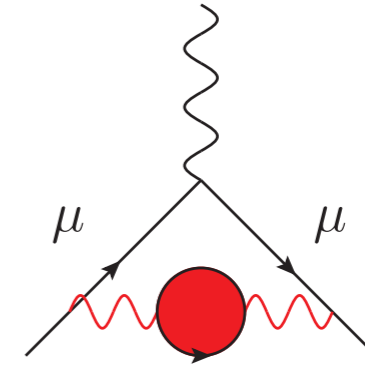# The HVP from μ-e scattering and the lattice QCD data

Marina Krstić Marinković

CERN & Trinity College Dublin

a_μ as...

$a_\mu = ($

$a_\mu^{\exp} - a_\mu^{\rm SM} \sim 3\,\sigma$

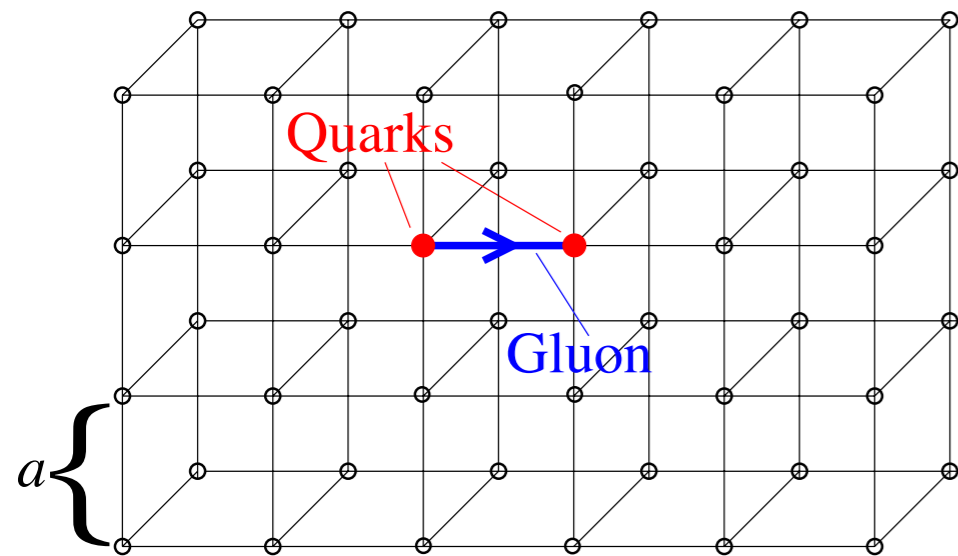|	$a_\mu [10^{-11}]$	$\Delta a_\mu [10^{-11}]$
experiment	116 592 089.	63.
QED $\mathcal{O}(\alpha)$	116 140 973.21	0.03
QED $\mathcal{O}(\alpha^2)$	413 217.63	0.01
QED $\mathcal{O}(\alpha^3)$	30 141.90	0.00
QED $\mathcal{O}(\alpha^4)$	381.01	0.02
QED $\mathcal{O}(\alpha^5)$	5.09	0.01
QED total	116 584 718.95	0.04
electroweak, total	153.6	1.0
HVP (LO) [Hagiwara et al. 11]	6 949.	43.
HVP (NLO) [Hagiwara et al. 11]	−98.	1.
HLbL [Jegerlehner-Nyffeler 09]	116.	40.
HVP (NNLO) [Kurz, Liu, Marquard, Steinhauser 14]	12.4	0.1
HLbL (NLO) [GC, Hoferichter, Nyffeler, Passera, Stoffer 14]	3.	2.
theory	116 591 855.	59.

~ $3\,\sigma$

- Current TH estimate affected by
  ➡ the experimental uncertainties;
  ➡ perturbation theory/models

- Lattice QCD estimate —> for a final cross-check of the SM result and to keep up with the planned experimental improvements
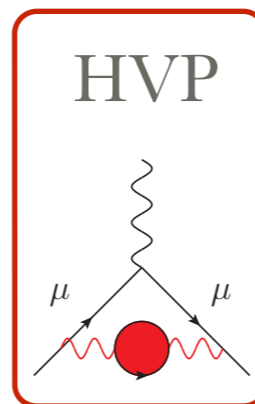
$$a_\mu^{HVP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left\{ \int_{m_\pi^2}^{E_{cut}^2} ds \frac{R_{\rm had}^{\rm data}(s)\hat{K}(s)}{s^2} + \int_{E_{cut}^2}^{\infty} ds \frac{R_{\rm had}^{\rm pQCD}(s)\hat{K}(s)}{s^2} \right\}$$

- HVP leading order: **largest uncertainty!** (around 50% of total th. error)

- **Lattice QCD provides a way to compute this contribution in a model-independent way**

# Non - perturbative computation of a_μ

Quarks

Gluon

a

$a_\mu = (g-2)_\mu/2$

Intro   HLbL: gauge & crossing   HLbL dispersive   Conclusions   Status of $(g-2)_\mu$ Approaches to HLbL

Status of $(g-2)_\mu$, experiment vs SM

	$a_\mu [10^{-11}]$	$\Delta a_\mu [10^{-11}]$
experiment	116 592 089.	63.
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$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} \sim 3\,\sigma$

HVP

μ μ

- Recipe fo

1. Generate ensembles of field configurations using *Monte Carlo*

2. Average over a set of configurations:

   ➡ Compute correlation function of fields, extract Euclidean matrix elements or amplitude

   ➡ Computational cost dominated by quarks: inverses of large, sparse matrix

3. Extrapolate to continuum, infinite volume, physical quark masses (now directly accessible)

Activity in the lattice community

Lattice Symposium talks on muon g-2
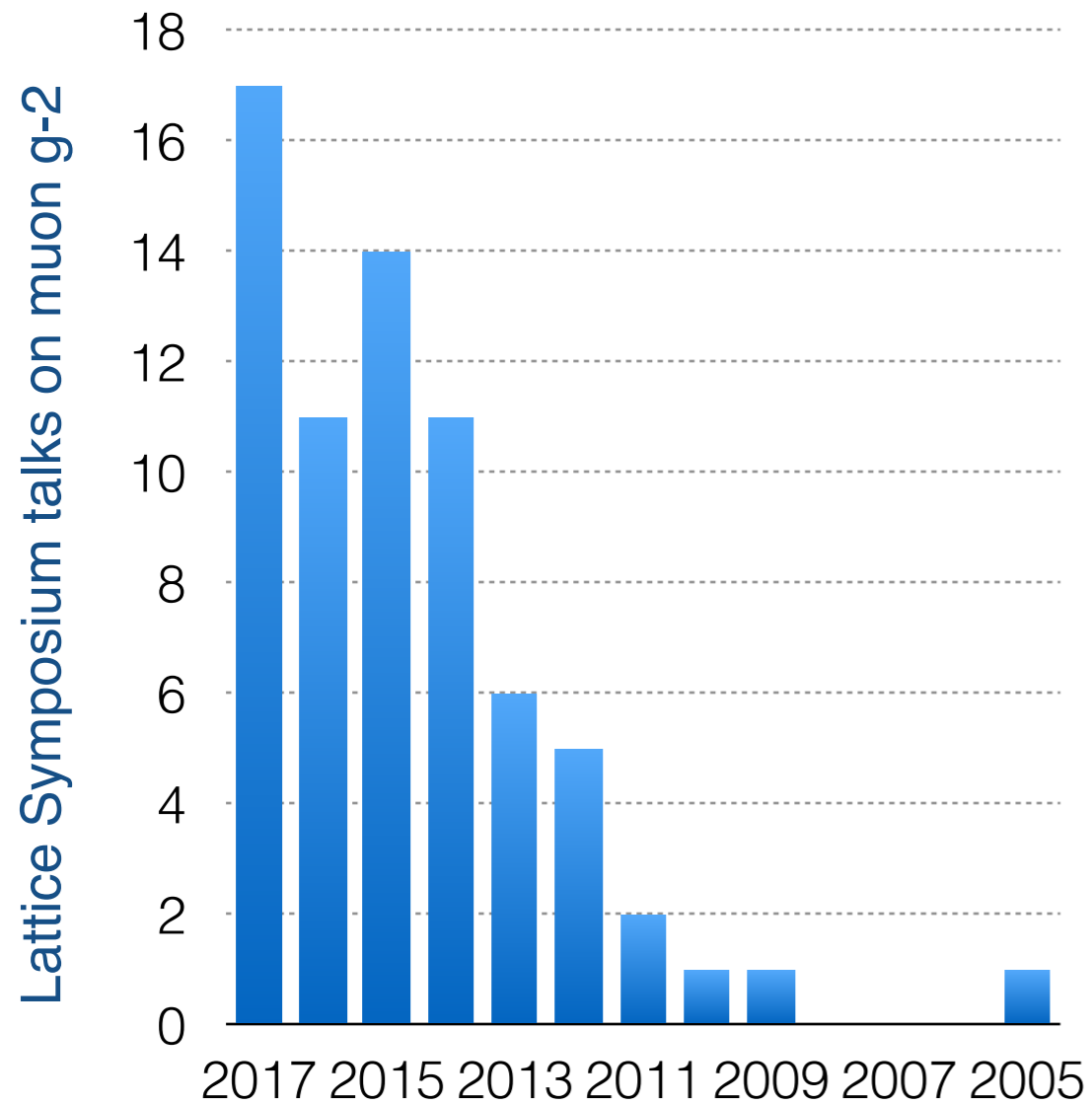
- HVP from the lattice:
  - ➡ RBC/UKQCD, Mainz U.[CLS], HPQCD[MILC], BMW, MILC, ABGP, Regensburg U., …
- HLbL from the lattice
  - ➡ RBC, Mainz U.(2 approaches)

