakly and strongly interacting systems
- 1D, 2D, 3D geometry & (optical) lattice
- access to time dependent phenomena & out of equilibrium situations
- and more…

Towards quantum simulations with cold atoms « a’ la Feynman »,i.e., the first idea of quantum computation


The End