- HVP from the lattice+experiment (R-ratio data):
  - ➡ Bernecker&Meyer [arXiv:1107.4388]
  - ➡ ETM, MILC, RBC/UKQCD …
- HVP from the lattice+experiment (space-like data):
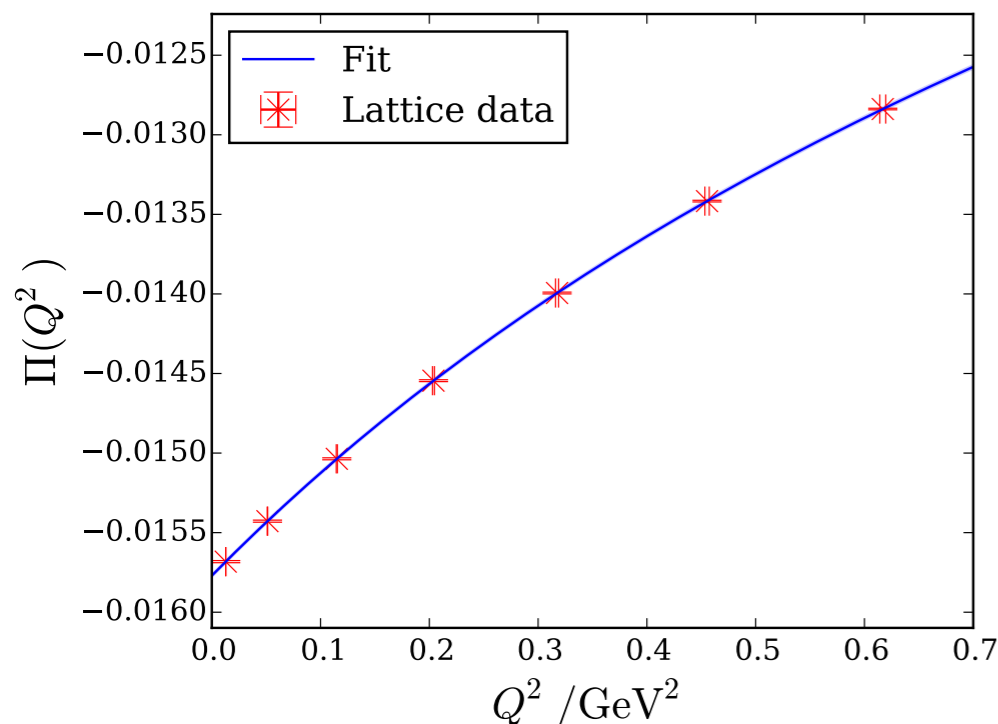 - ➡ **this talk…**

The leading hadronic contribution - HVP
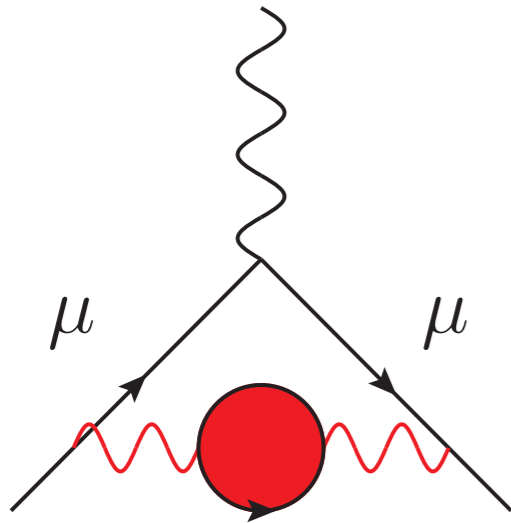
μ μ
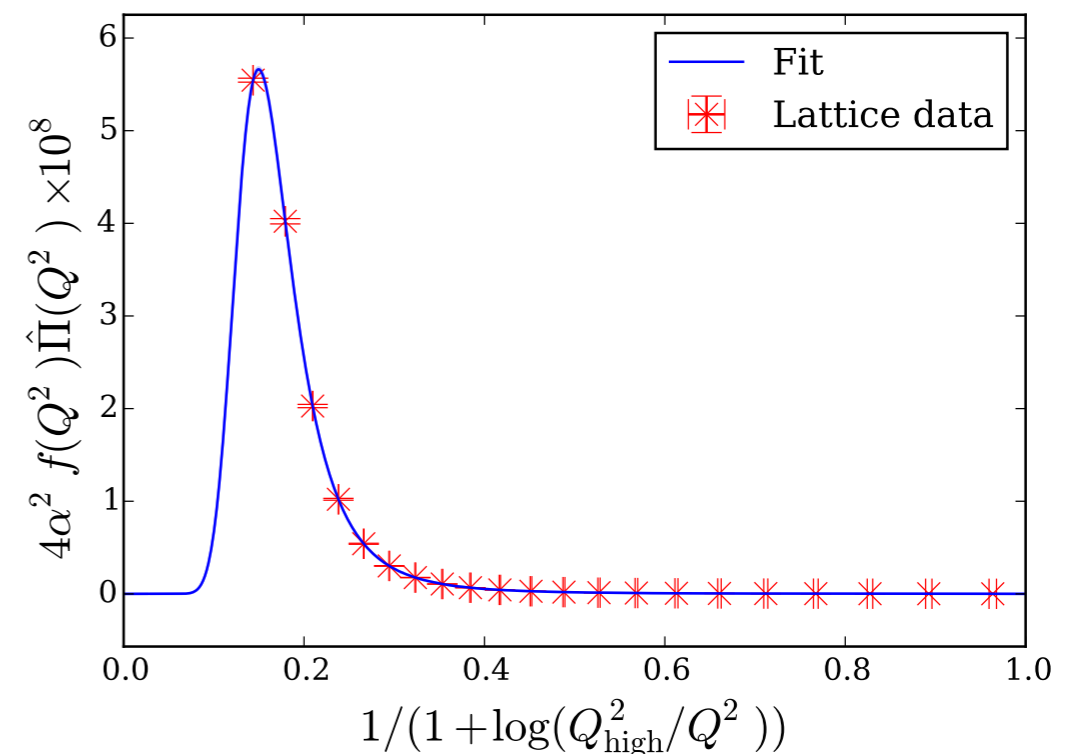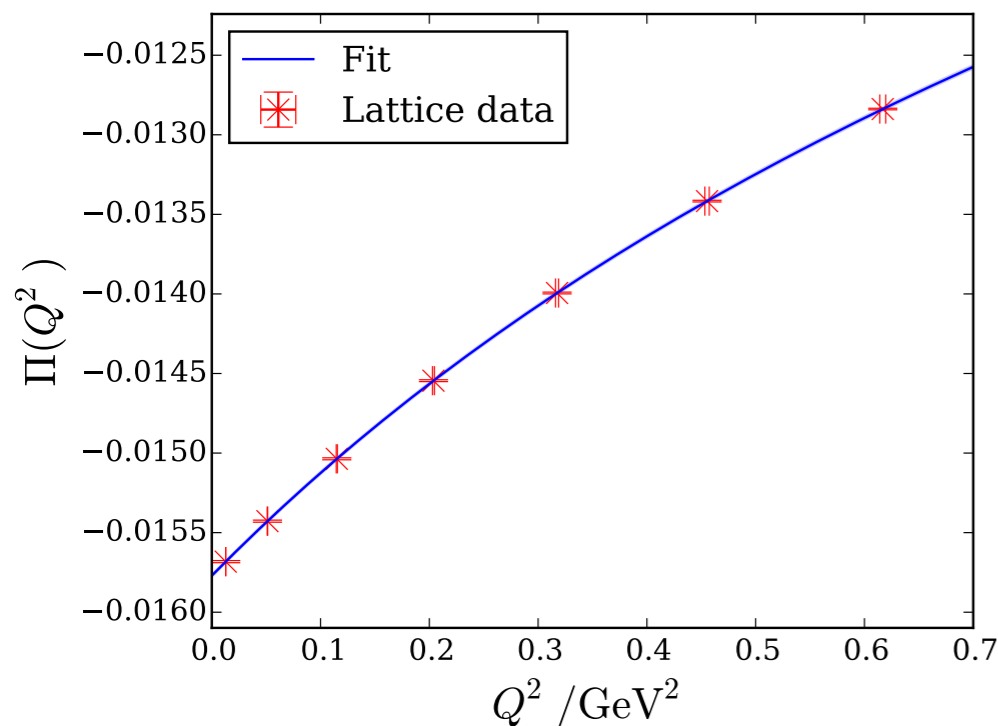
Vacuum polarisation inserted in

the photon propagator

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$$

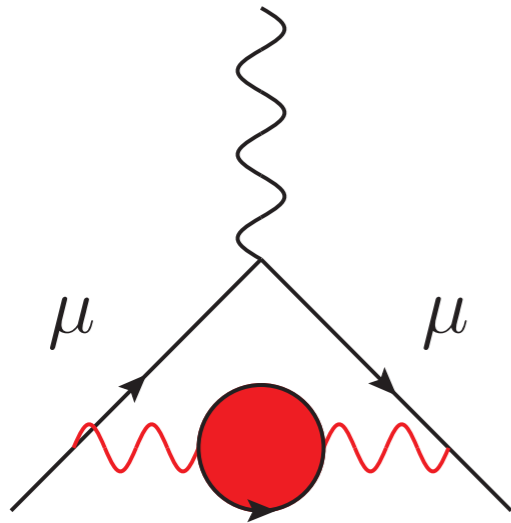$$f(Q^2) = m_\mu^2 Q^2 Z^3(Q^2) \frac{1 - Q^2 Z(Q^2)}{1 + m_\mu^2 Q^2 Z^2(Q^2)}$$

$$Z(Q^2) = \frac{\sqrt{Q^4 + 4m_\mu^2 Q^2} - Q^2}{2m_\mu^2 Q^2}$$

$$\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$$

$$\Pi_{\mu\nu}(Q) = \sum_f Q_f^2 \sum_x e^{iQx} \langle J_\mu^f(x) J_\nu^f(0) \rangle$$

$$\Pi_{\mu\nu}(Q) = (Q_\mu Q_\nu - g_{\mu\nu} Q^2)\Pi(Q^2)$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim 3\sigma$$

The leading hadronic contribution - HVP

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 \boxed{f(Q^2) \times \hat{\Pi}(Q^2)}$$

$$f(Q^2) = m_\mu^2 Q^2 Z^3(Q^2) \frac{1 - Q^2 Z(Q^2)}{1 + m_\mu^2 Q^2 Z^2(Q^2)}$$

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μ μ

Vacuum polarisation inserted in

the photon propagator

JHEP 1604 (2016) 063 [T.Blum, P.A.Boyle, L. Del Debbio, R.J. Hudspith,T. Izubuchi, A.Juettner, C.Lehner, R. Lewis, K. Maltman, M.K.M., A. Portelli, M.Spraggs]

strange quark HVP, RBC-UKQCD '16
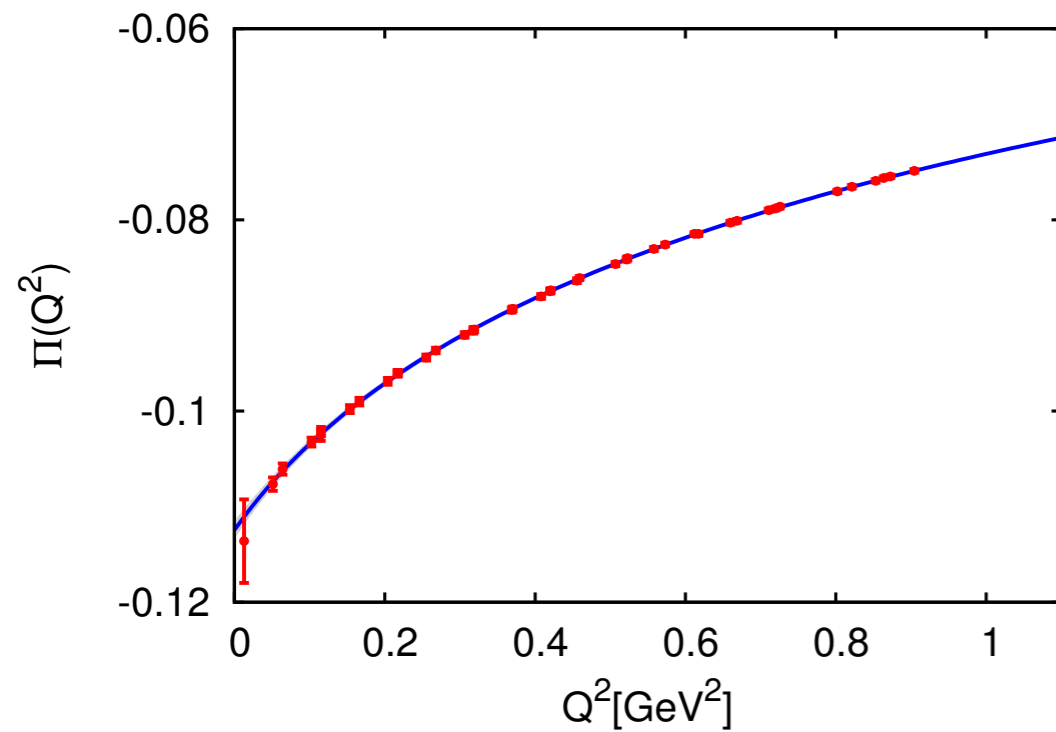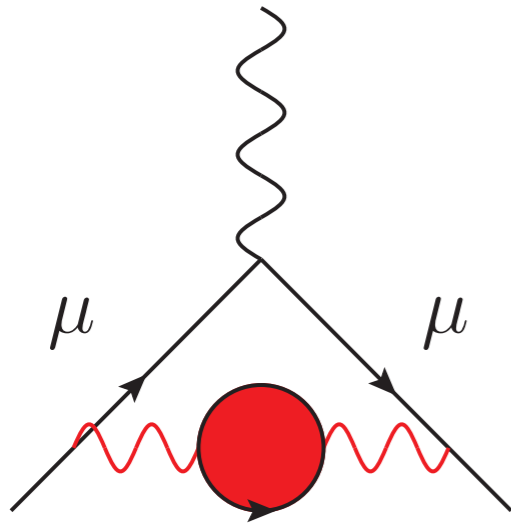
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The leading hadronic contribution - HVP

μ μ

Vacuum polarisation inserted in
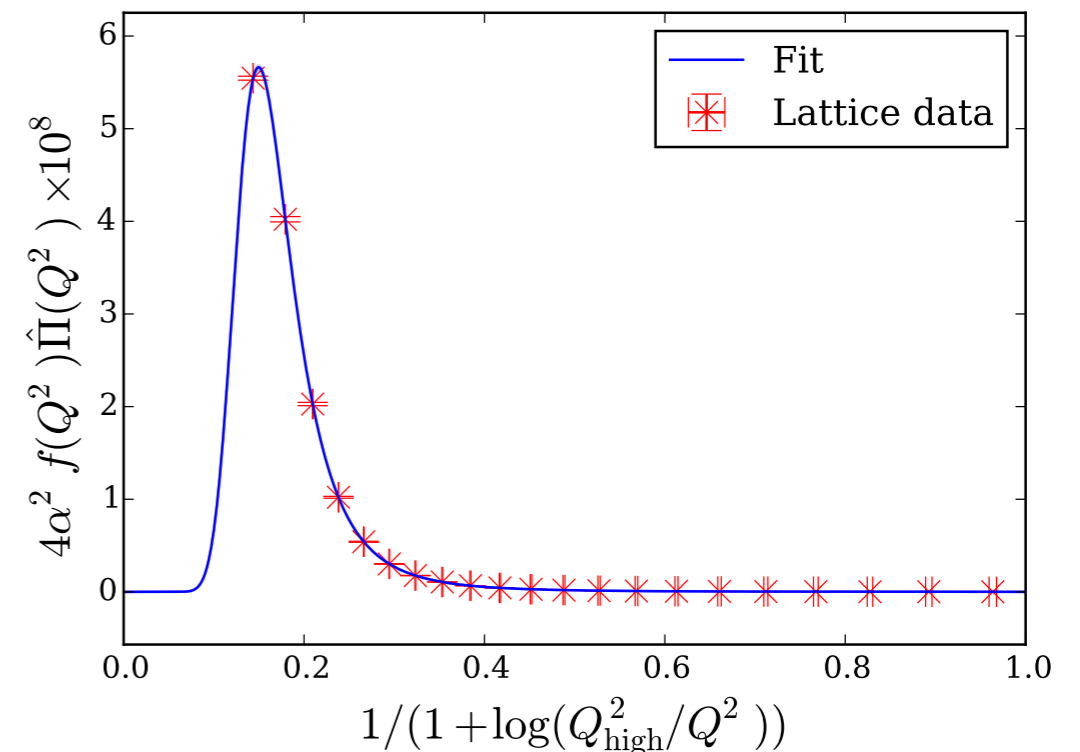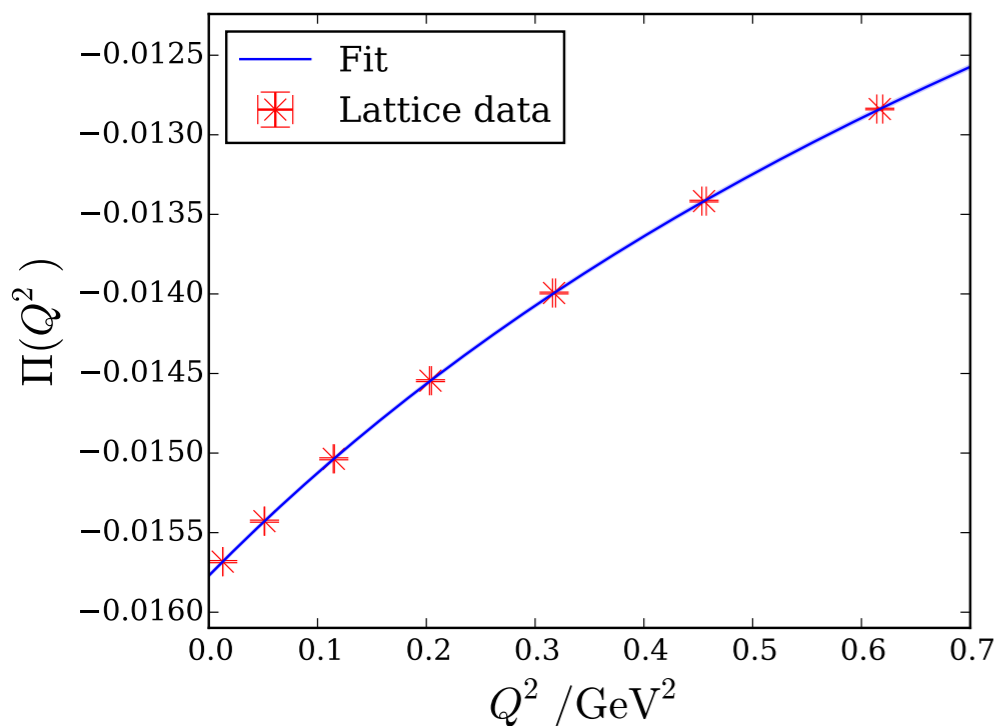
the photon propagator

light quark HVP, RBC-UKQCD '12
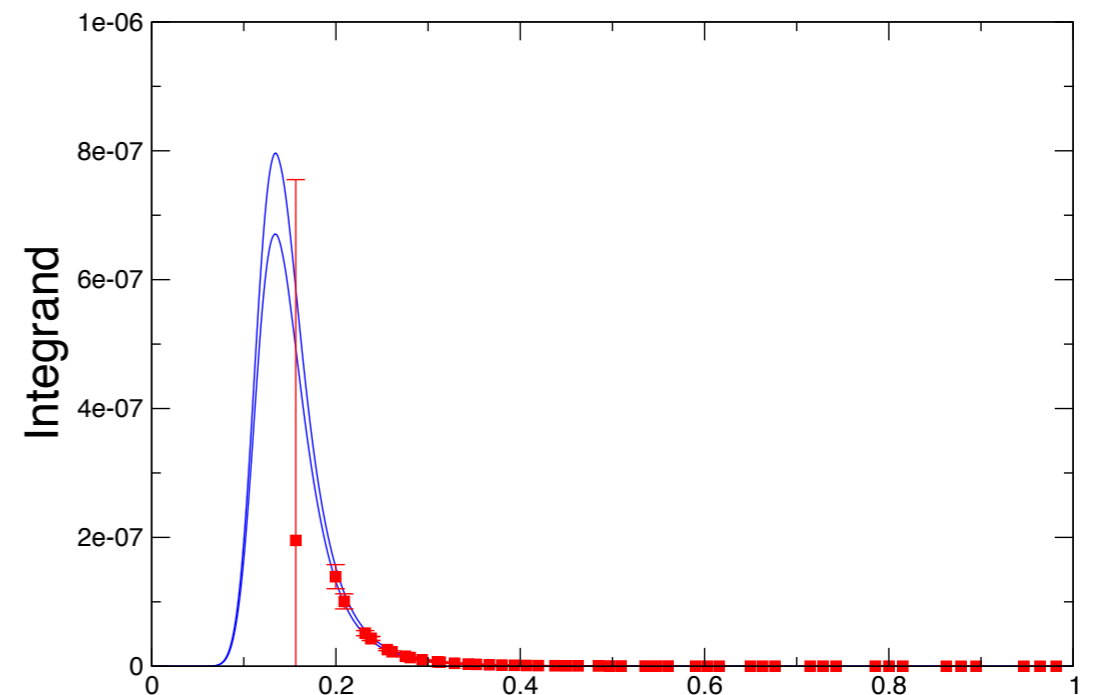
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Phys. Rev. D85 (2012) [P.A.Boyle, L. Del Debbio, E.Kerrane, J.Zanotti]

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} \sim 3\sigma$$

The leading hadronic contribution - HVP

μ μ

Vacuum polarisation inserted in

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$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$$

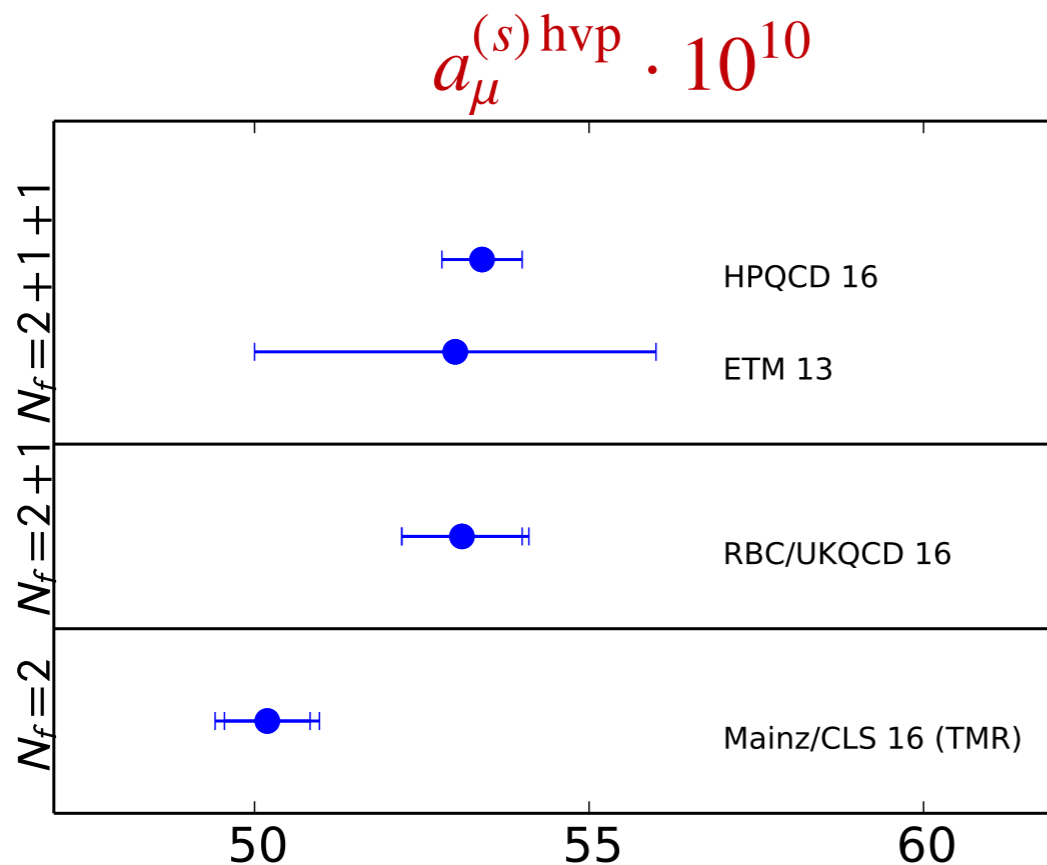$$f(Q^2) = m_\mu^2 Q^2 Z^3(Q^2) \frac{1 - Q^2 Z(Q^2)}{1 + m_\mu^2 Q^2 Z^2(Q^2)}$$

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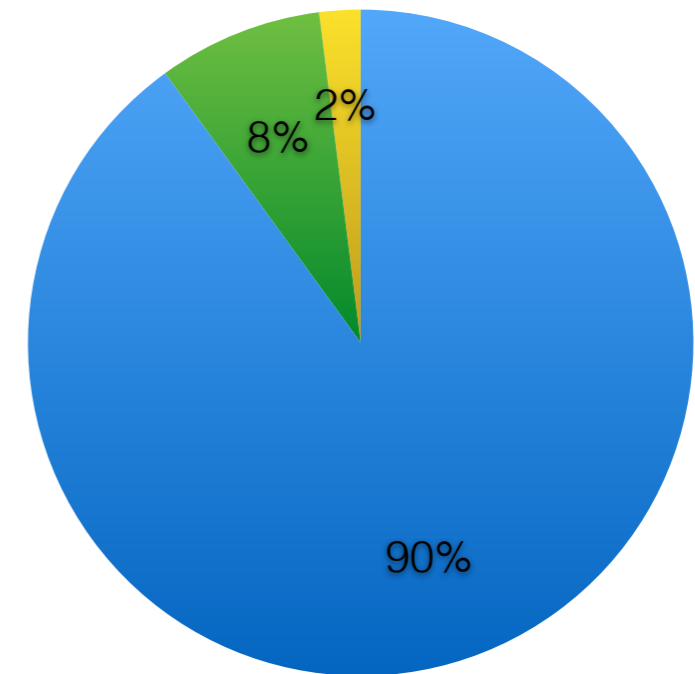
strange quark HVP, RBC-UKQCD '16

Summary: strange quark HVP

a_μ^{hvp}

$a_\mu^{(s)\,\text{hvp}} \cdot 10^{10}$

HPQCD 16

ETM 13

RBC/UKQCD 16

Mainz/CLS 16 (TMR)

Plot from H. Wittig @ Lattice 2016

light: u/d   strange   charm

2%
8%
90%

[**HPQCD: arXiv:1601.03071,**
Mainz: arXiv:1705.01775,
ETM: arXiv:1505.03283]

$a_\mu^{(c)\,\text{hvp}} \cdot 10^{10}$

1. Generate ensembles of field configurations using *Monte Carlo*

2. Average over a set of configurations:

   HPQCD 16
   ➡ Compute correlation function of fields, extract Euclidean matrix elements or amplitude
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   ➡ Computational cost dominated by quarks: inverses of large, sparse matrix

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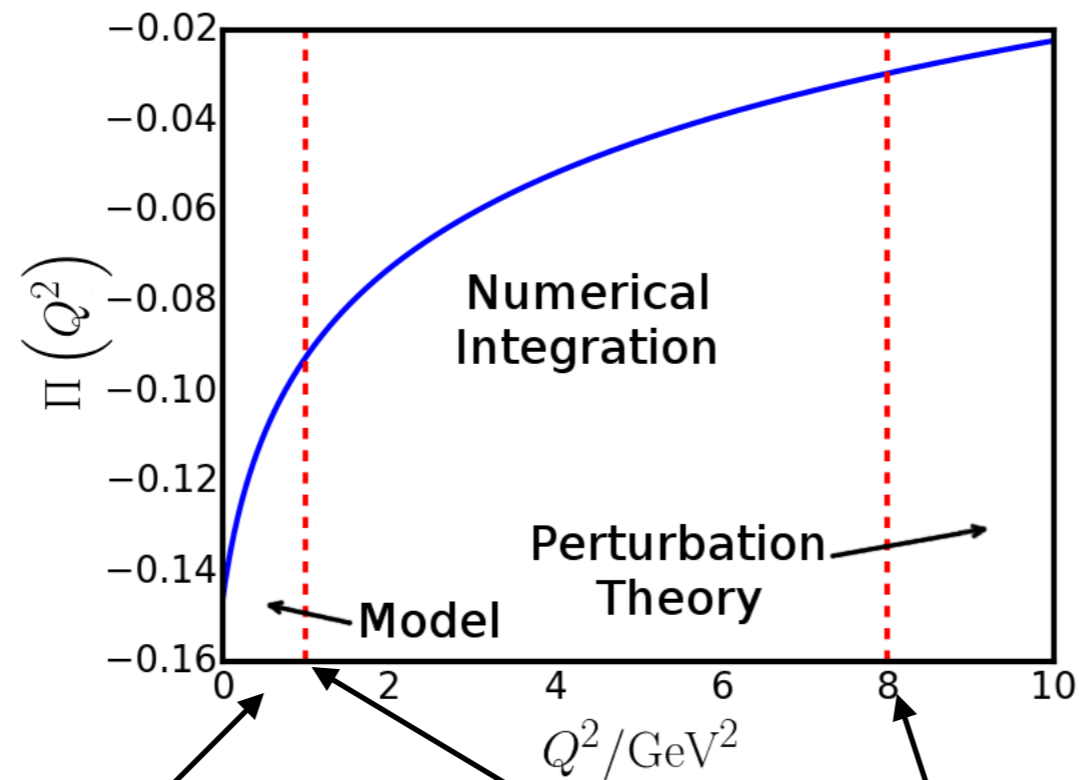
Dominant sources of errors

$$a_\mu^{\mathrm{hvp}} \cdot 10^{10}$$

- Three complete computations of a_μ^{HVP} **(u/d+s+c)**

  ➡ **recent HPQCD: ~1.8%** precision for (u/d+s+c+b)

    [**HPQCD arXiv:1601.03071**]

  ➡ **recent Mainz:** [**arXiv:1705.01775**]

  ➡ **ETM '15:** [**JHEP 1511(2015) 215, arXiv: 1505.03283**]

- Understanding the systematics is extremely important

  and more challenging:

  ➡ deterioration of signal at **Q² —> 0**

  ➡ disconnected diagrams, isospin breaking effects

  ➡ scale setting error [**arXiv:1705.01775**]
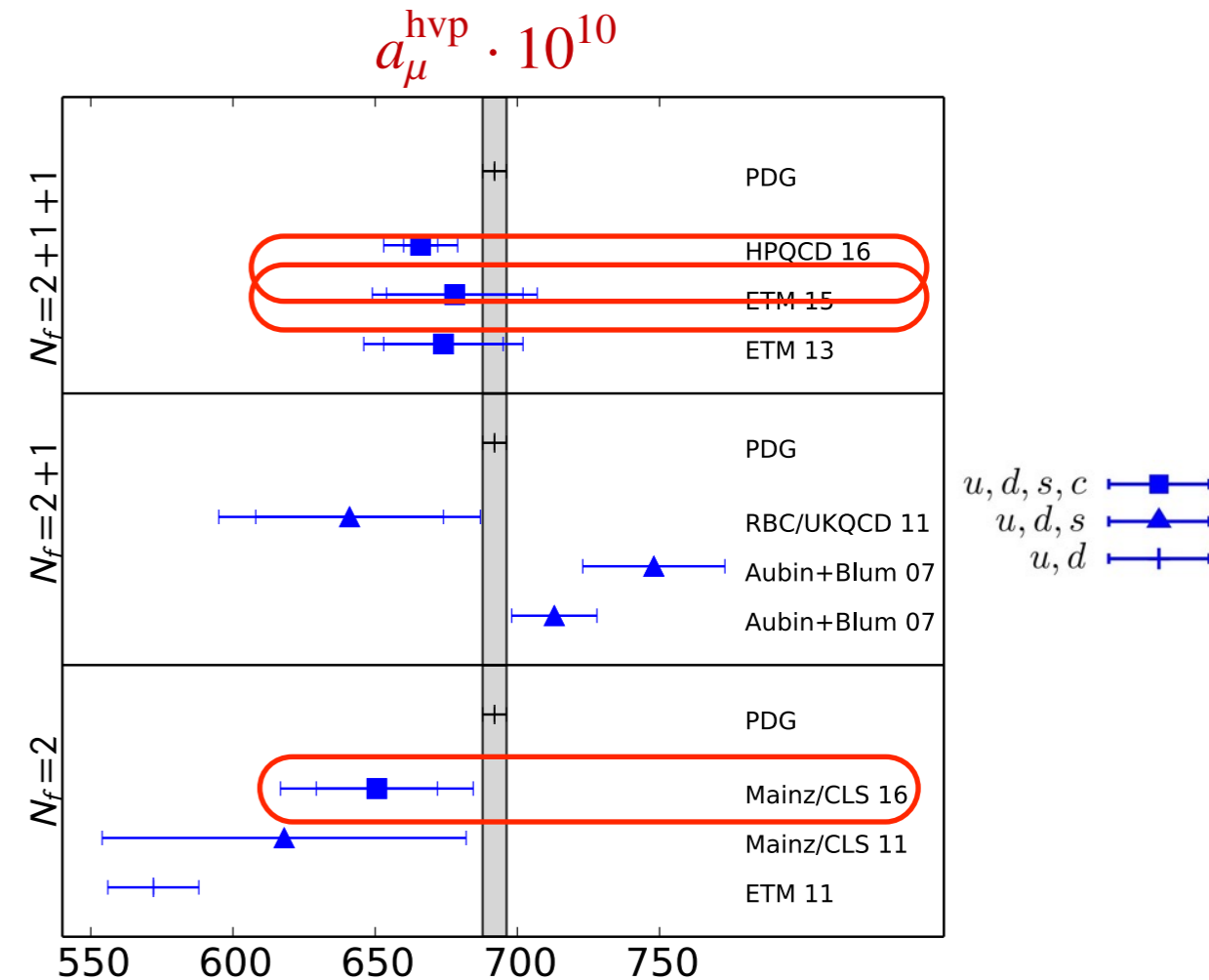
[Plot: H. Wittig @ LATTICE 2016]

✱ Individual flavour contributions:

light (*u,d*)	≈ 90%
strange (*s*)	≈ 8%
charm (*c*)	≈ 2%

Hybrid method

- **Low momentum region**

  ➡ **Pade approximants**

  ➡ **time moments**

  ➡ **conformal polynomials**

- **Vary low and high Q² cut**

strategy applied for the strange quark contribution to the HVP [RBC/UKQCD]

Hybrid method

- **Low momentum region**

  ➡ **Experiment (NLO, NNLO, radiative corrections …)**

- **Vary low and high Q^2 cut**

strategy proposed for the hybrid determination
of the total HVP (u+d+s+c+b)

# Proposals for new experimental measurements of a_μ^{HVP}

- Goal precision for HVP contribution to is <1%

➡ New proposals for the **space-like** experimental measurements of HVP

    ➡ [**Phys.Lett. B746 (2015) 325-329 by Carloni, Passera,Trentadue, Venanzoni**] **@KLOE2**

    ➡ [**Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.**] **@CERN**

- Estimated precision for the HVP from the **μe scattering** experiment is **0.3%** [**see slides by G. Venanzoni and U. Marconi**]

- **Relevance for lattice QCD determinations of HVP:**

    1. "hybrid method" [**Phys. Rev. D 90, 074508 (2014) Golterman,Maltman,Peris**] with experimental+lattice QCD data

        a) to complete the exp. result
        b) to cross-check lattice data

    2. continuum limit of $\Pi(Q^2)$ at fixed **Q²**

    3. help in choosing the parametrization for $\Pi(Q^2)$ with less FV/cutoff effects

Hybrid method: a_μ^{HVP} **from experimental + lattice QCD data**

- Estimated precision for the HVP from the **μe** exp. is **0.3%** in **[0,0.138]GeV²** **[see slides by G. Venanzoni and U. Marconi]**

- Due to the experimental constraints: region **[0.138, ∞] GeV²** cannot be covered by this exp.
  ➡ complementary to the lattice QCD data

➡ $x_{\max} = 0.93$

➡ $Q^2 = \dfrac{x^2 m_\mu^2}{1 - x}$

➡ $Q^2_{\exp,\max} = 0.138 \text{GeV}^2$

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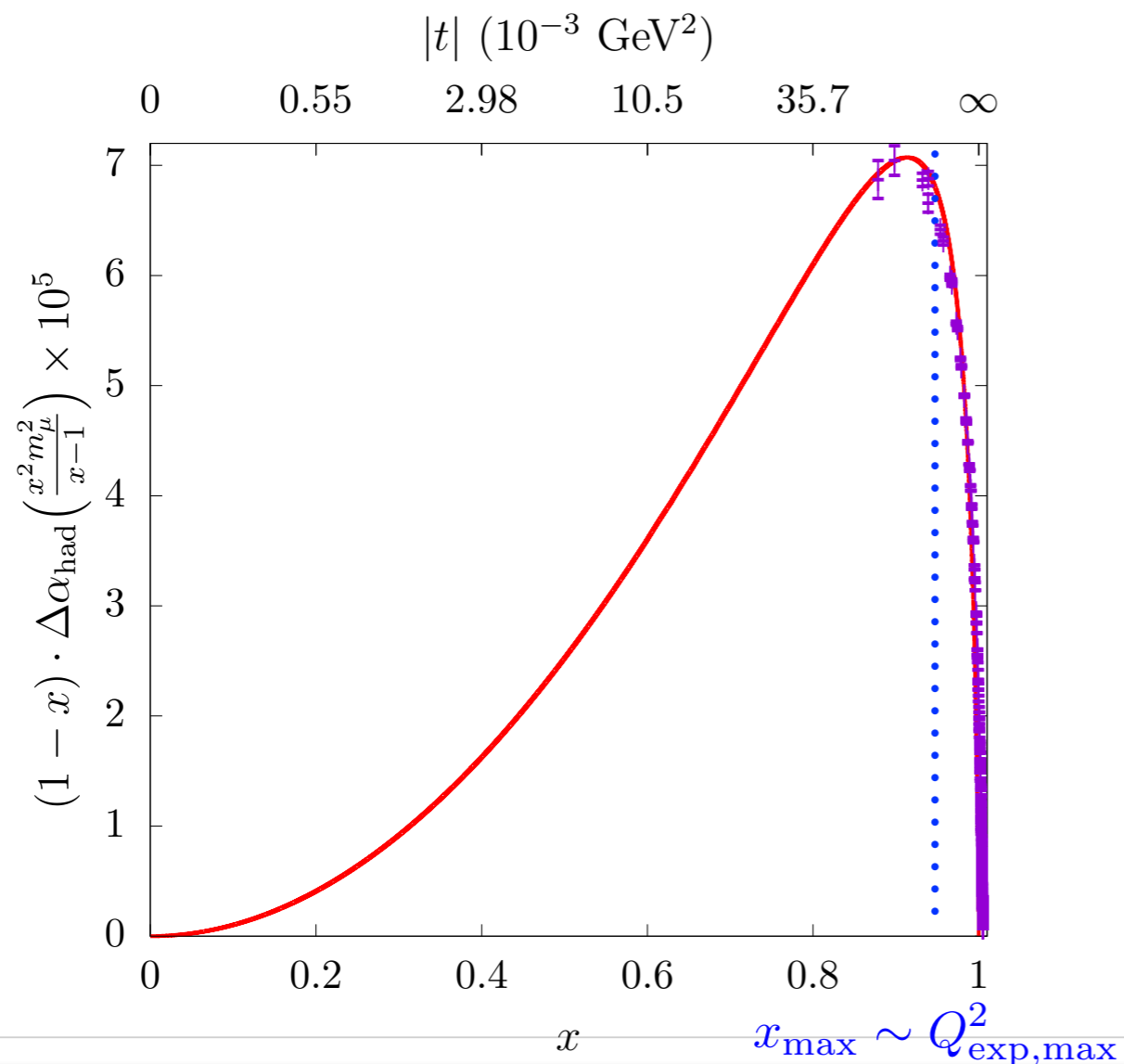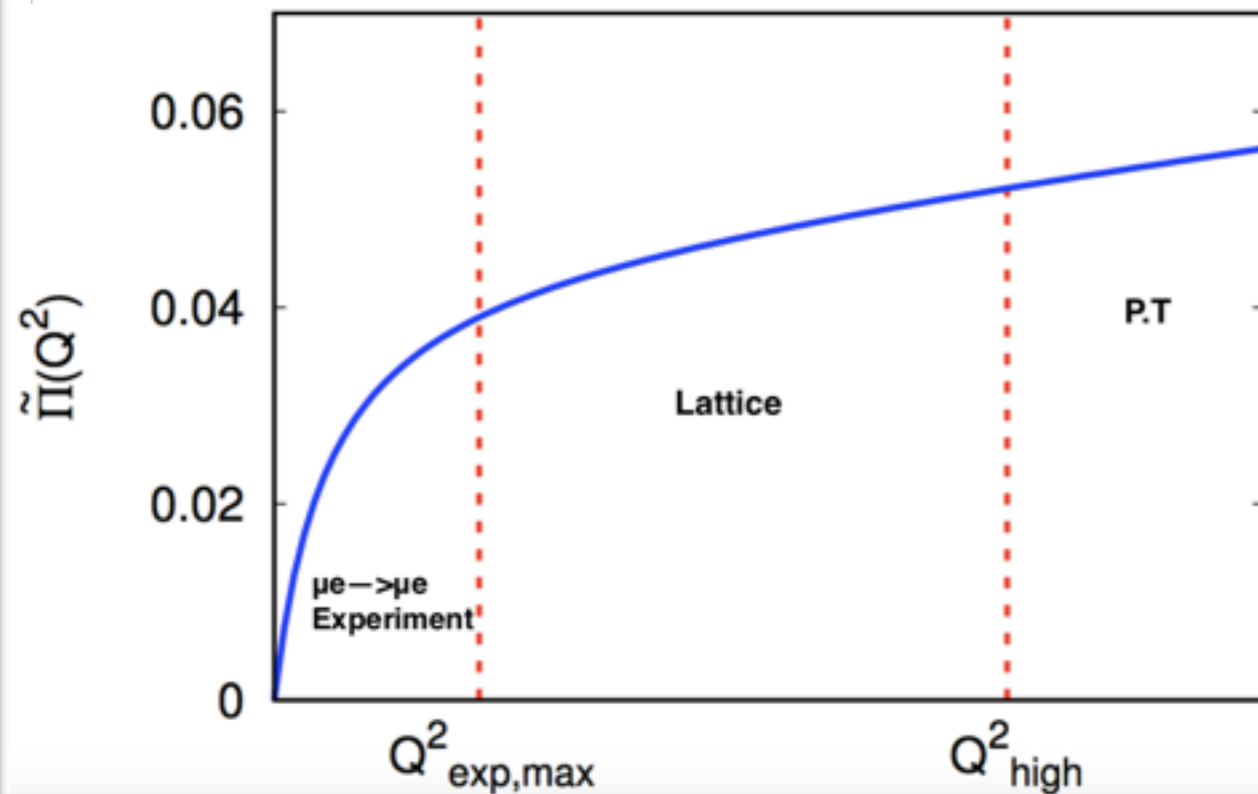 ➡ complementary to the lattice QCD data
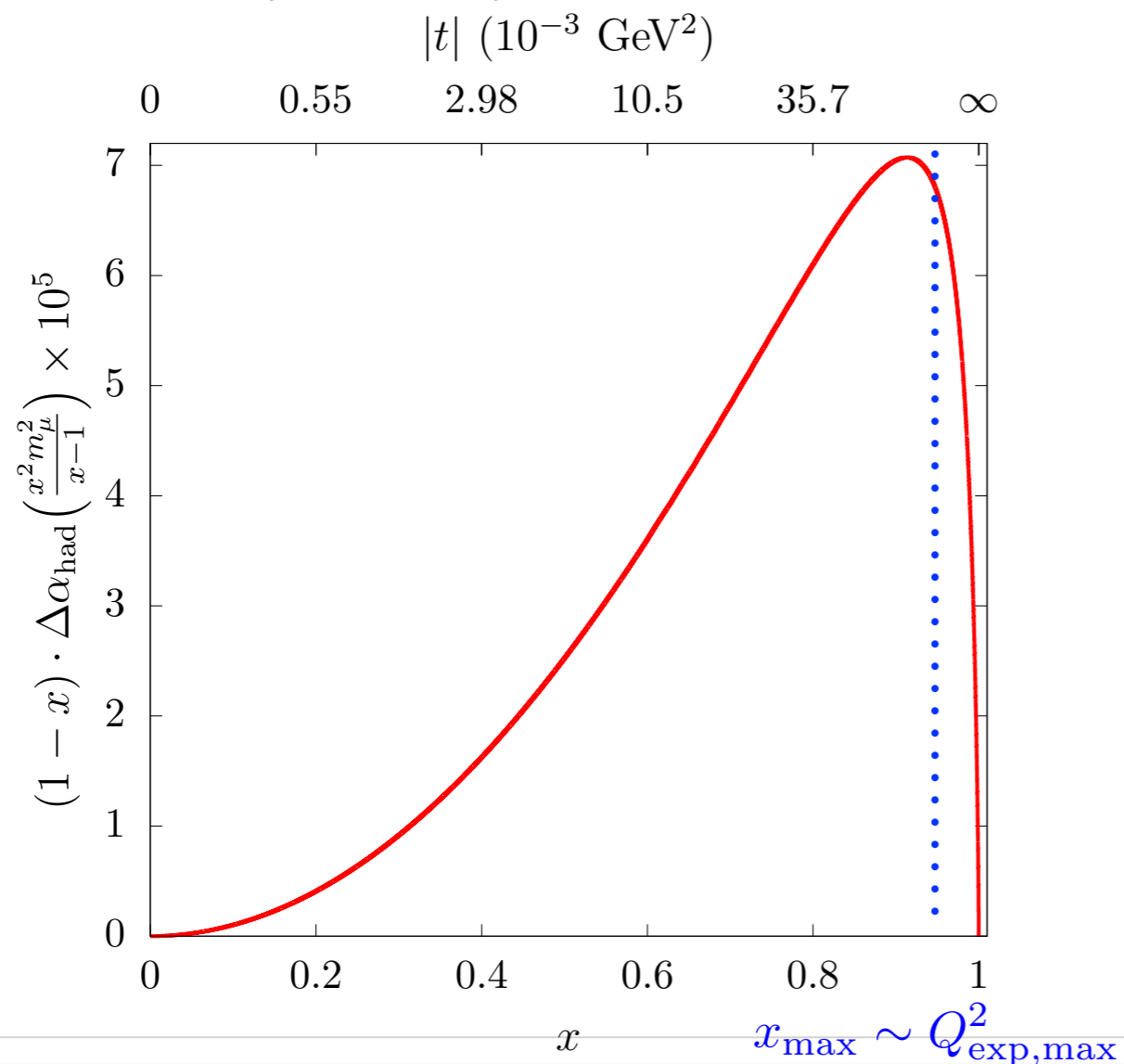
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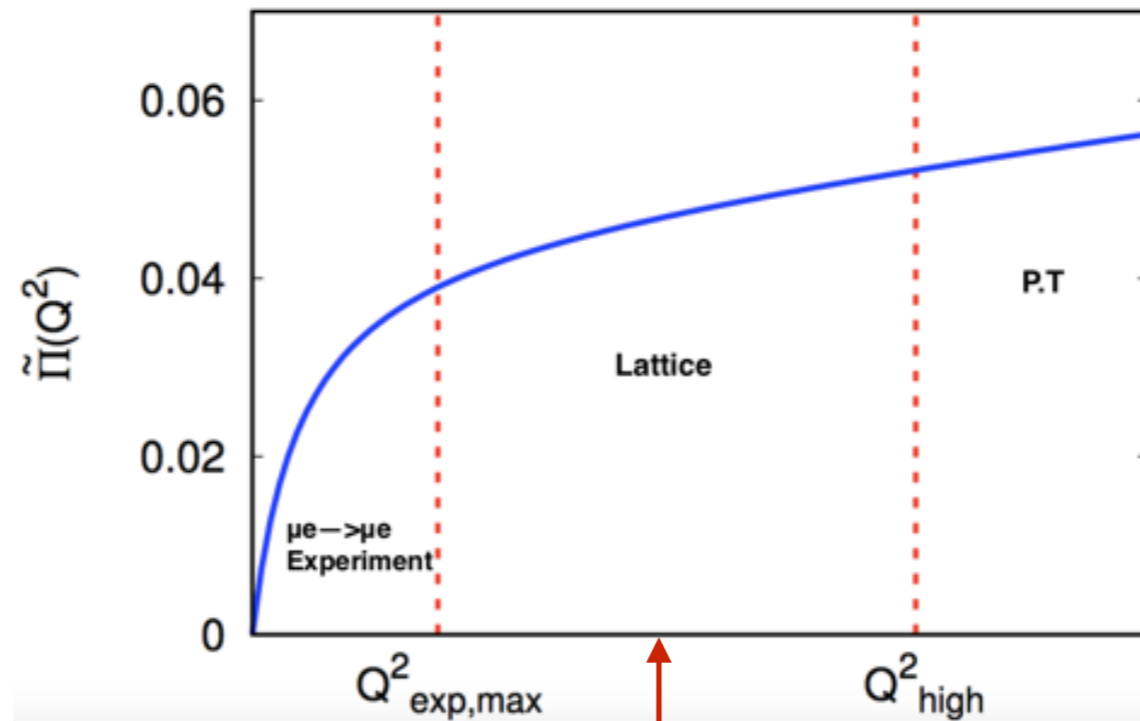- ➡ Nf=2, E5, L/a=32 (CLS), m_π≈440MeV
- ➡ Pade [1,1]
- ➡ $a_\mu^{HVP,uds} = 3.61(10) \times 10^{-8}$
- ➡ [0,$Q^2_{exp,max}$] —>87% of total $a_\mu^{HVP,uds}$
- ➡ [$Q^2_{exp,max}$,Q^2_{high}]—>12% total $a_\mu^{HVP,uds}$
- ➡ [Q^2_{high},∞] —> <1% of total $a_\mu^{HVP,uds}$

- ● ABGP Pade approximants [**Aubin,Blum,Golterman,Peris, Phys.Rev. D86 (2012) 054509**]:
  ➡ guaranteed to converge on the interval **[$Q^2_{exp,max}$,Q^2_{high}]**
  ➡ possible to combine with the numerical integration

Hybrid method

- **Low momentum region**
  - ➡ **Experiment (NLO, NNLO, radiative corrections …)**

- **Vary low and high Q^2 cut**

- ➡ continuum limit: $a \rightarrow 0$
- ➡ infinite volume limit: $V \rightarrow \infty$
- ➡ physical quark masses
- ➡ isospin breaking corrections ($m_u \neq m_d$ and $\alpha_{em} \neq 0$)

strategy proposed for the hybrid determination of the total HVP (u+d+s+c+b)

Cross-check **experimental** $\Pi(Q^2)$ **vs. continuum limit from the lattice**

- Take individual $\Pi(Q^2)$ values **[0,0.108]GeV²**

- Continuum limit at fixed Q² (previously extrapolated or measured at **m$_{\pi,\text{phys}}$**)

- Compare to the slope and curvature for HVP function [**see arXiv:1612.02364**]

- **For the continuum limit of** $\Pi(Q^2)$ **at fixed Q²:**

  ➡ twisted bc's / SCI

  ➡ interpolate between the values measured by conventional methods

---

1. **The HVP integral on a range [Q²$_{\text{min}}$,Q²$_{\text{max}}$] has continuum&FV limit:**

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{exp,max}}^{\infty} dQ^2 \ f(Q^2) \times \tilde{\Pi}(Q^2)$$

  ➡ <u>radiative corrections might be relevant</u> (≈ 1‰) **[c.f. slides by C. Carloni Calame for region [0,Q²$_{\text{exp,max}}$])**
  ➡ cutoff effects need to be assessed systematically

2. **Plan to engage whole lattice community, look in the momentum range [0.138, ∞]GeV²**

  ➡ Ideally, perform continuum limit (&infinite volume limit)
  ➡ Help us put together yet another estimate for a_μ^{HVP} joining th. and exp. efforts

Work in progress: QED+QCD simulations with C* bc's

- Generating configurations for N_f=2+1 O(a) improved Wilson fermions (QCD, QCD+QED)

- Next 1-2 years, expect to have first results on a_μ^{HVP}

- Particularly convenient for computing isospin breaking effects

  ➡ local formulation of QED+QCD

  ➡ different (smaller and better controlled?) F.V. effects

- **RC* collaboration:** **http://rcstar.web.cern.ch/**

- [**A.Patella, M.K.M @ Lattice 2017**] openQCD code —> added C* bc's and dynamical SU(3)+U(1)

- [**M. Hansen @ Lattice 2017**] —> first physics results with C* bc's

RC* Collaboration http://rcstar.web.cern.ch/
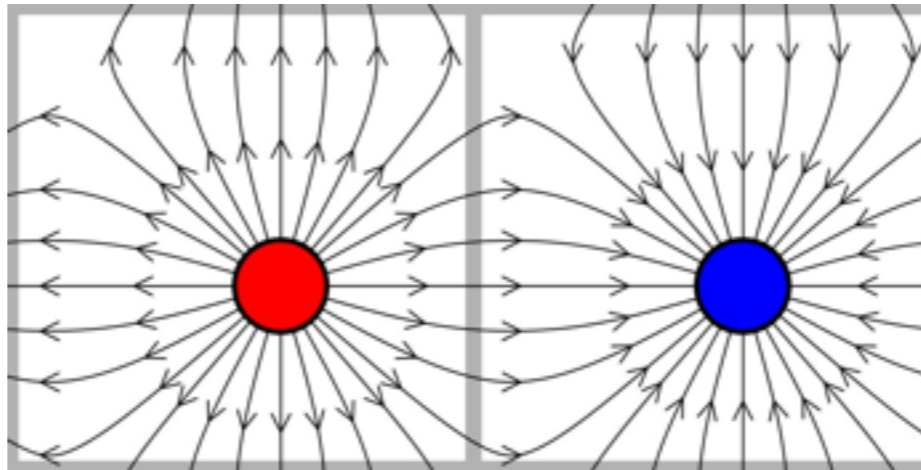
Rome II - University of Rome Tor Vergata

- N. Tantalo

- G.M. de Divitiis

IFT/UAM Madrid

- Isabel Campos

CP3 - University of Southern Denmark

- Martin Hansen

CERN

- Patrick Fritzsch

- Agostino Patella (&Plymouth University)

- Alberto Ramos

- Marina Krstic Marinkovic (&TCD)

Leading Isospin Breaking Effects of the HVP

- R123 method [arXiv:1303.4896] for computing leading isospin breaking corrections (**LIBE**)

  ➡ Expanding an observable (in the full theory) with respect to the isosymmetric (**m$_u$=m$_d$ and α$_{em}$=0**) QCD result

- For a start: applying it to the connected part of the HVP

- Main advantage w. respect to simulating QED+QCD:

  ➡ Diagrams obtained individually [before multiplying with $O(\alpha_{em})$, $O(m_u - m_d)$ coeff.]

  ➡ No extrapolation in α_{em}

- <u>Example:</u>

electroquenched approximation

considering sea quarks as electrically neutral

21

LIBE of the HVP in the electro-quenched approx.

- Expanding the connected part of the HVP

$$= Tr\{\gamma_\mu S_f \gamma_\nu S_f\}$$

- Electro-quenched approximation: $\Pi(q^2) = \Pi^0(q^2) + \Delta\Pi(q^2)$

HVP in isosymmetric th.

$(\ e_q^2 \ ) = (\ e_q^2 \ )^0 - e_q^4 e^2$

photon exchange

tadpole

self-energy

$- e_q^4 e^2$

mass-insertion

$+ e_q^2 (m_f - m_f^0) + \ldots$

Summary & Outlook

- Lattice gives an independent theory prediction of hadronic contributions to a_μ

- Lattice goals: for HVP is <1% and goal for HLbL is <10%

- Full control of the systematics is needed — the first one (HVP) might be achieved by utilising

  experimental data (R-ratios, space-like)

- Proposal to do a "hybrid determination" from µ-e scattering and lattice data (+p.t.)

- Preliminary estimate: 12% of the total $a_\mu^{HVP,uds}$ coming from the intermediate region $[Q^2_{exp,max}, Q^2_{high}]$
  ➡ continuum limit, infinite volume limit, isospin breaking corrections are the next challenges

---

- Isospin breaking corrections:
  ➡ HVP: gather statistics, continuum limit, chiral limit, estimate FV effects
  ➡ HVP: repeat the study on QCD configurations with C* bc's
  ➡ Full QCD+QED simulations with C* bc's: stability with TM preconditioning, generate set of
     ensembles for a pilot measurement phase (optimisation), meson mass splittings, HVP, …